Day-ahead decision making in electricity markets

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Adrien Le Franc, Pierre Carpentier, Jean-Philippe Chancelier, Michel De Lara

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Outline of the presentation

1. **Introductory example from the French non-interconnected zones (NIZ)**
2. **Resolution methods**
3. **Numerical results from the French NIZ example**
4. **Conclusion and Perspectives**

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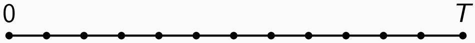
1. **Introductory example from the French non-interconnected zones (NIZ)**

   Rules for solar plant management in the French NIZ

   Optimization problem formulation

Market rules for solar plants in the French NIZ

- We operate a solar plant over **one day** with discrete time steps $t \in \{0, 1, \ldots, T\}$



- For every operating day
  - In the **day-ahead** stage, producers must supply a power production profile $p \in \mathbb{R}^T$
  - In the **intraday** stage, producers manage the power plant and deliver a power profile $d \in \mathbb{R}^T$

Cost structure for the intra-day problem

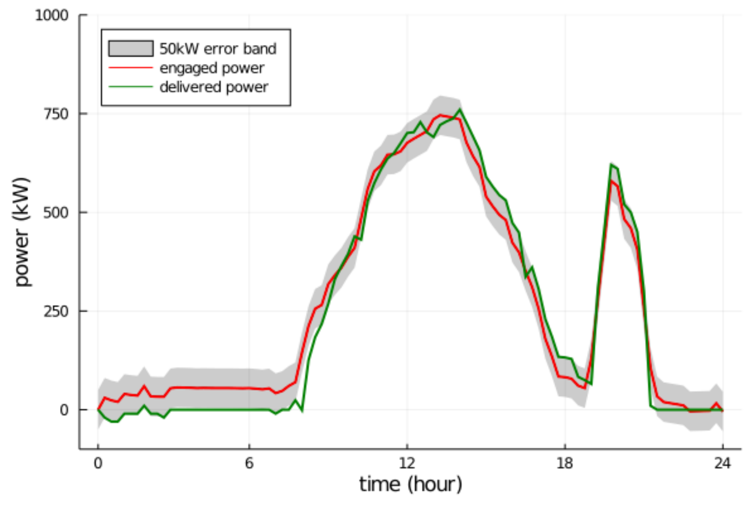
During the **intraday** stage, the **delivered power** d
is compared with the **engaged power** p
to compute the stage cost

$$C_t(d_t, p_t) = \underbrace{-c_t \Delta_t d_t}_{\text{reward}} + \underbrace{C_t^{\mathrm{p}}(d_t, p_t)}_{\text{penalty}}$$

where

$$C_t^{\mathrm{p}}(d_t, p_t) = \begin{cases} c_t \Delta_t \left[\frac{\left(d_t - \underline{d}(p_t)\right)^2}{\overline{p}} - 0.2\left(d_t - \underline{d}(p_t)\right) \right] , & \text{if } d_t < \underline{d}(p_t) \\ 0 , & \text{if } \underline{d}(p_t) \leq d_t \leq \overline{d}(p_t) \\ c_t \Delta_t p_t , & \text{if } \overline{d}(p_t) < d_t \end{cases}$$

Engaged power vs delivered power



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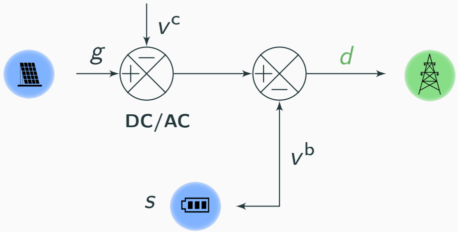
Schematic organization of the solar plant



- $s \in [0, \overline{s}]^{T+1}$ state of charge (state)
- $g \in [0, \overline{p}]^T$ generated power (uncertainty)
- $v^c \in [0, g]^T$ curtailed power (control)
- $v^b \in [\underline{v}, \overline{v}]^T$ battery power (control)
- $d = g - v_b - v_c$ **delivered power**

Stochastic optimal control framework

- We introduce the the **state**, **control** and **noise** variables

$$x = \begin{pmatrix} s \\ g \end{pmatrix} \ , \ \ u = \begin{pmatrix} v^{\mathrm{b}} \\ v^{\mathrm{c}} \end{pmatrix} \ , \ \ w = \epsilon$$

- The state process $\mathbf{X}$ is ruled by the dynamics

$$\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) = \begin{pmatrix} \mathbf{S}_t + \rho_c {\mathbf{V}_t^{\mathrm{b}}}^{+} - \frac{1}{\rho_d} {\mathbf{V}_t^{\mathrm{b}}}^{-} \\ \alpha_t \mathbf{G}_t + \beta_t + \boldsymbol{\epsilon}_{t+1} \end{pmatrix}$$

- The stage costs formulate as

$$L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}, p_t) = C_t(\mathbf{G}_{t+1} - \mathbf{V}_t^{\mathrm{b}} - \mathbf{V}_{t+1}^{\mathrm{c}}, p_t)$$

Intraday NIZ problem

Minimizing the **intraday** operating cost
formulates as a **multistage stochastic optimization problem**
parametrized by p

$$\min_{\mathbf{U}_0,\ldots,\mathbf{U}_{T-1}} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}, p_t) + K(\mathbf{X}_T, p_T)\Big]$$

$$\mathbf{X}_0 = x_0$$

$$\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) \ , \ \ \forall t \in \{0, \ldots, T-1\}$$

$$\mathbf{U}_t \in \mathcal{U}_t(\mathbf{X}_t, p_t) \ , \ \ \forall t \in \{0, \ldots, T-1\}$$

$$\sigma(\mathbf{U}_t) \subseteq \sigma(\mathbf{W}_1, \ldots, \mathbf{W}_t) \ , \ \ \forall t \in \{0, \ldots, T-1\}$$

Coupled day-ahead and intraday problem

The optimal management of the solar plant
over one operating day formulates as

$$\overbrace{\min_{p \in \mathcal{P}}}^{\text{day-ahead}} \overbrace{\min_{\mathbf{U}_0, \dots, \mathbf{U}_{T-1}} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}, p_t) + K(\mathbf{X}_T, p_T) \Big]}^{\text{intra-day value } \Phi(p)}$$

$$\mathbf{X}_0 = x_0$$

$$\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) \;, \;\; \forall t \in \{0, \dots, T-1\}$$

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Computing the value $\Phi(p)$ of the intraday problem

For $p \in \mathcal{P}$ we may compute by **dynamic programming**

$$\Phi(p) = V_0(x_0, p)$$

where for $t \in \{0, \ldots, T\}$ and $x \in \mathbb{X}_t$
we define the **parametric value functions**

$$V_T(x, p) = K(x, p)$$

$$V_t(x, p) = \min_{u \in \mathcal{U}_t(x, p_t)} \mathbb{E}\Big[L_t(x, u, \mathbf{W}_{t+1}, p_t) + V_{t+1}\big(f_t(x, u, \mathbf{W}_{t+1}), p\big)\Big]$$

Descent method for the day-ahead problem

- We consider applications where the value function Φ and the constraint set $\mathcal{P}$ are **convex**
- We want to apply a first order descent algorithm

Projected (sub)gradient algorithm

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input: $p^0 \in \mathcal{P}$, $\{\alpha_k\}_{k=1\ldots K} \in \mathbb{R}_+^K$
for $k = 1 \ldots K$ **do**
  compute y^k as a (sub)gradient of Φ at p^k
  update $p^{k+1} = \Pi_{\mathcal{P}}(p^k - \alpha^k y^k)$
end
output: p^K

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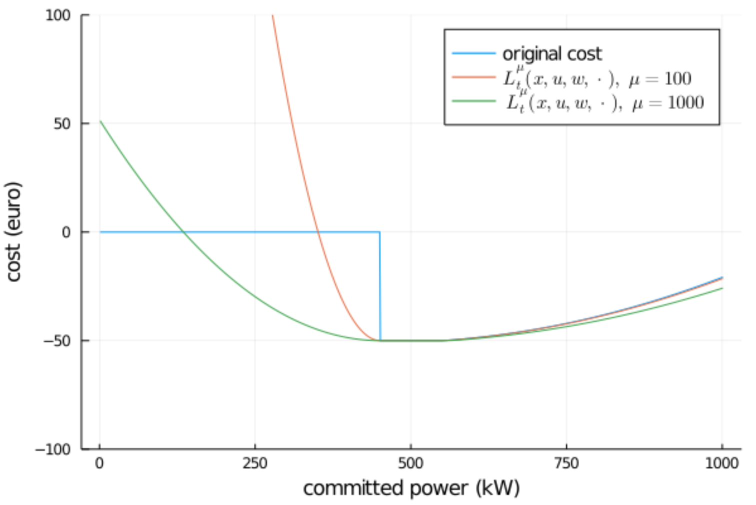
how do we compute a (sub)gradient of Φ at p^k ??

2. **Resolution methods**

   Lower smooth approximations

   Lower polyhedral approximations

Smoothing the cost function with the Moreau envelope



Gradient of a convex differentiable value function

When the value functions are **convex** and **differentiable** with respect to p we compute the gradient $\nabla_p \Phi(p^k)$ where for $t \in \{0, \dots, T\}$ and $x \in \mathbb{X}_t$

$$u^* \in \operatorname*{arg\,min}_{u \in \mathcal{U}_t(x)} \mathbb{E}\Big[L_t^\mu(x, u, \mathbf{W}_{t+1}, p_t^k) + V_{t+1}\big(f_t(x, u, \mathbf{W}_{t+1}), p^k\big)\Big]$$

$$\nabla_p V_t(x, p^k) = \mathbb{E}\Big[\nabla_p L_t^\mu(x, u^*, \mathbf{W}_{t+1}, p_t^k) + \nabla_p V_{t+1}\big(f_t(x, u^*, \mathbf{W}_{t+1}), p^k\big)\Big]$$

Outline of the section

2. Resolution methods

Lower smooth approximations

Lower polyhedral approximations

Polyhedral approximation with SDDP

When the value functions are **convex** and **non-differentiable** with respect to p we apply the SDDP algorithm to obtain **polyhedral lower apporximations**

- After each **forward-bakward** iteration $n \in \{1, \ldots, N\}$ we add a new cut $\langle \cdot \, , \alpha_t^n \rangle + \beta_t^n$
- Under convexity assumptions we have convergence guarantees of the polyhedral approximate

$$\underline{V}_t^N(x, p^k) = \max_{1 \leq n \leq N} \Big(\langle (x, p^k) \, , \alpha_t^n \rangle + \beta_t^n \Big) \leq V_t(x, p^k)$$

Subgradient of a convex non-differentiable value functions

We can evaluate an approximate subgardient of $\partial\Phi(p^k)$
by taking $y^k \in \partial\underline{V}_0^N(x, p^k)$ as a **dual variable**
of the constrained problem

$$\min_{x,p} \quad \underline{V}_0^N(x, p^k) \qquad \text{s.t.} \;\; p = p^k \quad [y^k]$$

Since $\underline{V}_0^N(x, p^k)$ is polyhedral the above problem is a LP

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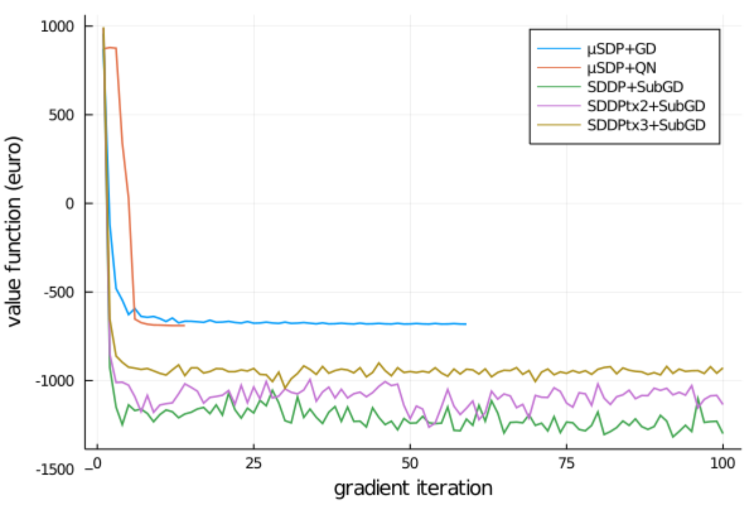
3. Numerical results from the French NIZ example

Numerical results for a single day

Numerical results for one year

Numerical results of the day-ahead optimization $\min_{p \in \mathcal{P}} \Phi(p)$



Computing time performance

	Steps	**Avg. time / (sub)gradient call** (seconds)	**Avg. time / iteration** (seconds)	**Avg. number of cuts / iteration**
μSDP+GD	35	2.55	2.71	-
μSDP+QN	12	2.55	10.17	-
SDDP+SubGD	100	2.55	2.68	17
SDDPt$\times$2+SubGD	100	5.10	5.23	37
SDDPt$\times$3+SubGD	100	7.65	8.02	55

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Numerical results for a single day

Numerical results for one year

Intraday simulation: experimental context

- We use PV forecast and observed data from **Schneider Electric's EMSx dataset**
- We use 1 year of data for **calibration**
- We another 1 year of data for **simulation**
- We simulate the management of 365 **consecutive days**
- We apply the French ZNI market rules **but** we do not consider intraday profile re-submission

Intraday simulation: methods

We consider two methods

- **Stochastic method based on μSDP+QN**
  - **day-ahead** : we use our gradient method
    with a smoothing of the cost to compute daily profiles
  - **intraday** : we perform intraday simulation
    with Stochastic Dynamic Programming
- **Deterministic method based on MPC**
  - **day-ahead** : we use forecasts and a deterministic MIQP solver
    to compute daily profiles
  - **intraday** : we perform intraday simulation
    with Model Predictive Control

Intraday simulation: numerical results

Method	**Total yearly gain** (€)
Deterministic (MPC)	560 410
Stochastic (μSDP+QN)	611 681

Our stochastic method gives 8% of gain versus a deterministic one

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Conclusion

- Our approach gives promising results: 8% of gain on the (simplified) NIZ use case
- **Question 1 :** how does it perform on the complete NIZ use case ?
- **Question 2 :** extension of the method to other energy markets ?