

# Two-scale stochastic dynamic optimization

## Energy storage investment and operation in subway stations

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May 16, 2017



# Optimization for subway stations

Paris subway stations consumption  $\equiv$  40.000 houses

Energy transition of cities requires significant investments

Is electrical storage affordable for subway stations?

We use stochastic optimization for short term control and long term management of batteries



# Outline

- 1 Electrical storage management issues
  - Why electrical storage in subway stations?
  - Managing storage short term operations
  - Battery operation impacts long term aging!
- 2 Long term management of batteries aging and renewal
  - Two scales management: investment/operation
  - Short term management problem formulation
  - Long term management problem formulation
  - We formulate a two-scale stochastic optimization problem
- 3 Resolution method and first results
  - Long term value functions and Bellman equation
  - Preliminary numerical results



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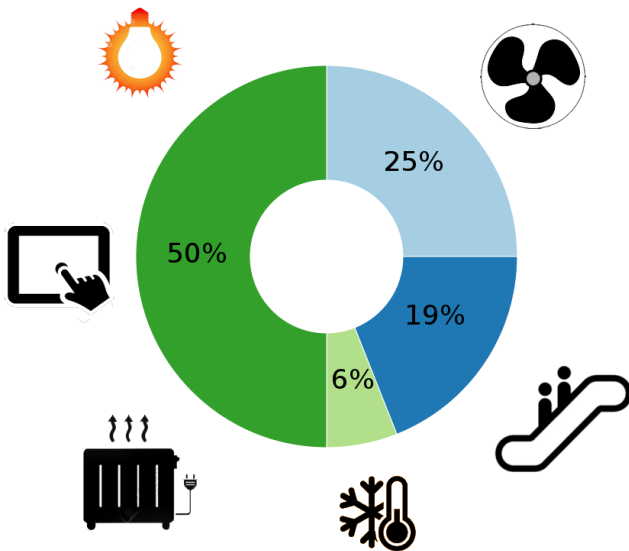
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# Why electrical storage in subway stations?



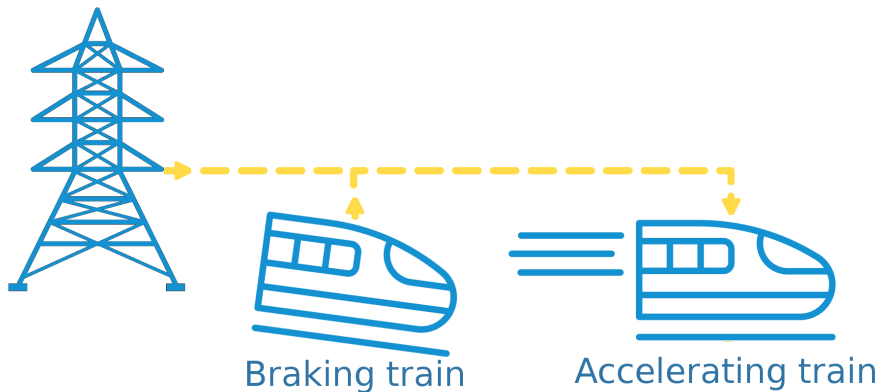
# Subway stations typical energy consumption



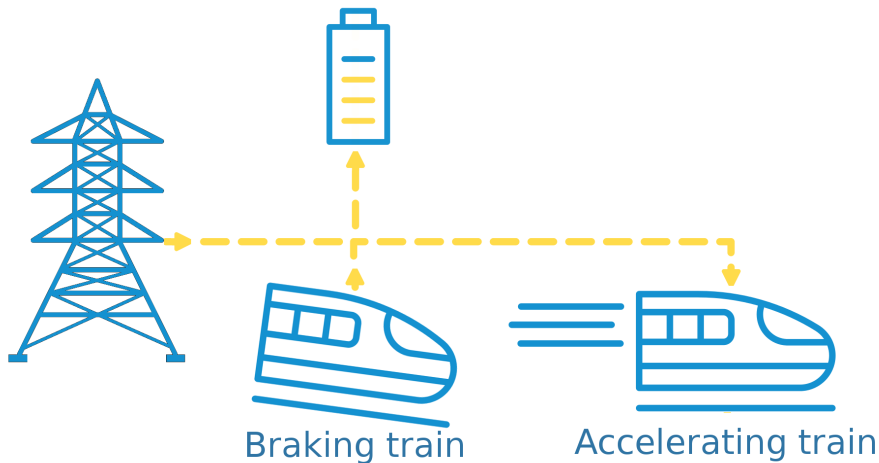
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# Subway stations have unexploited energy resources



# Energy recovery requires a buffer

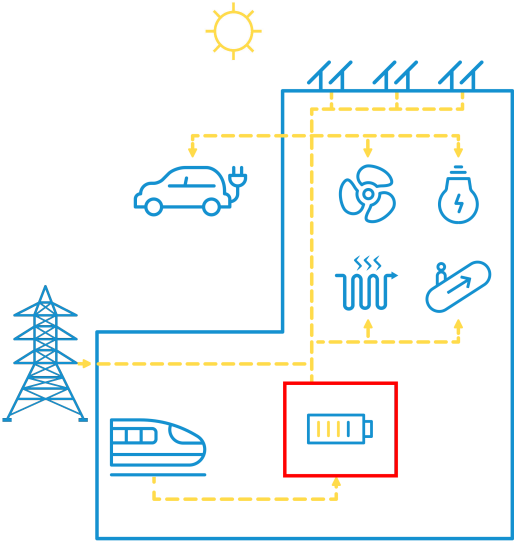




# Managing storage short term operations



# Microgrid concept for subway stations

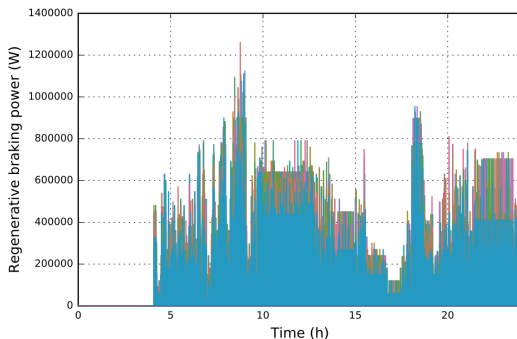


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# Stochastic optimization is relevant

Subways braking energy is unpredictable



We can optimize battery operations using  
Stochastic Dynamic Programming

# Battery operation impacts long term aging!



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# Two scales management: investment/operation

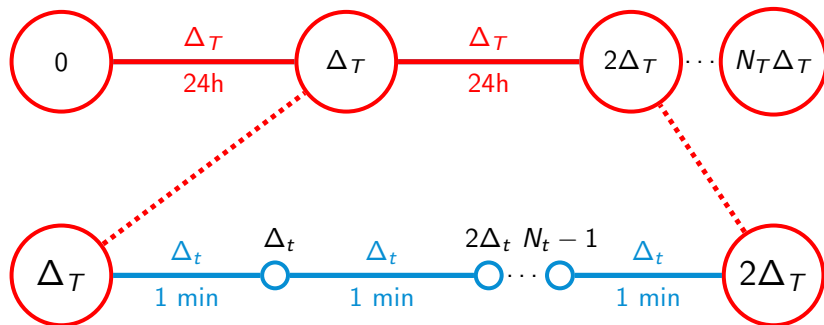


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# Two time scales

## Long term aging and renewal



## Short term operation

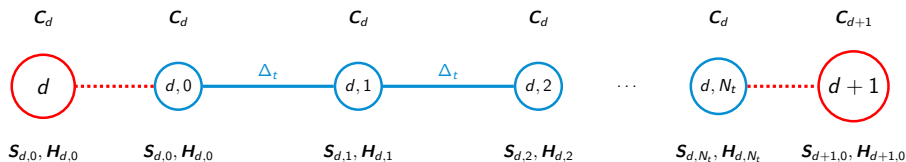


# What happens every minute $m$ of every day $d$ ?

Day:  $d$ , Minute:  $m$

- Battery capacity:  $C_d = C(d\Delta_T)$
- Battery state of charge:  $S_{d,m} = S(d\Delta_T + m\Delta_t)$
- Battery state of health:  $H_{d,m} = H(d\Delta_T + m\Delta_t)$

$S_{d,m}$  and  $H_{d,m}$  change every minute  $m$



Battery capacity  $C_d$  changes with health state at end of day  $d$

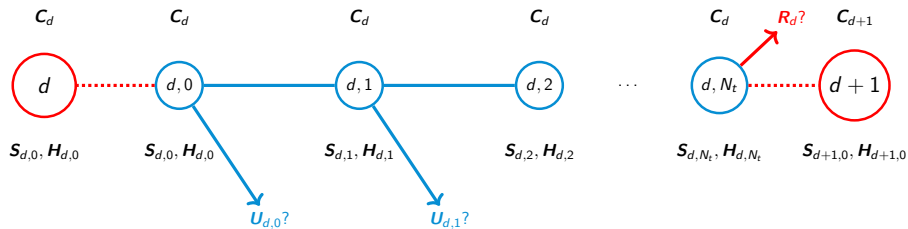
Battery  $C_d$  can be replaced every day  $d$



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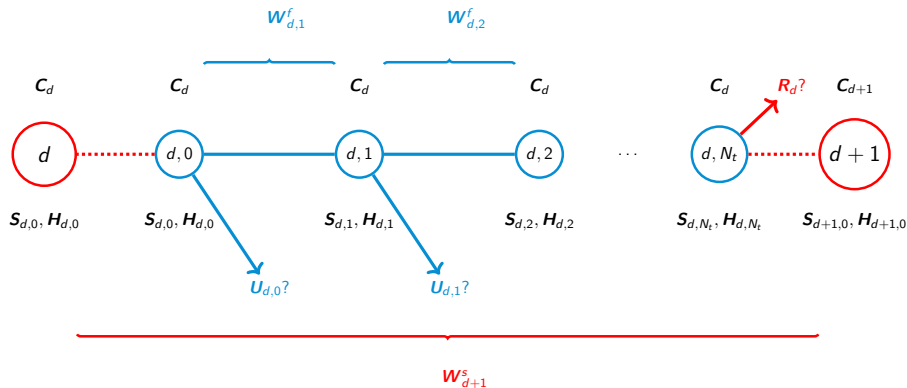


# We make both short and long term decisions



# Uncertain events occur

$W_{d,2}^f$ : electricity demand every minute

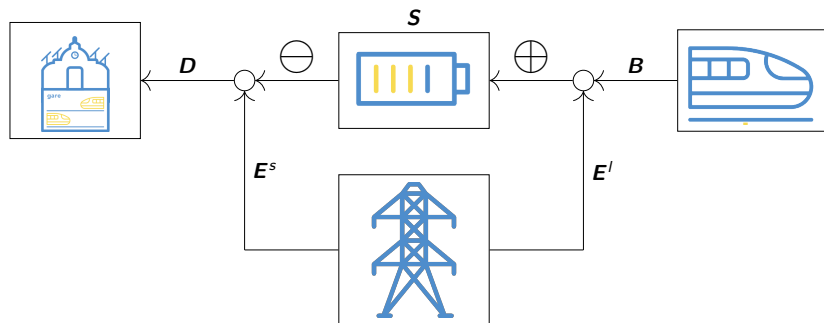


$W_{d+1}^s$ : battery prices every day

# Short term management problem formulation



# Electrical network representation



## Station node

- $D$ : Demand station
- $E^s$ : From grid to station
- $\ominus$ : Discharge battery

## Subways node

- $B$ : Braking
- $E'$ : From grid to battery
- $\oplus$ : Charge battery



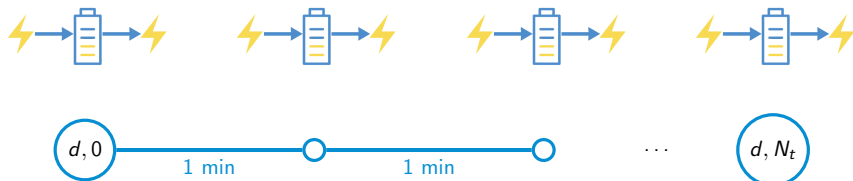
# Battery state of charge dynamics

For a given charge/discharge strategy  $\mathbf{U}$  over a day  $d$ :

$$\mathbf{S}_{d,m+1} = \mathbf{S}_{d,m} - \underbrace{\frac{1}{\rho_d} \mathbf{U}_{d,m}^-}_{\ominus} + \underbrace{\rho_c \text{sat}(\mathbf{S}_{d,m}, \mathbf{U}_{d,m}^+, \mathbf{B}_{d,m+1})}_{\oplus}$$

with

$$\text{sat}(x, u, b) = \min\left(\frac{S_{\max} - x}{\rho_c}, \max(u, b)\right)$$



# Battery aging dynamics

For a given charge/discharge strategy  $\mathbf{U}$  over a day  $d$

$$\mathbf{H}_{d,m+1} = \mathbf{H}_{d,m} - \frac{1}{\rho_d} \mathbf{U}_{d,m}^- - \rho_c \text{sat}(\mathbf{S}_{d,m}, \mathbf{U}_{d,m}^+, \mathbf{B}_{d,m+1})$$



# Every minute we save energy and money

If we have a battery on day  $d$  and minute  $m$  we save:

$$p_{d,m}^e \left( \underbrace{E_{d,m+1}^s + E_{d,m+1}^l - D_{d,m+1}}_{\text{Saved energy}} \right)$$

$p_{d,m}^e$  is the cost of electricity on day  $d$  at minute  $m$



# Summary of short term/Fast variables model

We call, at day  $d$  and minute  $m$ ,

- fast state variables:  $\mathbf{X}_{d,m}^f = \begin{pmatrix} \mathbf{S}_{d,m} \\ \mathbf{H}_{d,m} \end{pmatrix}$
- fast decision variables:  $\mathbf{U}_{d,m}^f = \begin{pmatrix} \mathbf{U}_{d,m}^- \\ \mathbf{U}_{d,m}^+ \end{pmatrix}$
- fast random variables:  $\mathbf{W}_{d,m}^f = \begin{pmatrix} \mathbf{B}_{d,m} \\ \mathbf{D}_{d,m} \end{pmatrix}$
- fast cost function:  $L_{d,m}^f(\mathbf{X}_{d,m}^f, \mathbf{U}_{d,m}^f, \mathbf{W}_{d,m+1}^f)$
- fast dynamics:  $\mathbf{X}_{d,m+1}^f = F_{d,m}^f(\mathbf{X}_{d,m}^f, \mathbf{U}_{d,m}^f, \mathbf{W}_{d,m+1}^f)$

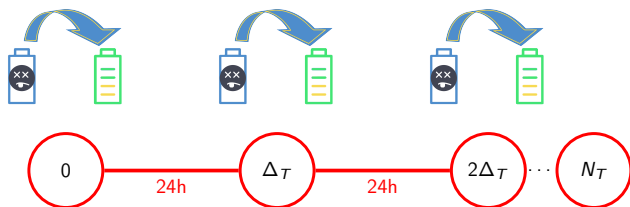




# Long term problem formulation



# We decide our battery purchases at the end of each day



Should we replace our battery  $\mathbf{C}_d$  by buying a new one  $\mathbf{R}_d$  or not?

$$\mathbf{C}_{d+1} = \begin{cases} \mathbf{R}_d, & \text{if } \mathbf{R}_d > 0 \\ f(\mathbf{C}_d, \mathbf{H}_{d,N_t}), & \text{otherwise} \end{cases}$$

paying renewal cost  $\mathbf{P}_d^b \mathbf{R}_d$  at uncertain market prices  $\mathbf{P}_d^b$

# Summary of long term/Slow variables model

We call, at day  $d$ ,

- slow state variables:  $\mathbf{X}_d^s = (\mathbf{C}_d)$
- slow decision variables:  $\mathbf{U}_d^s = (\mathbf{R}_d)$
- slow random variables:  $\mathbf{W}_d^s = (\mathbf{P}_d^b)$
- slow cost function:  $L_d^s(\mathbf{X}_d^s, \mathbf{U}_d^s, \mathbf{W}_{d+1}^s) = \mathbf{P}_d^b \mathbf{R}_d$
- slow dynamics:  $\mathbf{X}_{d+1}^s = F_d^s(\mathbf{X}_d^s, \mathbf{U}_d^s, \mathbf{W}_{d+1}^s)$



## Linking the two scales

Long term (slow) decisions may impact short term (fast) initial state

$$\mathbf{x}_{d,0}^f = \phi_d(\mathbf{x}_d^s, \mathbf{x}_{d-1}^f, N_t)$$

This is not the case here but, in general,  
short term (fast) variables may impact long term (slow) dynamics

$$\mathbf{x}_{d+1}^s = F_d^s(\mathbf{x}_d^s, \mathbf{u}_d^s, \mathbf{w}_{d+1}^s, \mathbf{x}_{d,0}^f, \mathbf{u}_{d,:}^f, \mathbf{w}_{d,:}^f)$$

as well as the long term (slow) cost

$$L_d^s(\mathbf{x}_d^s, \mathbf{u}_d^s, \mathbf{w}_{d+1}^s, \mathbf{x}_{d,0}^f, \mathbf{u}_{d,:}^f, \mathbf{w}_{d,:}^f)$$



# We formulate a two-scale stochastic optimization problem



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# Two-scale stochastic optimization problem

We minimize fast and slow costs over the long term

$$\min_{\mathbf{x}^f, \mathbf{x}^s, \mathbf{u}^f, \mathbf{u}^s} \mathbb{E} \left[ \sum_{d=0}^{N_T-1} \left( \sum_{m=0}^{N_t-1} L_{d,m}^f(\mathbf{x}_{d,m}^f, \mathbf{u}_{d,m}^f, \mathbf{w}_{d,m+1}^f) \right) + L_d^s(\mathbf{x}_d^s, \mathbf{u}_d^s, \mathbf{w}_{d+1}^s, \mathbf{x}_{d,0}^f, \mathbf{u}_{d,:}^f, \mathbf{w}_{d,:}^f) \right]$$

$$\mathbf{x}_{d,m+1}^f = F_{d,m}^f(\mathbf{x}_{d,m}^f, \mathbf{u}_{d,m}^f, \mathbf{w}_{d,m+1}^f)$$

$$\mathbf{x}_{d,0}^f = \phi_d(\mathbf{x}_d^s, \mathbf{x}_{d-1,N_t}^f)$$

$$\mathbf{x}_{d+1}^s = F_d^s(\mathbf{x}_d^s, \mathbf{u}_d^s, \mathbf{w}_{d+1}^s, \mathbf{x}_{d,0}^f, \mathbf{u}_{d,:}^f, \mathbf{w}_{d,:}^f)$$

$$\mathbf{u}_{d,m}^f \preceq \mathcal{F}_{d,m}$$

$$\mathbf{u}_d^s \preceq \mathcal{F}_{d,N_t}$$



# Information model

$$\mathcal{F}_{d,m} = \sigma \left( \begin{array}{l} \mathbf{W}_{d',m'}^f, d' < d, m' \leq N_t + 1 \\ \mathbf{W}_{d'}^s, d' \leq d \\ \mathbf{W}_{d,m'}^f, m' \leq m \end{array} \right) = \sigma \left( \begin{array}{l} \text{previous days fast noises} \\ \text{previous days slow noises} \\ \text{current day previous minutes fast noises} \end{array} \right)$$

Fast information accumulation

$$\mathcal{F}_{d,m} = \mathcal{F}_{d,0} \vee \sigma(\mathbf{W}_{d,1:m}^f)$$

Slow information implies a jump between  $d, N_t$  and  $d + 1, 0$

$$\mathcal{F}_{d+1,0} = \mathcal{F}_{d,N_t} \vee \sigma(\mathbf{W}_{d+1}^s)$$



# Two-scale stochastic optimal control reformulation

With the notations

$$\mathbf{X}_d = (\mathbf{X}_{d-1, N_t}^f, \mathbf{X}_d^s)$$

$$\mathbf{U}_d = (\mathbf{U}_{d, :}^f, \mathbf{U}_d^s)$$

$$\mathbf{W}_d = (\mathbf{W}_{d-1, :}^f, \mathbf{W}_d^s)$$

we can reformulate the problem as

$$\min_{\mathbf{X}, \mathbf{U}} \mathbb{E} \left[ \sum_{d=0}^{N_T-1} L_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) \right]$$

$$\mathbf{X}_{d+1} = F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1})$$

$$\mathbf{U}_{d, m}^f \preceq \mathcal{F}_{d, m}$$

$$\mathbf{U}_d^s \preceq \mathcal{F}_{d, N_t}$$

where the non-anticipativity constraints are not standard





How to decompose  
a two-scale stochastic optimization problem  
into  
a short term optimization problem  
and  
a long term optimization problem?



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# Long term value functions and Bellman equation



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# We can define long term (slow) value functions

Every day  $d_0$ , we can define a long term (slow) value function

$$V_{d_0}(x_{d_0}) = \min_{\mathbf{X}, \mathbf{U}} \mathbb{E} \left[ \sum_{d=d_0}^{N_T-1} L_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) \right]$$
$$\mathbf{X}_{d+1} = F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1})$$
$$\mathbf{U}_{d,m}^f \preceq \mathcal{F}_{d,m}$$
$$\mathbf{U}_d^s \preceq \mathcal{F}_{d,N_t}$$
$$\mathbf{X}_{d_0} = x_{d_0}$$



# Long term (slow) value functions satisfy a Bellman equation

Assuming independence of the noises  $\mathbf{W}_d$

$$V_d(x_d) = \min_{\mathbf{U}_d} \mathbb{E} \left[ L_d(x_d, \mathbf{U}_d, \mathbf{W}_{d+1}) + V_{d+1} \left( F_d(\mathbf{x}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) \right) \right]$$
$$\text{s.t } \mathbf{U}_d = (\mathbf{U}_d^s, \mathbf{U}_{d,:}^f)$$
$$\mathbf{U}_{d,m}^f \preceq \mathbf{W}_{d,1:m}^f$$
$$\mathbf{U}_d^s \preceq \mathbf{W}_{d,1:N_t}^f$$

with non standard non-anticipativity constraints as well



# How we can decompose the two scales by Two Scale Stochastic Dynamic Programming

The long term (slow) value function  
is the final cost of a daily stochastic optimization problem  
which can be solved by different methods (SP, DP, SDDP)

Two Scale Stochastic Dynamic Programming (TSSDP)

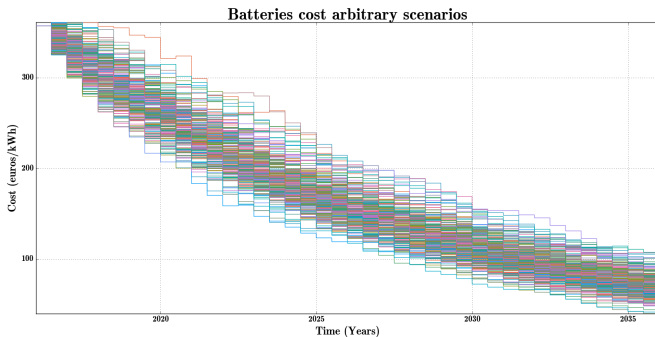


# Preliminary numerical results



# Synthetic data

- Maximum exangeable energy: model proposed in Haessig et al. [1]
- Discount rate: 4.5%
- Batteries cost stochastic model: **synthetic scenarios** that approximately coincide with **market forecasts**





# Comparison of 3 investment strategies over 20 years

We compare 3 investment strategies over 20 years,  
100  $C^b$  scenarios, 1 single capacity (80 kWh) **Strategies:**

- Strategy NPV : Buy now, replace battery when dead, no aging control
- Strategy NPVA: Buy now, replace battery when dead, control aging
- Strategy FNPVA: Buy anytime, replace battery anytime, control aging

**Objective:** maximize discounted revenues over 20 years



# Preliminary results

- NPV = -7000 euros  $\Rightarrow$  do not invest!
- NPVA = +12000 euros  $\Rightarrow$  do not strain your first batteries!
- FNPVA = +33000 euros  $\Rightarrow$  start investment in 2020 and do not strain your first batteries!

	SDP	TSSDP
Offline comp. time	$\infty$ (out of memory)	16min
Online comp. time	?	[0s,1s]
Simulation comp. time	?	[20s,30s]
Lower bound	?	+38k

In Julia with a Core I7, 2.6 Ghz, 8Go ram + 12Go swap SSD



# Conclusion

Our study leads to the following conclusions:

- Controlling aging is relevant
- TSSDP provides encouraging results
- TSSDP can be used for aging aware intraday control
- Classical Net Present Value and Free Net Present Value strategies lead to different conclusions
- Free Net Present Value strategy correspond to a more accurate model of the investment management



# Ongoing work

We are now focusing on

- Confirming, developing and **improving TSSDP results**
- Improving **risk modelling**
- Improving **batteries cost stochastic model**
- Improving **aging model**
- Include **environmental incentives** (particulate matters)
- Apply the method to more complex **energy efficiency investments**



# References



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