Combining decomposition methods to solve large scale multi-stage stochastic optimization problems

Scientific training period proposal

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1 Organism, supervision and material conditions

Organism

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Supervision and material conditions

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Number of students: 1

Material conditions: a financial gratification is offered

Dates: to be discussed

2 Proposal

Research domain

Mathematics, stochastic optimization, computer science.

Context

The internship deals with the optimization of dynamical systems in a stochastic setting, that is, in the realm of Stochastic Optimal Control (SOC) problems in discrete time. The standard way to numerically solve such problems is the celebrated Dynamic Programming method, due to Richard Bellman. The main difficulty of the method is the so-called curse of dimensionality: the computational burden exponentially increases with the number of state variables of the dynamical system. Several ways to circumvent this difficulty have been proposed. Among them, decomposition and approximation methods such as Stochastic Dual Dynamic Programming, Progressive Hedging, Dual Approximate Dynamic Programming appear to be effective on large classes of problems.

Subject

Different decomposition-coordination schemes are available to tackle large scale multi-stage stochastic optimization problems. Among them, two methods are complementary.

- The *Progressive Hedging* method [6, 7], related to the stochastic programming branch of stochastic optimization [5], provides a decomposition of the problem by scenarios, by dualizing the so-called not-anticipativity constraints of the problem. This approach allows to handle problems displaying a large number of dynamical subsystems, but its complexity grows exponentially with the number of stages.

- The *Dual Approximate Dynamic Programming* algorithm [1, 2, 3], related to the stochastic optimal control branch of stochastic optimization [4], provides a spatial decomposition of the problem by dualizing the constraints that couple the dynamical subsystems. The complexity of this approach grows linearly with the number of stages, but the method relies upon an approximation allowing to solve the subproblems by the dynamic programming method, and the quality of the solution depends on this approximation.

The proposal consists to explore tracks to mix these two decomposition methods. A first way is to split the horizon of optimization in two parts. The first part represents the proximate future, whereas the second part tackles the ultimate future. On the second part, one uses the Dual Approximate Dynamic Programming algorithm to yield a final cost for the optimization horizon corresponding to the proximate future, properly handled by Progressive Hedging.

Expected work

The student will familiarize with decomposition-coordination methods in the stochastic framework in discrete time. Then, on a dam management problem, he will formulate the two subproblems that appear during the splitting of the stages in two parts, and the way to interface the methods of Progressive Hedging and of Dual Approximate Dynamic Programming. At last, he will put the method to the test on a simple case.

References

[1] K. Barty, P. Carpentier and P. Girardeau. Decomposition of large-scale stochastic optimal control problems. *RAIRO Recherche opérationnelle, 44(3), 167-183, 2010.*

[2] P. Girardeau. Resolution of grands problems in optimization stochastic dynamic and synthèse of laws of control. *PhD dissertation, École des Ponts ParisTech, Université Paris-Est, France, 2010.*

[3] V. Leclère. Contributions to Decomposition Methods in Stochastic Optimization. *PhD dissertation, École des Ponts ParisTech, Université Paris-Est, France, 2014.*

[4] D. Bertsekas. Dynamic Programming and Optimal Control. *Athena Scientific, Belmont, Massachusets, second edition, 2000.*

[5] A. Shapiro, D. Dentcheva, and A. Ruszczynski. Lectures on stochastic programming: modeling and theory. *Society for Industrial and Applied Mathematics, volume 9, 2009.*

[6] R.T. Rockafellar and R. J-B. Wets. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research, 16(1), 119-147, 1991.*

[7] J.-P. Watson and D. L Woodruff. Progressive hedging innovations for a class of stochastic mixed-integer resource allocation problems. *Computational Management Science, 8(4), 355-370, 2011.*