Decentralized energy problems with large number of customers

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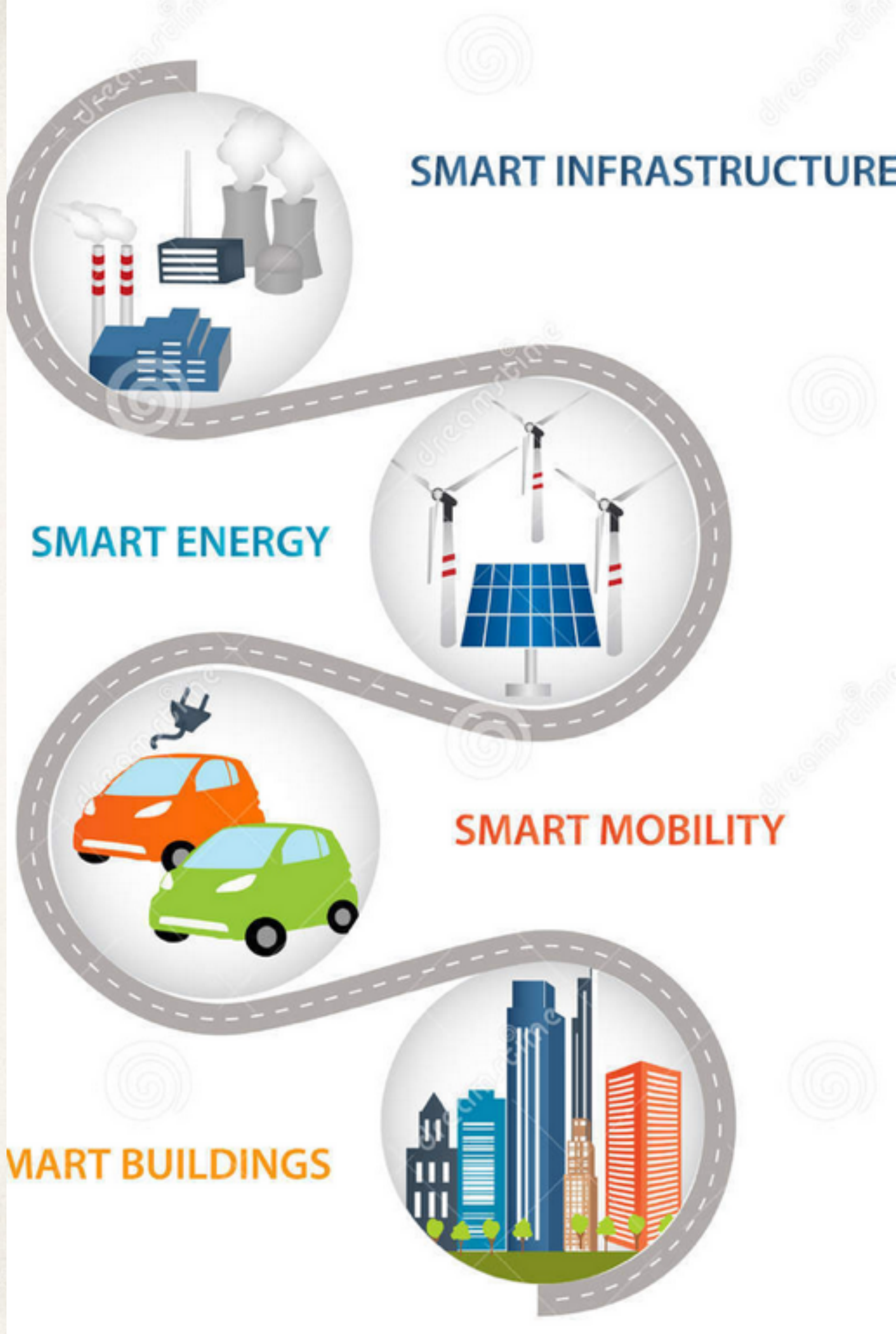
Context of smart cities

- Smart grids/new energy context
- Decentralized decisions
- Large number of decision makers in interaction:
    - many local producers of energy
    - Electrical Vehicles
    - devices, etc …



Outline

- Games with large number of players
  - Wardrop equilibrium
  - Network Congestion games

- Optimization methods and game frameworks
  - Stackelberg games
  - Mathematical Programming with Equilibrium Constraints

- A coupled energy/traffic problem

- Conclusion and perspectives

Wardrop equilibrium

- If the number of players tends to be important, standard Nash equilibrium concept becomes difficult to determine explicitly. We talk of **Non-atomic** Games.
- A player has no influence on the average strategy of all the players.
- There exists from the 70s, in the routing community, a notion of equilibrium that takes into account the infinitesimal number of players. This is the **Wardrop** Equilibrium.
- This equilibrium concept has important links with particular types of games: potential games, congestion games and population games.

Concept of Wardrop Equilibrium

- A large number of vehicles choose their travel path every day.
- Each vehicle has a source and a destination.
- A typical goal for each vehicle is to minimize his delay.
- The optimality concept is the **Wardrop Equilibrium**.

J. Wardrop, *Some theoretical aspects of road traffic research communication networks*, Proc. Inst. Civ. Eng., Part 2, 1:325-378, 1952.

Principles and concept

- *First Wardrop principle* : « The journey time on all routes actually used are equal, and less those which would be experienced by a single vehicle on any unused route. »
- *Second Wardrop principle* : « The journey time is a minimum. »
- This definition is very close to the first principle of a Nash Equilibrium : « a flow patten is in Nash equilibrium if no individual decision maker on the network can change to a less costly strategy or route. »
- The main difference is the impact of an individual on the other players. In the Wardrop context, a unique individual has a **negligible** impact on the performances of the other players.

Each flow on each path *r* from a commodity (demand for a pair O-D) *w* is even null, or even his cost is equal to the minimum cost on the path.

Notations:
h_{wr} the flow on the path *r*,
R_w the set of paths associated to the commodity *w*,
W the set of commodities,
c_{wr} is the cost of path *r*, π_{wr} is the minimum cost over all the paths for this commodity and d_w the demand for this commodity.

The first Wardrop principle gives the following system:

$$h_{wr}(c_{wr} - \pi_{wr}) = 0, \quad r \in R_w, w \in W, \tag{1}$$

$$c_{wr} - \pi_{wr} \geq 0, \quad r \in R_w, w \in W, \tag{2}$$

$$\sum_{r \in R_w} h_{wr} = d_w, \quad w \in W. \tag{3}$$

- The Wardrop equilibrium is a good approximation of the Nash equilibrium for a game in which the number of players is finite but important (A. Haurie, P. Marcotte, *On the relationship between Nash-Cournot and Wardrop Equilibria*, Networks, 15:295-308,1985.)
- The Wardrop equilibrium is generally simpler to compute than the Nash equilibrium.
- This equilibrium has important links with specific games like potential games, congestion games and population games.
- In terms of applications: network congestion games, road networks and electricity markets.

Network Congestion Games

Network Congestion Games

- Directed graph G=(V,E)
- Multiple source-destination pairs (s_k,t_k), demand d_k for commodity k
- Players are nonatomic (infinitesimally small)
- Strategy set: paths P_k between (s_k,t_k) for all *k*
  Players' decisions: flow vector *x*
- Edge delay (latency) functions: $l_e(x_e)$ typically assumed continuous and non-decreasing.

Wardrop's First Principle

- « Travel times on used routes are equal and no greater than travel times on unused routes. »

- A flow x is a Wardrop Equilibrium if for every source-destination pair k and for every path p with positive flow between this pair:

$$l_p(x) \leq l_{p'}(x), \quad \text{for all } p'$$

where $l_p(x) = \sum_{e \in p} l_e(x_e)$

- Alternative definition: A Flow vector x is a Wardrop Equilibrium if it solves:

$$\min \sum_{e \in E} h_e(x_e)$$

- where $h'_e(x_e) := l_e(x_e)$, then we get: $\min \sum_{e \in E} \int_0^{x_e} l_e(z)dz$
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- Then algorithms based on convex combination methods (like Franck-Wolf and variants) can be applied to find a Wardrop Equilibrium.
- Many extensions to the concept of Wardrop Equilibrium exist.
- Other concepts of large games are: population games, mean-field type games, …

Optimization methods and game frameworks

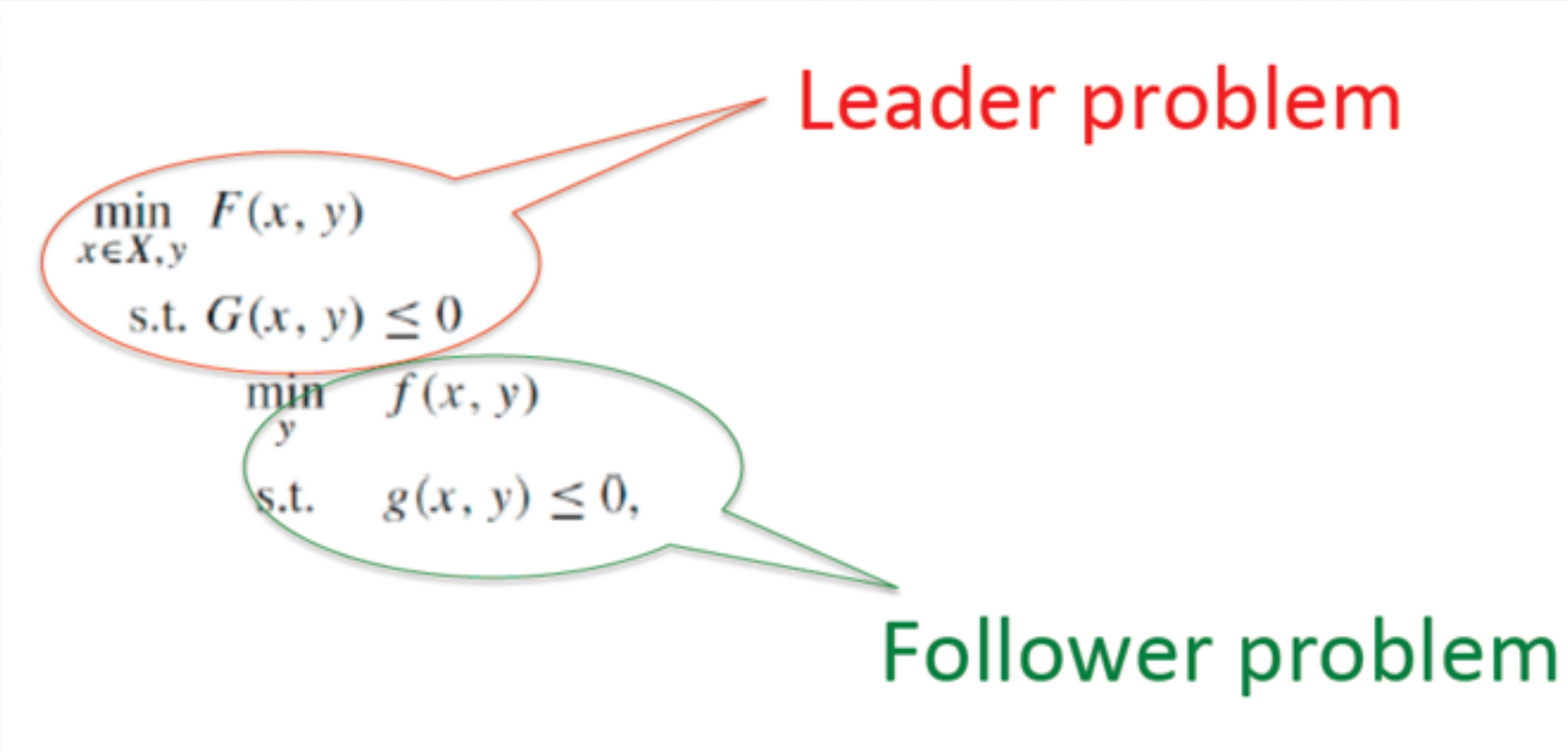
- How to control a Wardrop Equilibrium?
- Two frameworks when the « «controller » has his own objective:
  - Stackelberg Games
  - Mathematical Programming with Equilibrium Constraints

Stackelberg Games

- Two types of players with their own objective functions: leader and follower.
- Leader plays first and the follower plays after observing the action of the leader.

Stackelberg Games

Link with bilevel programming problem:



Mathematical Programming with Equilibrium Constraints (MPEC)

- In this framework, there are several followers that interact into a game setting.
    - The lower level solution concept is an equilibrium (which depends on the leader's strategy).
    - Numerous Applications: Toll pricing, control of EV, etc.
- MPEC are difficult to solve and many different methods are proposed in the literature, depending on the type of lower-level problem considered.

Mathematical Programming with Stochastic Equilibrium Constraints (MPSEC)

- The decisions of the followers are non-deterministic, i.e. not fully rational.
- Logit based discrete choice models can be proposed to illustrate such behavior.
- This framework is more suitable for realistic problems, particularly for individual energy consumption and behavior.
- It is also possible to extend beckman's formula for such framework (minimization of a convex function).

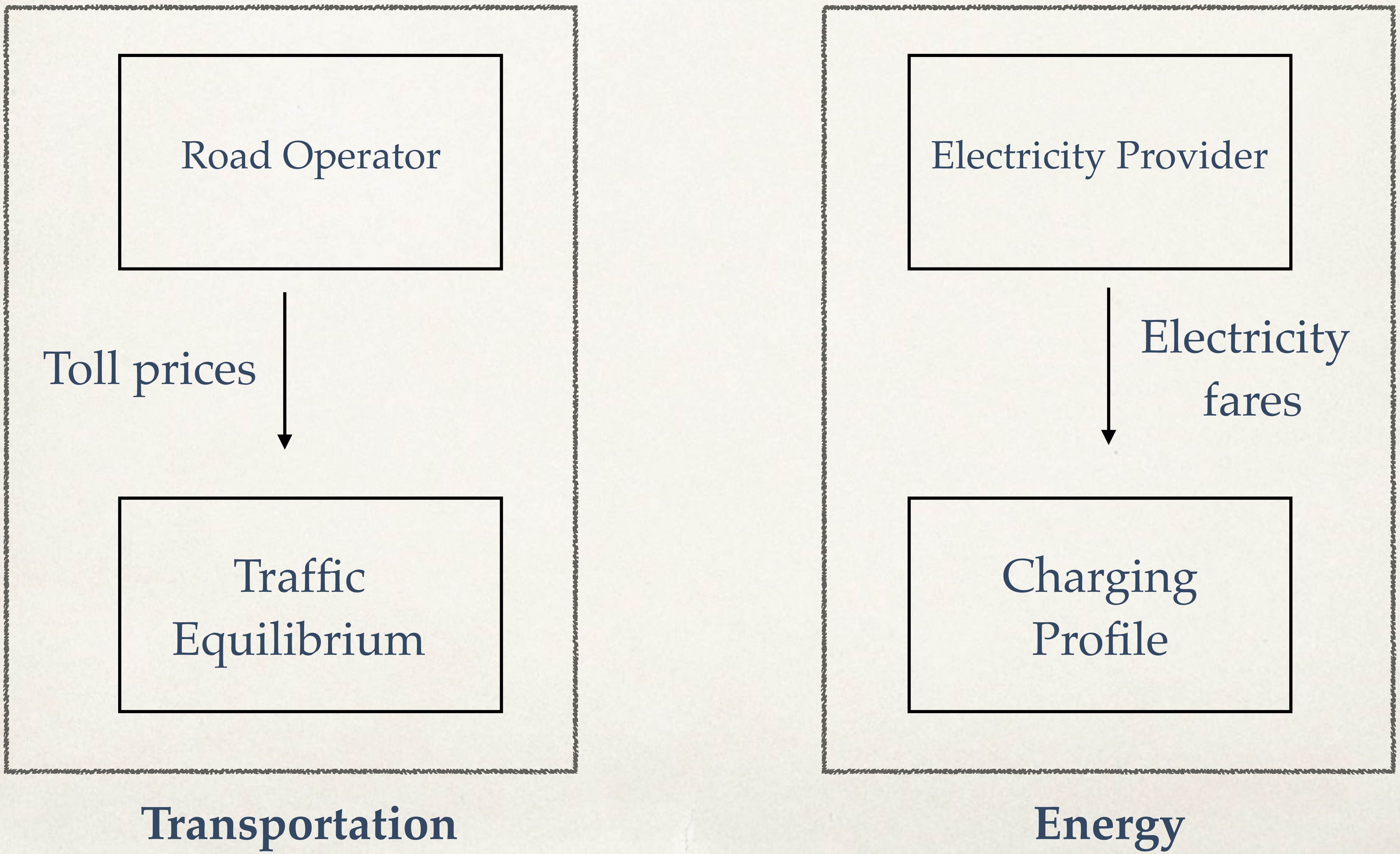
An energy problem

- Coupling bilevel optimization problems with large number of customers.
- Electrical Vehicles can be considered as large number of players in interaction:
    - on the road (driving problem)
    - on electricity demand (charging problem)

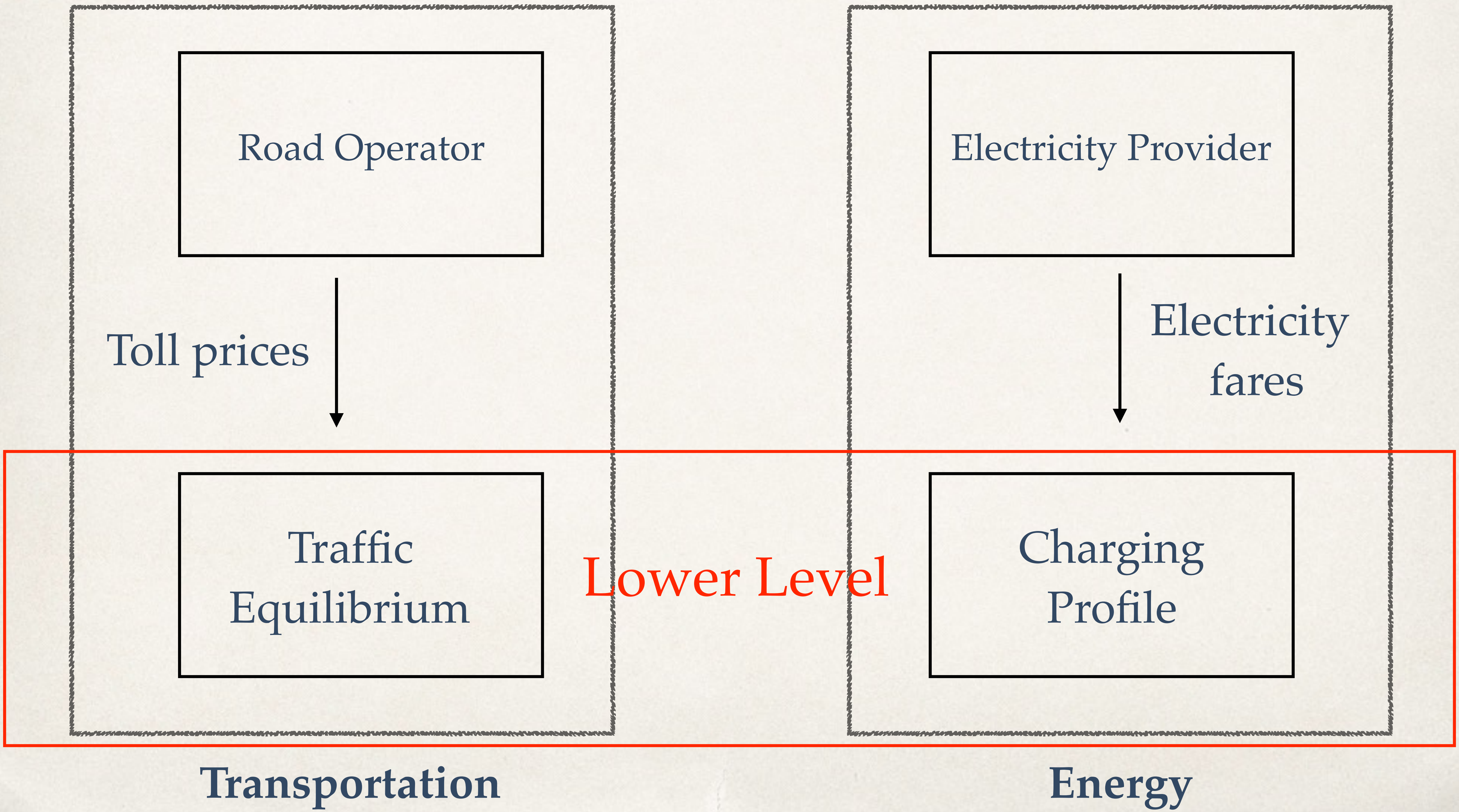
Context

- Integrated network management and energy planning
- Coupled two main actors: transportation planner and energy company
- **GOAL**: to design a model which integrates the couple decision processes of these actors, taking into account customers (particularly EV) behaviors.
- Joint work with EDF Labs, founded by PGMO.

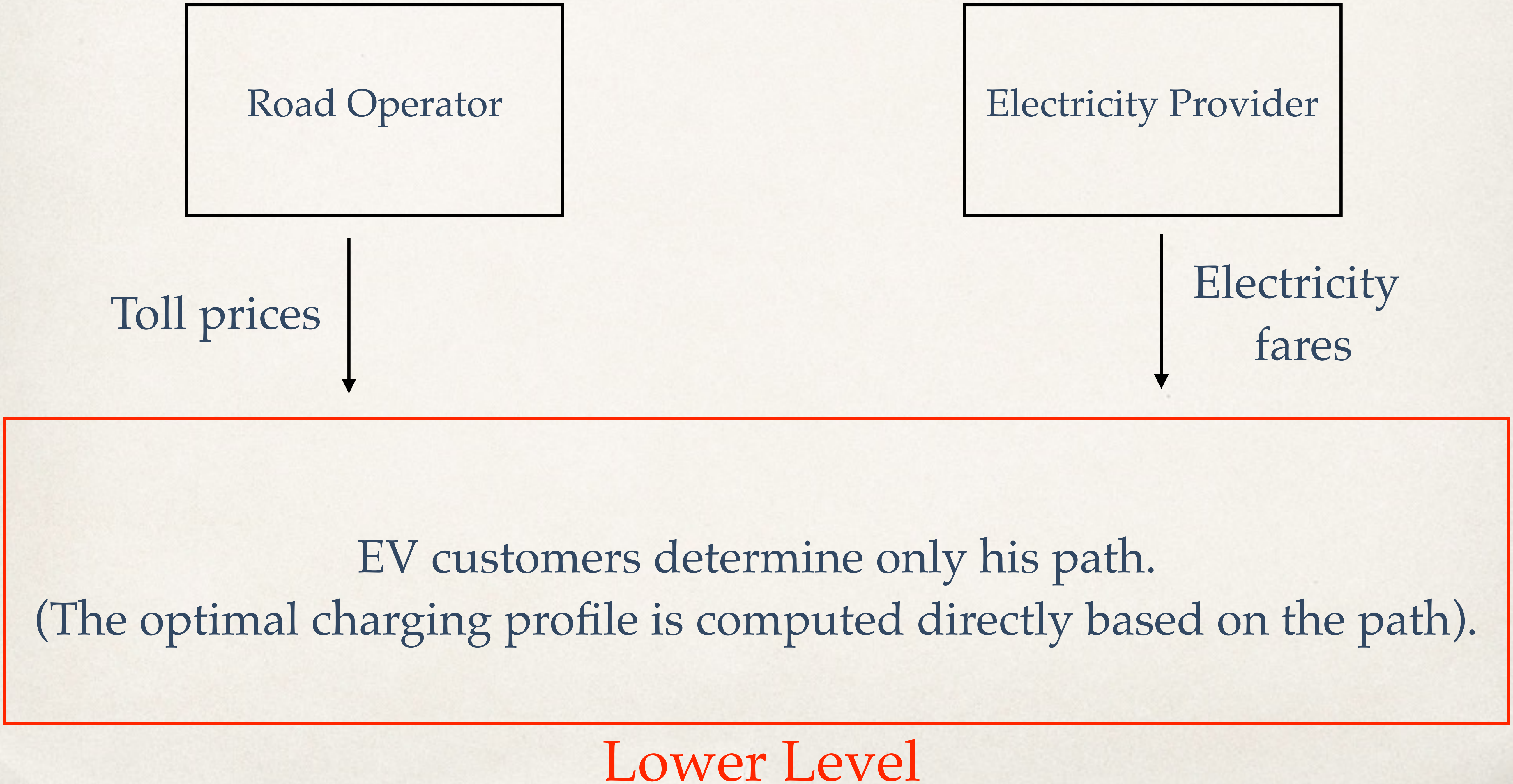
Global Problem



Global Problem



Global Problem



Some ongoing results

- The optimal charging profile is explicitly given depending on the traffic equilibrium flow, considering non flexible part (Valley-filling method).
- Cost functions that include tolls, delays and energy are not symmetric
  - ➡ Beckmann's formulae is not directly applicable
  - ➡ we consider generalization of network congestion games

THANK YOU

Questions?