Long term battery ageing control: introducing the Adaptative Weights Algorithm

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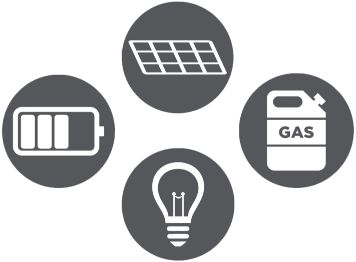
Motivation: the Microgrid



- The microgrid is **disconnected** from the main network
- **Constraints:** on the control and the state.
- **Operating cost:** cost of diesel
- We have a forecast for PV production and load
- Goal: minimize the operating cost (such that production meets demand)

Dynamics and continuous time optimal control formulation

- State of charge: c
- diesel cost: $\ell(u) = \beta u^2$
- state of charge dynamics: $F_c(c,u,t) = (\rho_i P_i(u,t) - P_o(u,t)/\rho_o)/C$
- with
  - $P_i(u,t) = (-u - P_s(t) + P_l(t))^+$ being the power that gets into the battery
  - $P_o(u,t) = (-u - P_s(t) + P_l(t))^-$ being the power that gets out of the battery.

Resolution by dynamic programming

In R. Bellman words: *"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."*

BocopHJB

- The value function of the optimal control problem is the viscosity solution of an Hamilton-Jacobi-Bellman (HJB) equation
- We solve this equation with a semi-Lagrangian scheme:

$$V^k(x) = \min_u \underbrace{h\ell^u(x)}_{\text{running cost}} + \underbrace{V^{k+1}(x + hf^u(x))}_{\text{value at the next point}}$$

- We then use the value function to build a decision
- The scheme is implemented in BocopHJB (available online)
- Bonnans, Giorgi, B. H. , Martinon, and Tissot, Bocophjb 1.0.1 user guide, tech. rep., 2015.

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The ageing dilemma

Why grad students are always tired?

Cafeine makes you take a suboptimal trajectory.

Battery ageing models

- loss of efficiency:

$$\rho_i(a) = (1-a)\rho,$$

  where ρ is the initial coefficient for $a = 0$.

- The aging dynamics corresponds to a severity factor model :

$$F_a(a(t), c(t), u(t), t) = \eta(c)\frac{P_o(a, c, u, t)}{K}$$

  where $\eta(c) = \frac{(-4c^2+5)}{5}$

Claim

By combining pre-computation and dimension reduction, the algorithm reduces the computing time by several orders and allows for online implementations.

Problem formulation

The age a and the charge c follow a T-periodic dynamics controlled by a time dependent parameter u:

$$P(a_0, c_0, t_0) \begin{cases} V(a_0, c_0, t_0) := \inf_{u \in U} \int_{t_0}^{NT} \ell(u(t), t)\mathrm{d}t + \psi(a_{NT}) \\ (a(t_0), c(t_0)) = (a_0, c_0), \quad a(NT) \leq a_{max} \\ (\dot{a}, \dot{c}) = (F_a(a, c, u, t), F_c(a, c, u, t)) \end{cases}$$

Problem structure

Assumption (Slow aging)

There exists a constant $L > 0$ such that F_c is L-Lipschitz and uniformly bounded by L, and F_a is L/N-Lipschitz and uniformly bounded by L/N.

Assumption (Monotonicity)

The value functions V^μ and V are non decreasing in a_0 and non increasing in c_0.

Assumption (Regularity of the ageing process)

For any $\epsilon > 0$, $\Delta > 0$, there exists $\epsilon_1 > 0$ such that if $x_0 = (a_0, c_0) \in \mathcal{A} \times \mathcal{C}$, $u \in U_{x_0}$ and $\Delta \leq \int_0^T F_a(X(t), u(t), t)\mathrm{d}t \leq \Delta + \epsilon_1$, then there exists $u' \in U_{x_0}$ such that $\int_0^T F_a(X(t), u'(t), t)\mathrm{d}t = \Delta$ and $\left|\int_0^T [\ell(u(t), t) - \ell(u'(t), t)]\mathrm{d}t\right| \leq \epsilon$.

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First step: Bilevel decomposition

$$P^\mu(a_0,\delta,c_0,c_F)\begin{cases} V^\mu(a_0,\delta,c_0,c_F) := \inf_{u\in U_T}\int_0^T \ell(u(t),t)\mathrm{d}t \\ (a(0),c(0)) = (a_0,c_0), \quad a(T)\leq a_0+\delta, \quad c(T)\geq c_F \\ (\dot{a},\dot{c}) = (F_a(a,c,u,t),F_c(a,c,u,t)). \end{cases}$$

First step: Bilevel decomposition

Lemma

$$V(a_0, c_0, t_k) = \inf_{\delta, c_f} V^{\mu}(a_0, \delta, c_0, c_f) + V(a_0 + \delta, c_f, t_{k+1})$$

Second step: relaxation

Let us introduce the relaxed micro-problem:

$$P_r^{\mu}(.., \alpha) \begin{cases} V_r^{\mu}(.., \alpha) := \inf_{u \in U} \int_0^T [\ell(u(t), t) + \alpha F_a(X(t), u(t), t)] \mathrm{d}t \\ (a(0), c(0)) = (a_0, c_0) \text{ and } c(T) \geq c_F. \\ (\dot{a}(t), \dot{c}(t) = F(a(t), c(t), u(t), t) \end{cases}$$

Lemma

For all $(a_0, c_0, c_f, \alpha) \in \mathcal{A} \times \mathcal{C}^2 \times \mathrm{IR}_+$,

$$V_r^\mu(a_0, c_0, c_f, \alpha) = \inf_\delta V^\mu(a_0, \delta, c_0, c_f) + \alpha\delta.$$

In particular $\forall \delta \in \mathrm{IR}_+, \mathrm{V}^\mu(\mathrm{a}_0, \delta, \mathrm{c}_0, \mathrm{c}_\mathrm{f}) \geq \mathrm{V}_\mathrm{r}^\mu(\mathrm{a}_0, \mathrm{c}_0, \mathrm{c}_\mathrm{f}, \alpha) - \alpha\delta$

Notations

- $\Delta(u) := \int_0^T F_a(X(t), u(t), t)\mathrm{d}t$
- $\mathcal{L}(u) := \int_0^T \ell(u(t), t)\mathrm{d}t$
- For any $(a_0, c_0, c_F, \alpha) \in \mathcal{A} \times \mathcal{C}^2 \times \mathrm{IR}_+$, let

$$\Gamma(a_0, c_0, c_F, \alpha)$$

be the set of $\lim_n \Delta(u_n)$ where u_n is a minimizing sequence of $P_r^\mu(a_0, c_0, c_F, \alpha)$

Result in the absence of jumps

Theorem

If there exists α such that

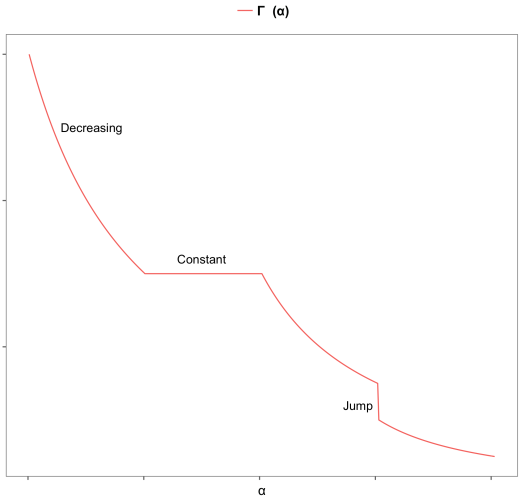
$$(a_1 - a_0) \in \Gamma(a_0, c_0, c_1, \alpha)$$

Then

$$V(a_0, c_0, T) = \inf_{(\alpha, c_F, \delta)} \{V_r^{\mu}(a_0, c_0, c_F, \alpha) - \alpha\delta + V(a_0 + \delta, c_F, T)\}$$

where the optimization is performed over the (α, δ, c_F) such that $\delta \in \Gamma(a_0, c_0, c_F, \alpha)$.

Jumps



Result

"The error we make is controlled by the size of the jumps."

Third step: approximation

$$\tilde{P}_r^{\mu}(..,\alpha) \begin{cases} \tilde{V}_r^{\mu}(..,\alpha) := \inf_{u\in U} \int_0^T [\ell(u(t),t) + \alpha F_a(a_0, c(t), u(t), t)]\mathrm{d}t \\ \dot{c}(t) = F_c(a_0, c(t), u(t), t) \\ (a(0), c(0)) = (a_0, c_0),\ c(T) \geq c_F \end{cases}$$

Algorithms for the periodic case

- Offline: Compute $\mathcal{L}$ and Δ for each possible values of (α, a). Compute the global value function at T.
- Online: Get the optimal weight α, solve the corresponding short term approximate problem.

Numerical results

- Bruteforce >10 hours, AWA approx. 12 minute, online approx. 1 sec
- Some % of relative error

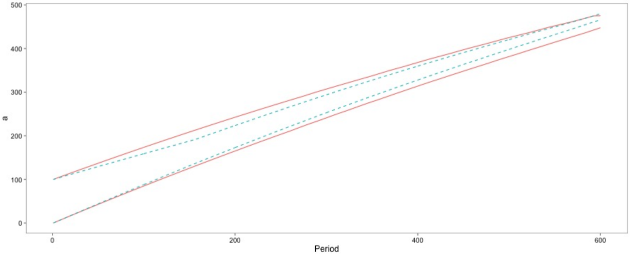


Figure: The age profile computed with AWA (solid line) and bruteforce dynamic programming (dotted line) for two initial age values

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Next

- other applications (inventory management and budget control ?)
- extensions (stochastic, impulse, elliptic, multidimensional...)
- finer error analysis

Bibliography

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