Risk and Sustainability: a Stochastic Stewardship Criterion

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Outline of the presentation

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Aggregating flows of goods or services over time

- Flows of goods or services over time
  - consumption $\mathcal{C}(t)$,
  - environment $\mathcal{E}(t)$.
- How policy-makers aggregate over consequences
  - (i) within generations,
  - (ii) over time,
  - (iii) according to risk

  will be crucial to policy design and choice.

[Stern, 2006]

Expected intertemporal discounted utility

$$\mathbb{E}\left[\sum_{t=t_0}^{+\infty}(\frac{1}{1+r})^{t-t_0}\overbrace{U(\mathcal{C}(t),\mathcal{E}(t))}^{\text{utility}}\right]$$

is built upon two well axiomatized theories,

- the discounted intertemporal utility [Koopmans, 1965]
- and the expected utility [von Neuman and Morgenstern, 1947].

This approach is widely used and offers interesting applicability properties as time consistency and dynamic programming.

Outline of the presentation

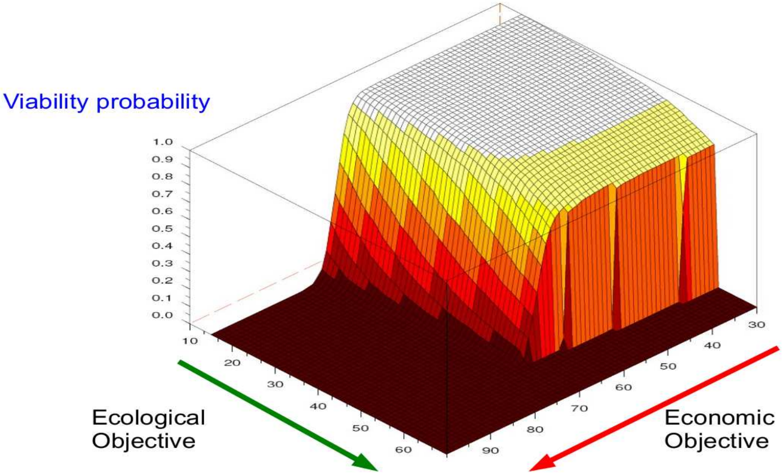
The notion of "stewardship"

[Stern, 2006]
A concept related to the idea of the rights of future generations is that of sustainable development: future generations should have a right to a standard of living no lower than the current one.

The notion of "stewardship" can be seen as a special form of sustainability.

It points to particular aspects of the world, which should themselves be passed on in a state at least as good as that inherited from the previous generation.

$$\text{consumption } \mathcal{C}(t) \geq \iota_{\mathcal{C}} \text{ and environment } \mathcal{E}(t) \geq \iota_{\mathcal{E}} \ .$$


Viability probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10
20
30
40
50
60
30
40
50
60
70
80
90
Ecological Objective
Economic Objective

Catastrophe insurance vs. consumption smoothing

[Weitzman, 2007] But I think progress begins by recognizing that the hidden core meaning of Stern vs. Critics may be about $(\cdots)$

- catastrophe insurance

$$SSC = \mathbb{E}\left[\prod_{t=t_0}^{+\infty} \underbrace{\mathbf{1}_{\mathcal{C}(t)\geq \iota_{\mathcal{C}}} \mathbf{1}_{\mathcal{E}(t)\geq \iota_{\mathcal{E}}}}_{\text{indicators} \geq \text{thresholds}}\right]$$

- *versus* consumption smoothing

$$EU = \mathbb{E}\left[\sum_{t=t_0}^{+\infty} \left(\frac{1}{1+r}\right)^{t-t_0} \underbrace{U\big(\mathcal{C}(t), \mathcal{E}(t)\big)}_{\text{utility}}\right]$$

Control system

Three types of variables

- state x: capital, CO_2 concentration, etc.
- decision a: investment, abatement, etc.
- uncertainty ω: costs uncertainties, climate parameters, etc.

Discrete-time decision dynamical system with uncertainty

$$
\begin{cases}
x(t+1) = \mathrm{dyn}\big(t, x(t), a(t), \omega(t)\big) , \quad t = t_0, \ldots, T-1 \\
x(t_0) = x_0
\end{cases}
$$

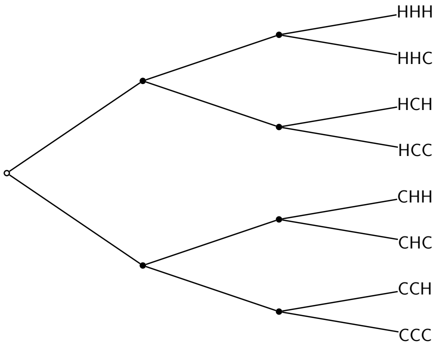
with

- time $t \in \{t_0, \ldots, T\}$, and horizon T.
- uncertainty perturbations of the dynamics.

Indicators and thresholds

- Indicators
  - consumption $\mathcal{I}_{\mathcal{C}}(t, x, a, \omega)$
  - environment $\mathcal{I}_{\mathcal{E}}(t, x, a, \omega)$: for instance, - CO_2 concentration
- Thresholds $\iota_{\mathcal{C}}$, $\iota_{\mathcal{E}}$ (for instance, - 450 ppm)
- Driving the system so that for all time $t = t_0, t_0 + 1, \ldots$
  - consumption $\geq$ threshold: $\mathcal{I}_{\mathcal{C}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{C}}$
  - environment $\geq$ threshold: $\mathcal{I}_{\mathcal{E}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{E}}$
- Particular aspects of the world are passed on in a state at least as good as that inherited from the previous generation.

Scenarios



A scenario is a sequence of uncertainties:

$$\omega(\cdot) := \big(\omega(t_0), \ldots, \omega(T)\big) \ .$$

Decision rule

- A decision rule $\mathfrak{a}$:
  'Do thus-and-thus if you find yourself in this portion of state space with this amount of time left.' (Richard E. Bellman)
- A scenario + a decision rule + a dynamics $\Longrightarrow$
  - a state path $x(\cdot) := \big(x(t_0), \ldots, x(T)\big)$
  - a decision path $a(\cdot) := \big(a(t_0), \ldots, a(T-1)\big)$
  - given by

$$x(t+1) = \mathrm{dyn}\big(t, x(t), \mathfrak{a}\big(t, x(t)\big), \omega(t)\big) \ , \quad a(t) = \mathfrak{a}\big(t, x(t)\big)$$

Viable scenarios

$$\Omega_{\mathfrak{a},t_0,x_0,T} := \left\{ \begin{array}{l} \text{scenarios } \omega(\cdot) \text{ along which} \\ \text{the state } x(\cdot) \text{ and decision } a(\cdot) \text{ trajectories} \\ \text{generated by dynamics dyn and decision rule } \mathfrak{a} \\ \text{starting from initial state } x_0 \text{ at initial time } t_0 \\ \text{satisfy the indicators objectives} \\ \mathcal{I}_{\mathcal{C}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{C}} \\ \mathcal{I}_{\mathcal{E}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{E}} \\ \text{from initial time } t_0 \text{ to horizon } T \end{array} \right\}$$

Scenarios along which particular aspects of the world are passed on in a state at least as good as that inherited from the previous generation.

Probability on the set of scenarios

- The larger the set $\Omega_{\mathfrak{a},t_0,x_0,T}$ of viable scenarios, the better.
- Suppose a probability $\mathbb{P}$ is given on the set of scenarios. This is a delicate issue!

Viability probability

The viability probability is $\mathbb{P}\left[\Omega_{\mathfrak{a},t_0,x_0,T}\right] =$

Probability $\left\{\right.$ scenarios $\omega(\cdot)$ along which
the state $x(\cdot)$ and decision $a(\cdot)$ trajectories
generated by dynamics dyn and decision rule $\mathfrak{a}$
starting from initial state x_0 at initial time t_0
satisfy the indicators objectives
$\mathcal{I}_{\mathcal{C}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{C}}$,
$\mathcal{I}_{\mathcal{E}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{E}}$,
from initial time t_0 to horizon T $\left.\right\}$.

This is the stochastic stewardship criterion (SSC): probability that particular aspects of the world are passed on in a state at least as good as that inherited from the previous generation.

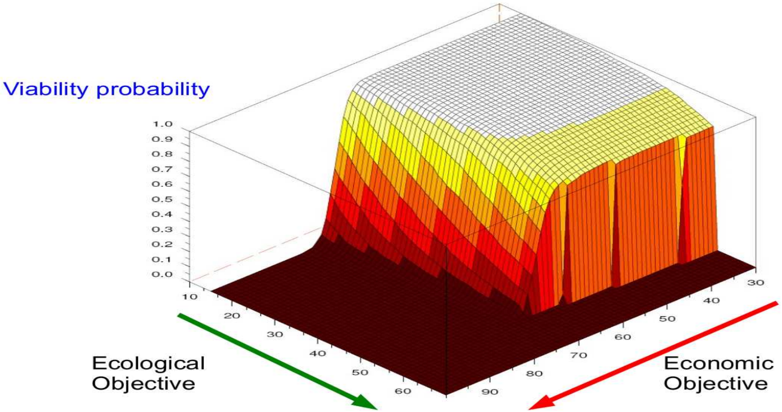
Maximal viability probability

- The maximal viability probability is

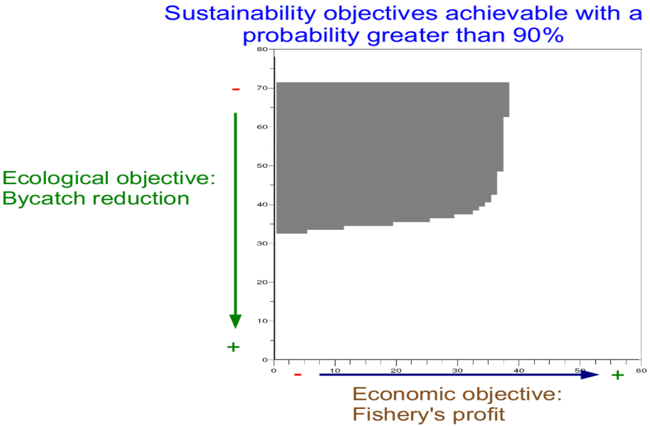
$$\mathrm{PV}(t_0, x_0) := \sup_{\mathfrak{a}} \mathbb{P}\left[\Omega_{\mathfrak{a}, t_0, x_0, T}\right] = \sup_{\mathfrak{a}} SSC \ .$$

- Maximizing the stochastic stewardship criterion (SSC) is maximizing the probability that particular aspects of the world are passed on in a state at least as good as that inherited from the previous generation.
- What about trade-offs?

Maximal viability probability function of objectives



Trade-offs in controlling the tails



Outline of the presentation

- Indicators $\mathcal{I}_1(t,x,a,\omega), \ldots, \mathcal{I}_K(t,x,a,\omega)$
- Thresholds $\iota_1, \ldots, \iota_K$

As the viability probability can be written in the following expected intertemporal form

$$\mathbb{P}\left[\Omega_{\mathfrak{a},t_0,x_0,T}\right] = \mathbb{E}\left[\prod_{t=t_0}^{T-1}\prod_{k=1}^{K}\mathbf{1}_{[\iota_k,+\infty[}\Big(\mathcal{I}_k\big(t,x(t),a(t),\omega(t)\big)\Big)\right] ,$$

a stochastic dynamic programming equation can be derived for $\mathrm{PV}\big(t,x\big) = \sup_{\mathfrak{a}}\mathbb{P}\left[\Omega_{\mathfrak{a},t,x,T}\right]$, whenever uncertainties are independent under probability $\mathbb{P}$.

[De Lara and Doyen, 2008]

Dynamic programming and time consistency

- Today, at time t_0, I formulate an optimization problem with criterion $\mathrm{Crit}\big(t_0, x(\cdot), a(\cdot), \omega(\cdot)\big)$ This yields a sequence of optimal decision rules $\mathfrak{a}^{t_0}(t_0, x), \mathfrak{a}^{t_0}(t_0+1, x), \ldots$
- Tomorrow, at time t_0+1, I will formulate an optimization problem with criterion $\mathrm{Crit}\big(t_0+1, x(\cdot), a(\cdot), \omega(\cdot)\big)$. This will yield a sequence of optimal decision rules $\mathfrak{a}^{t_0+1}(t_0+1, x), \mathfrak{a}^{t_0+1}(t_0+2, x), \ldots$
- Time consistency holds true whenever my today rule for tomorrow coincides with my tomorrow rule for tomorrow:

$$\mathfrak{a}^{t_0+1}\big(t_0+1, x\big) = \mathfrak{a}^{t_0}\big(t_0+1, x\big) \; .$$

Outline of the presentation

Uncertainty over whether or not the world will exist

(...) following distinguished economists from Frank Ramsey in the 1920s to Amartya Sen and Robert Solow more recently, the only sound ethical basis for placing less value on the utility (as opposed to consumption) of future generations was the uncertainty over whether or not the world will exist, or whether those generations will all be present.

[Stern, 2006]

Intertemporal discounted utility

$$\mathbb{E}\left[\overbrace{\sum_{t=t_0}^{+\infty}\left(\frac{1}{1+r}\right)^{t-t_0} U\big(\mathcal{C}(t),\mathcal{E}(t)\big)}^{\text{discounting}}\right] = \mathbb{E}\left[\overbrace{\sum_{t=t_0}^{t_0+\theta-1} U\big(\mathcal{C}(t),\mathcal{E}(t)\big)}^{\text{equal treatment}}\right]$$

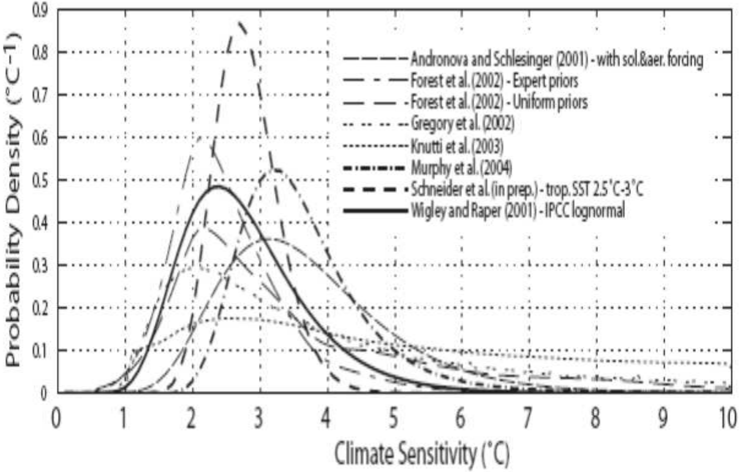
- the random duration θ follows a Geometric distribution $\mathbb{P}[\theta > n] = (1-p)^n$ with parameter $1 - p = \frac{1}{1+r}$, where p is interpreted as the probability that the world may disappear during each time period,
- and θ is independent of the consumption and environment stream (quite debatable in the climate change context).

Random horizon

Make the horizon $T = t_0 + \theta$ random, where θ follows a Geometric distribution with parameter $1 - p = \frac{1}{1+r}$, to capture the uncertainty over whether or not the world will exist:

$$\text{Probability} \left\{ \begin{array}{c|l} & x(t_0) = x_0 \\ & x(t+1) = \mathrm{dyn}\big(t, x(t), a(t), \omega(t)\big) \\ \omega(\cdot) & a(t) = \mathfrak{a}\big(t, x(t)\big) \\ \theta & \mathcal{I}_{\mathcal{C}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{C}} \\ & \mathcal{I}_{\mathcal{E}}(t, x(t), a(t), \omega(t)) \geq \iota_{\mathcal{E}} \\ & \forall t = t_0, \ldots, t_0 + \theta - 1 \end{array} \right\} .$$

Ambiguity: probability distributions over climate sensitivity



Multi-prior approach

- Different probabilities $\mathbb{P}$ belonging to a set $\mathcal{P}$

$$\sup_{\mathfrak{a}} \underbrace{\inf_{\mathbb{P}\in\mathcal{P}} \overbrace{\mathbb{P}\left[\Omega_{\mathfrak{a},t_0,x_0,T}\right]}^{\text{viability probability}}}_{\text{pessimistic over probabilities}} .$$

- Time consistency when indicators $\mathcal{I}_k(t,x,a)$ do not explicitely depend upon uncertainty ω.

Outline of the presentation

The negative consequences of not acting

$(\cdots)$ the Review estimates that if we don't act, the overall costs and risks of climate change will be equivalent to losing at least 5% of global GDP each year, now and forever.

[Stern, 2006]

Loss-frame *vs* gain-frame

Beth E. Meyerowitz and Shelly Chaiken. The effect of message framing on breast self-examination attitudes, intentions, and behavior. *Journal of Personality and Social Psychology*, 52(3): 500–510, March 1987.

- You can lose several potential health benefits by failing to spend only five minutes each month doing breast self-examination

$$\vee$$

- You can gain several potential health benefits by spending only five minutes each month doing breast self-examination

Subjects who read a pamphlet with arguments framed in loss language manifested more positive BSE attitudes, intentions, and behaviors.

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