

Multistage stochastic optimization of a hydrogen supply chain

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Why hydrogen ?



- Hydrogen displays promising features for **decarbonization** industry, transportation and building sectors
- Transition towards a hydrogen economy requires hydrogen costs to come down, through **optimal** choices of infrastructure **design** and **operation**
- The optimal choices of design and operation rely on **multistage stochastic** optimization

Problem statement for the management of a hydrogen supply chain

Solving the problem by decomposition method

Numerical results

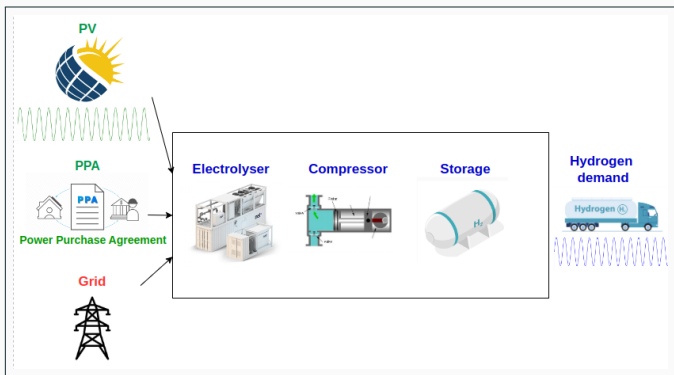
Optimizing the stock capacity

Future works and extensions

Problem statement for the management of a hydrogen supply chain

Problem statement

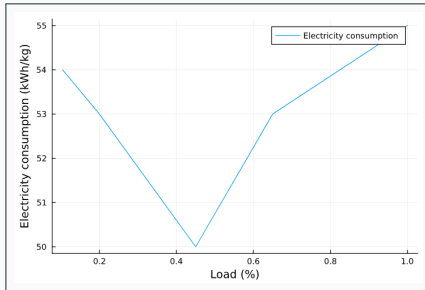
- Schiever company: diesel trucks → hydrogen trucks



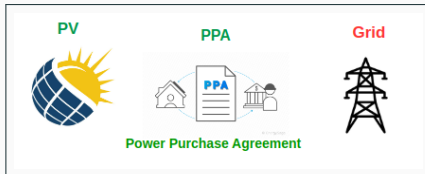
The objective is to **minimize the operation cost** of this supply chain by making decisions every **hour** during one week

Special characteristics of the problem

- A **nonlinear** electricity consumption for the electrolyser



- An **energy** mix to power the supply chain



Decisions and uncertainties

Decision	Description
E_h^{PPA}	Electricity from PPA contract (kWh)
E_{h+1}^{G}	Electricity from the grid (kWh)
I_h^{E}	Load at which the electrolyser is functioning (%)
M_h^{E}	Turn the electrolyser to cold, idle or start mode
H_h^{D}	Quantity of hydrogen extracted from the storage (kg) to satisfy demand

Uncertainty	Description
E_{h+1}^{PV}	Renewable (PV) electricity (kWh) during $[h, h + 1[$
D_{h+1}	Demand of hydrogen (kg) during $[h, h + 1[$

Optimization problem formulation

$$\begin{aligned}
 & \min_{(\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{M}_h^{\text{E}^{\text{H}}}, \mathbf{I}_h^{\text{E}}, \mathbf{H}_h^{\text{H} \rightarrow \text{D}})_{h \in \mathbb{H}}} \mathbb{E}_{(\mathbf{D}_{h+1}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}} \left[\right. \\
 & \quad \sum_{h \in \mathbb{H}} \underbrace{c^{\text{ppa}} \mathbf{E}_h^{\text{PPA}} + c_h^{\text{g}} \mathbf{E}_{h+1}^{\text{G}}}_{\text{Electricity cost}} + \underbrace{c^{\text{d}} (\mathbf{D}_{h+1} - \mathbf{H}_h^{\text{H} \rightarrow \text{D}})_+}_{\text{Demand dissatisfaction cost}} \\
 & \quad \left. + K((\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}) \right]
 \end{aligned}$$

subject to

- **nonanticipativity** constraints
- operational constraints
- renewable energy constraints
- a **coupling constraint** between the energy mix and the hydrogen equipments

$$\underbrace{\mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^{\text{G}} + \mathbf{E}_{h+1}^{\text{PV}}}_{\text{electricity supply}} = \underbrace{g(\mathbf{M}_h^{\text{E}^{\text{H}}}, \mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}})}_{\text{electricity consumption}}$$

Optimization problem formulation

$$\begin{aligned}
 & \min_{(\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{M}_h^{\text{E}^{\leftarrow}}, \mathbf{I}_h^{\text{E}}, \mathbf{H}_h^{\rightarrow \text{D}})_{h \in \mathbb{H}}} \mathbb{E}_{(\mathbf{D}_{h+1}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}} \left[\right. \\
 & \quad \sum_{h \in \mathbb{H}} \underbrace{c^{\text{ppa}} \mathbf{E}_h^{\text{PPA}} + c_h^{\text{g}} \mathbf{E}_{h+1}^{\text{G}}}_{\text{Electricity cost}} + \underbrace{c^{\text{d}} (\mathbf{D}_{h+1} - \mathbf{H}_h^{\rightarrow \text{D}})_+}_{\text{Demand dissatisfaction cost}} \\
 & \quad \left. + K((\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}) \right]
 \end{aligned}$$

subject to

- **nonanticipativity** constraints
- operational constraints
- **renewable energy constraints**
- a **coupling constraint** between the energy mix and the hydrogen equipments

$$\underbrace{\mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^{\text{G}} + \mathbf{E}_{h+1}^{\text{PV}}}_{\text{electricity supply}} = \underbrace{g(\mathbf{M}_h^{\text{E}^{\leftarrow}}, \mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}})}_{\text{electricity consumption}}$$

Renewable energy constraints of the problem

- Constraint on energy from the grid

Electricity from the grid is less than p percent of the total electricity

$$\frac{\sum_{h \in \mathbb{H}} \mathbf{E}_{h+1}^G}{\sum_{h \in \mathbb{H}} \mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^G + \mathbf{E}_{h+1}^{\text{PV}}} \leq p$$
$$\iff \sum_{h \in \mathbb{H}} \left((1-p) \mathbf{E}_{h+1}^G - p(\mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^{\text{PV}}) \right) \leq 0$$

reformulated as a cost function

$$K\left(\left(\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^G, \mathbf{E}_{h+1}^{\text{PV}}\right)_{h \in \mathbb{H}}\right) = \alpha \max\left(0, \sum_{h \in \mathbb{H}} \left((1-p) \mathbf{E}_{h+1}^G - p(\mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^{\text{PV}}) \right)\right)$$

- Constraint on PPA contract

Electricity from the PPA contract is bounded

$$\sum_{h \in \mathbb{H}} \mathbf{E}_h^{\text{PPA}} \leq \overline{E}^{\text{PPA}}$$

State dynamics for the Schiever case

To solve the problem by **dynamic programming**, we introduce the mode of electrolyser and three stock variables

State variable	Description
M_h^E	mode of the electrolyser ($\{start, idle, cold\}$)
S_h	quantity of hydrogen in the storage (kg)
Q_h	“cumul of electricity”
P_h	remaining stock of available PPA (kWh)

State dynamics for the Schieve case

Letting $X_h = (M_h^E, S_h, P_h, Q_h)$ be the four dimensional state, we have a state dynamics $\mathbf{X}_{h+1} = f(\mathbf{X}_h, \mathbf{I}_h^E, \mathbf{M}_h^{E\hat{~}}, \mathbf{H}_h^{\rightarrow D}, \mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^G, \mathbf{E}_{h+1}^{\text{PV}}, \mathbf{D}_{h+1})$ with

$$\mathbf{M}_{h+1}^E = \mathbf{M}_h^{E\hat{~}} \quad (\text{Mode of the electrolyser})$$

$$\mathbf{S}_{h+1} = \mathbf{S}_h + \underbrace{\mathbf{I}_h^E \mu (\mathbf{M}_h^E, \mathbf{M}_h^{E\hat{~}}) \overline{m^E}}_{\text{H}_2 \text{ produced}} - \underbrace{\mathbf{H}_h^{\rightarrow D}}_{\text{H}_2 \text{ extracted}} \quad (\text{Stock of hydrogen})$$

$$\mathbf{P}_{h+1} = \mathbf{P}_h - \mathbf{E}_h^{\text{PPA}} \quad (\text{Stock of PPA})$$

$$\mathbf{Q}_{h+1} = \mathbf{Q}_h + p \mathbf{E}_{h+1}^G - (1-p)(\mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^{\text{PV}}) \quad (\text{Cumul of electricity})$$

Thanks to the state Q_h , the cost $K((\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^G, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}})$ can be written as $J(\mathbf{Q}_T) = \alpha \max(0, \mathbf{Q}_T)$

Numerical considerations

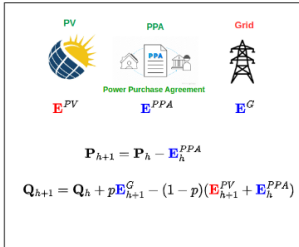
A difficult numerical problem:

- **one week** horizon with **hourly** decisions (168 time steps)
- **four** dimensional state
 - using Stochastic Dynamic Programming, we **discretize** the states
 - the stock of PPA (P) and of the “cumul of electricity” (Q) take large values and require a **fine** discretization which is numerically demanding
- **two** random variables
- **five** decisions at each hour

Solving the problem by decomposition method

A decomposition approach

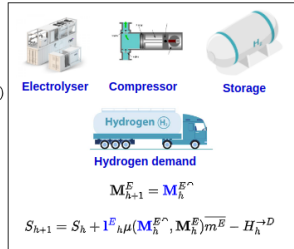
Electricity allocation problem



Coupling constraint

$$E_{h+1}^{PV} + E_h^{PPA} + E_{h+1}^G = g(M_h^{E^{\wedge}}, M_h^E, I_h^E)$$

Operational problem



A decomposition approach

$$\text{val}(\mathcal{P}) = \min_{(\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}}, \mathbf{H}_h^{\rightarrow\text{D}})_{h \in \mathbb{H}}} \mathbb{E}_{(\mathbf{D}_{h+1}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}} \left[\underbrace{\sum_{h \in \mathbb{H}} c^{\text{ppa}} \mathbf{E}_h^{\text{PPA}} + c_h^{\text{g}} \mathbf{E}_{h+1}^{\text{G}} + K((\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}})}_{\text{electricity cost}} + \underbrace{\sum_{h \in \mathbb{H}} c^{\text{d}} (\mathbf{D}_{h+1} - \mathbf{H}_h^{\rightarrow\text{D}})_+}_{\text{demand dissatisfaction cost}} \right]$$

subject to nonanticipativity constraints, operational constraints, electricity constraints and the following **coupling constraint**

$$\underbrace{\mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^{\text{G}} + \mathbf{E}_{h+1}^{\text{PV}}}_{\text{electricity supply}} = \underbrace{g(\mathbf{M}_h^{\text{E}}, \mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}})}_{\text{electricity consumption}}$$

Decoupling the operational and electricity problems

Dualize the coupling constraints with Lagrange multipliers: $\lambda = \{\lambda_h\}_{h \in \mathbb{H}}$

- electrolyser/compressor/storage operational (O) problem

$$\phi^O(\lambda) = \min_{(\mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}}, \mathbf{H}_h^{\text{D}})_{h \in \mathbb{H}}} \mathbb{E}_{(\mathbf{D}_{h+1})_{h \in \mathbb{H}}} \left[\sum_{h \in \mathbb{H}} c^d (\mathbf{D}_{h+1} - \mathbf{H}_h^{\text{D}})_+ - \sum_{h \in \mathbb{H}} \lambda_h (g(\mathbf{M}_h^{\text{E}}, \mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}})) \right]$$

s.t. (undetailed) operational constraints

- electricity (E) allocation problem

$$\phi^E(\lambda) = \min_{(\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}})_{h \in \mathbb{H}}} \mathbb{E}_{(\mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}} \left[\sum_{h \in \mathbb{H}} c^{\text{ppa}} \mathbf{E}_h^{\text{PPA}} + c_h^{\text{g}} \mathbf{E}_{h+1}^{\text{G}} + K((\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}) + \sum_{h \in \mathbb{H}} \lambda_h (\mathbf{E}_h^{\text{PPA}} + \mathbf{E}_{h+1}^{\text{G}} + \mathbf{E}_{h+1}^{\text{PV}}) \right]$$

s.t. electricity constraints

Decoupling the operational and electricity allocation problems

- For a fixed **deterministic** multiplier $\lambda = \{\lambda_h\}_{h \in \mathbb{H}}$, independently:
 - compute $\phi^O(\lambda)$ (**operational problem**) by **Stochastic Dynamic Programming (SDP)**
 $\phi^O(\lambda) = V_0^{O,\lambda}(S_0, M_0^E)$
given by induction $V_h^{O,\lambda} = B^{O,\lambda}(V_{h+1}^{O,\lambda})$
 - compute $\phi^E(\lambda)$ (**electricity allocation problem**) by **Stochastic Dual Dynamic Programming (SDDP)**
 $\phi^E(\lambda) = V_0^{E,\lambda}(P_0, Q_0)$
given by induction $V_h^{E,\lambda} = B^{E,\lambda}(V_{h+1}^{E,\lambda})$
- Update the multiplier λ by a **gradient based** optimization algorithm to maximize the dual function $\phi^O(\lambda) + \phi^E(\lambda)$

$$\phi^O(\lambda) + \phi^E(\lambda) \leq \underbrace{\max_{\lambda'} \phi^O(\lambda') + \phi^E(\lambda')}_{\text{dual problem}} \leq \underbrace{\text{val}(\mathcal{P})}_{\text{primal problem}}$$

Policy design

For a fixed multiplier $\lambda = \{\lambda_h\}_{h \in \mathbb{H}}$

- we obtain a **feasible policy** $\pi^\lambda = \{\pi_h^\lambda\}_{h \in \mathbb{H}}$ for the problem (\mathcal{P}) by

$$\underbrace{\text{policy}}_{\pi_h^\lambda}(S_h, M_h^E, P_h, Q_h) = \arg \min_{(\mathbf{E}_h^{\text{PPA}}, \mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}}, \mathbf{H}_h^{\rightarrow \text{D}})} \mathbb{E}_{(\mathbf{D}_{h+1}, \mathbf{E}_{h+1}^{\text{PV}})} \left[\begin{aligned} & \min_{\mathbf{E}_{h+1}^{\text{G}}} L_h(\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{E}_{h+1}^{\text{PV}}, \mathbf{H}_h^{\rightarrow \text{D}}) \\ & + \underbrace{V_{h+1}^{O, \lambda}(\mathbf{S}_{h+1}, \mathbf{M}_{h+1}^{\text{E}}) + V_{h+1}^{E, \lambda}(\mathbf{P}_{h+1}, \mathbf{Q}_{h+1})}_{\text{surrogate additive value function}} \end{aligned} \right]$$

under nonanticipativity constraints, operational constraints, renewable energy constraints and the **coupling constraint**

- we have $\text{val}(\mathcal{P}) \leq \text{val}(\mathcal{P}_{\pi^\lambda})$,
where $\text{val}(\mathcal{P}_{\pi^\lambda})$ denotes the cost when applying the policy π^λ

PV and demand data

The independent random variables $(\mathbf{D}_{h+1}, \mathbf{E}_{h+1}^{PV})_{h \in \mathbb{H}}$ are given by

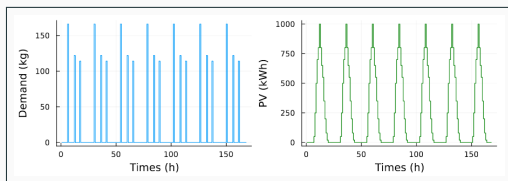
Table 1: Probability distribution of \mathbf{D}_{h+1}

Outcome	$0.8\mu_{h+1}^d$	$0.9\mu_{h+1}^d$	μ_{h+1}^d	$1.1\mu_{h+1}^d$	$1.2\mu_{h+1}^d$
Probability	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Table 2: Probability distribution of \mathbf{E}_{h+1}^{PV}

Outcome	$0.8\mu_{h+1}^{PV}$	$0.9\mu_{h+1}^{PV}$	μ_{h+1}^{PV}	$1.1\mu_{h+1}^{PV}$	$1.2\mu_{h+1}^{PV}$
Probability	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

where $(\mu_{h+1}^d, \mu_{h+1}^{PV})_{h \in \mathbb{H}}$ are deterministic and given in the following figure



Monte-Carlo evaluation of a policy

- Let $(d, e^{\text{PV}}) = (d_{h+1}, e_{h+1}^{\text{PV}})_{h \in \mathbb{H}}$ be a sample of hydrogen demand and PV production

We simulate a policy as follows:

$$x_0 \rightsquigarrow \pi_0^\lambda(x_0) \rightsquigarrow (d_1, e_1^{\text{PV}}) \rightsquigarrow x_1 = f(x_0, \pi_0^\lambda(x_0), (d_1, e_1^{\text{PV}})) \rightsquigarrow \pi_1^\lambda(x_1) \rightsquigarrow (d_2, e_2^{\text{PV}}) \rightsquigarrow x_2 = f(x_1, \pi_1^\lambda(x_1), (d_2, e_2^{\text{PV}})) \dots \rightsquigarrow (d_{h+1}, e_{h+1}^{\text{PV}}) \rightsquigarrow x_h = f(x_{h-1}, \pi_h^\lambda(x_{h-1}), (d_{h+1}, e_{h+1}^{\text{PV}}))$$

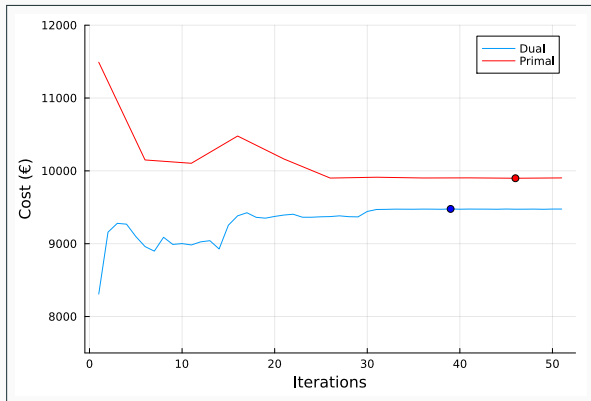
- Let $\{(d, e^{\text{PV}})_i\}_{i \in \llbracket 1, N \rrbracket}$ be a set of noise trajectories, we use the approximation

$$\text{val}(\mathcal{P}_{\pi^\lambda}) \approx \frac{1}{N} \sum_{i=1}^N \text{val}(\mathcal{P}_{\pi^\lambda}^i)$$

where $\text{val}(\mathcal{P}_{\pi^\lambda}^i)$ is the cost of the simulation for the noise trajectory $(d, e^{\text{PV}})_i$

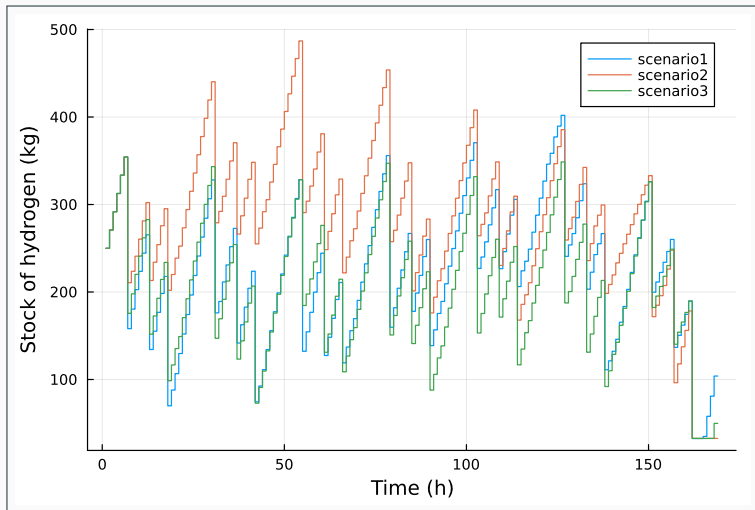
Numerical results

Maximizing the dual function



maximization of the dual function with decomposition algorithm

Evolution of the hydrogen stock over time



Electricity consumption over time

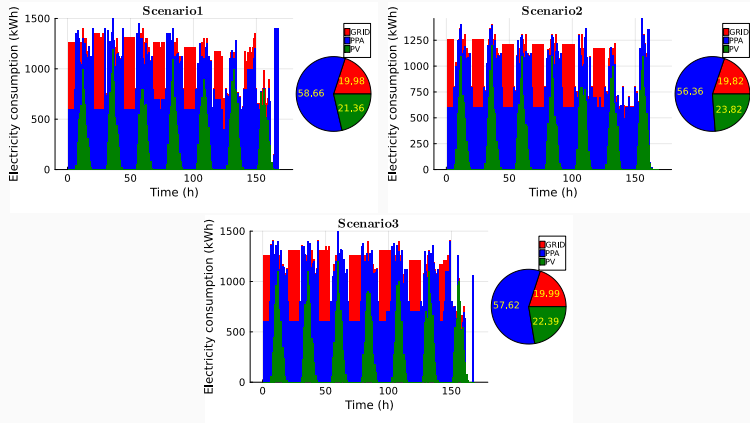


Figure 1: Time evolution of the electricity consumption for the three different scenarios

Optimizing the stock capacity

The design optimization problem

The stock design problem can be written as

$$\min_{i \in \mathbb{I}} \min_{(\mathbf{U}_h)_{h \in \mathbb{H}} \in Y_i} \mathbb{E}_{(\mathbf{D}_{h+1}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}} \left[\sum_{h \in \mathbb{H}} L_h(\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{E}_{h+1}^{\text{PV}}, \mathbf{H}_h^{\rightarrow \text{D}}) + K((\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{E}_{h+1}^{\text{PV}})_{h \in \mathbb{H}}) \right] + c_i,$$

where \mathbb{I} is the set of designs, $\mathbf{U}_h = (\mathbf{E}_h^{\text{PPA}}, \mathbf{E}_{h+1}^{\text{G}}, \mathbf{M}_h^{\text{E}}, \mathbf{I}_h^{\text{E}}, \mathbf{H}_h^{\rightarrow \text{D}})$, Y_i is the set of nonanticipativity constraints, operational constraints (that depends on i), renewable energy constraints and the coupling constraints between the energy mix and the hydrogen equipments and c_i is the cost of the design i .

An alternative design optimization problem

We consider the following optimization problem

$$\min_{i \in \mathbb{I}} \max_{\lambda \in \mathbb{R}^T} \phi^E(\lambda) + \phi_i^O(\lambda) + c_i ,$$

where

- $\phi^E(\lambda)$ is the electricity allocation problem
that does not depend on i
- $\phi_i^O(\lambda)$ is the operational problem that depends on i

The computation of $\phi_i^O(\lambda)$ is faster than the computation of $\phi^E(\lambda)$, which is the key ingredient used in the following algorithm to solve the minmax optimization problem

Solving the minmax problem

1. set $\mathbb{J} \leftarrow \mathbb{I}$, choose an initial design $i \in \mathbb{J}$, set $\mathbb{J} \leftarrow \mathbb{J} \setminus \{i\}$, choose an initial multiplier $\lambda^0 \in \mathbb{R}^T$
2. maximize the dual function $\phi^E + \phi_i^O + c_i$ associated to the design problem i . The multiplier obtained at the end of the optimization is denoted by λ^*
3. store the oriented pair $(i, \phi^E(\lambda^*) + \phi_i^O(\lambda^*) + c_i)$ in the list \mathbb{O}
4. for all $i' \in \mathbb{J}$, if $\phi_i^O(\lambda^*) + c_i < \phi_{i'}^O(\lambda^*) + c_{i'}$, it follows that $\max_{\lambda \in \mathbb{R}^T} \phi^E(\lambda) + \phi_i^O(\lambda) + c_i < \max_{\lambda \in \mathbb{R}^T} \phi^E(\lambda) + \phi_{i'}^O(\lambda) + c_{i'}$ and therefore that the design i' is not optimal, set $\mathbb{J} \leftarrow \mathbb{J} \setminus \{i'\}$
5. if $\mathbb{J} \neq \{\emptyset\}$, choose $i \in \mathbb{J}$, set $\mathbb{J} \leftarrow \mathbb{J} \setminus \{i\}$, set $\lambda^0 \leftarrow \lambda^*$ and goto2
6. if $\mathbb{J} = \{\emptyset\}$, search for the best design in the list \mathbb{O}

In one numerical experiment with six different stock capacity, this algorithm entered step 2 only once, the other five designs were removed at step 4, which gives promising perspectives

Future works and extensions

Future works and extensions

- We modeled a hydrogen supply chain with three sources of energy
- We solved the optimal management of the Schiever supply chain with **decomposition method** involving two subproblems:
 - an operational problem solved by SDP
 - an electricity allocation problem solved by SDDP
- We proposed a simple algorithm to optimize the capacity of the hydrogen storage
- The next step will be to consider the design of the hydrogen equipments (storage and electrolyser) and the PPA contracts

Any questions ?

This project received the support of ADEME

