Assessment of Cost-Effective Strategies (Under Uncertainty)

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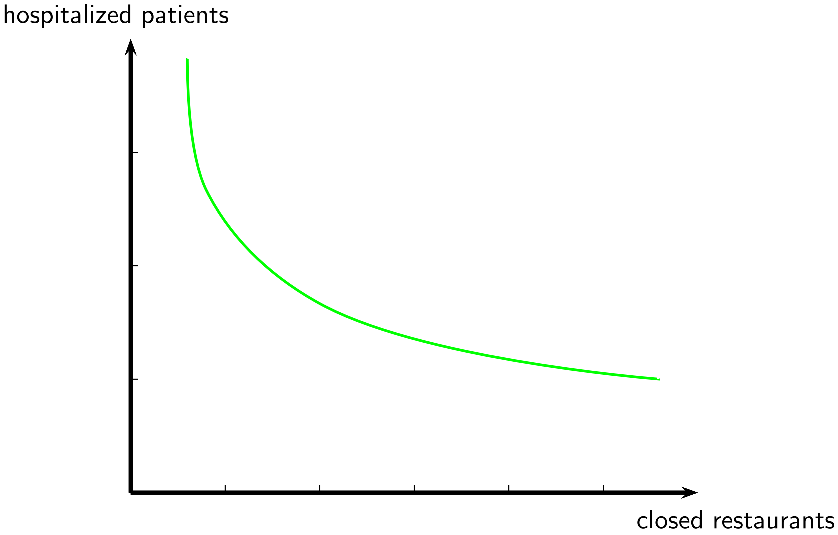
Tradeoffs between health and economic activity



Questions about epidemic control

- Possibly conflicting objectives
  - maintaining the population of hospitalized individuals below a critical threshold
  - avoiding virulent mutations of the infectious agent
  - ensuring economic activity, controlling costs, etc.
- Menu of possible decisions
  - quarantine, lockdown
  - screening tests
  - opening of new medical units, etc.
- What is at stake?
  - timing and amplitude of decisions
  - design of strategies (that are robust w.r.t.uncertainties)
  - role of models
    (uncertainties within models, discrepancies between models)
  - assessment of strategies under uncertainty

Outline of the presentation

Cost-effectiveness analysis

Formulation of optimal control problems

How to handle uncertainties in the decision process?

Design and assessment of cost-effective strategies under uncertainty

Conclusion

Outline of the presentation

What is "optimization"?

Optimizing is obtaining the best compromise between needs and resources
Marcel Boiteux (président d'honneur d'Électricité de France)

- Resources: portfolio of assets, menu of possible decisions
  - quarantine, lockdown
  - screening tests
  - medical units (and opening of new medical units), etc.
- Needs: health (but not only)
  - maintaining the population of hospitalized individuals below a critical threshold
  - avoiding virulent mutations of the infectious agent
  - ensuring economic activity, etc.
- Best compromise: Pareto optimum, minimize socio-economic costs (including externalities)

Outline of the presentation

Decision variables

- Decision variables u, belonging to a decision set $\mathbb{U}$

$$u \in \mathbb{U}$$

  - $\mathbb{U}$ may encompass scalar $\mathbb{R}$, vectorial $\mathbb{R}^d$, integer $\mathbb{Z}$ variables
  - u_t may be possibly indexed by time
  - u_ω may be possibly indexed by randomness/uncertainty
  - u_i may be possibly indexed by agents/units/nodes
- Constraints restrict the wiggle room of decision variables

$$u \in \mathbb{U}^{ad} \subset \mathbb{U}$$

  - bounds constraints (capacities) $\underline{u} \leq u \leq \overline{u}$
  - dynamic relations between successive times (stocks evolution)
  - information relations (who knows what)
  - relations between agents/units/nodes

Here are the ingredients for a multicriteria optimization problem

- A set $\mathbb{U}^{ad} \subset \mathbb{U}$ comprising decisions (over which there will be bargaining)
- A finite set $\mathbb{A}$ (stakeholders, viewpoints, multiple selves)
- Each stakeholder expresses her/his objective, need, preference by means of an indicator, criterion, objective function

$$\mathbb{U} \ni u \mapsto J_a(u) \in \mathbb{R}\ , \ \ \forall a \in \mathbb{A}$$

  - Each criterion $J_a : \mathbb{U} \to \mathbb{R}(\cup\{+\infty\})$ takes (possibly extended) real numerical values, but expressed in its own unit
  - A large value is bad

Blanket assumption: when needed, the set $\mathbb{U}^{ad}$ is a convex subset of $\mathbb{R}^d$, all functions J_a are convex and qualification of constraints holds true

The economic posture: defining a social optimum respecting that you and I do not have the same tastes

Martin L. Weitzman

> *An enormously important part of the "discipline" of economics is supposed to be that economists understand the difference between their own personal preferences for apples over oranges and the preferences of others for apples over oranges*

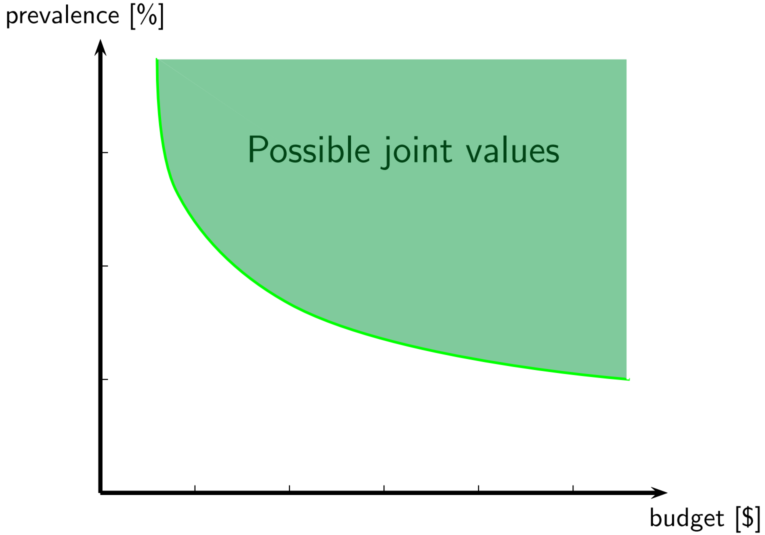
The space of outcomes

- In multicriteria optimization, stakeholders $a \in \mathbb{A}$ bargain over a common decision $u \in \mathbb{U}$
- For this purpose, they consider the image of the mapping

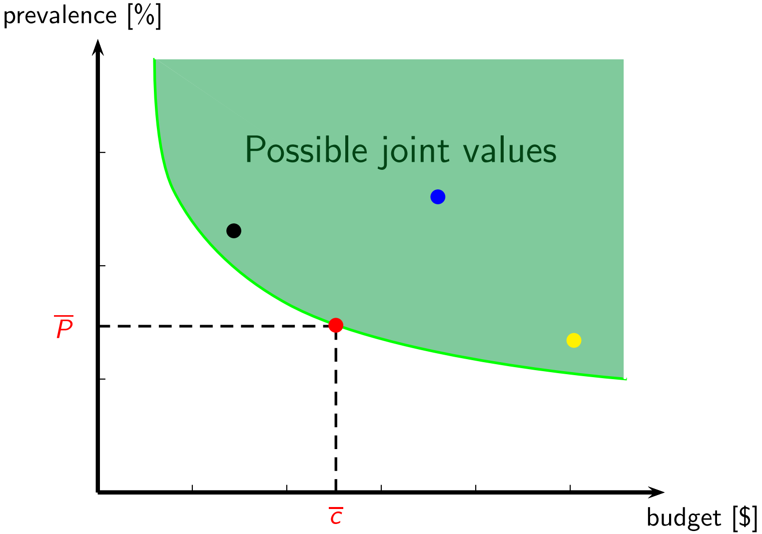
$$\{J_a\}_{a \in \mathbb{A}} : \mathbb{U}^{ad} \to \mathbb{R}^{\mathbb{A}}$$

in the space $\mathbb{R}^{\mathbb{A}}$ of joint outcomes

Tradeoffs between budget and prevalence constraints



Tradeoffs between budget and prevalence constraints



In a multicriteria optimization problem, a solution is a Pareto optimum

Efficiency=Pareto optimum= you cannot rob Peter to pay Paul

- A decision $u^\flat \in \mathbb{U}$ is dominated by a decision $u^\sharp \in \mathbb{U}$ if
  - all stakeholders prefer $u^\sharp$ to $u^\flat$, that is,

$$J_a(u^\sharp) \leq J_a(u^\flat) \ , \ \ \forall a \in \mathbb{A}$$

  - at least one stakeholder strictly prefers $u^\sharp$ to $u^\flat$, that is,

$$\exists a \in \mathbb{A} \ , \ \ J_a(u^\sharp) < J_a(u^\flat)$$

- A decision is a Pareto optimum
  if it is not dominated by any other decision

Outline of the presentation

Pareto optima can be obtained by (monocriterion) optimization in two ways

- Weights (prices)
  Pick a family $\{\lambda_a\}_{a \in \mathbb{A}} \in \mathbb{R}_+^{\mathbb{A}}$ of weights,
  and then solve the optimization problem

$$\min_{u \in \mathbb{U}^{ad}} \sum_{a \in \mathbb{A}} \lambda_a J_a(u)$$

- Focal agent and thresholds (quantities)
  - Pick a focal agent $\bar{a} \in \mathbb{A}$ (whatever)
  - Pick a family $\theta_{-\bar{a}} = \{\theta_a\}_{a \in \mathbb{A} \setminus \{\bar{a}\}} \in \mathbb{R}^{\mathbb{A} \setminus \{\bar{a}\}}$ of thresholds (each in its own unit)

  and then solve the optimization problem

$$\begin{array}{rl} J_{\bar{a}}^*(\theta_{-\bar{a}}) = & \min_{u \in \mathbb{U}^{ad}} J_{\bar{a}}(u) \\ \text{under the constraints} & J_a(u) \leq \theta_a \;, \;\; \forall a \in \mathbb{A} \setminus \{\bar{a}\} \end{array}$$

From thresholds to weights

- Solving the optimization problem (cost-effectiveness)

$$J^*_{\bar{a}}(\theta_{-\bar{a}}) = \begin{array}{l} \min_{u \in \mathbb{U}^{ad}} J_{\bar{a}}(u) \\ J_a(u) \leq \theta_a \ , \ \ \forall a \in \mathbb{A} \setminus \{\bar{a}\} \end{array}$$

  one obtains
  - an optimal solution $u^* \in \mathbb{U}$
  - a family $\lambda^*_{-\bar{a}} = \{\lambda^*_a\}_{a \in \mathbb{A} \setminus \{\bar{a}\}} \in \mathbb{R}_+^{\mathbb{A} \setminus \{\bar{a}\}}$ of Lagrange multipliers (provided as multipliers of the constraints)
- The optimal solution $u^* \in \mathbb{U}$ also solves

$$\min_{u \in \mathbb{U}^{ad}} \underbrace{1 \times J_{\bar{a}}(u) + \sum_{a \neq \bar{a}} \lambda^*_a \times J_a(u)}_{\text{socio-economic costs}}$$

Weights are (shadow) prices

- Starting from thresholds expressed in their own units, we obtain Lagrange multipliers, that is, dual variables in the duality between quantities and prices
- Historically, dual variables have moved from geometric (Lagrange) to economic (Kantorovich) flavor
  - Lagrange multipliers of inequality constraints are geometric dual variables
  - Kantorovich "resolving multipliers" of constrained primal quantities (or "objectively determined estimators") are economic dual variables
- The price of a resource is the sensitivity of the optimal payoff with respect to marginal changes $\theta_a \to \theta_a + \epsilon_a$

$$\lambda_a^* = \frac{\partial J_{\bar{a}}^*(\theta_{-\bar{a}})}{\partial \theta_a} \; , \; \; \forall a \in \mathbb{A} \setminus \{\bar{a}\}$$

Outline of the presentation

Cost-effectiveness analysis
- Social optimum between (possibly) conflicting objectives
- Social optimum and optimization
- Additional material (*valeur de la vie humaine*)

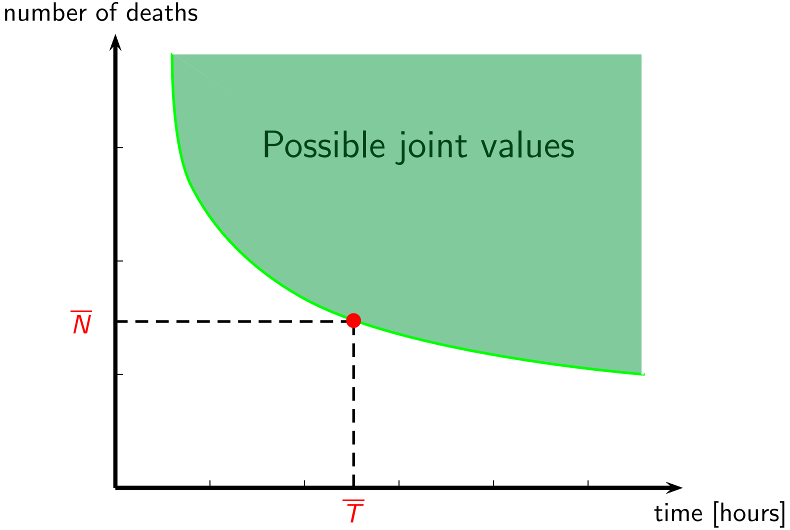
Formulation of optimal control problems
- Controlled dynamical systems, state feedback strategies
- Mathematical formulation of goals and costs
- Discounting

How to handle uncertainties in the decision process?
- Controlled dynamical models with uncertainties
- Formulation of optimal control problems under uncertainty

Design and assessment of cost-effective strategies under uncertainty
- Simulation or knowledge *versus* decision models
- Assessing the cost-effectiveness of strategies

Conclusion

Tradeoffs between deaths and time spent on the roads



La “valeur” de la vie humaine

- La “valeur” de la vie humaine = coût d’évitement d’une mort (statistique) supplémentaire
- Cette valeur est révélée par les actions entreprises dans différents secteurs
  - route (carrefour giratoire)
  - sûreté nucléaire
  - hôpital
- Si ce coût d’évitement n’est pas le même sur la route, dans les hôpitaux et au voisinage d’une centrale, alors, à dépense égale pour la collectivité, on pourrait sauver beaucoup plus de gens $\Longrightarrow$

égalisation marginale des coûts d’évitement

Décider c'est choisir, choisir c'est pondérer et pondérer c'est donner des prix à toute chose (Marcel Boiteux)

- Décider c'est choisir
- choisir c'est pondérer
- et pondérer c'est donner des prix à toute chose,
  - matérielle ou immatérielle,
  - marchande ou non marchande

"Pondération de chacune des raretés primaires dans leur infinie diversité, bilan consolidé de tous les cheminements, les uns dans les autres imbriqués, jusqu'à remonter à chacune de ces ressources rares, cela parait a priori tout à fait inextricable"

Un vieux “truc” qui ne marche pas si mal (Marcel Boiteux)

- “Et pourtant, il y a, pour ce faire, un vieux ’truc’ que l’on utilise depuis des siècles et qui ne marche pas si mal.
- Cela consiste à affecter à chaque ressource élémentaire un coefficient plus ou moins élevé suivant sa rareté. . . coefficient que l’on appelle un prix.
- En multipliant par ce coefficient-prix la quantité de telle ressource rare que l’on mobilise, on obtient un coût ;
- ces coûts se cumulent tout le long des processus de fabrication pour aboutir au *prix de revient* du produit final. . .
- et la solution la meilleure, celle qui épargne au mieux les raretés élémentaires pondérées par leur importance relative, c’est celle qui coûte le moins cher !”
- “Je suis un peu confus d’avoir retenu votre attention jusqu’à maintenant pour en arriver à une telle banalité”.

Marcel Boiteux, Du Culte de l’énergie, Foi et Vie, n. 23, avril 1977, 76e année

Brûler du pétrole, c'est comme brûler sa commode Louis XV (Marcel Boiteux)

"Les prix qui règnent dans nos économies traduisent-ils correctement, et durablement, tous les aspects des raretés dont la menace pèse sur l'humanité ?" (Marcel Boiteux)

Wrapping up

How do you find a compromise between possibly conflicting objectives?

- Efficiency = Pareto optimum among conflicting objectives
  each expressed in its own unit
- Cost-efficiency = minimizing costs under "physical" constraints
  each expressed in its own unit
- Efficiency and cost-efficiency = more or less equivalent
  as making decisions reveals implicit prices and values,
  unveiling numerical values in the trade-offs
  achieved through decision-making

Outline of the presentation

Cost-effectiveness analysis

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Conclusion

When time is of the essence

- What is the proper timing of decisions? Bold play or smoothing?
- When the notion of "solution" moves from decisions to strategies
- Why do we discount (utility or cost) over time?

Outline of the presentation

Discrete-time nonlinear state-control system

$$x_{t+1} = f_t(x_t, u_t), \quad t \in \mathbb{T} = \{t_0, t_0 + 1, \ldots, T - 1\}$$

- the time $t \in \overline{\mathbb{T}} = \{t_0, t_0 + 1, \ldots, T - 1, T\} \subset \mathbb{N}$ is discrete with initial time t_0 and horizon T ($T < +\infty$ or $T = +\infty$) (*the time period* $[t, t+1[$ *may be a year, a month, a day, etc.*)
- the state variable x_t belongs to the *state space* $\mathbb{X}_t = \mathbb{R}^{n_{\mathbb{X}_t}}$ (*stocks, biomasses, abundances, capital*)
- the control variable u_t is an element of the *control space* $\mathbb{U}_t = \mathbb{R}^{n_{\mathbb{U}_t}}$ (*inflows, outflows, catches, harvesting effort, investment*)
- the dynamics f_t maps $\mathbb{X}_t \times \mathbb{U}_t$ into $\mathbb{X}_{t+1}$ (*storage, age-class model, population dynamics, economic model*)

Stages and controls

- We use a discrete (sequential) time index (stage) that corresponds to the timing of decisions: the time index can represent a day, or a week, depending at what rythm decisions are made
- Controls can take continuous or discrete values when they represent the amplitude of different measures: number of screening tests, number of openings of new medical units,
- Controls can take binary values — lauching of lockdown, etc.
- In practice, some parameters of a model will become functions of new control variables; for instance, contact parameters between compartments in a SIR model can become functions of the severity of lockdown measures

States

In so-called SIR models, the state is usually made of

- the abundances of individuals in different age classes and in different health states — like susceptible, infected (hospitalized or not), recovered
- In the simplest SIR model, the state is (S, I, R)
- In a SIR model with age classes, the state is $\left(\{S^a\}_{a \in A_S}, \{I^a\}_{a \in A_I}, \{R^a\}_{a \in A_R}\right)$
- The state can also include the abundances of infectious agents and their vectors, the number of beds in specialized medical units, etc.
- In joint economics-epidemics models, the state could include economic compartments

State feedback strategies

A state feedback strategy (policy) is a sequence

$$\lambda = \{\lambda_t\}_{t=t_0,\dots,T-1}$$

of mappings, each of them being a strategy at time t

$$\lambda_t : \mathbb{X}_t \to \mathbb{U}_t$$

By this definition, we encapsulate the notion that a strategy can *adapt* to the current state,
as a the decision u_t furnished by the strategy λ_t at time t is

$$u_t = \lambda_t(x_t)$$

- Among the strategies, we distinguish the open loop strategies made of $\{\lambda_t\}_{t=t_0,\dots,T-1}$ such that each λ_t takes a constant value (hence generally nonstationary)
- In epidemics control, an elementary example of strategy at time t would be the number of new hospital beds
  as a function of the value of the number of infected individuals
- It can also be a binary variable correponding to the opening or closure of schools as a function of the whole state and of the past value of epidemiological uncertain variables

Outline of the presentation

Desirable sets (indicators and thresholds)

- Goals can be formulated as a sequence of subsets

$$\mathbb{D}_t \subset \mathbb{X}_t \times \mathbb{U}_t \ , \quad t = t_0, \ldots, T-1$$

called desirable sets, that capture "effectiveness" in that states and controls are constrained by

$$(x_t, u_t) \in \mathbb{D}_t \ , \quad t = t_0, \ldots, T-1$$

- Desirable sets are usually defined as

$$\mathbb{D}_t = \Big\{ (x,u) \in \mathbb{X}_t \times \mathbb{U}_t \ \Big| \ \underbrace{\mathcal{I}^j(x,u)}_{\text{indicator}} \leq \underbrace{\theta^j}_{\text{threshold}} \ , \ j = 1, 2, \ldots, p \Big\}$$

Capping the epidemic peak for Ross-Macdonald Model

- The dynamics of the system is given by

$$\text{infected mosquito proportion} \quad \frac{dm}{dt} = A_m h_t (1 - m_t) - u_t m_t$$

$$\text{infected human proportion} \quad \frac{dh}{dt} = A_h m_t (1 - h_t) - \gamma h_t$$

- Determine, if it exists, a piecewise continuous function (fumigation policy rate) $u(\cdot)$,

$$u(\cdot) : t \mapsto u_t \ , \ \underline{u} \leq u_t \leq \overline{u} \ , \ \forall t \geq 0 \ ,$$

such that the following so-called viability constraint is satisfied

$$h_t \leq \overline{H} \ , \ \forall t \geq 0$$

- For instance, in the simplest SIR model, a stationary desirable set

$$\mathbb{D} = \Big\{ \big((S, I, R), u\big) \in \mathbb{X} \times \mathbb{U} \,\Big|\, \alpha I \leq b \Big\}$$

represents the state constraint $\alpha I_t \leq b$, for all times t, that is, make in sort that the fraction α of those infected I_t (corresponding to those requiring hospitalization) be less than the number b of beds

- In a more elaborate model, the number b_t of beds would be part of the state, together with a new control variable u_t^b corresponding to the opening/closing of beds at time t, giving an expression like

$$\mathbb{D} = \Big\{ \big((S, I, R, b), (u, u^b)\big) \in \mathbb{X} \times \mathbb{U} \,\Big|\, \alpha I \leq b \Big\}$$

Costs

Costs are represented by functions

- called instantaneous costs

$$L_t : \mathbb{X}_t \times \mathbb{U}_t \to \mathbb{R} \; , \quad t = t_0, \ldots, T-1$$

- and by a final cost

$$K : \mathbb{X}_T \to \mathbb{R}$$

The total costs over the whole time span are given by

$$j\underbrace{\big(x(\cdot), u(\cdot)\big)}_{\text{trajectory}} = \sum_{t=t_0}^{T-1} L_t(x_t, u_t) + K(x_T)$$

The optimal control problem

Minimize intertemporal costs

$$\min_{(x(\cdot),u(\cdot))} \sum_{t=t_0}^{T-1} L_t(x_t, u_t) + K(x_T)$$

under dynamical constraints

$$x_{t+1} = f_t(x_t, u_t) \ , \quad t = t_0, \ldots, T-1$$

over strategies

$$u_t = \lambda_t(x_t) \ , \quad t = t_0, \ldots, T-1$$

under constraints (effectiveness)

$$(x_t, u_t) \in \mathbb{D}_t \ , \quad t = t_0, \ldots, T-1$$

Outline of the presentation

Discounting erases the future

$$\sum_{t=t_0}^{T-1} \left(\frac{1}{1+r_e}\right)^t L(x_t, u_t) + K(x_T)$$

The French public discount rate

En France, le rapport *Révision du taux d'actualisation des investissements publics* (Commissariat général du Plan, groupe d'experts présidé par Daniel Lebègue, janvier 2005) a conduit à diviser par deux (de 8% à 4%) le taux d'actualisation à retenir pour évaluer la rentabilité des choix d'investissements publics

$$\frac{1}{1+r_e} = \frac{1}{1+0.04} \approx 0.96$$

The future in one hundred years is valued, seen from today, 2%

$$\left(\frac{1}{1+0.04}\right)^{10} \approx 0.68 \ , \quad \left(\frac{1}{1+0.04}\right)^{50} \approx 0.14 \ , \quad \left(\frac{1}{1+0.04}\right)^{100} \approx 0.02$$

The discount rate is not necessarily an interest rate

M. Boiteux, À propos de la "critique de la théorie de l'actualisation telle qu'employée en France", Revue d'Économie Politique, 1976.

- "Le taux d'actualisation optimal pour orienter les choix d'intérêt général" n'est pas "nécessairement égal dans la réalité au taux d'intérêt d'un quelconque marché monétaire et financier"
- "l'actualisation, instrument de cohérence des choix"

Discounting may be related to random final time

- Nicholas Stern. *The Economics of Climate Change*. Cambridge University Press, 2006.
  (...) following distinguished economists from Frank Ramsey in the 1920s to Amartya Sen and Robert Solow more recently, the only sound ethical basis for placing less value on the utility (as opposed to consumption) of future generations was the uncertainty over whether or not the world will exist, or whether those generations will all be present
- The discounted utility criterion can be written without discounting as a mathematical expectation

$$\int_0^{\infty} L(c(t)) e^{-\delta t} dt = \mathbb{E}\left[\int_0^{\tau} \underbrace{L(c(t))}_{\text{utility}} dt \right]$$

where the random final time τ follows a memoryless exponential distribution with mean duration time $1/\delta$

“The pure time discount rate” and chance of extinction

Nicholas Stern. *The Economics of Climate Change.*
Cambridge University Press, 2006.
(...) we should interpret the factor $e^{-\delta t}$ in $W = \int_0^\infty L(c(t))e^{-\delta t}dt$ as the probability that the world exists at that time

Pure time preference δ	Probability of human race not surviving 10 years	Probability of human race not surviving 100 years
0,1	0,010	0,095
0,5	0,049	0,393
1,0	0,095	0,632
1,5	0,139	0,777

“Self-promotion, nobody will do it for you” ;-)

Lilian Sofia Sepulveda Salcedo, Michel De Lara,
Robust Viability Analysis of a Controlled Epidemiological Model,
Theoretical Population Biology, Volume 126, pp 51–58, April 2019

Michel De Lara, Lilian Sofia Sepulveda Salcedo,
Viable Control of an Epidemiological Model,
Mathematical Biosciences, Volume 280, pp 24–37, 2016

M. De Lara, L. Doyen, Sustainable Management of Natural Resources. Mathematical Models and Methods, *Springer*, 2008

Outline of the presentation

Perspectives

- Sensitivity analysis *versus* stochastic and robust optimization
- Strategies are the proper concept of solution

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- the state variable x_t belongs to the *state space* $\mathbb{X}_t = \mathbb{R}^{n_{\mathbb{X}_t}}$ (*stocks, biomasses, abundances, capital*)
- the control variable u_t is an element of the *control space* $\mathbb{U}_t = \mathbb{R}^{n_{\mathbb{U}_t}}$ (*inflows, outflows, catches, harvesting effort, investment*)
- the uncertainty $w_t \in \mathbb{W}_t = \mathbb{R}^{n_{\mathbb{W}_t}}$ (*recruitment or mortality uncertainties, climate fluctuations*)
- the dynamics f_t maps $\mathbb{X}_t \times \mathbb{U}_t \times \mathbb{W}_{t+1}$ into $\mathbb{X}_{t+1}$ (*storage, age-class model, population dynamics, economic model*)

Uncertainties and scenarios

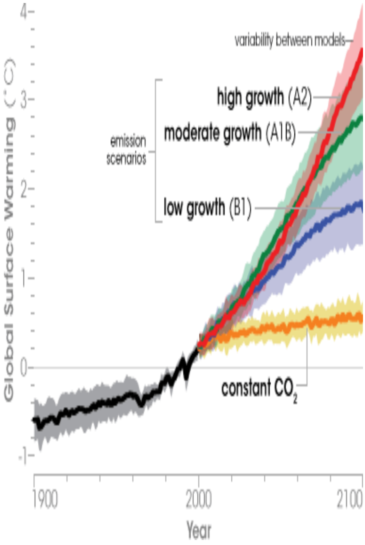
In practice, some parameters of a model will become functions of new uncertain variables, hence will become time-varying

- rates of contact of different compartments of a population model (young infected individuals with older hospitalized ones, for instance)
- Unknown factors in the evolution of the infectious agents, in its transmission, etc.

A scenario (pathway, chronicle) is a sequence of uncertainties

$$w(\cdot) = \big(w_{t_0+1}, w_{t_0+2}, \dots, w_T\big) \in \mathbb{W}^{T-t_0}$$

Beware! Scenario holds a different meaning in other scientific communities



- In practice, what modelers call a "scenario" is a mixture of
  - a sequence of uncertain variables (also called a pathway, a chronicle)
  - a policy `Pol`
  - and even a static or dynamical model
- In what follows

  scenario = pathway = chronicle

Strategies

A strategy is a sequence

$$\lambda = \{\lambda_t\}_{t=t_0,\dots,T-1}$$

of mappings, each of them being a strategy at time t

$$\lambda_t : \mathbb{X}_{t_0} \times \prod_{s=t_0+1}^{T} \mathbb{W}_s \times \mathbb{X}_t \to \mathbb{U}_t$$

By this definition, we encapsulate the notion that a strategy can *adapt* to the current state and to the current cumulated knowledge about uncertain variables,
as a the decision u_t furnished by the strategy λ_t at time t is

$$u_t = \lambda_t(x_{t_0}, w_{t_0+1}, \dots, w_t, x_t)$$

Outline of the presentation

Costs

Costs are represented by functions,

- called instantaneous costs,

$$L_t : \mathbb{X}_t \times \mathbb{U}_t \times \mathbb{W}_{t+1} \to \mathbb{R} \ , \quad t = t_0, \ldots, T-1$$

- and by a final cost

$$K : \mathbb{X}_T \to \mathbb{R}$$

The total costs over the whole time span is given by

$$j \underbrace{\big(x(\cdot), u(\cdot), w(\cdot)\big)}_{\text{trajectory}} = \sum_{t=t_0}^{T-1} L_t(x_t, u_t, w_{t+1}) + K(x_T)$$

Minimizing worst costs under robust constraints

Minimize worst intertemporal costs

$$\min_{\{\lambda_t\}_{t=t_0,\dots,T-1}} \sup_{\{w_t\}_{t=t_0+1,\dots,T}} \sum_{t=t_0}^{T-1} L_t(x_t, u_t, w_{t+1}) + K(x_T)$$

under dynamical constraints

$$x_{t+1} = f_t(x_t, u_t, w_{t+1}) \ , \quad t = t_0, \dots, T-1$$

over strategies

$$u_t = \lambda_t(x_{t_0}, w_{t_0+1}, \dots, w_t, x_t) \ , \quad t = t_0, \dots, T-1$$

under robust constraints

$$(x_t, u_t) \in \mathbb{D}_t \ , \quad t = t_0, \dots, T-1$$

for all uncertainty chronicle $w(\cdot) = (w_{t_0+1}, w_{t_0+2}, \dots, w_T) \in \prod_{t=t_0+1}^{T} \mathbb{W}_t$

Minimizing expected costs under chance constraints

$$\min_{\{\lambda_t\}_{t=t_0,\ldots,T-1}} \mathbb{E}\Big[\sum_{t=t_0}^{T-1} L_t(x_t, u_t, w_{t+1}) + K(x_T)\Big]$$

under dynamical constraints

$$x_{t+1} = f_t(x_t, u_t, w_{t+1}) \ , \quad t = t_0, \ldots, T-1$$

over strategies

$$u_t = \lambda_t(x_{t_0}, w_{t_0+1}, \ldots, w_t, x_t) \ , \quad t = t_0, \ldots, T-1$$

under chance constraints

$$\mathbb{P}\Big((x_t, u_t) \in \mathbb{D}_t \ , \quad t = t_0, \ldots, T-1\Big) \leq 1 - \epsilon$$

With risk measures $\mathbb{G}$, $\mathbb{G}_1, \ldots, \mathbb{G}_T$

$$\min_{\{\lambda_t\}_{t=t_0,\ldots,T-1}} \mathbb{G}\Big[\sum_{t=t_0}^{T-1} L_t(x_t, u_t, w_{t+1}) + K(x_T)\Big]$$

under dynamical constraints

$$x_{t+1} = f_t(x_t, u_t, w_{t+1}) \ , \quad t = t_0, \ldots, T-1$$

over strategies

$$u_t = \lambda_t(x_{t_0}, w_{t_0+1}, \ldots, w_t, x_t) \ , \quad t = t_0, \ldots, T-1$$

under constraints

$$\mathbb{G}_t\Big(\mathcal{I}^j(x_t, u_t)\Big) \leq \theta^j \ , \quad t = t_0, \ldots, T-1$$

Outline of the presentation

Cost-effectiveness analysis

Formulation of optimal control problems

How to handle uncertainties in the decision process?

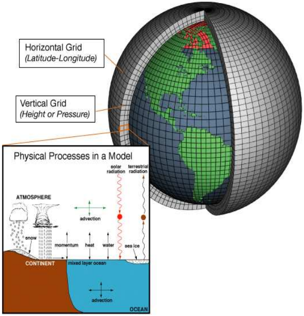
Design and assessment of cost-effective strategies under uncertainty

Conclusion

Outline of the presentation

First, we start by laying out
a far-reaching distinction between
knowledge/assessment/simulation models
versus
decision models
(for control/optimization problems)

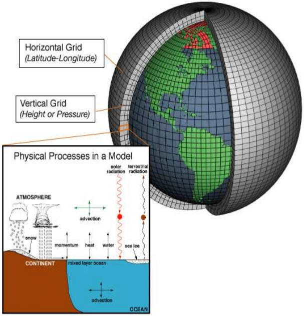
We distinguish two polar classes of models: knowledge models *versus* decision models

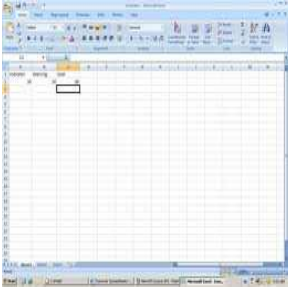


Knowledge models:
$1/1\ 000\ 000 \to 1/1\ 000 \to 1/1$
maps

Office of Oceanic and Atmospheric Research (OAR) climate model

We distinguish two polar classes of models: knowledge models *versus* decision models





Knowledge models:
$1/1\ 000\ 000 \rightarrow 1/1\ 000 \rightarrow 1/1$
maps

Office of Oceanic and Atmospheric Research (OAR) climate model

Action/decision models:
economic models are fables designed to provide insight

William Nordhaus economic-climate model

This talk is *not* about crafting dynamical models

Elaborating a dynamical model is a delicate venture

- Peter Yodzis, *Predator-Prey Theory and Management of Multispecies Fisheries*, Ecological Applications 4:51–58, 1994

  > *In population modelling the functional forms of models are at least as important as are parameter values in expressing the underlying biology and in determining the outcome. (...) For instance, May et al. (1979) assumed, without comment, a particular form of predator-prey interaction; and this particular form was carried over, again without comment, by Flaaten. It turns out that this "invisible" but powerful assumption is responsible in large part for the conclusion reached by Flaaten (1988). (...) Flaaten's work is controversial because of his conclusion that "sea mammals should be heavily depleted to increase the surplus production of fish resources for man" (Flaaten 1988:114).*

- Carlos Castillo Chavez

Outline of the presentation

Cost-effectiveness analysis
- Social optimum between (possibly) conflicting objectives
- Social optimum and optimization
- Additional material (*valeur de la vie humaine*)

Formulation of optimal control problems
- Controlled dynamical systems, state feedback strategies
- Mathematical formulation of goals and costs
- Discounting

How to handle uncertainties in the decision process?
- Controlled dynamical models with uncertainties
- Formulation of optimal control problems under uncertainty

Design and assessment of cost-effective strategies under uncertainty
- Simulation or knowledge *versus* decision models
- Assessing the cost-effectiveness of strategies

Conclusion

Ingredients

- Assessment scenarios consist of a finite set $\mathbb{S}$ of scenarios and
  - a family $\pi = \{\pi^s\}_{s\in\mathbb{S}}$ of nonnegative numbers summing up to one, (might be $\pi^s = 1/|\mathbb{S}|$)
  - a family $\big\{(x^s_{t_0}, w^s(\cdot))\big\}_{s\in\mathbb{S}}$ of uncertainty chronicles
- With any policy $\lambda = \{\lambda_t\}_{t=t_0,\ldots,T-1}$ and any $s \in \mathbb{S}$, we associate the state trajectory $x^{\lambda,s}(\cdot)$ and the control history $u^{\lambda,s}(\cdot)$, given by the following so-called closed loop system

$$
\begin{aligned}
x^{\lambda,s}_{t+1} &= f_t(x^{\lambda,s}_t, u^{\lambda,s}_t, w^s_{t+1}) \ , \quad t = t_0, \ldots, T-1 \ , \\
x_{t_0} &= x^s_{t_0} \ , \\
u^{\lambda,s}_t &= \lambda_t(x^s_{t_0}, w^s_{t_0+1}, \ldots, w^s_t, x^{\lambda,s}_t) \ , \quad t = t_0, \ldots, T-1 \ .
\end{aligned}
$$

Assessing the effectiveness of a given strategy

The critical scenarios (or critical uncertainty chronicles) associated with the strategy $\lambda = \{\lambda_t\}_{t=t_0,\dots,T-1}$ are the

$$\left\{ s \in \mathbb{S} \,\middle|\, \exists t \in \{t_0, \dots, T-1\} \, , \ (x_t^{\lambda,s}, u_t^{\lambda s}) \notin \mathbb{D}_t \right\}$$

- Such scenarios are critical because there exists at least one time t for which at least one constraint in $\mathbb{D}_t$ is violated
- A strategy is "effective" when it passes the test to display no critical scenarios

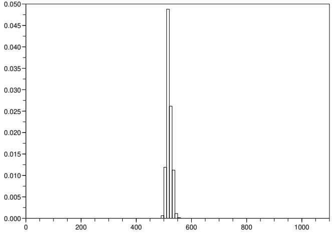
Assessing the costs of a given strategy

Assessing the costs of a given strategy $\lambda = \{\lambda_t\}_{t=t_0,\ldots,T-1}$ amounts to assessing the probability distribution of the random variable

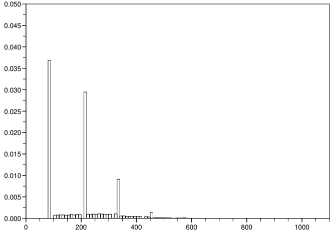
$$s \in \mathbb{S} \mapsto j\big(x^{\lambda,s}(\cdot), u^{\lambda,s}(\cdot), w^s(\cdot)\big)$$

that is, in practice, its histogram

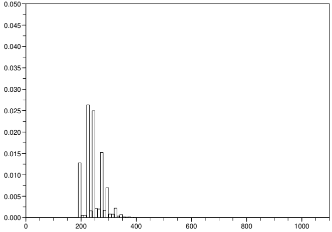
Histogram of the costs for groups of 2



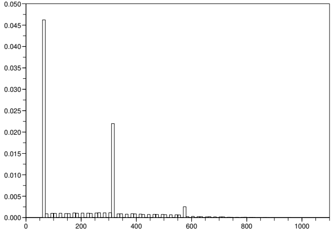
Histogram of the costs for groups of 11



Histogram of the costs for groups of 5



Histogram of the costs for groups of 16

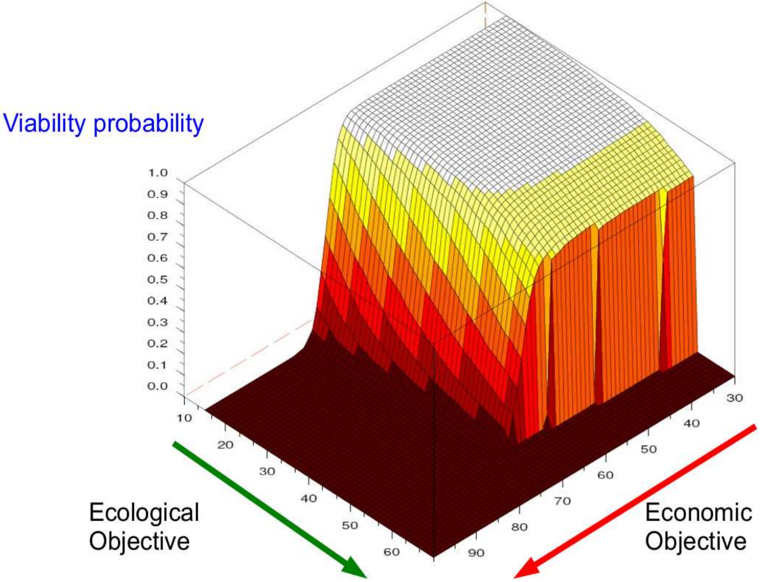


Outline of the presentation

Perspectives

- Use decision models to propose and design strategies
- Use simulation models to assess strategies
- Use risk measures (conditional value at risk)
  and tune parameters (by trials and errors)
  to capture the risk aversion of decision-makers
- Add new recourse variables to overcome hard constraints
  so as to ensure effectiveness
- Provide graphical outputs: histograms, sustainable thresholds

Thank you :-)


Viability probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10
20
30
40
50
60
30
40
50
60
70
80
90
Ecological Objective
Economic Objective