Biodiversity and Climate: a Mathematical Perspective on Sustainability and Resilience

Michel De Lara
Cermics, École des Ponts, France

Chicago, IMSI, June 17 to 21, 2024
The Architecture of Green Energy Systems:
The Underlying Problem and Its Challenges

Two international panels tackle biodiversity (IPBES) and climate change (IPCC)

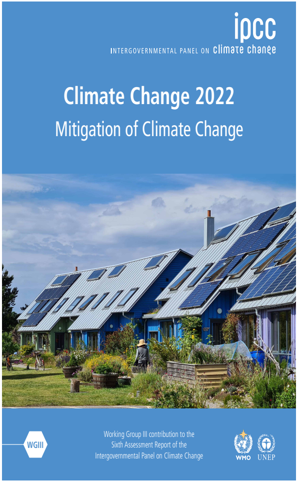
ipBes Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services

ipcc INTERGOVERNMENTAL PANEL ON climate change

WMO UNEP

Intergovernmental Panel on Climate Change (IPCC)

Climate Change 2022: Mitigation of Climate Change, WGIII AR6 assessment (2022)

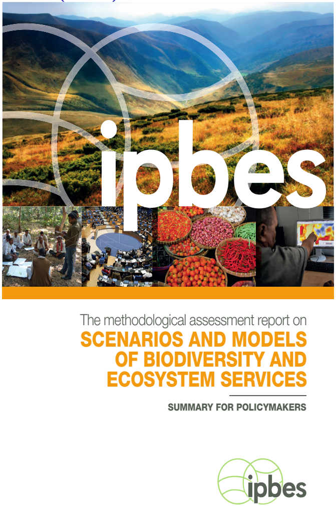


Annex III: Scenarios and Modelling Methods (p. 1841)

- Part I: Modelling Methods (p. 1843–1847)
  simulation models
  optimisation models
  perfect foresight
  recursive-dynamic
  general equilibrium
  strategic interaction
- Part II: Scenarios (p. 1870)
  Scenarios are descriptions of alternative future developments
  - A.III.II.2.2 Treatment of Scenario Uncertainty (p. 1876)

Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services (IPBES)

Methodological Assessment Report on Scenarios and Models of Biodiversity and Ecosystem Services (2016)



Many words speak to the stochastic optimization community

- scenarios
- models
- policy choices/targets/evaluation
- robustness

Lack of robustness is identified as a weakness

- `Decision makers in Governments, private sector and civil society want more robust information`
- *[the impact of uncertainty on results is underlined]*
  `Key finding 3.4: Uncertainty associated with models is often poorly evaluated and reported in published studies, which may lead to serious misconceptions -- both overly optimistic and overly pessimistic`
- *[out-of-sample assessment is poor]*
  `most studies do not provide a critical evaluation of the robustness of their findings by comparing their projections to fully independent data sets (i.e., data not used in model construction or calibration) or to other types of models`

Notational conventions

- `teletypefont family`: to denote excerpts from the reports
- *[emphasize in brackets]*: my comments

Outline of the presentation

Sustainability: illustration in climate change economic models [10']

Resilience: mathematical formalism and examples [25']

Perspectives for stochastic optimization [15']

"Self-promotion, nobody will do it for you" ;-) [2']

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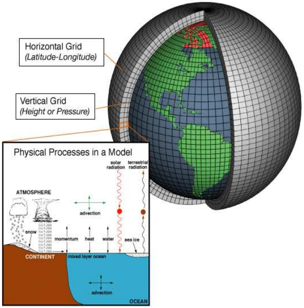
A few words on the purpose of modelling

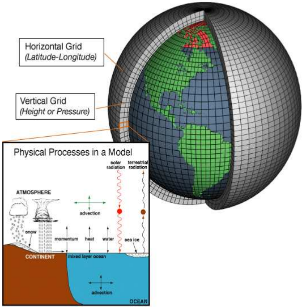
We distinguish two polar classes of models: knowledge models *versus* decision models



Knowledge models:
1/1 000 000 $\rightarrow$ 1/1 000 $\rightarrow$ 1/1
maps

Office of Oceanic and Atmospheric Research (OAR) climate model

We distinguish two polar classes of models: knowledge models *versus* decision models



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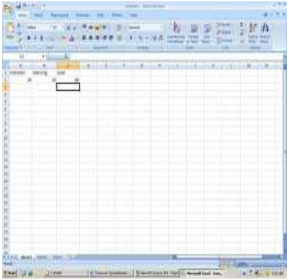

Action/decision models:
economic models are fables designed to provide insight

William Nordhaus economic-climate model

This talk is about

FRAMING DECISION PROBLEMS

(and *not* about crafting relevant models,
although this is crucial)

[Yodzis, 1994][1]

(additional material in appendix)

[1] P. Yodzis. Predator-prey theory and management of multispecies fisheries. *Ecological Applications*, 4(1):51–58, Feb. 1994

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- A stylized decision model for climate change mitigation
- Sustainability: hard *versus* soft? aggregating or not?

Resilience: mathematical formalism and examples [25']
- Climate resilient development (IPCC) and beyond
- On the meaning of "scenarios" in biodiversity and climate change
- Viable scenarios and stochastic/robust viability
- Resilience as belonging to a viability kernel
- Resilience as cost distance to a viability kernel

Perspectives for stochastic optimization [15']
- A digression on the mathematical handling of risk
- Framing: axiomatics of acceptable "bioeconomics" sets
- Solving: mixing multiple decompositions

"Self-promotion, nobody will do it for you" ;-) [2']

A carbon cycle model "à la Nordhaus" is an example of decision model

- Time t in years (the *tempo* of decisions)
  $t \in \{t_0, t_0+1, \dots, T-1, T\}$ (T horizon)
- Two state variables
  - Economic production Q_t (GWP)

$$Q_{t+1} = \overbrace{(1+g)}^{\text{economic growth}} Q_t$$

  - Environmental CO_2 concentration M_t

$$M_{t+1} = M_t \underbrace{-\delta(M_t - M_{-\infty})}_{\text{natural sinks}} + \alpha \overbrace{\texttt{Emiss}(Q_t)}^{\text{emissions}} \underbrace{(1-u_t)}_{\substack{\text{after}\\ \text{abatement}}}$$

- Decision $u_t \in [0,1]$ is the abatement rate of CO_2 emissions

Mixing dynamics, optimization and constraints yields a cost-effectiveness problem

- Minimize abatement costs

$$\min_{u_{t_0},\dots,u_{T-1}} \sum_{t=t_0}^{T-1} \delta^{t-t_0} \underbrace{\mathcal{C}\big(u_t, Q_t\big)}_{\text{abatement costs}}$$

- under the GWP-CO_2 dynamics

$$\begin{cases} Q_{t+1} &= (1+g)Q_t \\ M_{t+1} &= M_t - \delta(M_t - M_{-\infty}) + \alpha \mathtt{Emiss}\big(Q_t\big)(1-u_t) \end{cases}$$

- and under target constraint policy target

$$\underbrace{M_T \leq M^\sharp}_{\text{CO2 concentration}}$$

"We have entered the Climate Casino and are rolling the global-warming dice", warns economist William Nordhaus

- On top of time t come (*contaminating*) uncertainties, also called states of Nature $w = (w_t)_{t=0,\dots,T-1} \in \mathcal{W}$
- Minimize stochastic? robust? abatement costs

$$\min_{\text{over what?}} \boxed{\text{how to get rid of } \ w?} \sum_{t=t_0}^{T-1} \delta^{t-t_0} \underbrace{\mathcal{C}\big(u_t(w), Q_t(w), w_t\big)}_{\text{abatement costs}}$$

- under the GWP-CO_2 dynamics

$$Q_{t+1}(w) = \big(1 + g(w_t)\big) Q_t(w)$$
$$M_{t+1}(w) = M_t(w) - \delta(M_t(w) - M_{-\infty}) + \alpha\big(w_t\big)\texttt{Emiss}\big(Q_t, w_t\big)(1 - u_t)$$

- and under target constraint

$$\boxed{\text{how to handle } \ w?} \underbrace{M_T(w) \leq M^\sharp}_{\text{CO2 concentration}}$$

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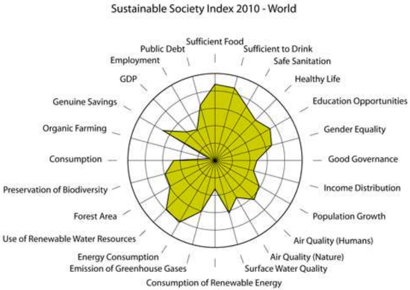
Sustainable development in one slide:
a disaggregated perspective

Sustainable development, goals, indicators: a disaggregated "spiderweb" perspective



- Sustainable development as development
  - that meets the needs of the present $\boxed{t}$
  - without compromising the ability of future generations $\boxed{\forall t' > t}$ to meet their own needs
- materialized with goals and indicators
  - the 17 goals of the UN Sustainable Development Agenda
  - and quantitative indicators (metrics) together with targets $\boxed{\text{indicator} \geq \text{target}}$

The standard economic risk analysis
is challenged by sustainability

Aggregating or not?
This is the question

Regarding climate change economics, the question of aggregation is raised

Weak *versus* strong sustainability

[Stern, 2006][2] raises the question of aggregation

- Are the services of
  - consumption $C_t(w)$
    (for instance, a fraction $\gamma Q_t(w)$ of GWP)
  - environment $E_t(w)$
    (for instance, the opposite $-M_t(w)$ of the CO_2 concentration)

  aggregated or not?
- And then, *how policy-makers aggregate over consequences*
  - *(i) within generations*
  - *(ii) over time (t)*
  - *(iii) according to risk (states of Nature w)*

  will be crucial to policy design and choice

[2] Nicholas Stern. *The Economics of Climate Change*, Cambridge University Press, 2006

The question of aggregation: between economy ($C_t(w)$) and environment ($E_t(w)$)

standard economic analysis

$$\text{utility } L\big(C_t(w), E_t(w)\big) \propto \overbrace{\underbrace{C_t(w)^{\alpha} E_t(w)^{\beta}}_{\text{substitutable needs within generation}}}^{\text{smooth utility}}$$

versus

sustainability

$$\overbrace{\underbrace{C_t(w) \geq C^{\flat}, \; E_t(w) \geq E^{\flat}}_{\text{separate needs within generation}}}^{\text{indicators } \geq \text{ thresholds}}$$

The question of aggregation: between risks (w) and between times (t)

The standard economic risk analysis aims at maximizing the expected intertemporal discounted utility

$$\underbrace{\mathbb{E}_w}_{\text{expected}} \left[\sum_{t=t_0}^{+\infty} \overbrace{\delta^{t-t_0}}^{\text{discounted}} \underbrace{C_t(w)^{\alpha} E_t(w)^{\beta}}_{\text{smooth utility}} \right]$$

Expected intertemporal discounted utility is grounded in smooth trade-offs

$$\sum_{w} \overbrace{\underbrace{\pi(w)}_{\text{probability}}}^{\substack{\text{trade-offs}\\\text{between}\\\text{states of Nature}}} \quad \sum_{t=t_0}^{+\infty} \overbrace{\underbrace{\delta^{t-t_0}}_{\text{discount}}}^{\substack{\text{trade-offs}\\\text{between times}}} \quad C_t(w)^{\alpha} E_t(w)^{\beta}$$

Expected intertemporal discounted utility
is built upon two well-known axiomatized theories,
where "continuity of preferences" plays a major role

- discounted intertemporal utility[3]
- expected utility[4]

---

[3] T. Koopmans. Representation of preference orderings over time. In C.B. McGuire and R. Radner, editors, *Decision and Organization*, pages 79–100. North-Holland, 1972

[4] J. von Neuman and O. Morgenstern. *Theory of games and economic behaviour*. Princeton University Press, Princeton, 1947

Aggregating or not?

Economics of risk and time
versus
catastrophe insurance

Consumption smoothing *versus* catastrophe insurance

[Weitzman, 2007][5] *But I think progress begins by recognizing that the hidden core meaning of Stern vs. Critics may be about* $(\cdots)$

- consumption smoothing

$$\max \sum_{w} \pi(w) \sum_{t=t_0}^{+\infty} \delta^{t-t_0} C_t(w)^{\alpha} E_t(w)^{\beta}$$

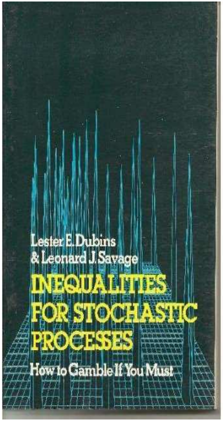
versus

- catastrophe insurance (a flavor of stochastic viability)

$$\max \mathrm{Prob}\{w \mid \underbrace{C_t(w) \geq C^{\flat},\ E_t(w) \geq E^{\flat}}_{\text{indicators} \geq \text{thresholds}}, \quad \forall t = t_0, \ldots, +\infty\}$$

[5] M. L. Weitzman. A review of the Stern review on the economics of climate change. *Journal of Economic Literature*, 45(3):703–724, Sept. 2007

Maximizing the probability of success may be an objective



How to gamble if you must, L.E. Dubbins and L.J. Savage, 1965

Imagine yourself at a casino with \$1,000. For some reason, you desperately need \$10,000 by morning; anything less is worth nothing for your purpose.

The only thing possible is to gamble away your last cent, if need be, in an attempt to reach the target sum of \$10,000.

- The question is how to play, not whether. What ought you do? How should you play?
  - Diversify, by playing 1 \$ at a time?
  - Play boldly and concentrate, by playing 1,000 \$ only one time?
- What is your decision criterion?

What is hard and what is soft?
This is the question

In optimization, the discussion above boils down to: what is hard and what is soft?

- The modelling question of distinguishing hard *versus* soft

$$
\begin{array}{lcc}
\text{objective function} & \inf & \overbrace{f(u)}^{\text{soft, trade-offs}} \\
\text{constraints} & & \underbrace{u \in \mathcal{U}}_{\text{hard, thresholds}}
\end{array}
$$

- becomes even more delicate with both time t and uncertainty w (risk factor, state of Nature)

$$
\begin{array}{lcccc}
\text{objective function} & \inf & \begin{cases} \sum_t \\ \sup_t \end{cases} & \begin{cases} \sum_w \\ \sup_w \end{cases} & f_t\big(u_t(w)\big) \\
\text{constraints} & & \begin{cases} ???t \\ \forall t \end{cases} & \begin{cases} ???w \\ \forall w \end{cases} & u_t(w) \in ???
\end{array}
$$

A summary table of different aggregations/compensations over time and risk factors

soft: possible aggregation $\perp$ hard: no possible aggregation

	time compensatory $\sum_t$	time non-compensatory $\forall t$
risk compensatory $\sum_w$	expected discounted utility	stochastic viability (more on that later)
risk non-compensatory $\forall w$	robust discounted utility	robust viability (more on that later)

Where have we gone till now? And what comes next

- A glimpse at sustainability in climate change
- A first hint at the stochastic optimization community "know-how" on proposing different aggregations over time and risk factors
- Now, we turn to resilience

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Intergovernmental Panel on Climate Change (IPCC)

Climate Change 2022: Impacts, Adaptation and Vulnerability, WGII AR6 assessment (2022)

Climate Change 2022:
Impacts, Adaptation and Vulnerability

Working Group II Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change

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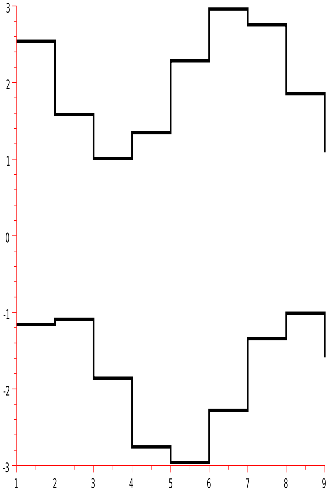
Working Group II Technical Support Unit

- Technical Summary — Box TS.1
  Core Concepts of the Report (p. 43)
  - risk (and risk management)
  - vulnerability (and exposure)
  - adaptation, <u>resilience</u>
- Chapter 1: Point of Departure
  and Key Concepts
  - Executive Summary (p. 123–124)
    Sustainable Development Goals (SDGs),
    solution space
  - Chapter 1 (p. 131)
    - 1.2 (...) Impacts, Adaptation and Vulnerability (p. 131)
    - 1.3 (...) Climate Risks (p. 143)
- Chapter 17: Decision-Making Options for Managing Risk (p. 2539)
- Chapter 18: (p. 2655)

  Climate Resilient Development Pathways

What is resilience?



[Holling, 1973][a]

> *Resilience is the capacity of a system to continually change and adapt yet remain within critical thresholds*
>
> *(Stockholm Resilience Centre)*

Tribute to
Jean-Pierre Aubin, Patrick Saint-Pierre, Luc Doyen, Sophie Martin

From viability to
stochastic and robust viability

[a] C. S. Holling. Resilience and stability of ecological systems. *Annual Review of Ecology and Systematics*, 4:1–23, 1973

Handling uncertainty in control theory
An example in fishery management

[De Lara and Martinet, 2009][6]

[6] M. De Lara and V. Martinet. Multi-criteria dynamic decision under uncertainty: A stochastic viability analysis and an application to sustainable fishery management. *Mathematical Biosciences*, 217(2):118–124, February 2009

Here is a model of European Hake and Nephrops (lobsters) in technical interaction (Bay of Biscay)

- The control u is the relative fishing effort multiplier for the trawlers fleet targeting Nephrops
- The states N are abundances for age classes ranging from 1 to $A = 9$

$$N^h_{1,t+1} = {w^h}_{t+1} \text{ uncertain hake recruitment}$$
$$N^n_{1,t+1} = {w^n}_{t+1} \text{ uncertain nephrops recruitment}$$

$$N^h_{a,t+1} = N^h_{a-1,t}\left(1 - M^h_{a-1} - \overbrace{u_t F^{nh}_{a-1}}^{\text{hake bycatch}} - F^{hh}_{a-1}\right)$$

$$N^n_{a,t+1} = N^n_{a-1,t}\left(1 - M^n_{a-1} - \overbrace{u_t F^{nn}_{a-1}}^{\text{nephrops fishing mortality}}\right)$$

$$N^h_{A,t+1} = N^h_{A-1,t}\left(1 - M^h_{A-1} - u_t F^{nh}_{A-1} - F^{hh}_{A-1}\right) + N^h_{A,t}\left(1 - M^h_A - u_t F^{nh}_A - F^{hh}_A\right)$$
$$N^n_{A,t+1} = N^n_{A-1,t}\left(1 - M^n_{A-1} - u_t F^{nn}_{A-1}\right) + N^n_{A,t}\left(1 - M^n_A - u_t F^{nn}_A\right)$$

An example of "disaggregated" approach for sustainable management

- Economic objective:
  gross return is greater than a threshold

$$\underbrace{\texttt{Payoff}\big(N^n_t, u_t\big) \geq \texttt{Payoff}^\flat}_{\text{control constraint}}$$

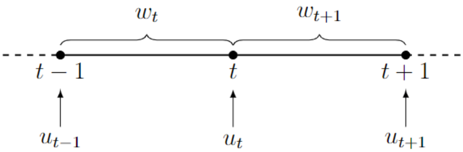
- Ecological objective:
  sufficient recruitment of mature hakes

$$\underbrace{N^h_{4,t} \geq (N^h_4)^\flat}_{\text{state constraint}}$$

Discrete time nonlinear state-control system with uncertainties

$$x_{t+1} = \overbrace{F_t}^{\text{dynamics}} \Big(\underbrace{x_t}_{\text{state } \in \mathcal{X}} , \underbrace{u_t}_{\text{control } \in \mathcal{U}} , \overbrace{w_{t+1}}^{\text{uncertainty } \in \mathcal{W}} \Big)$$

In discrete time $t \in \mathcal{T} = \{ t_0, t_0 + 1, \ldots, T - 1, \underbrace{T}_{\substack{\text{horizon} \\ \text{(finite or infinite)}}} \}$



A (state) policy is a mapping $\pi : \underbrace{(t, x) \in \mathcal{T} \times \mathcal{X}}_{\text{(time, state)}} \mapsto u = \underbrace{\pi_t(x) \in \mathcal{U}}_{\text{control}}$

(a specific way to handle *nonanticipativity constraints*)

Outline of the presentation

Scenarios: same word, different meanings

In STOCHASTIC OPTIMIZATION, decisions and scenarios are "orthogonal"

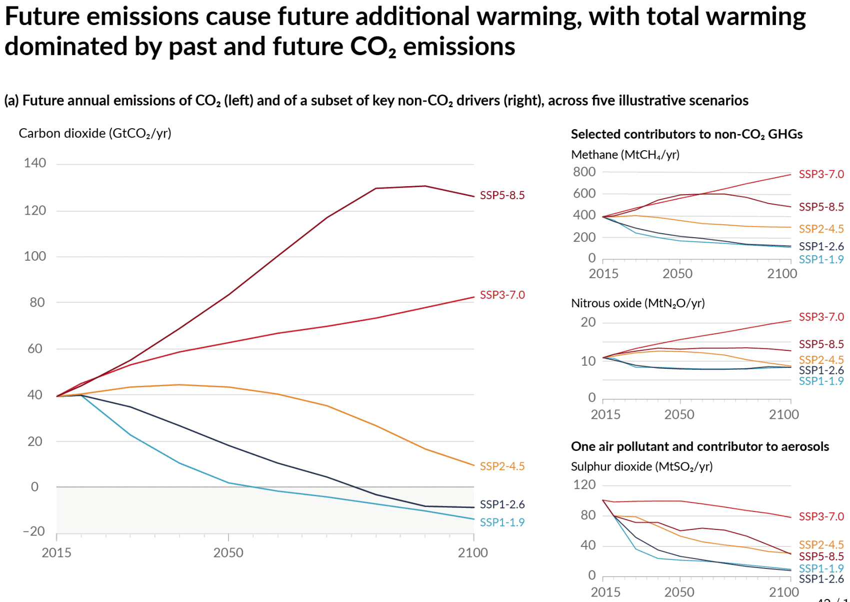
$$\overbrace{\text{decisions}}^{\text{in the decision-maker's hands}} \quad \perp \quad \underbrace{\overbrace{\text{scenarios}}^{\text{out of the decision-maker's hands}}}_{\text{EXOGENEOUS}}$$

Time $t \in \{t_0, t_0 + 1, \dots, T-1, T\}$ (T horizon)

$$\underbrace{(u_{t_0}, u_{t_0+1}, \dots, u_{T-1})}_{\text{sequence of decisions/controls}} \perp \underbrace{(w_{t_0}, w_{t_0+1}, \dots, w_{T-1}, w_T)}_{\text{sequence of uncertainties=scenario}}$$

The letter u stands for the Russian word for control: *upravlenie*

But IPCC "scenarios" are the outputs of policies!



“Scenarios” and models in the IPBES and IPCC jargon

Scenarios and models play complementary roles, with

$$x_{t+1} = \overbrace{F_t}^{\text{dynamics}} \Big(\underbrace{x_t}_{\text{state}} , \overbrace{\underbrace{u_t}_{\text{control}} , \underbrace{w_{t+1}}_{\text{uncertainty}}}^{\boxed{\text{IPBES/IPCC "scenario"}}} \Big) , \quad u_t = \underbrace{\pi_t}_{\text{policy}} (x_t)$$

- scenarios describing possible futures for
  - drivers of change
    [uncertainties w_{t+1}?]
  - or policy interventions
    [controls/decisions u_t, policies $u_t = \pi_t(x_t)$]
- and models
  [dynamical system F_t]
  translating those scenarios into
  projected consequences

The confusion goes on with three types of scenarios within the policy cycle

(i) "exploratory scenarios", which represent different plausible futures, often based on storylines

$$\underline{\text{"exploratory scenario"}} = \big((x_t, u_t, w_{t+1})\big)_{t=t_0,t_0+1,\ldots,T-1}$$

(ii) "target-seeking scenarios", also known as "normative scenarios", which represent an agreed-upon future target and scenarios that provide alternative pathways for reaching this target

$$\underline{\text{"target-seeking scenarios"}} = \Big\{\big((x_t, u_t, w_{t+1})\big)_{t=t_0,t_0+1,\ldots,T-1} \Bigm| x_T \in \text{target}\Big\}$$

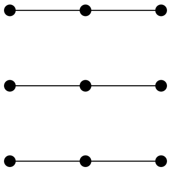
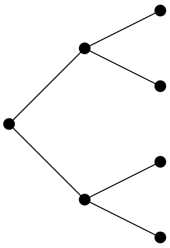
(iii) "policy screening scenarios", also known as "ex-ante scenarios", which represent various policy options under consideration

$$\underline{\text{"policy screening scenarios"}} = \{\text{policies}\} \;,\quad u_t = \underbrace{\pi_t}_{\text{policy}}(x_t)$$

Scenarios and optimization:

In theory, theory and practice are the same.
In practice, they are not.

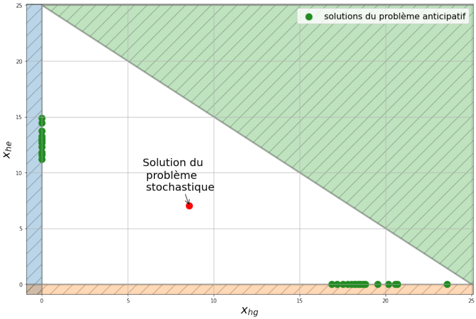
What does one know when making a decision?

perfect foresight optimization	$\mathbb{E}_\omega \left[\min_{u_0, u_1} f(u_0, u_1, \omega) \right]$	
dynamic stochastic optimization	$\min_{u_0} \mathbb{E}_\omega \left[\min_{u_1} f(u_0, u_1, \omega) \right]$	
open loop optimization	$\min_{u_0, u_1} \mathbb{E}_\omega \left[f(u_0, u_1, \omega) \right]$	

Call to the stochastic optimization community

Alternatives to the word scenario? (event/contingency tree?)

Find ways to carry and promote — to biased minds[7] ;-) —
the notion of nonanticipative solution
because, in many "scenario" constructions,
decisions are anticipative (perfect foresight) :-(



[7] J. Boutang and M. De Lara. *The Biased Mind. How Evolution Shaped our Psychology, Including Anecdotes and Tips for Making Sound Decisions.* Springer-Verlag, Berlin, 2015

Outline of the presentation

How to mix (mathematically) sustainability and uncertainty to achieve resilience?

- On the one hand,
  we have multiple objectives to be sustained over time
- On the other hand,
  uncertainties make it impossible
  to achieve all these objectives all the time

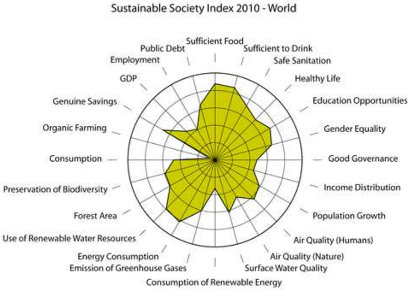
We propose the notion of viable scenarios

Sustainability in a decision setting

We mathematically express the objectives pursued as control and state constraints

$$\textit{sustainability} = \text{ ``}\textit{disaggregated}\text{'' } \textit{hard constraints}$$

Sustainability as “indicator $\geq$ threshold” (the higher, the better)



Indicators $\mathcal{I}_t^k : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$
and thresholds $\tau_t^k \in \mathbb{R}$,
$k = 1, \ldots, K^c + K^s$

- control constraints

$$\left.\begin{array}{rl} \mathcal{I}_t^1(x, u) & \geq \tau_t^1 \\ \cdots & \geq \cdots \\ \mathcal{I}_t^{K^c}(x, u) & \geq \tau_t^{K^c} \end{array}\right\} u \in \mathcal{B}_t(x)$$

- state constraints

$$\left.\begin{array}{rl} \mathcal{I}_t^{K^c+1}(x, \not{u}) & \geq \tau_t^{K^c+1} \\ \cdots & \geq \cdots \\ \mathcal{I}_t^{K^c+K^s}(x, \not{u}) & \geq \tau_t^{K^c+K^s} \end{array}\right\} x \in \mathcal{A}_t$$

Constraints may be explicit on the control variable

and are rather easily handled by reducing the decision set

Examples of control constraints

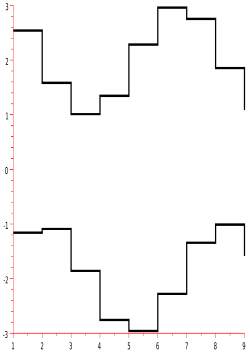
- Physical bounds
  $0 \leq u_t \leq 1$
- $\texttt{Payoff}\big(N_t^n, u_t\big) \geq \texttt{Payoff}^\flat$

Control constraints / admissible decisions

$$\underbrace{u_t}_{\text{control}} \in \underbrace{\mathcal{B}_t\big(x_t\big)}_{\text{admissible set}} \quad , \quad t = t_0, \ldots, T-1$$

Meeting constraints bearing on the state variable is delicate

due to the dynamics pipeline between controls and state



State constraints / admissible states

$$\underbrace{x_t}_{\text{state}} \in \underbrace{\mathcal{A}_t}_{\text{admissible set}} , \quad t = t_0, \ldots, T$$

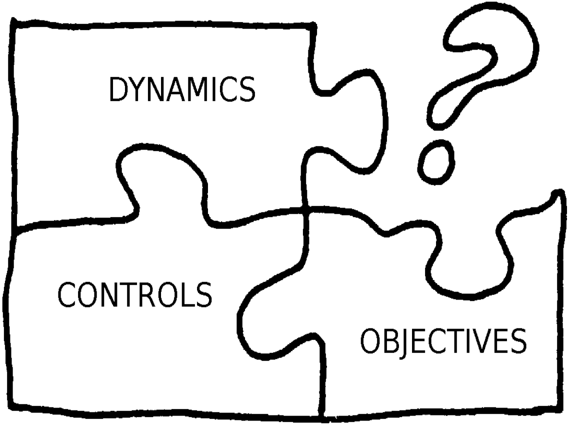
Examples ("tipping points")

- CO_2 concentration $M_t \leq M^\sharp$
- sustainability $C_t \geq C^\flat$, $E_t \geq E^\flat$
- $N^h_{4,t} \geq (N^h_4)^\flat$

State constraints are mathematically difficult because of "inertia"

$$x_t = \underbrace{\text{function}}_{\text{iterated dynamics}} \big(x_{t_0}, \underbrace{u_{t_0}, \ldots, u_{t-1}}_{\text{past controls}}, \underbrace{w_{t_0}, \ldots, w_t}_{\text{past uncertainies}}\big)$$

Can we solve the compatibility puzzle between dynamics and objectives by means of suitable controls?



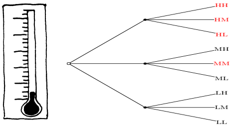
A formal definition of scenarios

Following usage in stochastic optimization, we call scenario a temporal sequence of uncertainties

Definition

A scenario is a temporal sequence of uncertainties

$$w(\cdot) = \big(w_{t_0}, \ldots, w_{T-1}\big) \in \mathcal{S} = \mathcal{W}^{T-t_0}$$



El tiempo se bifurca perpetuamente hacia innumerables futuros
(Jorge Luis Borges, *El jardín de senderos que se bifurcan*)

Choosing a set of scenarios is excluding
"things we don't know we don't know" (Donald Rumsfeld)
(additional material in appendix)

Viable scenarios

We propose the notion of *viable scenario under a given policy* as a step to formalize resilience

Definition

A scenario $w(\cdot) \in \mathcal{S}$ is said to be viable under policy $\pi : \mathcal{T} \times \mathcal{X} \to \mathcal{U}$ if the trajectories $x(\cdot) = \big(x_{t_0}, \ldots, x_T\big)$ and $u(\cdot) = \big(u_{t_0}, \ldots, u_{T-1}\big)$ generated by the dynamics

$$x_{t+1} = F_t\big(x_t, u_t, w_{t+1}\big) , \quad t = t_0, \ldots, T-1$$

driven by the policy

$$u_t = \pi_t\big(x_t\big)$$

satisfy the state and control constraints

$$\underbrace{u_t \in \mathcal{B}_t\big(x_t\big)}_{\text{control constraints}} \quad \text{and} \quad \underbrace{x_t \in \mathcal{A}_t}_{\text{state constraints}} \quad , \quad \forall t = t_0, \ldots, T$$

We look after policies that make the corresponding set of viable scenarios "large"

Definition

The set of viable scenarios — under policy $\pi : \mathcal{T} \times \mathcal{X} \to \mathcal{U}$, and starting from initial state x_0 at initial time t_0 — is denoted by

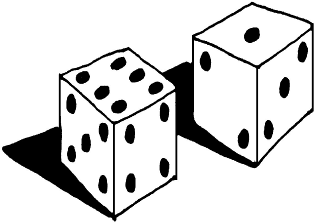
$$\mathcal{S}^{\pi}_{t_0,x_0} = \{w(\cdot) \in \mathcal{S} \mid \quad \text{the state constraints} \quad x_t \in \mathcal{A}_t \quad \text{and the control constraints} \quad u_t = \pi_t(x_t) \in \mathcal{B}_t(x_t) \quad \text{are satisfied for all times } t = t_0, \ldots, T\}$$

- The larger set $\mathcal{S}^{\pi}_{t_0,x_0}$ of viable scenarios, the better, because the policy π is able to maintain the system within constraints for a large "number" of scenarios
- But "large" in what sense? Probabilistic (stochastic)? Robust?

To measure subsets of scenarios,
we equip scenarios with an *a priori* structure:

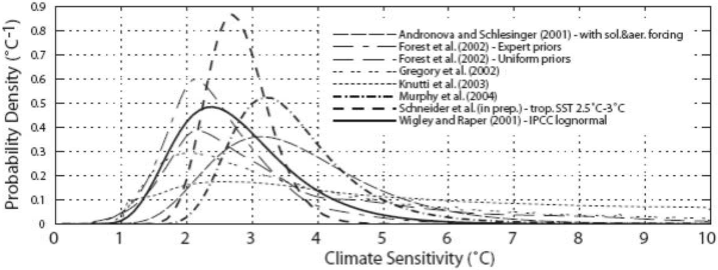
stochastic *versus* robust
(or casino *versus* parano)

In the stochastic approach, the set $\mathcal{S}$ of scenarios is equipped with a known probability $\mathbb{P}$



Equipping the set $\mathcal{S}$ of scenarios with a probability $\mathbb{P}$ is a delicate issue!

- The probabilistic distribution of the climate sensitivity parameter in climate models differs according to authors

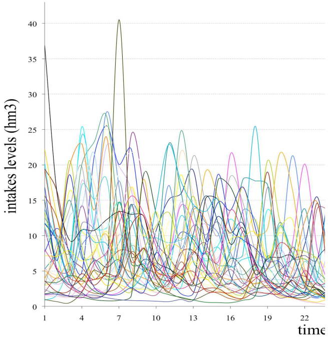


- Call to the stochastic optimization community: promote distributionaly robust optimization

In the set-membership approach, only a subset $\overline{\mathcal{S}}$ of the set $\mathcal{S}$ of scenarios is known

Selected scenarios belong to a known subset $\overline{\mathcal{S}}$

$$w(\cdot) \in \overline{\mathcal{S}} \subset \mathcal{S}$$



Historical water inflows scenarios in a dam

Parallel between robust and stochastic

Probability *versus* plausibility

$(+, \times)$	algebras	$(\max, +)$
$(\Omega, \mathcal{F})$	measurable space	$(\Omega, \mathcal{F})$
probability	measuring sets	plausibility
$\mathbb{P} : \mathcal{F} \to [0,1]$		$\mathbb{K} : \mathcal{F} \to [-\infty, 0]$
$\mathbb{P}(\bigcup_{n \in \mathcal{N}} A_n)$ $= \sum_{n \in \mathcal{N}} \mathbb{P}(A_n)$	countable disjoint union axiom	$\mathbb{K}(\bigcup_{n \in \mathcal{N}} A_n)$ $= \sup_{n \in \mathcal{N}} \mathbb{K}(A_n)$
$\mathbb{P}(\emptyset) = 0$ bottom $\bot$ $+$-neutral $\times$-absorbing	normalization (lower)	$\mathbb{K}(\emptyset) = -\infty$ bottom $\bot$ max-neutral $+$-absorbing
$\mathbb{P}(\Omega) = 1$ top $\top$ $\times$-neutral	normalization (upper)	$\mathbb{K}(\Omega) = 0$ top $\top$ $+$-neutral
$\mathbb{P}(A) \approx 0$	unlikely	$\mathbb{K}(A) \approx -\infty$
$\mathbb{P}(A) \approx 1$	likely	$\mathbb{K}(A) \approx 0$

Expectation *versus* fear[8] operators

$\chi_A = \begin{cases} 1 \\ 0 \end{cases}$	indicator function	$\chi_A = \begin{cases} 0 \\ -\infty \end{cases}$
$\chi_{\bigcap_{n\in\mathcal{N}} A_n} = \prod_{n\in\mathcal{N}} \chi_{A_n}$	countable intersection	$\chi_{\bigcap_{n\in\mathcal{N}} A_n} = \sum_{n\in\mathcal{N}} \chi_{A_n}$
expectation operator $\mathbb{E}[\chi_A] = \mathbb{P}(A)$	action on indicator	fear operator $\mathbb{F}[\chi_A] = \mathbb{K}(A)$
$\mathbb{P}(A) = \int_A p(\omega)\,\mathrm{d}\omega$ $p = \chi_A/\mathbb{P}(A)$	density uniform	$\mathbb{K}(A) = \sup_{\omega\in A} \kappa(\omega)$ $\kappa = -\chi_A$
Lebesgue integral expectation operator $\mathbb{E}[\mathbf{X}] = \int_\Omega \mathbf{X}(\omega)p(\omega)\,\mathrm{d}\omega$	action on functions	idempotent integral fear operator $\mathbb{F}[\mathbf{X}] = \sup_{\omega\in\Omega}\left[\mathbf{X}(\omega) \dot{+} \kappa(\omega)\right]$
$\mathbb{E}[\mathbf{X}+\mathbf{Y}] = \mathbb{E}[\mathbf{X}] + \mathbb{E}[\mathbf{Y}]$	"linearity"	$\mathbb{F}[\max\{\mathbf{X},\mathbf{Y}\}] = \max\{\mathbb{F}[\mathbf{X}],\mathbb{F}[\mathbf{Y}]\}$
$\mathbb{E}[\mathbf{X}\times\mathbf{Y}] = \mathbb{E}[\mathbf{X}]\times\mathbb{E}[\mathbf{Y}]$	independence	$\mathbb{F}[\mathbf{X}+\mathbf{Y}] = \mathbb{F}[\mathbf{X}]+\mathbb{F}[\mathbf{Y}]$

[8] P. Bernhard. A separation theorem for expected value and feared value discrete time control. Technical report, INRIA, Projet Miaou, Sophia Antipolis, Décembre 1995

Outline of the presentation

Stochastic viability kernels

Stochastic viability kernels

[De Lara and Doyen, 2008][9]

Definition

The stochastic viability kernel at confidence level $\beta \in [0,1]$ is

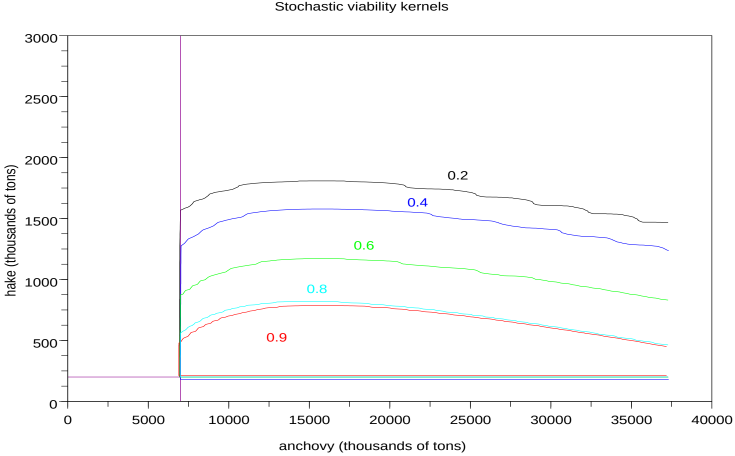
$$\mathrm{Viab}^{\beta}_{t_0} = \left\{ x_0 \in \mathcal{X} \;\middle|\; \begin{array}{l} \text{there exists a policy } \pi \text{ such that} \\ \mathbb{P}\Big(\mathcal{S} \setminus \mathcal{S}^{\pi}_{t_0,x_0}\Big) \leq 1-\beta \end{array} \right\}$$

$$\begin{aligned} & x_0 \in \mathrm{Viab}^{\beta}_{t_0} \\ \iff & \text{there exists a policy } \pi : \mathcal{T} \times \mathcal{X} \to \mathcal{U} \text{ such that} \\ & \mathbb{P}\Big(w(\cdot) \in \mathcal{S} \mid x_t \in \mathcal{A}_t \;,\;\; u_t = \pi_t(x_t) \in \mathcal{B}_t(x_t) \text{ for } t = t_0, \ldots, T \Big) \geq \beta \end{aligned}$$

[9] M. De Lara and L. Doyen. *Sustainable Management of Natural Resources. Mathematical Models and Methods*. Springer-Verlag, Berlin, 2008

Stochastic viability kernels $\mathrm{Viab}_{t_0}^{\beta}$ for a hake-anchovy fisheries model

[De Lara, Martinet, and Doyen, 2015][10]



[10] M. De Lara, V. Martinet, and L. Doyen. Satisficing versus optimality: Criteria for sustainability. *Bulletin of Mathematical Biology*, 77(2):281–297, 2015

The stochastic viability value function satisfies a (multiplicative) dynamic programming equation

For ant time t, define the probability-to-go as the function $V_t : \mathcal{X} \to [0,1]$ such that [Doyen and De Lara, 2010][11]

$$V_t(x) = \sup_{\pi} \mathbb{P}\big(\mathcal{S}^{\pi}_{t,x}\big) \ , \ \ \forall x \in \mathcal{X}$$

Proposition

If the primitive random variables $\big(w_{t_0}, w_{t_0+1}, \dots, w_{T-2}, w_{T-1}\big)$ are independent under the probability $\mathbb{P}$, we have that

$$\begin{aligned} V_T(x) &= \mathbf{1}_{\mathcal{A}_T}(x) \\ V_t(x) &= \mathbf{1}_{\mathcal{A}_t}(x) \max_{u \in \mathcal{B}_t(x)} \mathbb{E}_{w_{t+1}}\Big[V_{t+1}\Big(F_t\big(x, u, w_{t+1}\big)\Big)\Big] \end{aligned}$$

for all $x \in \mathcal{X}$, and where t runs from $T-1$ down to t_0

[11] L. Doyen and M. De Lara. Stochastic viability and dynamic programming. *Systems and Control Letters*, 59(10):629–634, Oct. 2010

Robust viability kernels

Robust viability kernels

Definition

The robust viability kernel at implausibility level $\eta \in [-\infty, 0]$ is

$$\mathrm{Viab}_{t_0}^{\eta} = \left\{ x_0 \in \mathcal{X} \;\middle|\; \begin{array}{l} \text{there exists a policy } \pi \text{ such that} \\ \mathbb{K}\Big(\mathcal{S} \setminus \mathcal{S}_{t_0,x_0}^{\pi}\Big) \leq \eta \end{array} \right\}$$

$$\begin{aligned} & x_0 \in \mathrm{Viab}_{t_0}^{\eta} \\ \iff & \text{there exists a policy } \pi : \mathcal{T} \times \mathcal{X} \to \mathcal{U} \text{ such that} \\ & \mathbb{K}\Big(w(\cdot) \in \mathcal{S} \mid x_t \in \mathcal{A}_t \; , \;\; u_t = \pi_t(x_t) \in \mathcal{B}_t(x_t) \text{ for } t = t_0, \ldots, T \Big) \leq \eta \end{aligned}$$

Robust viable epidemics control
[Sepulveda Salcedo and De Lara, 2019][12]

[12] L. S. Sepulveda Salcedo and M. De Lara. Robust viability analysis of a controlled epidemiological model. *Theoretical Population Biology*, 126:51–58, 2019

“Canal Endémico” stands as the reference to control dengue

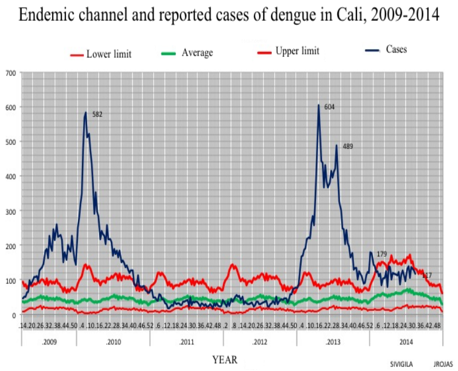


Figure: Cases of dengue between 2009 and 2014. Source: Secretaría Municipal de Salud de Cali.

Program "Dengue Control" of SMS

Control mosquito breeding sites

Capping the human infected population with the Ross-Macdonald model [De Lara and Sepulveda, 2016]

- The dynamics of the system is given by

$$\text{infected mosquito proportion} \quad \frac{dm}{dt} = A_m h(t)(1 - m(t)) - u(t)m(t)$$

$$\text{infected human proportion} \quad \frac{dh}{dt} = A_h m(t)(1 - h(t)) - \gamma h(t)$$

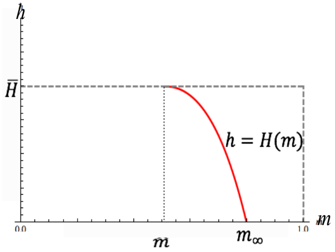
- Determine, if it exists, a piecewise continuous function (fumigation policy rates) $u(\cdot)$,

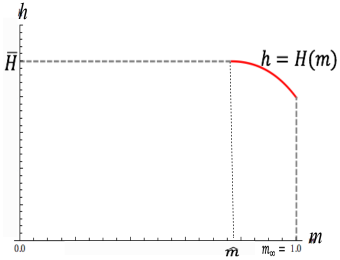
$$u(\cdot) : t \mapsto u(t) \, , \ \ \underline{u} \leq u(t) \leq \overline{u} \, , \ \ \forall t \geq 0$$

such that the following so-called viability constraint is satisfied

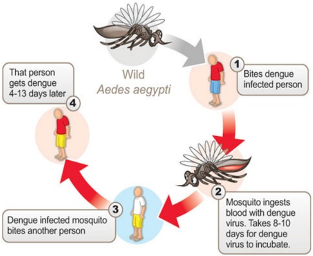
$$h(t) \leq \overline{H} \, , \ \ \forall t \geq 0$$

Capping the human infected population with the Ross-Macdonald model: viability kernels [De Lara and Sepulveda, 2016]





In epidemics transmission, sources of uncertainty abound



+

Uncertainties are captured by $\begin{cases} \text{mosquitoes transmission rate} & A_t^M \\ \text{human transmission rate} & A_t^H \end{cases}$

in the forthcoming model

New variables

- Time
  - Discrete-time $t = 0, 1, \ldots, T$
    with interval $[t, t+1[$ representing one day
- State variables
  - M_t denotes the proportion of infected mosquitoes
    at the beginning of the interval $[t, t+1[$
  - H_t denotes the proportion of infected humans
    at the beginning of the interval $[t, t+1[$
- Control variable
  - U_t denotes the mosquito mortality due to fumigation
    during the interval $[t, t+1[$

Discrete-time dynamic control model with uncertainties

- Let us denote by $F(M, H, U, A^M, A^H)$ the solution, at time $s = 1$, of the deterministic differential system with initial condition $\big(m_0, h_0\big) = (M, H)$ and stationary control U
- We obtain the sampled and controlled Ross–Macdonald model

$$\big(M_{t+1}, H_{t+1}\big) = F\big(M_t, H_t, U_t, A_t^M, A_t^H\big)$$

- The control constraints capture limited fumigation resources

$$\underline{U} \leq U_t \leq \overline{U} \ , \ \ \forall t = 0, \ldots, T-1$$

during a day

Robust viability problem statement

The robust viability kernel is the set of initial conditions (M_0, H_0) from which at least one admissible policy $\Lambda : \mathcal{T} \times [0,1] \to \mathbb{R}_+$ is such that

$$\Big(\overbrace{M_{t+1}}^{\substack{\text{proportion of} \\ \text{infected mosquitoes}}} , \overbrace{H_{t+1}}^{\substack{\text{proportion of} \\ \text{infected humans}}} \Big) = \underbrace{F}_{\text{dynamics}} \Big(M_t, H_t, \overbrace{U_t}^{\text{fumigation}}, \underbrace{A^M_{t+1}, A^H_{t+1}}_{\text{uncertainties}} \Big)$$

with $U_t = \overbrace{\Lambda_t}^{\text{policy}} \big(M_t, H_t\big)$ so that

$$\boxed{\text{infected humans} \quad H_t \leq \overline{H} \ , \ \forall t = 0, \ldots, T}$$

for all the scenarios $\Big(\big(A^M_1, A^H_1\big), \ldots, \big(A^M_T, A^H_T\big) \Big) \in \overline{\mathcal{S}} \subset (\mathbb{R}^2)^T$

Uncertainties are $\begin{cases} \text{mosquitoes transmission rate} & A^M_t \\ \text{human transmission rate} & A^H_t \end{cases}$

We make a strong assumption on the set of scenarios

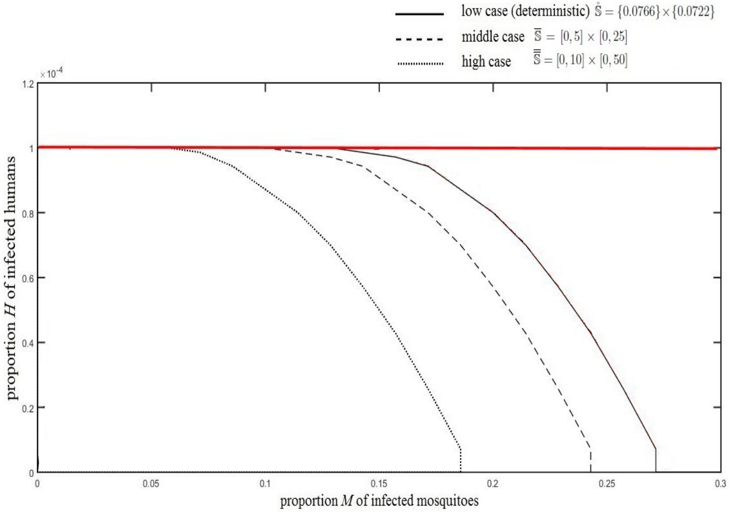
$$\text{scenario} \quad \big(A^M(\cdot), A^H(\cdot)\big) = \Big(\big(A^M_1, A^H_1\big), \ldots, \big(A^M_T, A^H_T\big)\Big)$$

- We make the strong independence assumption that

$$\big(A^M(\cdot), A^H(\cdot)\big) \in \underbrace{\overline{\mathcal{S}} = \mathcal{S}_1 \times \mathcal{S}_2 \times \cdots \times \mathcal{S}_T}_{\text{product} \equiv \text{independence}}$$

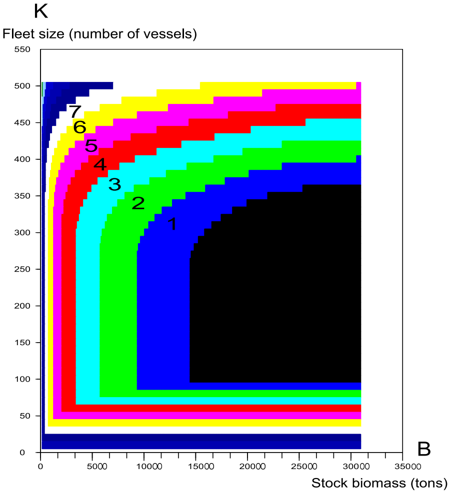
  Therefore, from one time t to the next $t+1$, uncertainties can be drastically different since (A^M_t, A^H_t) is not related to (A^M_{t+1}, A^H_{t+1})
- Such an assumption makes it possible to write a dynamic programming equation with (M, H) as state variable
- For the sake of simplicity, we take $\mathcal{S}_1 = \mathcal{S}_2 = \cdots = \mathcal{S}_T = \mathcal{S}$

Robust viability kernels shrink when uncertainties expand



Outline of the presentation

The minimum time of crisis and recovery measures the distance to a viability kernel in terms of time units



Minimum time of crisis
[Doyen and Saint-Pierre, 1997][a]

Relaxing some constraints
to try and enter into
the viability kernel
in minimum time

Example of fishery closure[b]

[a] L. Doyen and P. Saint-Pierre. Scale of viability and minimum time of crisis. *Set-valued Analysis*, 5:227–246, 1997

[b] V. Martinet, L. Doyen, and O. Thébaud. Defining viable recovery paths toward sustainable fisheries. *Ecological Economics*, 64(2):411–422, 2007

From time units to cost units

- La résilience est définie comme
  l'inverse du coût des perturbations envisagées[13]
- Resilience as the inverse of minimal expected or robust costs to reach a stochastic or robust viability kernel

[13] S. Martin. *La résilience dans les modèles de systèmes écologiques et sociaux.* Thèse École normale supérieure de Cachan - ENS Cachan, Juin 2005

Where have we gone till now? And what comes next

- We have developed — and illustrated with examples —
  a possible theory for resilience,
  that draws upon tools from control theory
  and robust/stochastic multistage stochastic optimization
- We now discuss perspectives for these fields
  regarding climate and biodiversity issues

Outline of the presentation

Sustainability: illustration in climate change economic models [10']

Resilience: mathematical formalism and examples [25']

Perspectives for stochastic optimization [15']

"Self-promotion, nobody will do it for you" ;-) [2']

Many possible extensions

- Imperfect/partial observation of the state
- Not only stochastic optimization,
  but bargaining[14], equilibrium and game theory
  (conflicting stakeholders)
- Value of information – cost of gathering information[15] – exploration/exploitation
- etc.

I will focus on

- framing sustainability and resilience problems
- solving
  - cope with non independent noises
  - cope with high dimensional states

[14] V. Martinet, P. Gajardo, and M. De Lara. Bargaining on monotonic economic environments. *Theory and Decision*, 2023

[15] L. E. Dee, M. De Lara, C. Costello, and S. D. Gaines. To what extent can ecosystem services motivate protecting biodiversity? *Ecology Letters*, 20(8):935–946, 2017

Outline of the presentation

Risk is about asymmetry

Better treat a stick as a snake than the reverse!

Variance and standart deviation fail the test as risk measures: they are measures of dispersion and variability

- The variance $\text{var}(\mathbf{X}) = \mathbb{E}\Big[\big(\mathbf{X} - \mathbb{E}[\mathbf{X}]\big)^2\Big]$ is not measured in the same units than $\mathbf{X}$, since $\text{var}(\theta\mathbf{X}) = \theta^2 \,\text{var}(\mathbf{X})$, but this can be corrected by using the standart deviation $\sigma(\mathbf{X}) = \sqrt{\text{var}(\mathbf{X})}$
- The variance is not monotonous: $\mathbf{X} \geq \mathbf{Y} \not\Rightarrow \text{var}(\mathbf{X}) \leq \text{var}(\mathbf{Y})$ (take $\mathbf{Y} = 0$ and any $\mathbf{X} \geq 0$ which is not constant)
- The variance weighs symmetrically what is above and what is below the mean, whereas the essence of risk is asymmetry between bad and good odds:

$$\underbrace{\sum_{s\in\mathcal{S}} \pi^s [x^s - \bar{x}]^2}_{\substack{\text{symetry} \\ \text{variability, dispersion}}} \quad versus \quad \underbrace{\sum_{s\in\mathcal{S}} \sum_{x^s \geq \bar{x}} \pi^s [x^s - \bar{x}]}_{\substack{\text{asymmetry} \\ \text{risk}}}$$

An anecdote on the difficulty in risk handling

Tourism issues impose constraints upon traditional economic management of a hydro-electric dam

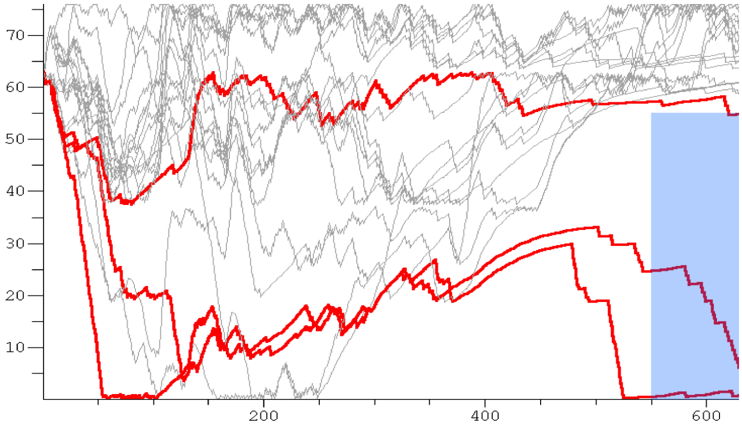
- Maximizing the revenue from turbinated water
- under a tourism constraint of having enough water in July and August 90% of the years (chance constraint)

[Alais, Carpentier, and De Lara, 2017][a]

[a] J.-C. Alais, P. Carpentier, and M. De Lara. Multi-usage hydropower single dam management: chance-constrained optimization and stochastic viability. *Energy Systems*, 8(1):7–30, Feb. 2017

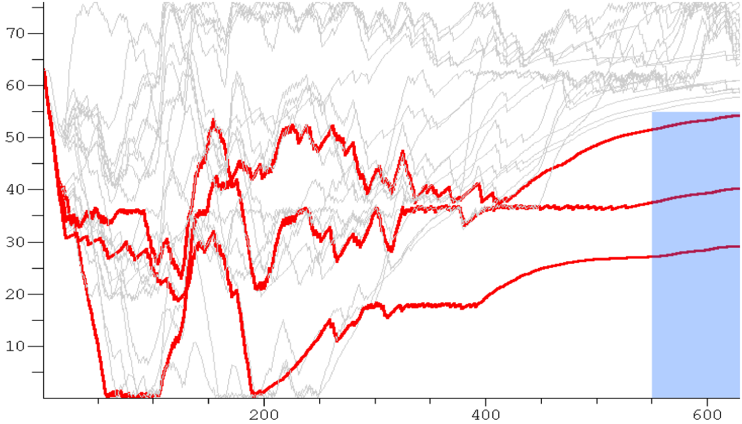
We formulated the management problem as chance-constrained multistage stochastic optimization

We found solutions where 90% of the stock trajectories meet the tourism constraint in July and August



But this was not what was really wanted (Did they want a conditional value-at-risk?)

So we had to change solutions



Measuring risk as a gauge

Going from home to the airport with a safety margin

- When you go from home to the airport, you consider possible transportation delay (road accident, bus delay), represented by a (stochastic) transport time $\mathbf{X}$
- You take a safety margin, and add some (deterministic) extra time $\rho(\mathbf{X})$
- This extra time $\rho(\mathbf{X})$ depends
  - on the randomness ($\mathbf{X}$) that affects transportation
  - on how you perceive (ρ) the importance of being "just in time"
- This deterministic extra time is an example of (gauge) risk measure

$$\underbrace{\overbrace{\rho(\mathbf{X})}^{\text{deterministic extra time}} + \overbrace{\mathbf{X}}^{\text{stochastic transportation time}}}_{\text{acceptable stochastic time from home to airport}} \in \mathcal{A}$$

Risk measures as capital requirement

A measure of risk associates to each cost $\mathbf{X}$

- the minimum extra capital $\rho(\mathbf{X})$, a deterministic number,
- required to make it "acceptable" to a regulator
- that is, such that when you substract $\rho(\mathbf{X})$ from the cost $\mathbf{X}$, the shifted cost $\mathbf{X} - \rho(\mathbf{X})$ becomes acceptable

The lower $\rho(\mathbf{X})$, the better (the less risk)

Interpreting the mathematical expectation as a gauge risk measure

- Define the following set of acceptable random variables

$$\mathcal{A} = \{\mathbf{Z} \in \mathbb{L}(\Omega, \mathcal{F}, \mathbb{R}) \mid \mathbb{E}[\mathbf{Z}] \leq 0\}$$

  When the random variable $\mathbf{Z}$ is interpreted as a cost, a cost with negative mean is acceptable

- The mathematical expectation $\mathbb{E}[\mathbf{X}]$ of a random cost $\mathbf{X}$ is the smallest amount x you can substract to $\mathbf{X}$ to make $\mathbf{X} - x$ acceptable

$$\mathbb{E}[\mathbf{X}] = \inf\{x \in \mathbb{R} \mid \mathbf{X} - x \in \mathcal{A}\} = \inf\{x \in \mathbb{R} \mid \mathbb{E}[\mathbf{X} - x] \leq 0\}$$

Risk in practice
violation, Value at Risk, quantile

Eurocode/1990/Method of partial coefficients

> *The EN Eurocodes are a series of 10 European Standards, EN 1990 - EN 1999, providing a common approach for the design of buildings and other civil engineering works and construction products.*

For a mechanical structure, the (reliability) condition

$$\text{Probability}\big(\text{sollicitation } \mathbf{E} \leq \text{resistance } \mathbf{R}\big) \text{ is high}$$

between the random variables **E** (sollicitation, action) and **R** (resistance)
— that are uncertain values due to numerous approximations —
is replaced by a deterministic relation of the form

$$\text{partial coefficient } \times E \leq R/\text{partial coefficient}$$

between the outputs E and R of codes of calculation for structures

Eurocode/1990/Method of partial coefficients

Table: Coefficients pour les matériaux en béton et en acier

Matériau	**Durable et Transitoire**	**Accidentel**
Béton (sauf pieux)	$\gamma_C = 1,50$	$\gamma_C = 1,20$
Béton pour pieux	$\gamma_C = 1,65$	$\gamma_C = 1,32$
Acier de béton armé	$\gamma_S = 1,15$	$\gamma_S = 1,00$
Acier de précontrainte	$\gamma_S = 1,15$	$\gamma_S = 1,00$
Module d'Young de l'acier de béton armé	$\gamma_{cE} = 1,20$	$\gamma_{cE} = 1,20$
Accroissement de contrainte $\Delta\sigma_p$ ou $\gamma_{\Delta P,\text{sup}} = \gamma_{\Delta P,\text{inf}} = 1,00$ (non fissuré)	$\gamma_{\Delta P,\text{sup}} = 1,20$ et $\gamma_{\Delta P,\text{inf}} = 0,80$	
Accroissement de contrainte $\Delta\sigma_p$ (version française)	$\gamma_{\Delta P,\text{sup}} = \gamma_{\Delta P,\text{inf}} = 1,00$	

Nuclear accidents prevention

- Three Mile Island accident:
  before the fact, the core meltdown was considered as excluded
- Nuclear accidents with probability per reactor per year
  - between 10^{-6} and 10^{-4} are considered as hypothetical,
  - whereas below 10^{-6} they are not envisaged
- Fukushima nuclear plants had a 10^{-9} nuclear accident probability per reactor per year

Transmission System Operator's (N-1) criterion

- "(N-1) criterion" is the rule according to which the elements remaining in operation within a Transmission System Operator's (TSO's) control area after occurrence of a contingency are capable of accommodating the new operational situation without violating operational security limits (Article 3(2)(14) of the Network Code on System Operation)
- Each TSO shall assess the risks associated with the contingencies after simulating each contingency from its contingency list and after assessing whether it can maintain its transmission system within the operational security limits in the (N-1) situation
- In case of an (N-1) situation caused by a disturbance, each TSO shall activate a remedial action in order to ensure that the transmission system is restored to a normal state as soon as possible and that this (N-1) situation becomes the new N-Situation

Danish Transmission System Operator's P10 rule

- Requirements for the prognosis at the time of bidding for reserves (Ex-ante) Energinet requires that there must at maximum be bid in capacity corresponding to the 10% percentile with delivery of capacity reserves from fluctuating renewables and flexible consumption. This means, that the participant's prognosis, which must be approved by Energinet, evaluates that the probability is 10% that the sold capacity is not available. This entails that there is a 90% chance that the sold capacity or more is available. This is when the prognosis is assumed to be correct.
- The probability is then also 10%, that the entire sold capacity is not available. If this were to happen, it does not entail that the sold capacity is not available at all, however just that a part of the total capacity is not available. The available part will with high probability be close to the sold capacity. Because of this Energinet uses the 10% percentile and not the e.g., 5% or 1% percentiles. Energinet will continuously evaluate the determined percentile based on experience.
- If a market participant repeatedly, in good faith, does not deliver the sold reserve-capacity, then the participant will be excluded from participating in the market, until an approved prognosis can be approved by Energinet. If a participant can not deliver the sold capacity because of a bid based on a capacity lower than the 10% percentile, the participant will be excluded instantly for an undetermined time. This will happen as part of Energinet'items regular monitoring. If a participant, in good faith, is not able to deliver the sold capacity, the payment will be repaid after the rules for the different ancillary service productions according to the "Ancillary services to be delivered in Denmark - Tender conditions".

Value at Risk

The Value at Risk (quantile)

Let $\lambda \in]0,1[$, that plays the role of a risk level

Value at Risk

The Value at Risk of the cost $\mathbf{X}$ at level $\lambda \in]0,1[$ is

$$VaR_\lambda(\mathbf{X}) = \inf\{x \in \mathbb{R} \mid \mathbb{P}(\mathbf{X} > x) < \lambda\}$$

with acceptance set

$$\mathcal{A} = \{\mathbf{Z} \in \mathbb{L}(\Omega, \mathcal{F}, \mathbb{R}) \mid \mathbb{P}(\mathbf{Z} \geq 0) < \lambda\}$$

- Intuitively, saying that the $VaR_{5\%}$ of a portfolio is 100 means that the loss will be more than 100 with probability at most 5%
- $VaR_{5\%}$ is the maximum loss in the 95% of the cases
- However, $VaR_{5\%}$ does not inform on the size of the loss

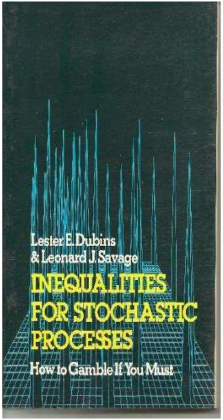
Value at Risk and diversification

Beware: here **X** and **Y** are minus costs!

ω	**X**	$\mathbb{P}$	ω	**Y**	$\mathbb{P}$	ω	$0.5\mathbf{X} + 0.5\mathbf{Y}$	$\mathbb{P}$
1	-100	4%	1	0	4%	1	-50	4%
2	0	4%	2	-100	4%	2	-50	4%
3	0	4%	3	0	4%	3	0	4%
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
25	0	4%	25	0	4%	25	0	4%

- The minimum m to be added to **X** in such a way that $\mathbb{P}(\mathbf{X} + m < 0) \leq 5\%$ is $m = 0$ since $\mathbb{P}(\mathbf{X} - \epsilon < 0) = 100\% > 5\%$ for all $\epsilon > 0$.
- Hence $VaR_{5\%}(\mathbf{X}) = VaR_{5\%}(\mathbf{Y}) = 0$.
- And...

$$VaR_{5\%}(\mathbf{X}) = VaR_{5\%}(\mathbf{Y}) = 0 < 50 = VaR_{5\%}(0.5\mathbf{X} + 0.5\mathbf{Y})$$



How to gamble if you must,
L.E. Dubbins and
L.J. Savage, 1965

Imagine yourself at a casino with \$1,000. For some reason, you desperately need \$10,000 by morning; anything less is worth nothing for your purpose.

The only thing possible is to gamble away your last cent, if need be, in an attempt to reach the target sum of \$10,000.

- The question is how to play, not whether. What ought you do? How should you play?
  - Diversify, by playing 1 \$ at a time?
  - Play boldly and concentrate, by playing 1,000 \$ only one time?
- What is your decision criterion?

Moving from violation (Value at Risk [quantile])
to severity (Conditional Value at Risk [superquantile])

The Tail Value at Risk (superquantile)

Let $\lambda \in]0,1[$, that plays the role of a risk level

Tail Value at Risk

The Tail Value at Risk of the cost $\mathbf{X}$ at level $\lambda \in]0,1[$ is

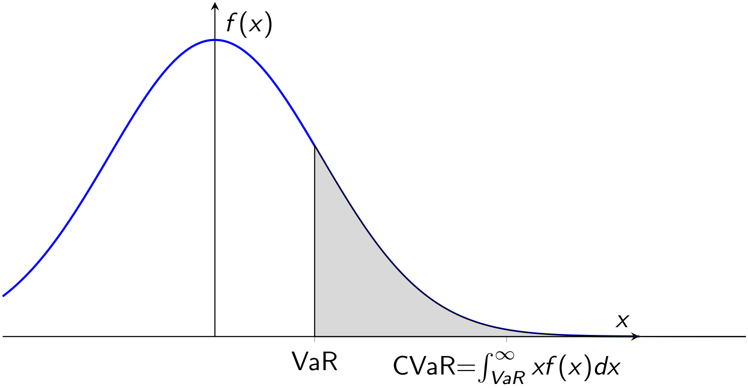
$$TVaR_\lambda(\mathbf{X}) = \frac{1}{1-\lambda} \int_\lambda^1 VaR_{\lambda'}(\mathbf{X}) d\lambda'$$

[Rockafellar and Uryasev, 2000]

$$TVaR_\lambda[\mathbf{X}] = \inf_{s \in \mathbb{R}} \left\{ \frac{\mathbb{E}[(\mathbf{X}-s)^+]}{1-\lambda} + s \right\} \ , \ \ \lambda \in [0,1[$$

Limit cases

$$TVaR_0[\mathbf{X}] = \mathbb{E}[\mathbf{X}]$$
$$TVaR_1[\mathbf{X}] = \lim_{\lambda \to 1} TVaR_\lambda[\mathbf{X}] = \sup_{\omega \in \Omega} \mathbf{X}(\omega)$$


$f(x)$
x
VaR
CVaR$=\int_{VaR}^{\infty} xf(x)dx$

More on the Tail Value at Risk

- The Average Value at Risk or Tail Value at Risk

$$TVaR_{\lambda}(\mathbf{X}) = \frac{1}{\lambda} \int_0^{\lambda} VaR_{\lambda'}(\mathbf{X}) d\lambda'$$

- The Worst Conditional Expectation

$$\sup\{\mathbb{E}[\mathbf{X} \mid A] \,,\; A \in \mathcal{F} \,,\; \mathbb{P}(A) < \lambda\}$$

  are the worst costs conditioned over events
  of probability less than the risk level $\lambda \in]0,1[$
- If $\mathbb{P}\left\{\mathbf{X} \leq Q^{-}_{1-\lambda}(\mathbf{X})\right\} = \lambda$,

$$TVaR_{\lambda}(\mathbf{X}) = \mathbb{E}[\mathbf{X} \mid \overbrace{\mathbf{X} \geq VaR_{\lambda}(\mathbf{X})}^{\text{costs greater than VaR}}]$$

  is the average of costs greater than the Value at Risk (severity)

Properties of the Tail Value at Risk

The Tail Value at Risk of a cost **X** is measured in the same units than **X**, and is

- invariant by translation

$$TVaR_{\lambda}(\mathbf{X}+x) = TVaR_{\lambda}(\mathbf{X})+x \ , \ \ \forall x \in \mathbb{R}$$

- monotonous

$$\mathbf{X} \geq \mathbf{Y} \Rightarrow TVaR_{\lambda}(\mathbf{X}) \geq TVaR_{\lambda}(\mathbf{Y})$$

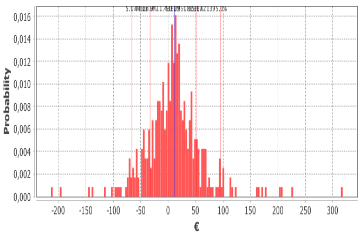
- positively homogeneous

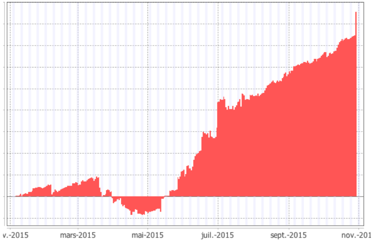
$$TVaR_{\lambda}(\theta\mathbf{X}) = \theta\, TVaR_{\lambda}(\mathbf{X}) \ , \ \ \forall \theta > 0$$

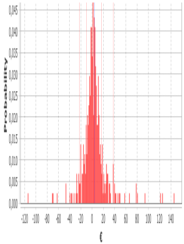
- convex, hence favors diversification :-)

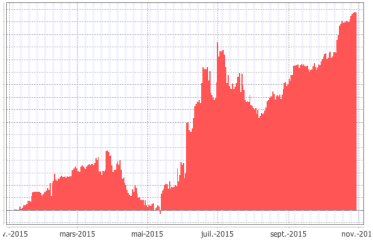
Illustration: the financial director gasp!

Management of a hydro-electric dam: random profits risk neutral (upper row) versus risk averse (lower row)









Convex risk measures

Axiomatics of risk measures

A risk measure is a mapping $\rho : \mathbb{L}(\Omega, \mathcal{F}, \mathbb{R}) \to \mathbb{R}$ (or $\mathbb{R} \cup \{-\infty\}$)

Risk measures

A risk measure ρ is

- (T) invariant by translation if $\rho(\mathbf{X} + x) = \rho(\mathbf{X}) + x$, for all $x \in \mathbb{R}$
- (M) monotonous whenever $\mathbf{X} \geq \mathbf{Y} \Rightarrow \rho(\mathbf{X}) \geq \rho(\mathbf{Y})$
- (C) convex if $\rho(\theta\mathbf{X} + (1-\theta)\mathbf{Y}) \leq \theta\rho(\mathbf{X}) + (1-\theta)\rho(\mathbf{Y})$
- (PH) positively homogeneous if $\rho(\theta\mathbf{X}) = \theta\rho(\mathbf{X})$ when $\theta > 0$
- (S) subadditive if $\rho(\mathbf{X} + \mathbf{Y}) \leq \rho(\mathbf{X}) + \rho(\mathbf{Y})$

One says that ρ is a monetary risk measure if it is monotonous (M) and invariant by translation (T)

Affine risk measures

The cost average under probability $\mathbb{Q}$ is

$$\rho(\mathbf{X}) = \mathbb{E}_{\mathbb{Q}}(\mathbf{X})$$

whereas the shifted cost average under probability $\mathbb{Q}$ is

$$\rho(\mathbf{X}) = \mathbb{E}_{\mathbb{Q}}(\mathbf{X}) - \gamma$$

Convex risk measures

Given

- a subset $\mathcal{Q}$ of probabilities on Ω, representing different priors about the randomness
- a function $\gamma : \mathcal{Q} \to \mathbb{R}$, with $\sup_{\mathbb{Q}\in\mathcal{Q}} \gamma(\mathbb{Q}) < +\infty$, representing cost shifts

we define

$$\rho(\mathbf{X}) = \sup_{\mathbb{Q}\in\mathcal{Q}} \Big(\mathbb{E}_{\mathbb{Q}}(\mathbf{X}) - \gamma(\mathbb{Q}) \Big)$$

which expresses

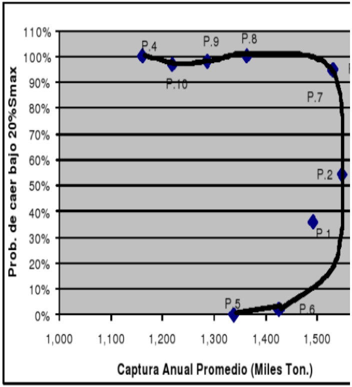
- first, an average of the cost $\mathbf{X}$ over different outcomes ponderations $\mathbb{Q} \in \mathcal{Q}$, each being *penalized* by $\gamma(\mathbb{Q})$
- second, a conservative attitude by taking the largest with the sup operation over priors

Outline of the presentation

Axiomatics for robust sustainability

A battery of assessment frameworks

- Integrated Ecosystem Assessment (IEA)
- Ecological Risk Assessment
- Ecosystem-based Management (EBM)
- Ecosystem Approach to Management
- Driver Pressure State Impact Response (DPSIR) Approach
- Management strategy evaluation (MSE)



[De Lara and Martinet, 2009][a] [Martinet, Peña-Torres, De Lara, and Ramírez, 2016][b]

[a] M. De Lara and V. Martinet. Multi-criteria dynamic decision under uncertainty: A stochastic viability analysis and an application to sustainable fishery management. *Mathematical Biosciences*, 217(2):118–124, February 2009

[b] V. Martinet, J. Peña-Torres, M. De Lara, and H. Ramírez. Risk and sustainability: Assessing fishery management strategies. *Environmental and Resource Economics*, 64(9): 683–707, Aug. 2016

Policies shape state-control random processes

- With a given policy π, we induce (shape) a random process

$$\overbrace{\mathcal{S} = \mathcal{W}^{T-t_0}}^{\substack{\text{(scenarios)}\\ \text{uncertainty trajectories}}} \to \overbrace{\mathcal{X}^{T-t_0+1}}^{\text{state trajectories}} \times \overbrace{\mathcal{U}^{T-t_0}}^{\text{control trajectories}}$$

$$w(\cdot) \mapsto \big(x(\cdot), u(\cdot)\big)_{\pi}[w(\cdot)]$$

by means of the closed-loop dynamics

$$x_{t+1} = F_t\big(x_t, \pi_t(x_t), w_{t+1}\big) \; , \; u_t = \pi_t\big(x_t\big) \; , \; t = t_0, \ldots, T-1$$

- Stochastic and robust viability correspond to

$$\underbrace{\Big\{ w(\cdot) \; \Big| \; \big(x(\cdot), u(\cdot)\big)_{\pi}[w(\cdot)] \notin \overbrace{\prod_{t=t_0}^{T-1} \big\{ (x_t, u_t) \mid u_t \in \mathcal{B}_t(x_t) \text{ and } x_t \in \mathcal{A}_t \big\} \times \mathcal{A}_T}^{\text{product set expresses robustness w.r.t. time}} \Big\}}_{\text{"small" (probabilistically or robustly)}}$$

Extension to more general acceptable sets of random processes

- Acceptable sets of random processes [De Lara, 2018][16]

$$\mathcal{A} \subset \left(\mathcal{X}^{T-t_0+1} \times \mathcal{U}^{T-t_0} \right)^{\mathcal{S}}$$

  like

$$\mathcal{A} = \Big\{ (x(\cdot), u(\cdot)) \,\Big|\, \underbrace{-\mathcal{R}_t^k}_{\text{risk measure}} \big(- \underbrace{\mathcal{I}_t^k(x_t, u_t)}_{\text{indicator}} \big) \geq \underbrace{\tau_t^k}_{\text{threshold}} \,,\ \forall k \,,\ \forall t \Big\}$$

- Axiomatics for bioeconomics acceptable sets?
  Inspired by mathematical finance and the role of convexity,
  but with the difficulty that the state space $\mathcal{X}$
  is a mix of physics, biology, society —
  hence not naturally equipped with convexity structure

[16] M. De Lara. A mathematical framework for resilience: Dynamics, uncertainties, strategies, and recovery regimes. *Environmental Modeling & Assessment*, 23(6): 703–712, Dec. 2018

Example: precautionary conditional expectation (the lower, the better)

As in fishery management, we consider — for each indicator $\mathcal{I}_t^k : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$ — corresponding reference points

$$\underbrace{\tau_\flat^k}_{\text{limit reference point}} \leq \underbrace{\tau_\sharp^k}_{\text{precautionary reference point}}$$

The condition of sustainability is that, when $\mathcal{I}_t^k(\mathbf{X}_t, \mathbf{U}_t) \leq \tau_\sharp^k$, then we need to ensure that $\tau_\flat^k \leq \mathcal{I}_t^k(\mathbf{X}_t, \mathbf{U}_t)$

- We define the precautionary conditional expectation by

$$\mathbb{E}_\mathbb{P}\big[\mathcal{I}_t^k(\mathbf{X}_t, \mathbf{U}_t) \mid \overbrace{\mathcal{I}_t^k(\mathbf{X}_t, \mathbf{U}_t) \leq \tau_\sharp^k}^{\text{precautionary zone}}\big]$$

- We translate the condition of sustainability into

$$\tau_\flat^k \leq \underbrace{\mathbb{E}_\mathbb{P}\big[\mathcal{I}_t^k(\mathbf{X}_t, \mathbf{U}_t) \mid \mathcal{I}_t^k(\mathbf{X}_t, \mathbf{U}_t) \leq \tau_\sharp^k\big]}_{\text{amplitude of the indicator in the precautionary zone}} \;, \; \forall t \;, \; \forall k$$

Axiomatics for robust sustainability

- sustainability = "disaggregation" w.r.t.
  - time (generations)
  - indicators (stakeholders)
- robustness = reflecting risk preferences

Acceptable set of random processes

$$\mathcal{A} = \Big\{ \big(x(\cdot), u(\cdot)\big) \,\Big|\, \overbrace{\underbrace{-\mathcal{R}_t^k}_{\text{risk measure}}}^{\text{robustness}} \overbrace{\Big(- \underbrace{\mathcal{I}_t^k(x_t, u_t)}_{\text{indicator}} \Big) \geq \underbrace{\tau_t^k}_{\text{threshold}} \;,\; \forall k \;,\; \forall t}^{\text{sustainability}} \Big\}$$

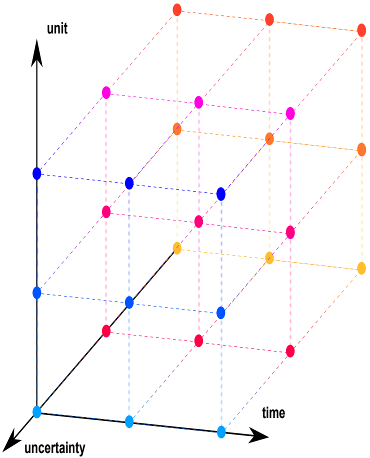
$$\subset \Big(\mathcal{X}^{T-t_0+1} \times \mathcal{U}^{T-t_0} \Big)^{\mathcal{S}}$$

Outline of the presentation

A bird's eye view of decomposition methods

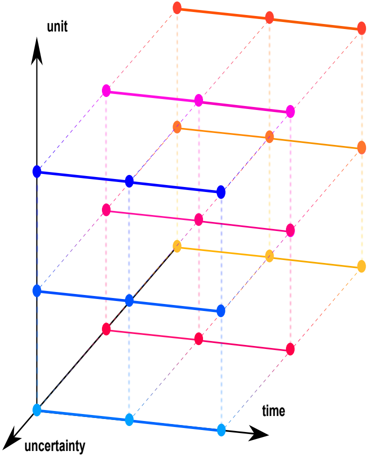
Tribute to
Guy Cohen, Pierre Carpentier, Jean-Philippe Chancelier
and ex-PhD students

Couplings for stochastic problems



$$\min \mathbb{E} \sum_i \sum_t L_t^i \big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1} \big)$$

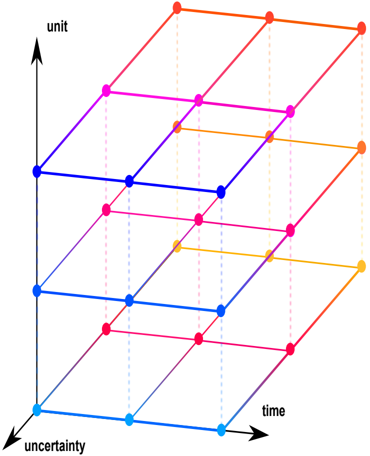
Couplings for stochastic problems: in time



$$\min \mathbb{E} \sum_i \sum_t L_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

s.t. $\mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$

Couplings for stochastic problems: in uncertainty

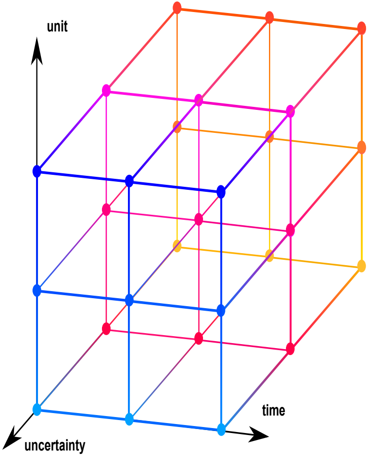


$$\min \mathbb{E} \sum_i \sum_t L_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\text{s.t. } \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\left[\mathbf{U}_t^i \mid \mathbf{W}_0, \dots, \mathbf{W}_t\right]$$

Couplings for stochastic problems: in space



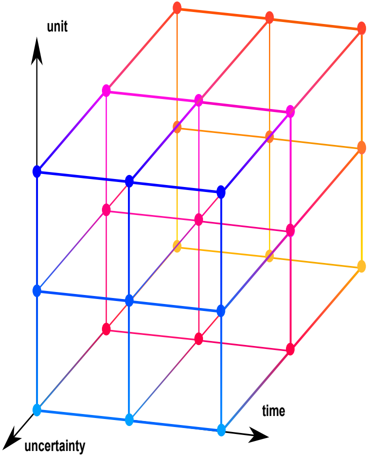
$$\min \mathbb{E} \sum_i \sum_t L_t^i\big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

s.t. $\mathbf{H}_{t+1}^i = \big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$

$$\mathbf{U}_t^i = \mathbb{E}\big[\mathbf{U}_t^i \,\big|\, \mathbf{W}_0, \dots, \mathbf{W}_t\big]$$

$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Can we decouple stochastic optimization problems?



$$\min \mathbb{E} \sum_i \sum_t L_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

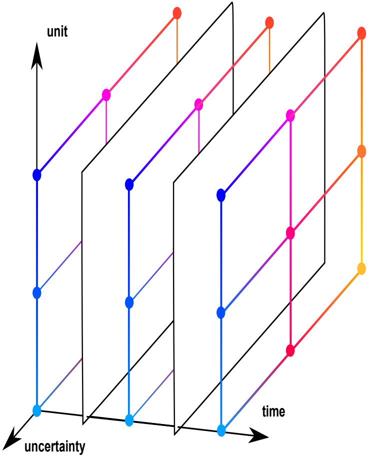
$$\text{s.t.} \quad \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\left[\mathbf{U}_t^i \,\middle|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\right]$$

$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Decomposition-coordination: divide and conquer

Sequential decomposition in time



$$\min \mathbb{E} \sum_i \sum_t L_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\text{s.t. } \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\left[\mathbf{U}_t^i \,\middle|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\right]$$

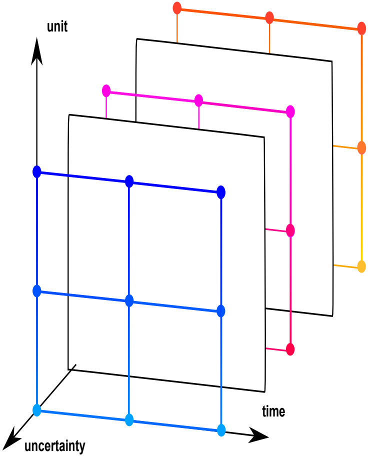
$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Dynamic Programming

Bellman (1956)[a]

[a] R. E. Bellman. *Dynamic Programming*. Princeton University Press, Princeton, N.J., 1957

Parallel decomposition in uncertainty/scenarios



$$\min \mathbb{E} \sum_i \sum_t L_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\text{s.t.} \quad \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\left[\mathbf{U}_t^i \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\right]$$

$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Progressive Hedging

Rockafellar - Wets (1991)[a]

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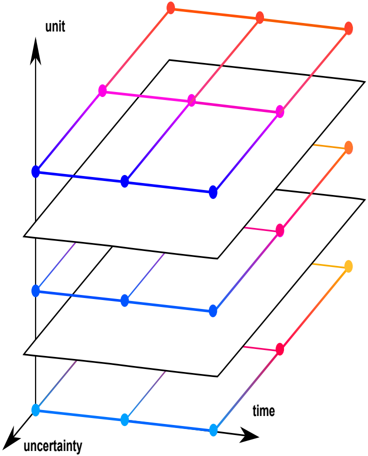
[a] R. Rockafellar and R. J.-B. Wets. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research*, 16(1):119–147, 1991

Parallel decomposition in space/units



$$\min \mathbb{E} \sum_i \sum_t L_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\text{s.t. } \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\left[\mathbf{U}_t^i \,\middle|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\right]$$

$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Price/ Resource decompositions[a]

[a] P. Carpentier, G. Cohen, and J.-C. Culioli. Stochastic optimal control and decomposition-coordination methods *In: Recent Developments in Optimization, Roland Durier and Christian Michelot (Eds.), Springer-Verlag, Berlin*, 1995

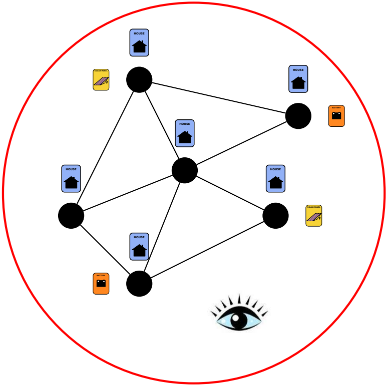
Combining sequential and parallel decomposition methods

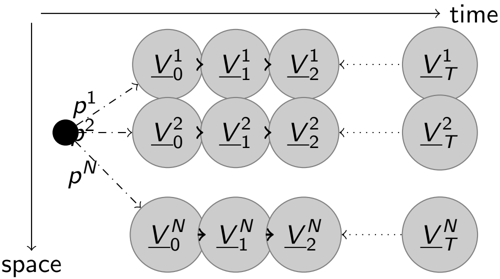
- Combining the three "pure" decomposition methods
  - time: Dynamic Programming
    and its variant by time block decomposition
    [Carpentier, Chancelier, De Lara, Martin, and Rigaut, 2023][17]
  - scenario: Progressive Hedging
  - space: decomposition by prices or by resources
- to produce blends and tackle large scale applications
  - time blocks + prices/resources
    - dynamic programming across time blocks
      + prices/resources decomposition by time block
      (application to two time scales battery management)
  - time + space
    - nodal decomposition by prices or by resources
      + dynamic programming within node
      (application to large scale microgrid management)

[17] P. Carpentier, J.-P. Chancelier, M. De Lara, T. Martin, and T. Rigaut. Time Block Decomposition of Multistage Stochastic Optimization Problems. *Journal of Convex Analysis*, 30(2), 2023

Mix of spatial and temporal decompositions

[Carpentier, Chancelier, De Lara, and Pacaud, 2020][18]
[Pacaud, De Lara, Chancelier, and Carpentier, 2022][19]





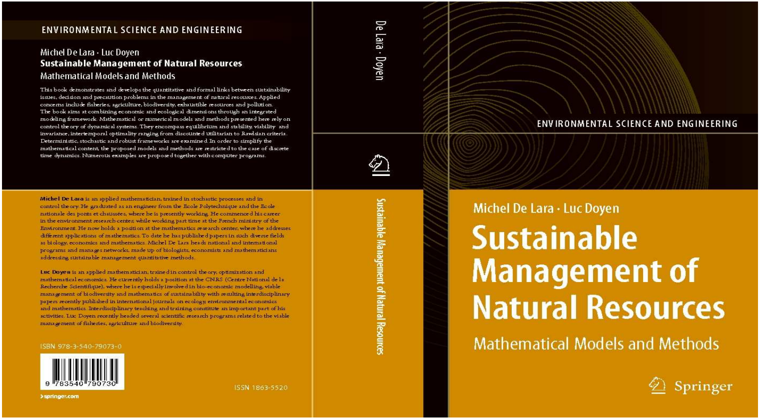
[18] P. Carpentier, J.-P. Chancelier, M. De Lara, and F. Pacaud. Mixed spatial and temporal decompositions for large-scale multistage stochastic optimization problems. *Journal of Optimization Theory and Applications*, 186(3):985–1005, 2020

[19] F. Pacaud, M. De Lara, J.-P. Chancelier, and P. Carpentier. Distributed multistage optimization of large-scale microgrids under stochasticity. *IEEE Transactions on Power Systems*, 37(1):204–211, 2022

Outline of the presentation

“Nul n’est mieux servi que par soi-même”
“Self-promotion, nobody will do it for you” ;-)

M. De Lara, L. Doyen, Sustainable Management of Natural Resources. Mathematical Models and Methods, *Springer*, 2008.



A wrap-up call to the stochastic optimization community

Maybe the stochastic optimization community could help by

- proposing various mathematical formalisms (robust, stochastic, distributionally robust, risk measures, etc.) to model hard (non aggregation) and soft (aggregation) constraints, especially in the presence of risk factors
- smoothing/softening the hard
  - computing Lagrange multipliers that turn hard constraints into soft ones (smooth trade-offs)
  - using plausability functions in robust optimization
  - promoting the use of suitable risk measures that play the role of "softeners" of almost sure constraints[20]
- developing axiomatics for risk in bioeconomics

[20] R. T. Rockafellar. Coherent approaches to risk in optimization under uncertainty. *INFORMS TutORials in Operations Research*, null(null):38–61, 2014

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THANK YOU!

Caveat: this talk is *not* about crafting dynamical models

[Yodzis, 1994][21] In population modelling the functional forms of models are at least as important as are parameter values in expressing the underlying biology and in determining the outcome. (...) For instance, May et al. (1979) assumed, without comment, a particular form of predator-prey interaction; and this particular form was carried over, again without comment, by Flaaten.

$$\text{Leslie/May et al./Flaaten predator dynamics} \quad \frac{dP}{dt} = r\Big(1 - \frac{P}{hN}\Big)$$

It turns out that this "invisible" but powerful assumption is responsible in large part for the conclusion reached by Flaaten (1988). (...) Flaaten's work is controversial because of his conclusion that "sea mammals should be heavily depleted to increase the surplus production of fish resources for man"

$$\text{Yodzis predator dynamics} \quad \frac{dP}{dt} = P\Big(-d + eF(N) - cP\Big)$$

[21] P. Yodzis. Predator-prey theory and management of multispecies fisheries. *Ecological Applications*, 4(1):51–58, Feb. 1994

To make a long story short...

Mathematical control theory, viability and stochastic optimization offer material for an operational definition of resilience

Theory. Mathematics provides concepts, tools and methods

- states, controls, uncertainties, dynamics (control theory)
- scenarios, policies, constraints (critical thresholds)
- (stochastic, robust) viability kernel = viable states
- minimal time of crisis, cost-efficiency (optimization)

Answers. Geometry + Optimization

- Resilient states = viable states
- Measuring resilience as the inverse of the minimal cost (expected, robust) to reach a viability kernel

Tribute to
Jean-Pierre Aubin, Patrick Saint-Pierre, Luc Doyen, Sophie Martin

Our emphasis is on the treatment of uncertainties:
stochastic and robust viability, and possible extensions

Caveat: this talk is *not* about crafting scenarios

Choosing a set of scenarios is excluding "things we don't know we don't know"

> *Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – the ones we don't know we don't know. And if one looks throughout the history of our country and other free countries, it is the latter category that tend to be the difficult ones.*

Donald Rumsfeld, former United States Secretary of Defense.
From Department of Defense news briefing, February 12, 2002

Can we formalize resilience? adaptive capacity?

- Being resilient: belonging to a viability kernel that captures compatibility between
  - controlled dynamics
  - acceptable set/viability constraints: possible values for output variables + critical thresholds (the spiderweb of sustainability)
  - tolerable risk (few nonviable scenarios)
- Adaptive capacity: set of viable policies?
  - = policies and enabling the system to remain within the acceptable set for a certain number of scenarios (expressing the level of risk tolerated)
  - exist only in a viable state
- Measuring resilience:
  - the more resilient, the lower the costs to reach a viable state
  - the less resilient, the higher the costs to reach a viable state

The three Rs of resilience

The '3Rs' of resilience[22]

- resistance
- recovery
- robustness/reliability

[22] R. Q. Grafton, L. Doyen, C. Béné, E. Borgomeo, K. Brooks, L. Chu, G. S. Cumming, J. Dixon, S. Dovers, D. Garrick, A. Helfgott, Q. Jiang, P. Katic, T. Kompas, L. R. Little, N. Matthews, C. Ringler, D. Squires, S. I. Steinshamn, S. Villasante, S. Wheeler, J. Williams, and P. R. Wyrwoll. Realizing resilience for decision-making. *Nature Sustainability*, 2(10):907–913, oct. 2019