Sustainable Yields for Ecosystems

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Ricardo OLIVEROS–RAMOS [3] and Jorge TAM [3]

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Outline of the presentation

1. How are fishing quotas fixed?
2. Anchovy–hake couple in the Peruvian upwelling ecosystem
3. Viable states and guaranteed yields
4. Sustainable yields for ecosystems

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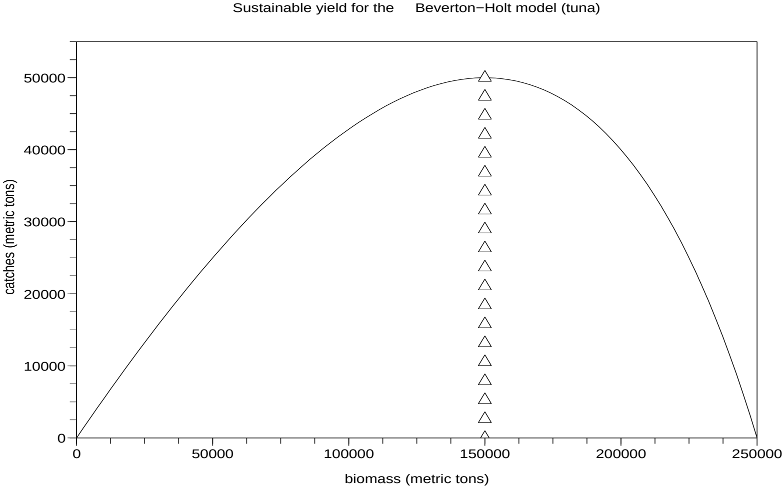
Sustainable yield for the Beverton−Holt model (tuna)
catches (metric tons)
50000
40000
30000
20000
10000
0
0
50000
100000
150000
200000
250000
biomass (metric tons)

Sustainable yields: species by species

- Maximum sustainable yield (MSY)
    - monospecific scalar dynamic model
    - steady state approach
    - ⇒ maximal yield which can be sustained at equilibrium
    - Following the World Summit on Sustainable Development (Johannesburg, 2002), the signatory States undertook to restore and exploit their stocks at MSY
- ICES precautionary approach
    - monospecific age-class dynamic model
    - short term:projects abundances one year ahead
    - ⇒ maximal yield which can be obtained without putting next year spawning stock biomass below its reference point

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- We propose a general approach
  - multi-specific dynamic model (age-class or not)
  - long-term: guaranteed yields and biological indicators
  - method: computing a set of viable states (viability kernel)
  - ⇒ species by species yields which can be guaranteed without putting biological indicators below their reference points
- Generic biomass ecosystem models with harvesting
  - ⇒ explicit expressions for viability kernel and guaranteed yields
- Specific case
  - Numerical results for a Lotka–Volterra model of the anchovy–hake couple in the Peruvian upwelling ecosystem between the years 1971 and 1981

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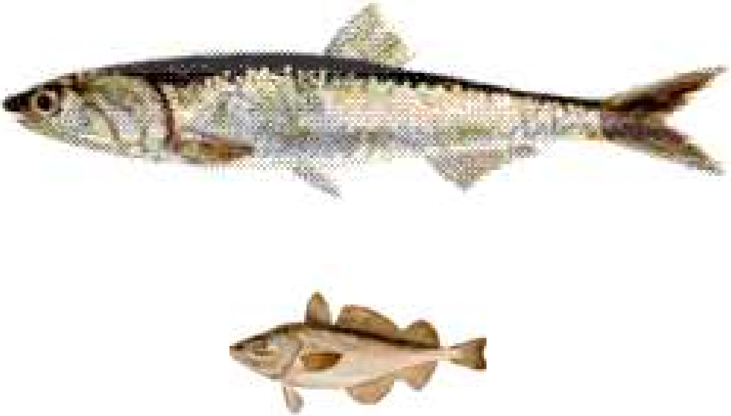
Anchoveta/Anchovy and Merluza/Hake



Anchoveta/Anchovy and Merluza/Hake

11 years of data from 1971 to 1981

Instituto del Mar del Perú (IMARPE)
In thousands of tonnes (10^3 tons)

- anchoveta_stocks=
  [4058 3116 3461 2649 4517 1232 3727 1812 1826 8793 3418]
- merluza_stocks=
  [347 437 455 414 538 735 636 738 408 312 148]
- anchoveta_captures=
  [5797 1600 2540 3191 2299 1323 353 1154 177 202 1209]
- merluza_captures=
  [27 13 133 109 85 93 107 303 93 159 69]

Conservation and catch thresholds

The following annual objectives
(IMARPE, taller internacional sobre la anchoveta peruana)

	Anchovy (prey, y)	Hake (predator, z)
minimal biomass	7 000 kt	200 kt
minimal catch	2 000 kt	5 kt

were theoretically jointly achievable every year but...

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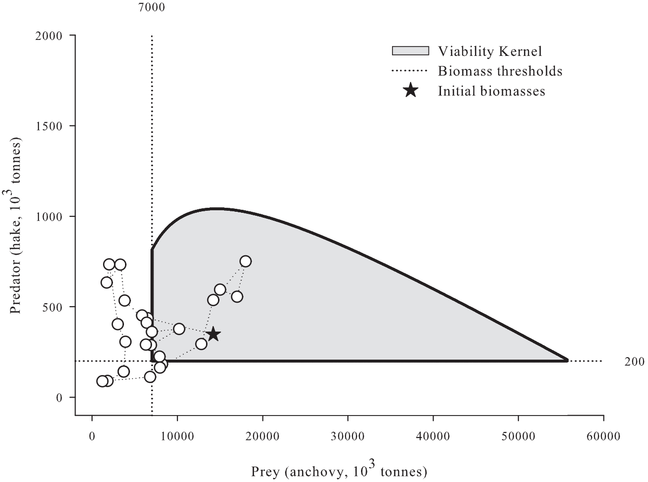

Figure: Viability kernel for minimal catches of 2 000 kt(anchovy) and 5 kt(hake)

Sustainable yields compatible with conservation

Theoretically, one could produce, year after year,

- anchovy yield of at least 2 000 kt
- hake yield of at least 5 kt

without harming the species in the sense that, every year

- anchovy biomass is at least 7 000 kt
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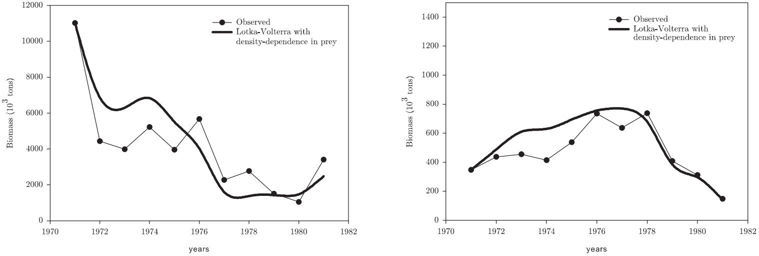
Hake–anchovy Peruvian fisheries between 1971 and 1981: a Lotka–Volterra model



(a) Anchovy (b) Hake

Figure: Comparison of observed and simulated biomasses of anchovy and hake using a Lotka–Volterra model with density-dependence in the prey. Model parameters are $R = 2.24$, $L = 0.98$, $\kappa = 64\,672 \times 10^3$ t ($K = 35\,800 \times 10^3$ t), $\alpha = 1.230 \times 10^{-6}\ t^{-1}$, $\beta = 4.326 \times 10^{-8}\ t^{-1}$.

Lotka–Volterra model with density–dependence

$$\begin{cases} y(t+1) = y(t)\overbrace{\left(R - \frac{R}{\kappa}y(t) - \alpha z(t) - v(t)\right)}^{R_y}, \\ z(t+1) = z(t)\underbrace{\left(L + \beta y(t) - w(t)\right)}_{R_z}, \end{cases}$$

- state vector (y, z) represents biomasses,
  - y prey biomass: anchovy
  - z predator biomass: hake
- control vector (v, w) is fishing effort of each species,
- catches are vy and wz (measured in biomass),
- R_y and R_z are annual growth factors.

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Generic nonlinear ecosystem models

For simplicity, we consider a two–dimensional state model

$$\begin{cases} y(t+1) = y(t)\,\overbrace{R_y\big(y(t),z(t),v(t)\big)}^{\text{growth factor}} \\ z(t+1) = z(t)\,\underbrace{R_z\big(y(t),z(t),w(t)\big)}_{\text{growth factor}} \end{cases}$$

where

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Trade-offs biology-economy

Viability kernel

C. Béné, L. Doyen, and D. Gabay. *A viability analysis for a bio-economic model.* Ecological Economics, 36:385–396, 2001.

The viability kernel is the set of initial states $\big(y(t_0), z(t_0)\big)$ from which appropriate controls $\big(v(t), w(t)\big)$, $t = t_0, t_0 + 1, \ldots$ produce a trajectory $\big(y(t), z(t)\big)$, $t = t_0, t_0 + 1, \ldots$ such that the following goals are satisfied

- preservation (minimal biomass thresholds)

$$\text{stocks:} \qquad y(t) \geq S_y^\flat \, , \quad z(t) \geq S_z^\flat$$

- and economic/social requirements (minimal catch thresholds)

$$\text{catches:} \qquad v(t)y(t) \geq C_y^\flat \, , \quad w(t)z(t) \geq C_z^\flat \, .$$

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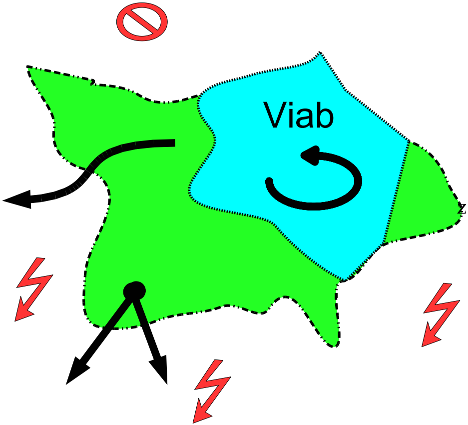
State constraint set and viability kernel



The state constraint set $\mathbb{A}$ is the large green set. It includes the smaller blue viability kernel $\mathbb{V}\mathrm{iab}(t_0)$.

Viability kernel

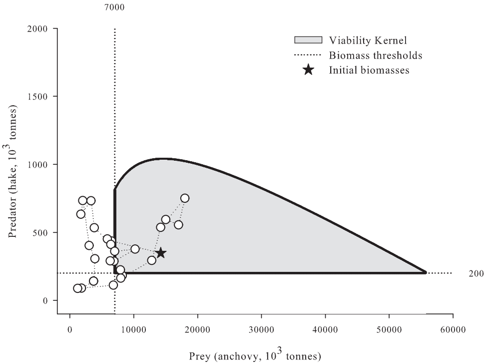


Figure: Viability kernel for minimal biomass thresholds $S_y^\flat = 7\ 000\ kt$ (anchovy) and $S_z^\flat = 200\ kt$ (hake), and minimal catches thresholds $C_y^\flat = 2\ 000\ kt$ (anchovy) and $C_z^\flat = 5\ kt$ (hake)

Explicit expression for the viability kernel

Proposition

- *If the growth factors are decreasing in the fishing effort*
- *and if the thresholds $S_y^\flat, S_z^\flat, C_y^\flat, C_z^\flat$ are such that the following growth factors are greater than one*

$$R_y(S_y^\flat, S_z^\flat, \frac{C_y^\flat}{S_y^\flat}) \geq 1 \text{ and } R_z(S_y^\flat, S_z^\flat, \frac{C_z^\flat}{S_z^\flat}) \geq 1 ,$$

the viability kernel is given by

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Explicit expression for the viability kernel

Proposition

- *If the growth factors are decreasing in the fishing effort*
- *and if the thresholds $S_y^\flat, S_z^\flat, C_y^\flat, C_z^\flat$ are such that the following growth factors are greater than one*

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From thresholds to initial states

- Given *a priori* conflicting requirements
  - ecological thresholds $S_y^\flat, S_z^\flat$ (minimal stocks),
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- we can tell whether or not they can be indefinitely maintained for initial biomasses $y(t_0)$ and $z(t_0)$:
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Outline of the presentation

The other way round: from initial state to thresholds

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Ecosystem sustainable yields

1. Considering that first are given minimal biomass conservation thresholds $S_y^\flat \geq 0 \,, \quad S_z^\flat \geq 0$
2. with initial biomasses $y(t_0) \geq S_y^\flat$ and $z(t_0) \geq S_z^\flat$
3. the following catches levels can be sustainably maintained

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C_y^{\flat,\star} = \quad & \min \Big\{ C_y \geq 0 \;\Big|\; \quad R_y(S_y^\flat, S_z^\flat, \frac{C_y}{S_y^\flat}) \geq 1 \ \text{ and} \\
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These sustainable yields $C_y^\flat(y(t_0), z(t_0))$ and $C_z^\flat(y(t_0), z(t_0))$

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A conceptual method towards ecosystem sustainable yields?

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Hake–anchovy Peruvian fishery: Peru official quotas and sustainable yields given by the viability approach

	Sustainable yields (kt)		Peru official quotas (kt)	
	Model 1	Model 2	2006	2007
Anchovy	5 152	5 399	4 250	5 300
Hake	49	$56,8$	55	35

Instituto del Mar del Perú shows interest for this rather transparent method

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Contribution to quantitative sustainable management

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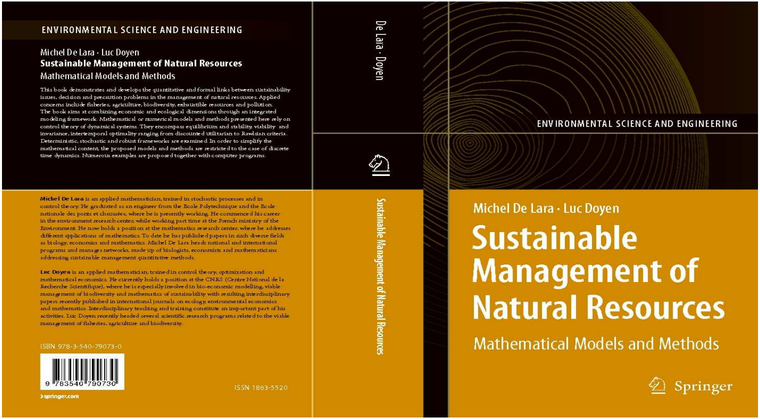
Credits

M. De Lara, L. Doyen, Sustainable Management of Natural Resources. Mathematical Models and Methods, *Springer*, 2008.



ENVIRONMENTAL SCIENCE AND ENGINEERING

Michel De Lara · Luc Doyen
Sustainable Management of Natural Resources
Mathematical Models and Methods

This book demonstrates and develops the quantitative and formal links between sustainability issues, decision and precaution problems in the management of natural resources. Applied concerns include fisheries, agriculture, biodiversity, exhaustible resources and pollution.
The book aims at combining economic and ecological dimensions through an integrated modeling framework. Mathematical or numerical models and methods presented here rely on control theory of dynamical systems. They encompass equilibrium and stability, viability and invariance, intertemporal optimality ranging from discounted utilitarian to Rawlsian criteria. Deterministic, stochastic and robust frameworks are examined. In order to simplify the mathematical content, the proposed models and methods are restricted to the case of discrete time dynamics. Numerous examples are proposed together with computer programs.

Michel De Lara is an applied mathematician, trained in stochastic processes and in control theory. He graduated as an engineer from the Ecole Polytechnique and the Ecole nationale des ponts et chaussées, where he is presently working. He commenced his career in the environment research center, while working part time at the French ministry of the Environment. He now holds a position at the mathematics research center, where he addresses different applications of mathematics. To date he has published papers in such diverse fields as biology, economics and mathematics. Michel De Lara heads national and international programs and manages networks, made up of biologists, economists and mathematicians addressing sustainable management quantitative methods.

Luc Doyen is an applied mathematician, trained in control theory, optimization and mathematical economics. He currently holds a position at the CNRS (Centre National de la Recherche Scientifique), where he is especially involved in bio-economic modelling, viable management of biodiversity and mathematics of sustainability with resulting interdisciplinary papers recently published in international journals on ecology, environmental economics and mathematics. Interdisciplinary teaching and training constitute an important part of his activities. Luc Doyen recently headed several scientific research programs related to the viable management of fisheries, agriculture and biodiversity.

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