Witsenhausen Model for Leader-Follower Problems in Energy Management

Thomas Buchholtzer, CERMICS,
École nationale des ponts et chaussées,
IP Paris, Marne-la-Vallée, France

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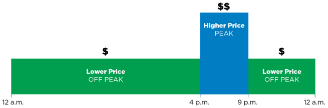
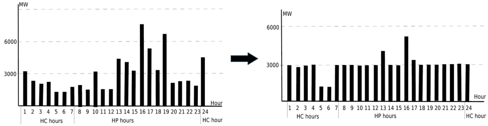
Example

Decides: (peak, off-peak) prices



Leader (agent)
electricity producer

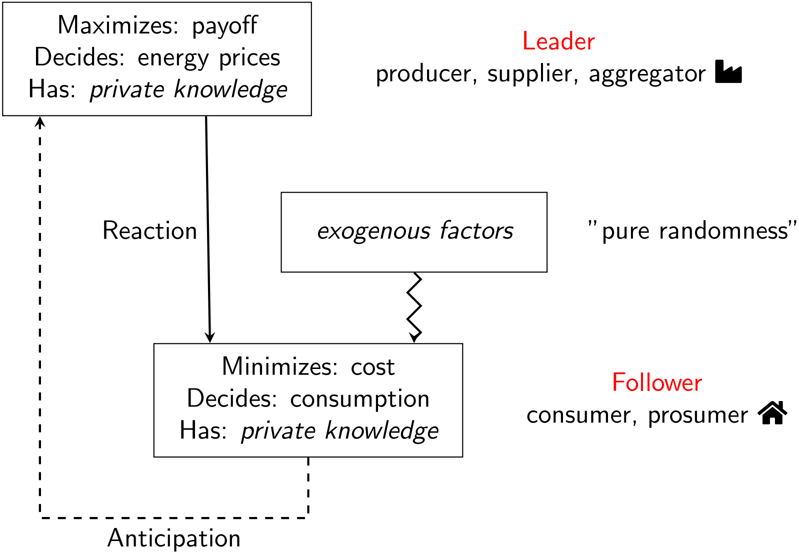
Decides: consumption shift



Follower (agent)
consumer

Sources: https://www.cleanpowersf.org/to (top), [Alekseeva, Brotcorne, Lepaul, and Montmeat, 2019] (bottom)

What kind of problem are we looking at?



Why are we interested in this kind of problem?

- Before
  - Consumers were mostly passive users of energy
  - Energy was mainly generated from controllable sources (e.g. nuclear, gas)
  - Supply could be smoothly adjusted to match demand at any time
- Now
  - Consumers can now produce their own energy (e.g. solar panels)
  - Renewable energy sources depend on weather and cannot be easily controlled (e.g. wind, solar)
  - Communication technology make it possible to adjust demand in real time

Demand response

Situations where customers change their consumption behaviors in response to price signals from the energy provider (e.g. time-of-use pricing)

A dynamic area of research

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FOCUS

A bi-level model for the design of dynamic electricity tariffs with demand-side flexibility

Patrizia Beraldi[1] · Sara Khodaparasti[1]

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Abstract
This paper addresses the electricity pricing problem with demand-side flexibility. The interaction between an aggregator and the prosumers within a coalition is modeled by a Stackelberg game and formulated as a mathematical bi-level program where the aggregator and the prosumer, respectively, play the role of upper and lower decision makers with conflicting goals. The aggregator establishes the pricing scheme by optimizing the supply strategy with the aim of maximizing the profit, prosumers react to the price signals by scheduling the flexible loads and managing the home energy system to minimize the electricity bill. The problem is solved by a heuristic approach which exploits the specific model structure. Some numerical experiments have been carried out on a real test case. The results provide the stakeholders with informative managerial insights underlining the prominent roles of aggregator and prosumers.

Keywords Pricing problem · Aggregator · Prosumers · Bi-level optimization

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A Bilevel Stochastic Programming Approach for Retailer Futures Market Trading

Miguel Carrión, *Member, IEEE,* José M. Arroyo, *Senior Member, IEEE,* and Antonio J. Conejo, *Fellow, IEEE*

Abstract—**This paper presents a bilevel programming approach to solve the medium-term decision-making problem faced by a power retailer. A retailer decides its level of involvement in the futures market and in the pool as well as the selling price offered to its potential clients with the goal of maximizing the expected profit at a given risk level. Uncertainty on future pool prices, client demands, and rival-retailer prices is accounted for via stochastic programming. Unlike in previous approaches, client response to retail price and competition among rival retailers are both explicitly considered in the proposed bilevel model. The resulting nonlinear bilevel programming formulation is transformed into an equivalent single-level mixed-integer linear programming problem by replacing the lower-level optimization by its Karush–Kuhn–Tucker optimality conditions and converting a number of nonlinearities to linear equivalents using some well-known integer algebra results. A realistic case study is solved to illustrate the efficient performance of the proposed methodology.**

Index Terms—**Bilevel programming, futures market, power retailer, risk, stochastic programming.**

$X^0_{e,s}$	Percentage of the demand of client group e initially supplied by retailer s.
α	Confidence level used in the calculation of the CVaR.
β	Weighting factor.
γ_e	Parameter representing the relationship between the pool price and the demand of client group e.
$\lambda^F_{f,j}$	Price of block j of the forward contracting curve of contract f [€/MWh].
$\lambda^P_t(\omega)$	Pool price in period t and scenario ω [€/MWh].
$\hat{\lambda}^P_t$	Expected pool price in period t [€/MWh].
$\lambda^S_{e,s}(\xi)$	Selling price offered by retailer s to client group e in scenario ξ [€/MWh].
$\pi(\omega)$	Probability of occurrence of pool price and client demand scenario ω.

A multi-leader-follower game for energy demand-side management

Didier Aussel [a], Sébastien Lepaul [b] and Léonard von Niederhäusern [c]

[a] Lab. PROMES UPR CNRS 8521, University of Perpignan Via Domitia, Perpignan, France; [b] EDF R&D, OSIRIS, Palaiseau, France; [c] Inria Lille-Nord Europe, Lille, France

ABSTRACT
A multi-leader-follower game (MLFG) corresponds to a bilevel problem in which the upper level and the lower level are defined by Nash non-cooperative competition among the players acting at the upper level (the leaders) and, at the same time, among the ones acting at the lower level (the followers). MLFGs are known to be complex problems, but they also provide perfect models to describe hierarchical interactions among various actors of real-life problems. In this work, we focus on a class of MLFGs modelling the implementation of demand-side management in an electricity market through price incentives, leading to the so-called *Bilevel Demand-Side Management problem (BDSM)*. Our aim is to propose some innovative reformulations/numerical approaches to efficiently tackle this difficult problem. Our methodology is based on the selection of specific Nash equilibria of the lower level through a precise analysis of the intrinsic characteristics of (BDSM).

ARTICLE HISTORY
Received 31 March 2019
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KEYWORDS
Bilevel optimization; demand-side management; energy markets

- **Goal**: provide a versatile framework for tackling complex demand response problems in energy management

How to model this kind of problem?

- The information structure is sequential
  - Leader (e.g. electricity producer) plays first
  - Follower (e.g. consumer) reacts
- We shed light on private knowledge
  - Leader's production cost
  - Follower's unwillingness to shift consumption
- We need to take "pure randomness" into account
  - Renewable energy production, demand, market prices
- We apply a versatile mathematical framework to handle problems with complex information structures
  - A W-model for decisions, uncertainty and information
  - A W-game for objective functions, beliefs and notions of equilibrium

Outline of the presentation

A W-game for producer-consumer electricity pricing

W-games for more advanced energy problems

Outline of the presentation

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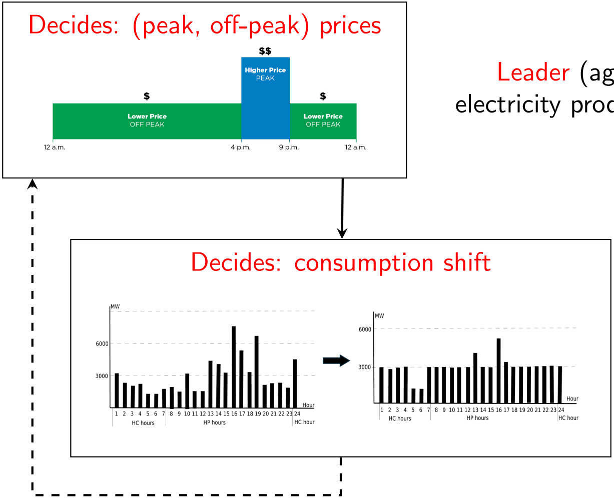
Example



Leader (agent)
electricity producer

Follower (agent)
consumer

Sources: https://www.cleanpowersf.org/to (top), [Alekseeva, Brotcorne, Lepaul, and Montmeat, 2019] (bottom)

Identification of the agents

An agent is a decision-maker taking only one action (or decision)

- We consider 2 agents
  - 1 leader agent (L): electricity producer
    decides the electricity prices
  - 1 follower agent (F): consumer
    decides to shift consumption
- We could consider a more complex case over a year
  with several agents
  - 12 leader agents: decide the electricity prices every month
  - 365 follower agents: decide to shift consumption every day

Details of agents' actions and action sets

Each agent makes an action u in a measurable space $(\mathcal{U}, \mathfrak{U})$
$\mathcal{U}$ is called the action set of an agent

- Leader's action: (peak, off-peak) prices (€)

$$u^{\mathrm{L}} = (\overline{u}^{\mathrm{L}}, \underline{u}^{\mathrm{L}}) \in \mathcal{U}^{\mathrm{L}} = \{(x, y) \in \mathbb{R}^2 \mid x \geq y\} \subset \mathbb{R}^2$$

We could have prices for each month (m)

$$u^{\mathrm{L}} = (\overline{u}^{\mathrm{L}}_m, \underline{u}^{\mathrm{L}}_m)_{m=1,\ldots,12}$$

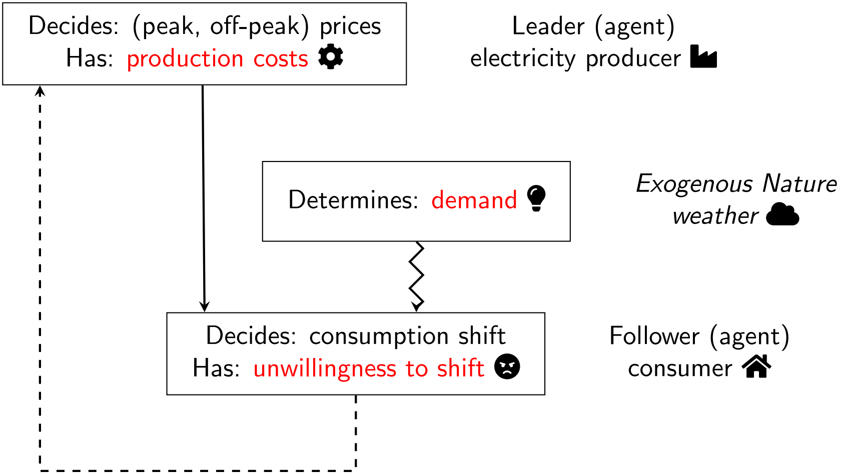
- Follower's action: consumption shift,
i.e. fraction of consumption during (peak, off-peak) hours (%)

$$u^{\mathrm{F}} = (\overline{u}^{\mathrm{F}}, \underline{u}^{\mathrm{F}}) \in \mathcal{U}^{\mathrm{F}} = \{(\alpha, \beta) \in \mathbb{R}^2_+ \mid \alpha + \beta = 1\} \subset \mathbb{R}^2_+$$

We could have consumption shift for each day (d)

$$u^{\mathrm{F}} = (\overline{u}^{\mathrm{F}}_d, \underline{u}^{\mathrm{F}}_d)_{d=1,\ldots,365}$$

There are three types of uncertainties (Nature)



Decomposition of Nature as a product

Nature contains everything that is not a decision

$$\Omega = \underbrace{\Omega^{\mathrm{e}}}_{\substack{\text{exogenous}\\ \text{Nature}}} \times \underbrace{\Omega^{\mathrm{L}}}_{\substack{\text{leader}\\ \text{type}}} \times \underbrace{\Omega^{\mathrm{F}}}_{\substack{\text{follower}\\ \text{type}}}$$

- Exogenous Nature: electricity demand (kWh)

$$\omega^{\mathrm{e}} \in \Omega^{\mathrm{e}} = \mathbb{R}_{+}$$

We could have electricity demand (kWh) for each day (d)

$$\omega^{\mathrm{e}} = (\omega^{\mathrm{e}}_{d})_{d=1,\ldots,365}$$

- Leader type: unitary production cost (€/kWh)

$$\omega^{\mathrm{L}} \in \Omega^{\mathrm{L}} = \mathbb{R}_{+}$$

- Follower type: unwillingness to shift to off-peak hours (€/kWh)

$$\omega^{\mathrm{F}} \in \Omega^{\mathrm{F}} = \mathbb{R}_{+}$$

Components of the upcoming objective functions

- Consumption (€)

$$\underbrace{\overline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}_{\substack{\text{peak}\\\text{demand}}} \cdot \underbrace{\overline{u}^{\mathrm{L}}}_{\substack{\text{peak}\\\text{price}}} + \underbrace{\underline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}_{\substack{\text{off-peak}\\\text{demand}}} \cdot \underbrace{\underline{u}^{\mathrm{L}}}_{\substack{\text{off-peak}\\\text{price}}}$$

- Production cost (€)

$$\underbrace{\omega^{\mathrm{e}}}_{\substack{\text{total}\\\text{demand}}} \cdot \underbrace{\omega^{\mathrm{L}}}_{\substack{\text{unitary}\\\text{production cost}}}$$

- Inconvenience cost (€)

$$\underbrace{\underline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}_{\substack{\text{off-peak}\\\text{demand}}} \cdot \underbrace{\omega^{\mathrm{F}}}_{\substack{\text{unwillingness}\\\text{to shift}}}$$

Details of the configuration space

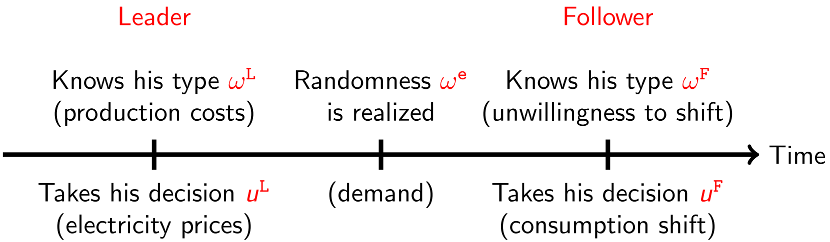
- Nature

$$\Omega = \underbrace{\mathbb{R}_+}_{\substack{\text{electricity}\\ \text{demand}}} \times \underbrace{\mathbb{R}_+}_{\substack{\text{unitary}\\ \text{production}\\ \text{cost}}} \times \underbrace{\mathbb{R}_+}_{\substack{\text{unwillingness}\\ \text{to shift}}} = \mathbb{R}_+^3$$

Configuration space is the product space $\mathcal{H} = \Omega \times \mathcal{U}^{\mathrm{L}} \times \mathcal{U}^{\mathrm{F}}$

$$\mathcal{H} = \underbrace{\mathbb{R}_+^3}_{\text{Nature}} \times \underbrace{\left\{(x, y) \in \mathbb{R}^2 \mid x \geq y\right\}}_{\substack{\text{(peak, off-peak)}\\ \text{prices}}} \times \underbrace{\left\{(\alpha, \beta) \in \mathbb{R}_+^2 \mid \alpha + \beta = 1\right\}}_{\substack{\text{consumption}\\ \text{shift}}}$$

Visualization of the information structure



Leader's information field and strategies

The leader information field $\mathfrak{I}^{\mathrm{L}}$ is a subfield of the σ-field associated with the configuration space $\mathfrak{H} = \mathfrak{G}^{\mathrm{e}} \otimes \mathfrak{G}^{\mathrm{L}} \otimes \mathfrak{G}^{\mathrm{F}} \otimes \mathfrak{U}^{\mathrm{L}} \otimes \mathfrak{U}^{\mathrm{F}}$

$$\underbrace{\mathfrak{I}^{\mathrm{L}}}_{\text{leader's information field}} = \underbrace{\{\emptyset, \Omega^{\mathrm{e}}\}}_{\text{cannot see consumer's demand}} \otimes \underbrace{\mathfrak{G}^{\mathrm{L}}}_{\text{knows his production cost}} \otimes \underbrace{\{\emptyset, \Omega^{\mathrm{F}}\}}_{\text{cannot see consumer's unwillingness to shift}} \otimes \underbrace{\{\emptyset, \mathcal{U}^{\mathrm{L}}\}}_{\text{absence of self-information}} \otimes \underbrace{\{\emptyset, \mathcal{U}^{\mathrm{F}}\}}_{\text{cannot see consumer's action}}$$

A leader's strategy is a mapping $\lambda^{\mathrm{L}} : (\mathcal{H}, \mathfrak{H}) \to (\mathcal{U}^{\mathrm{L}}, \mathfrak{U}^{\mathrm{L}})$ measurable with respect to his information field $\mathfrak{I}^{\mathrm{L}}$: $(\lambda^{\mathrm{L}})^{-1}(\mathfrak{U}^{\mathrm{L}}) \subset \mathfrak{I}^{\mathrm{L}}$

$$\underbrace{u^{\mathrm{L}}}_{\text{electricity prices}} = \underbrace{\lambda^{\mathrm{L}}}_{\text{leader's strategy}} \Big(\cancel{\omega^{\mathrm{e}}}, \underbrace{\omega^{\mathrm{L}}}_{\text{production costs}}, \cancel{\omega^{\mathrm{F}}}, \cancel{u^{\mathrm{L}}}, \cancel{u^{\mathrm{F}}} \Big)$$

Follower's information field and strategies

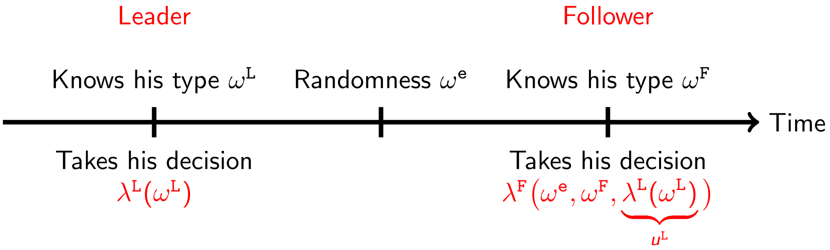
The follower information field $\mathfrak{I}^{\mathrm{F}}$ is a subfield of the σ-field associated with the configuration space $\mathfrak{H} = \mathfrak{G}^{\mathrm{e}} \otimes \mathfrak{G}^{\mathrm{L}} \otimes \mathfrak{G}^{\mathrm{F}} \otimes \mathfrak{U}^{\mathrm{L}} \otimes \mathfrak{U}^{\mathrm{F}}$

$$\underbrace{\mathfrak{I}^{\mathrm{F}}}_{\text{follower's information field}} = \underbrace{\mathfrak{G}^{\mathrm{e}}}_{\text{sees his demand}} \otimes \underbrace{\{\emptyset, \Omega^{\mathrm{L}}\}}_{\text{cannot see producer's cost}} \otimes \underbrace{\mathfrak{G}^{\mathrm{F}}}_{\text{knows his own unwillingness to shift}} \otimes \underbrace{\mathfrak{U}^{\mathrm{L}}}_{\text{sees the electricity prices}} \otimes \underbrace{\{\emptyset, \mathcal{U}^{\mathrm{F}}\}}_{\text{absence of self-information}}$$

A follower's strategy is a mapping $\lambda^{\mathrm{F}} : (\mathcal{H}, \mathfrak{H}) \to (\mathcal{U}^{\mathrm{F}}, \mathfrak{U}^{\mathrm{F}})$ measurable with respect to his information field $\mathfrak{I}^{\mathrm{F}}$: $(\lambda^{\mathrm{F}})^{-1}(\mathfrak{U}^{\mathrm{F}}) \subset \mathfrak{I}^{\mathrm{F}}$

$$\underbrace{u^{\mathrm{F}}}_{\text{consumption shift}} = \underbrace{\lambda^{\mathrm{F}}}_{\text{follower's strategy}} \Big(\underbrace{\omega^{\mathrm{e}}}_{\text{demand}}, \cancel{\omega^{\mathrm{L}}}, \underbrace{\omega^{\mathrm{F}}}_{\text{unwillingness to shift}}, \underbrace{u^{\mathrm{L}}}_{\text{electricity prices}}, \cancel{u^{\mathrm{F}}} \Big)$$

A sequential (hence playable) information structure



When playability holds true, the solution map
is the mapping $S_{\lambda^{\mathrm{L}},\lambda^{\mathrm{F}}} : \Omega \to \mathcal{H}$ which gives for every state of Nature
the unique outcome

$$S_{\lambda^{\mathrm{L}},\lambda^{\mathrm{F}}}(\omega^{\mathrm{e}},\omega^{\mathrm{L}},\omega^{\mathrm{F}}) = \Big(\omega^{\mathrm{e}},\omega^{\mathrm{L}},\omega^{\mathrm{F}},\underbrace{\lambda^{\mathrm{L}}(\omega^{\mathrm{L}})}_{u^{\mathrm{L}}},\underbrace{\lambda^{\mathrm{F}}\big(\omega^{\mathrm{e}},\omega^{\mathrm{F}},\lambda^{\mathrm{L}}(\omega^{\mathrm{L}})\big)}_{u^{\mathrm{F}}}\Big)$$

What land have we covered? What comes next?

- We have modeled the example as a W-model
  - = agents: producer, consumer
  - + action sets: electricity prices, consumption shift
  - + Nature: production costs, unwillingness, demand
  - + information fields: private knowledge, sequential information structure
- We have written the strategies and the solution map
- Now, we speak about W-games
  - = W-model
  - + players
  - + preferences (objective functions + beliefs on Nature)

Outline of the presentation

A W-game for producer-consumer electricity pricing
Formulation of a W-model
Formulation of a W-game
Notions of equilibria in W-games

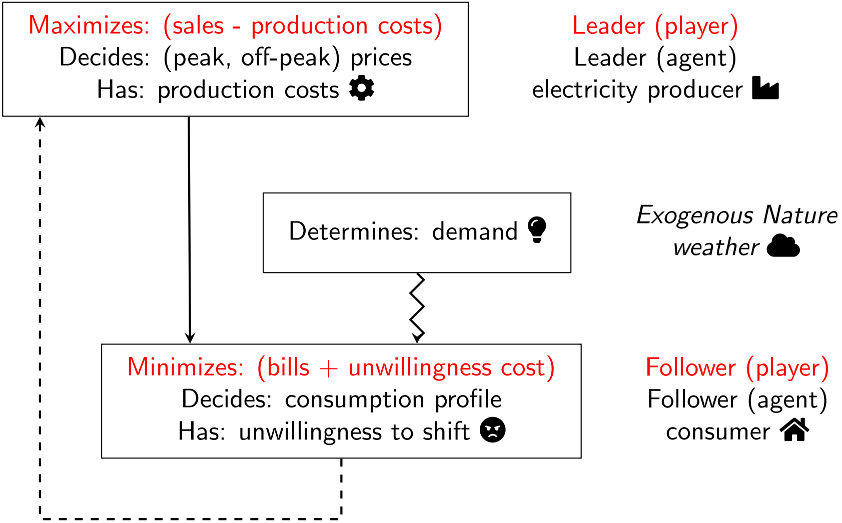
W-games for more advanced energy problems
Aggregator-prosumer energy pricing
Retailer futures market trading
Electricity market modeling

Identification of the players

A player is an individual or a corporation, possibly taking several decisions, endowed with a preference,
i.e. an objective function and a belief
We associate with each player her (executive) agents

- We have 2 players
  - Leader player: electricity producer
    associated with the leader agent
  - Follower player: consumer
    associated with the follower agent
- We could have considered a more complex case with multiple leaders and multiple followers
  - Leader players: a group of electricity producers
  - Follower players: a group of consumers

Players' objective functions



Expression of the objective functions

An objective function is a measurable function $j : \mathcal{H} \to \overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$ representing the player's preferences over the different outcomes

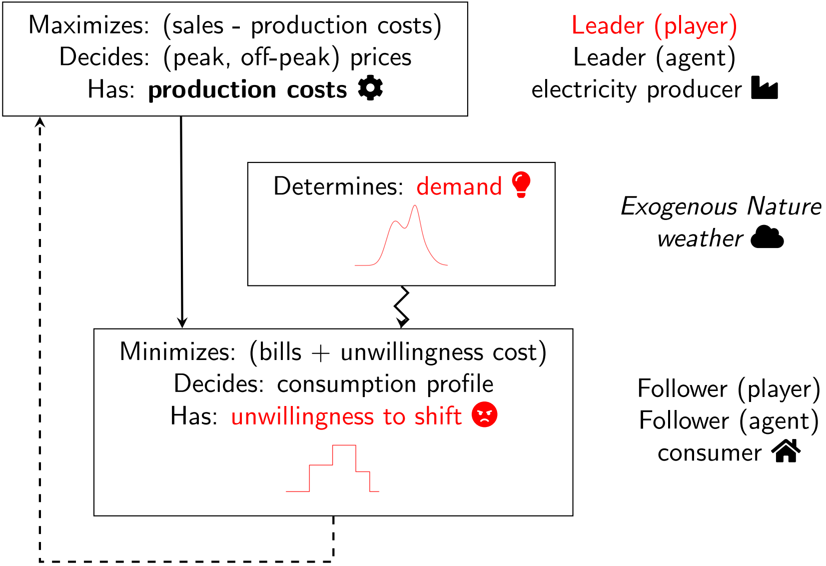
- Leader's payoff (maximization)

$$j^{\mathrm{L}}(\omega^{\mathrm{e}}, \omega^{\mathrm{L}}, \cancel{\omega^{\mathrm{F}}} u^{\mathrm{L}}, u^{\mathrm{F}}) = \underbrace{\overbrace{\overline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}^{\text{peak demand}} \overbrace{\overline{u}^{\mathrm{L}}}^{\text{peak price}} + \overbrace{\underline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}^{\text{off-peak demand}} \overbrace{\underline{u}^{\mathrm{L}}}^{\text{off-peak price}}}_{\text{sales}} - \underbrace{\overbrace{\omega^{\mathrm{e}}}^{\text{total demand}} \overbrace{\omega^{\mathrm{L}}}^{\text{unitary cost}}}_{\text{production cost}}$$

- Follower's cost (minimization)

$$j^{\mathrm{F}}(\omega^{\mathrm{e}}, \cancel{\omega^{\mathrm{L}}}, \omega^{\mathrm{F}} u^{\mathrm{L}}, u^{\mathrm{F}}) = \underbrace{\overbrace{\overline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}^{\text{peak demand}} \overbrace{\overline{u}^{\mathrm{L}}}^{\text{peak price}} + \overbrace{\underline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}^{\text{off-peak demand}} \overbrace{\underline{u}^{\mathrm{L}}}^{\text{off-peak price}}}_{\text{bills}} + \underbrace{\overbrace{\underline{u}^{\mathrm{F}}\omega^{\mathrm{e}}}^{\text{off-peak demand}} \overbrace{\omega^{\mathrm{F}}}^{\text{unwillingness to shift}}}_{\text{inconvenience cost}}$$

Leader's belief on Nature



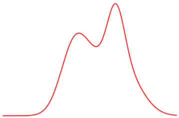
Writing the leader's belief

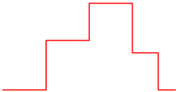
The leader's belief is a probability distribution on

$$\Omega = \Omega^{e} \times \Omega^{L} \times \Omega^{F}$$

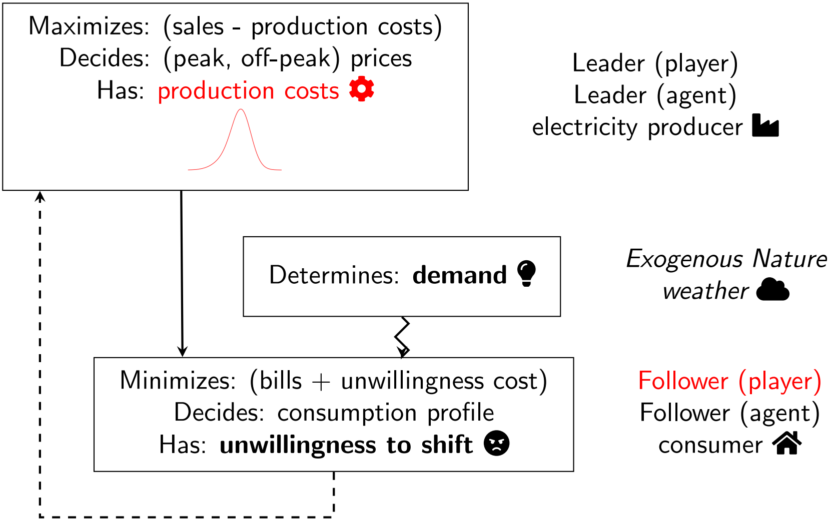
- Leader's belief

$$\beta^{L} = \underbrace{\beta^{L}_{e}}_{\substack{\text{distribution on}\\ \text{consumer's demand}}} \otimes \underbrace{\delta_{\{\omega^{L}\}}}_{\substack{\text{own type}\\ \text{known}}} \otimes \underbrace{\beta^{L}_{F}}_{\substack{\text{distribution on consumer's}\\ \text{unwillingness to shift}}}$$





Follower's belief on Nature



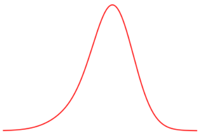
Writing the follower's belief

The follower's belief is a probability distribution on

$$\Omega = \Omega^{\mathrm{e}} \times \Omega^{\mathrm{L}} \times \Omega^{\mathrm{F}}$$

- Follower's belief

$$\beta^{\mathrm{F}} = \underbrace{\delta_{\{\omega^{\mathrm{e}}\}}}_{\text{demand known}} \otimes \underbrace{\beta^{\mathrm{F}}_{\mathrm{L}}}_{\text{distribution on producer's cost}} \otimes \underbrace{\delta_{\{\omega^{\mathrm{F}}\}}}_{\text{own type known}}$$



Focus on asymmetric knowledge: introducing W-game data

A player's data refers to her objective function and her belief
W-game data refers to the collection of the players' data

- W-game data
  - Leader's data $d^{L} = (j^{L}, \beta^{L})$
  - Follower's data $d^{F} = (j^{F}, \beta^{F})$
- A W-game is an additional layer upon a W-model

$$\text{W-game} = \text{W-model} + \text{W-game data}$$

W-games in normal form

- Strategies are the heart of normal form games
  - Λ^{L}: set of leader's strategies
  - Λ^{F}: set of follower's strategies

The normal form objective function is a function $J : \Lambda^{\mathrm{L}} \times \Lambda^{\mathrm{F}} \to \overline{\mathbb{R}}$ giving what a player can expect to gain (or lose) from a strategy profile

L , F	...	λ^{F}	...
...			
λ^{L}		$J^{\mathrm{L}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}})$, $J^{\mathrm{F}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}})$	
...			

Table: Normal form representation of a W-game

Expression of normal form objective functions

When working with beliefs, the normal form objective function is the average gain (or loss) of a strategy profile for a player

$$J(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}) = \mathbb{E}_{\beta}\Big[\underbrace{j \circ S_{\lambda^{\mathrm{L}},\lambda^{\mathrm{F}}}}_{\Omega \xrightarrow{S_{\lambda^{\mathrm{L}},\lambda^{\mathrm{F}}}} \mathcal{H} \xrightarrow{j} \overline{\mathbb{R}}} \Big] = \int_{\Omega} \big(j \circ S_{\lambda^{\mathrm{L}},\lambda^{\mathrm{F}}}\big)(\omega)\, \mathrm{d}\beta(\omega)$$

- Leader's normal form payoff (maximization)

$$J^{\mathrm{L}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}) = \int_{\Omega} j^{\mathrm{L}}\Big(\underbrace{\omega^{\mathrm{e}}, \omega^{\mathrm{L}}, \cancel{\omega^{\mathrm{F}}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}}), \lambda^{\mathrm{F}}\big(\omega^{\mathrm{e}}, \omega^{\mathrm{F}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}})\big)}_{S_{\lambda^{\mathrm{L}},\lambda^{\mathrm{F}}}(\omega)} \Big)\, \mathrm{d}\beta^{\mathrm{L}}(\omega)$$

- Follower's normal form cost (minimization)

$$J^{\mathrm{F}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}) = \int_{\Omega} j^{\mathrm{F}}\Big(\underbrace{\omega^{\mathrm{e}}, \cancel{\omega^{\mathrm{L}}}, \omega^{\mathrm{F}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}}), \lambda^{\mathrm{F}}\big(\omega^{\mathrm{e}}, \omega^{\mathrm{F}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}})\big)}_{S_{\lambda^{\mathrm{L}},\lambda^{\mathrm{F}}}(\omega)} \Big)\, \mathrm{d}\beta^{\mathrm{F}}(\omega)$$

Focus on W-game data

$$\text{W-game data} = \Big\{ \underbrace{(j^{\mathrm{L}}, \beta^{\mathrm{L}})}_{d^{\mathrm{L}}}, \underbrace{(j^{\mathrm{F}}, \beta^{\mathrm{F}})}_{d^{\mathrm{F}}} \Big\}$$

- Leader's normal form payoff (maximization)

$$J^{\mathrm{L}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}; \underbrace{d^{\mathrm{L}}}_{\text{data}}) = \int_{\Omega} \underbrace{j^{\mathrm{L}}\Big(\omega^{\mathrm{e}}, \omega^{\mathrm{L}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}}), \lambda^{\mathrm{F}}(\omega^{\mathrm{e}}, \omega^{\mathrm{F}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}}))\Big)}_{\text{objective function}} \, \mathrm{d}\, \underbrace{\beta^{\mathrm{L}}(\omega)}_{\text{belief}}$$

- Follower's normal form cost (minimization)

$$J^{\mathrm{F}}(\lambda^{\mathrm{F}}, \lambda^{\mathrm{L}}; \underbrace{d^{\mathrm{F}}}_{\text{data}}) = \int_{\Omega} \underbrace{j^{\mathrm{F}}\Big(\omega^{\mathrm{e}}, \omega^{\mathrm{F}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}}), \lambda^{\mathrm{F}}(\omega^{\mathrm{e}}, \omega^{\mathrm{F}}, \lambda^{\mathrm{L}}(\omega^{\mathrm{L}}))\Big)}_{\text{objective function}} \, \mathrm{d}\, \underbrace{\beta^{\mathrm{F}}(\omega)}_{\text{belief}}$$

What land have we covered? What comes next?

- We have expressed our example as a W-game
  - Objective functions: producer's payoff, consumer's cost
  - Decomposition of beliefs
- We have written the W-game in normal form
  - Normal form objective function
  - Everything in the strategies
- We have focused on W-game data to model asymmetric knowledge
- Now, we move to translating game theory equilibrium concepts in the language of W-games
  - Best response and Nash equilibrium
  - Stackelberg strategy and Nash-Stackelberg equilibrium

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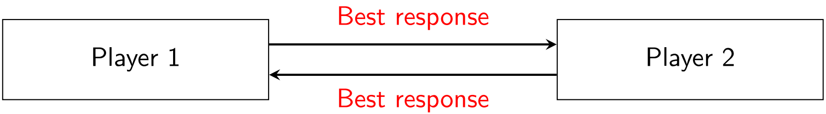
Recall: Nash equilibrium

A player plays a best response if she chooses a strategy that maximizes (resp. minimizes) her own payoff (resp. cost),
given the strategies selected by the others
A Nash equilibrium is when each player's strategy
is a best response to the strategies of the other players



- Most common notion for "solving" a game
- Stable situation: no player has an incentive to deviate unilaterally
- Example: a group of producers can play a Nash equilibrium

Nash equilibrium in leader-follower W-games

- Leader's best responses (maximization)

$$\Lambda^{\mathrm{L}}_{\mathcal{N}}(\lambda^{\mathrm{F}}; d^{\mathrm{L}}) = \operatorname*{arg\,max}_{\lambda^{\mathrm{L}} \in \Lambda^{\mathrm{L}}} J^{\mathrm{L}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}; d^{\mathrm{L}}) \subset \Lambda^{\mathrm{L}}$$

- Follower's best responses (minimization)

$$\Lambda^{\mathrm{F}}_{\mathcal{N}}(\lambda^{\mathrm{L}}; d^{\mathrm{F}}) = \operatorname*{arg\,min}_{\lambda^{\mathrm{F}} \in \Lambda^{\mathrm{F}}} J^{\mathrm{F}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}; d^{\mathrm{F}}) \subset \Lambda^{\mathrm{F}}$$

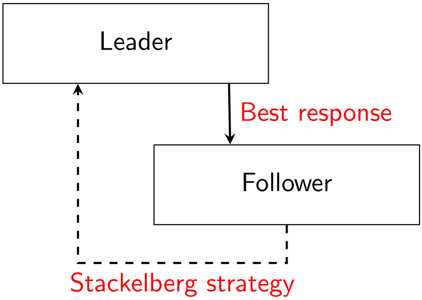
Nash equilibrium

A strategy profile $(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}) \in \Lambda^{\mathrm{L}} \times \Lambda^{\mathrm{F}}$ that satisfies

$$\begin{cases} \lambda^{\mathrm{L}} \in \Lambda^{\mathrm{L}}_{\mathcal{N}}(\lambda^{\mathrm{F}}; d^{\mathrm{L}}) : \text{the leader plays a best response} \\ \lambda^{\mathrm{F}} \in \Lambda^{\mathrm{F}}_{\mathcal{N}}(\lambda^{\mathrm{L}}; d^{\mathrm{F}}) : \text{the follower plays a best response} \end{cases}$$

Recall: Nash-Stackelberg equilibrium

A player plays a Stackelberg strategy if she chooses a strategy that maximizes (resp. minimizes) her own payoff (resp. cost), assuming the others play a best response
A Nash-Stackelberg equilibrium is when one player plays a best response and the other anticipates by choosing a Stackelberg strategy



Different types of Stackelberg strategies

- Stackelberg strategy is for the leader (maximization)
- Problem: multiplicity of best responses for the follower
- Optimistic Stackelberg strategies: the follower chooses the best response that is most advantageous for the leader

$$\Lambda^{\mathrm{L}}_{\mathcal{S}}(d^{\mathrm{L}}, \underbrace{d^{\mathrm{F}}}_{\text{follower's data}}) = \arg\max_{\lambda^{\mathrm{L}} \in \Lambda^{\mathrm{L}}} \sup_{\lambda^{\mathrm{F}} \in \Lambda^{\mathrm{F}}_{\mathcal{N}}(\lambda^{\mathrm{L}}; d^{\mathrm{F}})} J^{\mathrm{L}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}; d^{\mathrm{L}}) \subset \Lambda^{\mathrm{L}}$$

- Pessimistic Stackelberg strategies: the follower chooses the best response that is least advantageous for the leader

$$\Lambda^{\mathrm{L}}_{\mathcal{S}}(d^{\mathrm{L}}, \underbrace{d^{\mathrm{F}}}_{\text{follower's data}}) = \arg\max_{\lambda^{\mathrm{L}} \in \Lambda^{\mathrm{L}}} \inf_{\lambda^{\mathrm{F}} \in \Lambda^{\mathrm{F}}_{\mathcal{N}}(\lambda^{\mathrm{L}}; d^{\mathrm{F}})} J^{\mathrm{L}}(\lambda^{\mathrm{L}}, \lambda^{\mathrm{F}}; d^{\mathrm{L}}) \subset \Lambda^{\mathrm{L}}$$

- Existence of intermediate formulations (between optimistic and pessimistic)

Nash-Stackelberg equilibrium in leader-follower W-games

Nash-Stackelberg equilibrium

A strategy profile $(\lambda^{\mathtt{L}}, \lambda^{\mathtt{F}}) \in \Lambda^{\mathtt{L}} \times \Lambda^{\mathtt{F}}$ that satisfies

$$\begin{cases} \lambda^{\mathtt{L}} \in \Lambda^{\mathtt{L}}_{\mathcal{S}}(d^{\mathtt{L}}, d^{\mathtt{F}}) : \text{the leader plays a Stackelberg strategy} \\ \lambda^{\mathtt{F}} \in \Lambda^{\mathtt{F}}_{\mathcal{N}}(\lambda^{\mathtt{L}}; d^{\mathtt{F}}) : \text{the follower plays a best response} \end{cases}$$

Link with bilevel optimization

- We write the leader's problem
  as a bilevel optimization problem (optimistic formulation)

$$\max_{\lambda^{\mathrm{L}}\in\Lambda^{\mathrm{L}}} \quad \sup_{\lambda^{\mathrm{F}}\in\Lambda^{\mathrm{F}}_{\mathcal{N}}(\lambda^{\mathrm{L}};d^{\mathrm{F}})} \quad J^{\mathrm{L}}(\lambda^{\mathrm{L}},\lambda^{\mathrm{F}};d^{\mathrm{L}}) \qquad \text{(UL)}$$

$$\text{where} \quad \Lambda^{\mathrm{F}}_{\mathcal{N}}(\lambda^{\mathrm{L}};d^{\mathrm{F}}) = \operatorname*{arg\,min}_{\lambda^{\mathrm{F}}\in\Lambda^{\mathrm{F}}} J^{\mathrm{F}}(\lambda^{\mathrm{L}},\lambda^{\mathrm{F}};d^{\mathrm{F}}) \qquad \text{(LL)}$$

  - Upper-Level problem (UL): leader's problem (maximization)
  - Lower-Level problem (LL): follower's problem (minimization)
- We want to show the ambiguous knowledge of the follower's data d^{F} necessary for the computation of the leader's strategy

What land have we covered? What comes next?

- We have conducted the study on a simple example
- We have revisited key concepts of game theory in W-games
  - Best response
  - Nash equilibrium
- We have explored other concepts for leader-follower games
  - Stackelberg strategy
  - Nash-Stackelberg equilibrium: link with bilevel optimisation
- We have raised the question of the W-game data
- Now, we conclude by explaining how W-games can deal with more complex problems from the literature

Outline of the presentation

A W-game for producer-consumer electricity pricing

W-games for more advanced energy problems

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Retailer futures market trading
Electricity market modeling

Aggregator-prosumer energy pricing

A bi-level model for the design of dynamic electricity tariffs with demand-side flexibility

Patrizia Beraldi[1] **· Sara Khodaparasti**[1]

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Abstract
This paper addresses the electricity pricing problem with demand-side flexibility. The interaction between an aggregator and the prosumers within a coalition is modeled by a Stackelberg game and formulated as a mathematical bi-level program where the aggregator and the prosumer, respectively, play the role of upper and lower decision makers with conflicting goals. The aggregator establishes the pricing scheme by optimizing the supply strategy with the aim of maximizing the profit, prosumers react to the price signals by scheduling the flexible loads and managing the home energy system to minimize the electricity bill. The problem is solved by a heuristic approach which exploits the specific model structure. Some numerical experiments have been carried out on a real test case. The results provide the stakeholders with informative managerial insights underlining the prominent roles of aggregator and prosumers.

Keywords Pricing problem · Aggregator · Prosumers · Bi-level optimization

Figure: Abstract from [Beraldi and Khodaparasti, 2022]

A W-model with richer action sets

Decision variables	
Upper level decision variables	
p_t	Tariff set by the aggregator at time slot t
in_t^a	Energy charged to the aggregator's battery at time slot t
out_t^a	Energy discharged from the aggregator's battery at time slot t
soc_t^a	State of charge for the aggregator's battery at time slot t
γ_t^{ia}	Binary variables that indicates if the aggregator's battery is charged at time slot t
γ_t^{oa}	Binary variables that indicates if the aggregator's battery is discharged at time slot t
χ_t	Binary variable that indicates the status of the aggregator's production plant at time slot t
α_t	Amount of energy produced by the production plant at time slot t
β_t	Energy purchased from the DA market at time slot t
δ_t	Energy purchased from the bilateral contract at time slot t

Figure: Details of the leader's action u^{L} [Beraldi and Khodaparasti, 2022]

A W-model with richer Nature

Parameters	
C^a	Capacity of the aggregator's battery
$C^a_{\min}$	Lower bound on the state of charge in the aggregator's battery
$C^a_{\max}$	Upper bound on the state of charge in the aggregator's battery
soc^a_0	Initial energy level in the aggregator's battery
Δ	Average tariff
$\overline{\epsilon}$	Maximum energy production at time slot t
$\underline{\epsilon}$	Minimum energy production at time slot t
$\overline{p_t}$	Upper bound for tariff at time slot t
$\underline{p_t}$	Lower bound for tariff at time slot t
$\overline{\delta_t}$	Upper bound for energy purchased from bilateral contracts at time slot t
$\underline{\delta_t}$	Lower bound for energy purchased from bilateral contracts at time slot t
u^α_t	Unitary production cost at time slot t
u^β_t	The DA price at time slot t
u^δ_t	The price of energy purchased using bilateral contract at time slot t

Figure: Details of the leader's type ω^{L} [Beraldi and Khodaparasti, 2022]

What is implicit in the general formulation of a bilevel problem

$$\max_{x^U \in X^U \; x^L \in X^L} F(x^U, x^L) \tag{25}$$

$$H(x^U, x^L) \leq 0 \tag{26}$$

$$x^L \in arg \min_{x'^L \in X^L} \{f(x^U, x'^L) : h(x'^L) \leq 0\} \tag{27}$$

Figure: Problem formulation in [Beraldi and Khodaparasti, 2022]

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Electricity market modeling

Retailer futures market trading

A Bilevel Stochastic Programming Approach for Retailer Futures Market Trading

Miguel Carrión, *Member, IEEE*, José M. Arroyo, *Senior Member, IEEE*, and Antonio J. Conejo, *Fellow, IEEE*

***Abstract*—This paper presents a bilevel programming approach to solve the medium-term decision-making problem faced by a power retailer. A retailer decides its level of involvement in the futures market and in the pool as well as the selling price offered to its potential clients with the goal of maximizing the expected profit at a given risk level. Uncertainty on future pool prices, client demands, and rival-retailer prices is accounted for via stochastic programming. Unlike in previous approaches, client response to retail price and competition among rival retailers are both explicitly considered in the proposed bilevel model. The resulting nonlinear bilevel programming formulation is transformed into an equivalent single-level mixed-integer linear programming problem by replacing the lower-level optimization by its Karush–Kuhn–Tucker optimality conditions and converting a number of nonlinearities to linear equivalents using some well-known integer algebra results. A realistic case study is solved to illustrate the efficient performance of the proposed methodology.**

***Index Terms*—Bilevel programming, futures market, power retailer, risk, stochastic programming.**

NOMENCLATURE

Constants:

$X^0_{e,s}$	Percentage of the demand of client group e initially supplied by retailer s.
α	Confidence level used in the calculation of the CVaR.
β	Weighting factor.
γ_e	Parameter representing the relationship between the pool price and the demand of client group e.
$\lambda^F_{f,j}$	Price of block j of the forward contracting curve of contract f [€/MWh].
$\lambda^P_t(\omega)$	Pool price in period t and scenario ω [€/MWh].
$\hat{\lambda}^P_t$	Expected pool price in period t [€/MWh].
$\lambda^S_{e,s}(\xi)$	Selling price offered by retailer s to client group e in scenario ξ [€/MWh].
$\pi(\omega)$	Probability of occurrence of pool price and client demand scenario ω.
$\tau(\xi)$	Probability of occurrence of rival-retailer price scenario ξ.

Figure: Abstract from [Carrión, Arroyo, and Conejo, 2009]

W-games can deal with stochasticity

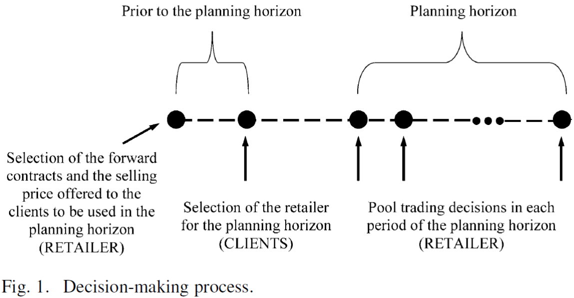

Figure: Timeline in [Carrión, Arroyo, and Conejo, 2009]

- Nature reveals over a time span $\mathcal{T}$, $\Omega^e = \prod_{t \in \mathcal{T}} \Omega^e_t$
- The retailer acts at several points $\mathcal{U}^L = \prod_{t \in \mathcal{T}} \mathcal{U}^L_t$
- Expected value replaced by a risk measure over the worst-case scenarios

What is implicit
in the extensive formulation of a bilevel problem

(BP)

$$\underset{\substack{P^F_{f,j},\lambda^R_e,E^P_t(\omega)\\E^R_{e,t}(\omega),\zeta,\eta(\omega)}}{\text{Maximize}} \sum_{\omega\in\Omega}\pi(\omega)\sum_{t\in T}\left[\sum_{e=1}^{N_E}E^R_{e,t}(\omega)\lambda^R_e - E^P_t(\omega)\lambda^P_t(\omega) - \sum_{f\in F_t}\sum_{j=1}^{N_J}P^F_{f,j}\lambda^F_{f,j}d_t\right] + \beta\left[\zeta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi(\omega)\eta(\omega)\right] \quad (3)$$

subject to

$$0 \le P^F_{f,j} \le \bar{P}^F_{f,j}, \quad \forall f\in F, j=1,\ldots,N_J \quad (4)$$

$$\sum_{e=1}^{N_E}E^R_{e,t}(\omega) = E^P_t(\omega) + \sum_{f\in F_t}\sum_{j=1}^{N_J}P^F_{f,j}d_t + E^{PC}_t \quad \forall t\in T, \quad \forall\omega\in\Omega \quad (5)$$

$$-\sum_{t\in T}\left[\sum_{e=1}^{N_E}E^R_{e,t}(\omega)\lambda^R_e - E^P_t(\omega)\lambda^P_t(\omega) - \sum_{f\in F_t}\sum_{j=1}^{N_J}P^F_{f,j}\lambda^F_{f,j}d_t\right] + \zeta - \eta(\omega) \le 0, \quad \forall\omega\in\Omega \quad (6)$$

$$\eta(\omega) \ge 0, \quad \forall\omega\in\Omega \quad (7)$$

$$E^R_{e,t}(\omega) = E^D_{e,t}(\omega)\sum_{\xi\in\Xi}\tau(\xi)x^S_{e,0}(\xi) \quad \forall e\in E, \quad \forall t\in T, \quad \forall\omega\in\Omega \quad (8)$$

where

$$x^S_{e,0}(\xi) \in \arg\left\{\underset{x^{S'}_{e,s}(\xi),y^S_{e,s,s'}(\xi)}{\text{Minimize}} \hat{E}^D_e\left[\lambda^R_e x^{S'}_{e,0}(\xi) + \sum_{\substack{s\in S\\s\neq 0}}\lambda^S_{e,s}(\xi)x^{S'}_{e,s}(\xi)\right] + \sum_{s\in S}\sum_{\substack{s'\in S\\s'\neq s}}\hat{E}^D_e C^{RS}_{e,s,s'}y^S_{e,s,s'}(\xi) \right. \quad (9)$$

subject to

$$x^{S'}_{e,s}(\xi) = X^0_{e,s} + \sum_{\substack{s'\in S\\s'\neq s}}y^S_{e,s',s}(\xi) - \sum_{\substack{s'\in S\\s'\neq s}}y^S_{e,s,s'}(\xi), \quad \forall s\in S \quad (10)$$

$$\sum_{s\in S}x^{S'}_{e,s}(\xi) = 1 \quad (11)$$

$$x^{S'}_{e,s}(\xi) \ge 0, \quad \forall s\in S \quad (12)$$

$$\left. y^S_{e,s,s'}(\xi) \ge 0, \quad \forall s,s'\in S, s\neq s' \right\} \quad \forall e\in E, \quad \forall\xi\in\Xi. \quad (13)$$

Figure: Problem formulation in [Carrión, Arroyo, and Conejo, 2009]

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- Retailer futures market trading
- Electricity market modeling

Electricity market modeling

A multi-leader-follower game for energy demand-side management

Didier Aussel [a], Sébastien Lepaul [b] and Léonard von Niederhäusern [c]

[a] Lab. PROMES UPR CNRS 8521, University of Perpignan Via Domitia, Perpignan, France; [b] EDF R&D, OSIRIS, Palaiseau, France; [c] Inria Lille-Nord Europe, Lille, France

ABSTRACT

A multi-leader-follower game (MLFG) corresponds to a bilevel problem in which the upper level and the lower level are defined by Nash non-cooperative competition among the players acting at the upper level (the leaders) and, at the same time, among the ones acting at the lower level (the followers). MLFGs are known to be complex problems, but they also provide perfect models to describe hierarchical interactions among various actors of real-life problems. In this work, we focus on a class of MLFGs modelling the implementation of demand-side management in an electricity market through price incentives, leading to the so-called *Bilevel Demand-Side Management problem (BDSM)*. Our aim is to propose some innovative reformulations/numerical approaches to efficiently tackle this difficult problem. Our methodology is based on the selection of specific Nash equilibria of the lower level through a precise analysis of the intrinsic characteristics of (BDSM).

ARTICLE HISTORY

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KEYWORDS

Bilevel optimization; demand-side management; energy markets

Figure: Abstract from [Aussel, Lepaul, and von Niederhäusern, 2022]

W-games can deal with multiple leaders and followers

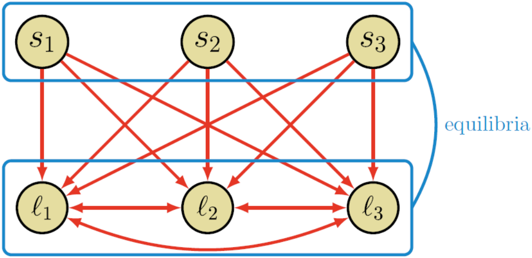

Figure 1. Problem scheme for three suppliers (above) and three local agents (below). Arrows represent (possible) energy flows, rectangles stand for Nash games.

Figure: Players' interactions in [Aussel, Lepaul, and von Niederhäusern, 2022]

- Set of leaders L and set of followers F
- Modularity of the product spaces $\mathcal{U}^{\mathtt{L}} = \prod_{\mathtt{l}\in\mathtt{L}} \mathcal{U}^{\mathtt{l}}$, $\Omega^{\mathtt{L}} = \prod_{\mathtt{l}\in\mathtt{L}} \Omega^{\mathtt{l}}$
- Extension of the notion of Nash-Stackelberg equilibrium

What is implicit in the formulation of a Nash equilibrium

Definition 3.2: A couple of strategies $(\mathbf{p}^*, \mathbf{e}^*)$ is said to be a *multi-optimistic equilibrium with common response* for (BDSM^{el}) if, for all $s \in \mathcal{S}$, $(\mathbf{p}_s^*, \mathbf{e}^*)$ is a solution of

$$(P_s^{el})\ (\mathbf{p}_{-s}^*) \quad \max_{\mathbf{p}_s} \max_{\mathbf{e}} \sum_{h \in \mathcal{H}} \left(\sum_{\ell \in \mathcal{L}} p_{s\ell}^h e_{\ell s}^h - c_s^h \left(\sum_{\ell \in \mathcal{L}} e_{\ell s}^h \right) \right)$$
$$\text{s.t.} \quad \{ \mathbf{e}_{\ell \cdot} \in \operatorname{argmin} (P_\ell^{el}) (\mathbf{p}_s, \mathbf{p}_{-s}^*), \quad \forall \ell \in \mathcal{L}.$$

Figure: Problem formulation in [Aussel, Lepaul, and von Niederhäusern, 2022]

Thank you for listening ;)

- A rich language
- A lot of open questions, and a lot of things not yet properly defined
- We aim to build a unified framework
  to ease the understanding of literature on energy management
- We want to propose a method
  to establish an energy management model from scratch
- We are looking for feedback

Ekaterina Alekseeva, Luce Brotcorne, Sébastien Lepaul, and Alain Montmeat. A bilevel approach to optimize electricity prices. *Yugoslav Journal of Operations Research*, 29:9,30, 2019. doi: 10.2298/yjor171115002a.

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