Decomposition/Coordination Methods for Multistage Stochastic Optimization Problems

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Motivation
Lecture outline

Decomposition and coordination
The three dimensions of stochastic optimization problems
A bird’s eye view of decomposition methods: the cube

A brief insight into scenario decomposition methods
Scenario decomposition methods “à la Progressive Hedging”
Handling risk with scenario decomposition methods

A brief insight into spatial decomposition methods
Spatial decomposition methods in the deterministic case
The stochastic case raises specific obstacles

Summary and research agenda
Outline of the presentation

Decomposition and coordination

A brief insight into scenario decomposition methods

A brief insight into spatial decomposition methods

Summary and research agenda
A long-term effort in our group (I)


A long-term effort in our group (II)


A long-term effort in our group (III)


2018 H. Gérard, “Décomposition de problèmes d’optimisation stochastique de grande dimension, avec mesure de risque”, Thèse de l’Université Paris-Est, octobre 2018
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Summary and research agenda
Decomposition-coordination: divide and conquer

- **Spatial** decomposition
  - Multiple players with their local information
  - Network with decision-makers located at nodes where they control local storage and flows through edges

- **Temporal** decomposition
  - A state is an information summary
  - Time coordination realized through dynamic programming, by value functions
  - Hard nonanticipativity constraints

- **Scenario** decomposition
  - Along each scenario, sub-problems are deterministic (powerful algorithms)
  - Scenario coordination realized through Progressive Hedging, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints
Let us fix problem and notations

\[
\min_{U, X} \mathbb{E} \left[ \sum_{i=1}^{N} \left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + K_i^i(X_T^i) \right) \right]
\]

subject to dynamics constraints

\[
X_{t+1}^i = f_t^i(X_t^i, U_t^i, W_{t+1}), \quad X_0^i = f_{-1}^i(W_0)
\]

state uncertainty

to measurability constraints on the control \(U_t^i\)

\[
\sigma(U_t^i) \subset \sigma(W_0, \ldots, W_t) \iff U_t^i = \mathbb{E}\left(U_t^i \middle| W_0, \ldots, W_t\right)
\]

and to instantaneous coupling constraints

\[
\sum_{i=1}^{N} \gamma_t^i(X_t^i, U_t^i) = 0
\]

(The letter \(U\) stands for the Russian word for control: \(u\)pravl\(e\)nie)
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Summary and research agenda
Couplings for stochastic problems

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega))
\]

\[
\sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0
\]
Couplings for stochastic problems: in time

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_t^i \left( X_t^i(\omega), U_t^i(\omega), W_{t+1}^i(\omega) \right)
\]

s.t. \( X_{t+1}^i = f_t^i \left( X_t^i, U_t^i, W_{t+1}^i \right) \)
Couplings for stochastic problems: in uncertainty

\[
\begin{align*}
\min & \sum_{\omega} \sum_i \sum_t \pi_\omega L_t^i (X_t^i(\omega), U_t^i(\omega), W_{t+1}(\omega)) \\
\text{s.t.} & \quad X_{t+1}^i = f_t^i(X_t^i, U_t^i, W_{t+1}) \\
& \quad U_t^i = \mathbb{E} \left( U_t^i \mid W_0, \ldots, W_t \right)
\end{align*}
\]
Couplings for stochastic problems: in space

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_\omega L^i_t(X^i_t(\omega), U^i_t(\omega), W_{t+1}(\omega))
\]

s.t. \( X^i_{t+1} = f^i_t(X^i_t, U^i_t, W_{t+1}) \)

\[
U^i_t = \mathbb{E}\left( U^i_t \mid W_0, \ldots, W_t \right)
\]

\[
\sum_i Y^i_t(X^i_t, U^i_t) = 0
\]
Can we decouple stochastic problems?

\[
\begin{align*}
\min & \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega)) \\
\text{s.t.} & \quad X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1}) \\
& \quad U_{t}^{i} = \mathbb{E}\left(U_{t}^{i} \mid W_{0}, \ldots, W_{t}\right) \\
& \quad \sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0
\end{align*}
\]
Sequential decomposition in time

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega))
\]

s.t. \( X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1}) \)

\[
U_{t}^{i} = \mathbb{E} \left( U_{t}^{i} \mid W_{0}, \ldots, W_{t} \right)
\]

\[
\sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0
\]

Dynamic Programming
Bellman (56)
Parallel decomposition in uncertainty/scenarios

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega))
\]

\[\text{s.t. } X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1})\]

\[U_{t}^{i} = \mathbb{E}\left(U_{t}^{i} \bigg| W_{0}, \ldots, W_{t}\right)\]

\[\sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0\]

Progressive Hedging
Rockafellar - Wets (91)
Parallel decomposition in space/units

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_t^i (X_t^i(\omega), U_t^i(\omega), W_{t+1}(\omega))
\]

s.t. \( X_{t+1}^i = f_t^i (X_t^i, U_t^i, W_{t+1}) \)

\( U_t^i = \mathbb{E} \left( U_t^i \bigg| W_0, \ldots, W_t \right) \)

\[
\sum_i Y_t^i(X_t^i, U_t^i) = 0
\]

Price and Quantity Decompositions with DP
Outline of the presentation

Decomposition and coordination

A brief insight into scenario decomposition methods

A brief insight into spatial decomposition methods

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Summary and research agenda
Non-anticipativity constraints are linear

- From tree to scenarios (fan)
- Equivalent formulations of the non-anticipativity constraints
  - pairwise equalities
  - all equal to their mathematical expectation
- Linear structure

\[ U_t = \mathbb{E} \left( U_t \middle| W_0, \ldots, W_t \right) \]
Progressive Hedging stands as a scenario decomposition method

We dualize the non-anticipativity constraints

- When the criterion is strongly convex, we use a Lagrangian relaxation (algorithm “à la Uzawa”) to obtain a scenario decomposition
- When the criterion is linear, Rockafellar - Wets (91) propose to use an augmented Lagrangian, and obtain the Progressive Hedging algorithm
Data: step $\rho > 0$, initial multipliers $\{\lambda_s^{(0)}\}_{s \in \mathcal{S}}$ and mean first decision $\bar{x}^{(0)}$;

Result: optimal first decision $x$;

repeat

forall scenarios $s \in \mathcal{S}$ do

Solve the deterministic minimization problem for scenario $s$, with a penalization $+\lambda_s^{(k)} \left( x_s^{(k+1)} - \bar{x}^{(k)} \right)$, and obtain optimal first decision $x_s^{(k+1)}$;

Update the mean first decisions

$$\bar{x}^{(k+1)} = \sum_{s \in \mathcal{S}} \pi_s x_s^{(k+1)};$$

Update the multiplier by

$$\lambda_s^{(k+1)} = \lambda_s^{(k)} + \rho \left( x_s^{(k+1)} - \bar{x}^{(k+1)} \right), \quad \forall s \in \mathcal{S};$$

until $x_s^{(k+1)} - \sum_{s' \in \mathcal{S}} \pi_{s'} x_{s'}^{(k+1)} = 0, \quad \forall s \in \mathcal{S};$
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Summary and research agenda
Suppose you had to manage a day-ahead energy market
You would have to fix reserves by night
and adjust in the morning with recourse energies
From linear to stochastic programming

- The linear program

\[
\min_{x \in \mathbb{R}^n} \langle c , x \rangle \\
Ax + b \geq 0 \quad (\in \mathbb{R}^m)
\]

- becomes a stochastic program

\[
\min_{x \in \mathbb{R}^n} \sum_{s \in S} \pi_s \langle c_s , x \rangle \\
A_s x + b_s \geq 0 , \quad \forall s \in S
\]

- We observe that there are as many (vector) inequalities as there are possible scenarios \( s \in S \)

\[
A_s x + b_s \geq 0 , \quad \forall s \in S
\]

and these inequality constraints can delineate an empty domain for optimization
Recourse variables need be introduced for feasibility issues

- We denote by $s \in S$ any possible value of the random variable $\xi$, with corresponding probability $\pi_s$
- and we introduce a recourse variable $y = (y_s)_{s \in S}$ and the program

\[
\min_{x, (y_s)_{s \in S}} \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right)
\]

\[
y_s \geq 0, \quad \forall s \in S
\]

\[
A_s x + b_s + y_s \geq 0, \quad \forall s \in S
\]

- so that the inequality $A_s x + b_s + y_s \geq 0$ is now possible, at (unitary recourse) price vector $p = (p_s, s \in S)$
- Observe that such stochastic programs are huge problems, with solution $(x, (y_s)_{s \in S})$, but remain linear
Minimizing the Tail Value at Risk of costs: linear programming formulation

- The risk-averse stochastic linear program with recourse

\[
\min_{x, (y_s)_{s \in S}} \min_{r \in \mathbb{R}} \left\{ r + \frac{1}{1 - \lambda} \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right) \right\}
\]

- can be written as the linear program

\[
\begin{align*}
\min_{x, (y_s)_{s \in S}} \min_{r} \min_{(v_s)_{s \in S}} & \quad r + \frac{1}{1 - \lambda} \sum_{s \in S} \pi_s v_s \\
 v_s - \langle c_s, x \rangle - \langle p_s, y_s \rangle & \geq 0, \quad \forall s \in S \\
v_s & \geq 0, \quad \forall s \in S \\
y_s & \geq 0, \quad \forall s \in S \\
A_s x + b_s + y_s & \geq 0, \quad \forall s \in S
\end{align*}
\]
Minimizing a mixture: linear programming formulation

▶ The risk-averse stochastic linear program with recourse

\[
\min_{x, (y_s)} \min_{r \in \mathbb{R}} \left\{ \theta \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right) + (1 - \theta) r + \frac{1 - \theta}{1 - \lambda} \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right) \right\}
\]

▶ can be written as the linear program

\[
\min_{x, (y_s)} \min_{r} \min_{(u_s, v_s)} \sum_{s \in S} \pi_s \left\{ \theta u_s + (1 - \theta) r + \frac{1 - \theta}{1 - \lambda} v_s \right\}
\]

\[
\begin{align*}
u_s - \langle c_s, x \rangle - \langle p_s, y_s \rangle & \geq 0, \quad \forall s \in S \\
v_s - u_s + r & \geq 0, \quad \forall s \in S \\
v_s & \geq 0, \quad \forall s \in S \\
y_s & \geq 0, \quad \forall s \in S \\
A_s x + b_s + y_s & \geq 0, \quad \forall s \in S
\end{align*}
\]
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Decomposition and coordination

- The system to be optimized consists of interconnected subsystems.
- We want to use this structure to formulate optimization subproblems of reasonable complexity.
- But the presence of interactions requires a level of coordination.
- Coordination iteratively provides a local model of the interactions for each subproblem.
- We expect to obtain the solution of the overall problem by concatenation of the solutions of the subproblems.
Example: the “flower model”

\[
\begin{align*}
\min_u & \quad \sum_{i=1}^{N} J_i(u_i) \\
\text{s.t.} & \quad \sum_{i=1}^{N} \theta_i(u_i) = 0
\end{align*}
\]

Unit Commitment Problem
Intuition of spatial decomposition

- **Unit 1**
- **Unit 2**
- **Unit 3**
- **Coordinator**

- **Purpose:** satisfy a demand with $N$ production units, at minimal cost
- **Price decomposition**
  - The coordinator sets a price $\lambda$
  - The units send their optimal decision $u_i$
  - The coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
  - And so on...
Intuition of spatial decomposition

Purpose: satisfy a demand with $N$ production units, at minimal cost.

Price decomposition:
- the coordinator sets a price $\lambda$
- the units send their optimal decision $u_i$
- the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly.
- and so on...
Intuition of spatial decomposition

- Purpose: satisfy a demand with $N$ production units, at minimal cost
- **Price decomposition**
  - the coordinator sets a price $\lambda$
  - the units send their optimal decision $u_i$

Diagram:
- Unit 1
- Unit 2
- Unit 3
- Coordinator

Arrows:
- $u_1^{(0)}$ from Unit 1 to Coordinator
- $u_2^{(0)}$ from Unit 2 to Coordinator
- $u_3^{(0)}$ from Unit 3 to Coordinator
Intuition of spatial decomposition

- **Purpose:** satisfy a demand with $N$ production units, at minimal cost

- **Price decomposition**
  - the coordinator sets a price $\lambda$
  - the units send their optimal decision $u_i$
  - the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly

Diagram:

- Coordinator
  - $\lambda^{(1)}$ to Unit 1
  - $\lambda^{(1)}$ to Unit 2
  - $\lambda^{(1)}$ to Unit 3
Intuition of spatial decomposition

- **Purpose:** satisfy a demand with $N$ production units, at minimal cost
- **Price decomposition**
  - the coordinator sets a price $\lambda$
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  - and so on...
Intuition of spatial decomposition

- Purpose: satisfy a demand with $N$ production units, at minimal cost
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  - the coordinator sets a price $\lambda$
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- and so on...
Intuition of spatial decomposition

- **Purpose**: satisfy a demand with $N$ production units, at minimal cost
- **Price decomposition**
  - the coordinator sets a price $\lambda$
  - the units send their optimal decision $u_i$
  - the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
  - and so on...
Price decomposition relies on dualization

\[ \min_{u_i \in \mathcal{U}_i, i=1 \ldots N} \sum_{i=1}^{N} J_i(u_i) \quad \text{subject to} \quad \sum_{i=1}^{N} \theta_i(u_i) = 0 \]

1. Form the Lagrangian and assume that a saddle point exists

\[ \max_{\lambda \in \mathcal{V}} \min_{u_i \in \mathcal{U}_i, i=1 \ldots N} \sum_{i=1}^{N} \left( J_i(u_i) + \langle \lambda, \theta_i(u_i) \rangle \right) \]

2. Solve this problem by the dual gradient algorithm “à la Uzawa”

\[ u_i^{(k+1)} \in \arg \min_{u_i \in \mathcal{U}_i} J_i(u_i) + \langle \lambda^{(k)}, \theta_i(u_i) \rangle, \quad i = 1 \ldots , N \]

\[ \lambda^{(k+1)} = \lambda^{(k)} + \rho \sum_{i=1}^{N} \theta_i(u_i^{(k+1)}) \]
Remarks on decomposition methods

- The theory is available for infinite dimensional Hilbert spaces, and thus applies in the stochastic framework, that is, when the $U_i$ are spaces of random variables.

- The minimization algorithm used for solving the subproblems is not specified in the decomposition process.

- New variables $\lambda^{(k)}$ appear in the subproblems arising at iteration $k$ of the optimization process

$$\min_{u_i \in U_i} J_i(u_i) + \langle \lambda^{(k)}, \theta_i(u_i) \rangle$$

- These variables are fixed when solving the subproblems, and do not cause any difficulty, at least in the deterministic case.
Price decomposition applies to various couplings
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**Summary and research agenda**
Stochastic optimal control (SOC) problem formulation

Consider the following SOC problem

\[ \min_{U, X} \mathbb{E} \left( \sum_{i=1}^{N} \left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + K_t^i(X_T^i) \right) \right) \]

subject to the constraints

\[ X_{0}^{i} = f_{-1}^{i}(W_0) , \]

\[ X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1}) , \quad t = 0 \ldots T-1 , \quad i = 1 \ldots N \]

\[ \sigma(U_t^i) \subset \sigma(W_0, \ldots, W_t) , \quad t = 0 \ldots T-1 , \quad i = 1 \ldots N \]

\[ \sum_{i=1}^{N} \theta_t^i(X_t^i, U_t^i) = 0 , \quad t = 0 \ldots T-1 \]
Stochastic optimal control (SOC) problem formulation

Consider the following SOC problem

$$\min_{U,X} \sum_{i=1}^{N} \left( \mathbb{E} \left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + K_t^i(X_T^i) \right) \right)$$

subject to the constraints

$$X_0^i = f_{-1}^i(W_0), \quad i = 1 \ldots N$$

$$X_{t+1}^i = f_t^i(X_t^i, U_t^i, W_{t+1}), \quad t = 0 \ldots T-1, \quad i = 1 \ldots N$$

$$\sigma(U_t^i) \subset \sigma(W_0, \ldots, W_t), \quad t = 0 \ldots T-1, \quad i = 1 \ldots N$$

$$\sum_{i=1}^{N} \theta_t^i(X_t^i, U_t^i) = 0, \quad t = 0 \ldots T-1$$
Dynamic programming yields centralized controls

- As we want to solve this SOC problem using dynamic programming (DP), we suppose to be in the Markovian setting, that is, $W_0, \ldots, W_T$ are a white noise

- The system is made of $N$ interconnected subsystems, with the control $U^i_t$ and the state $X^i_t$ of subsystem $i$ at time $t$

- The optimal control $U^i_t$ of subsystem $i$ is a function of the whole system state $(X^1_t, \ldots, X^N_t)$

$$U^i_t = \lambda^i_t(X^1_t, \ldots, X^N_t)$$

*Naive decomposition should lead to decentralized feedbacks*

$$U^i_t = \hat{\lambda}^i_t(X^i_t)$$

which are, in most cases, far from being optimal...
Straightforward decomposition of dynamic programming?

The crucial point is that the optimal feedback of a subsystem a priori depends on the state of all other subsystems, so that using a decomposition scheme by subsystems is not obvious...

As far as we have to deal with dynamic programming, the central concern for decomposition/coordination purpose boils down to

- how to decompose a feedback $\lambda_t$ w.r.t. its domain $X_t$ rather than its range $U_t$?

And the answer is

- impossible in the general case!
Price decomposition and dynamic programming

When applying price decomposition to the problem by dualizing the (almost sure) coupling constraint \( \sum \theta_t^i(X^i_t, U^i_t) = 0 \), multipliers \( \Lambda_t^{(k)} \) appear in the subproblems arising at iteration \( k \)

\[
\min_{U_t^i, X_t^i} \mathbb{E} \left[ \sum_t L_t^i(X_t^i, U_t^i, W_{t+1}) + \Lambda_t^{(k)} \cdot \theta_t^i(X_t^i, U_t^i) \right]
\]

- The variables \( \Lambda_t^{(k)} \) are fixed random variables, so that the random process \( \Lambda^{(k)} \) acts as an additional input noise in the subproblems
- But this process may be correlated in time, so that the white noise assumption has no reason to be fulfilled
- DP cannot be applied in a straightforward manner!

**Question:** how to handle the coordination instruments \( \Lambda_t^{(k)} \) to obtain (an approximation of) the overall optimum?
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Summary and research agenda
Let us move to broader stochastic optimization challenges

- **Stochastic** optimization requires to make risk attitudes explicit
  - robust, worst case, risk measures, in probability, almost surely

- **Stochastic dynamic** optimization requires to make online information explicit
  - State-based functional approach
  - Scenario-based measurability approach

**Numerical walls**

- in dynamic programming, the bottleneck is the dimension of the state
- in stochastic programming, the bottleneck is the number of stages
Here is our research agenda for stochastic decomposition

- Designing risk criteria compatible with decomposition
  - thèse d'Adrien Le Franc (2018—)
- Combining different decomposition methods
  - time: dynamic programming
  - scenario: Progressive Hedging
  - space: decomposition by prices or by quantities
  - into blends
    - time + space: Pierre Carpentier talk
      nodal decomposition by prices or by quantities
      + dynamic programming within node
    - time + scenario: Jean-Philippe Chancelier talk
      dynamic programming accross time blocks
      + Progressive Hedging within time blocks