An Overview of Decomposition/Coordination Methods for Multistage Stochastic Optimization Problems

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Lecture outline

Decomposition and coordination
    The three dimensions of stochastic optimization problems
    A bird’s eye view of decomposition methods: the cube

A brief insight into three decomposition methods
    Scenario decomposition methods
    Spatial (price/resource) decomposition methods
    Time decomposition methods

Summary and research agenda
Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

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Summary and research agenda
Temporal, scenario and spatial structures in multistage stochastic optimization problems

In multistage stochastic optimization problems, we consider that the control variable

$$U^i_t(\omega)$$

is indexed by

- Time/stages $t \in T (= \{0, \ldots, T-1\})$
- Scenarios $\omega \in \Omega$
- Space/units $i \in I (= \{1, \ldots, N\})$

The letter $U$ comes from the Russian word *upravlenie* for control
Let us fix problem and notations

\[
\min_{U, X} \mathbb{E}\left( \sum_{i \in I} \sum_{t \in T} L^i_t(X^i_t, U^i_t, W_{t+1}) \right)
\]
subject to

**dynamics constraints**

\[
X^{i}_{t+1} = g^i_t(X^i_t, U^i_t, W_{t+1}), \quad X^i_0 = g^{-1}_i(W_0)
\]

**measurability constraints** (nonanticipativity of the control \(U^i_t\))

\[
\sigma(U^i_t) \subset \sigma(W_0, \ldots, W_t) \iff U^i_t = \mathbb{E}(U^i_t | W_0, \ldots, W_t)
\]

**spatially coupling constraints**

\[
\sum_{i \in I} \Theta^i_t(X^i_t, U^i_t) = 0
\]
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Summary and research agenda
Couplings for stochastic problems

\[
\min \mathbb{E}\left( \sum_i \sum_t L_t^i(X_t^i, U_t^i, W_{t+1}) \right)
\]
Couplings for stochastic problems: in time

\[ \min \mathbb{E} \left( \sum_i \sum_t L^i_t (X^i_t, U^i_t, W_{t+1}) \right) \]

s.t. \( X^i_{t+1} = g^i_t (X^i_t, U^i_t, W_{t+1}) \)
Couplings for stochastic problems: in uncertainty

\[
\min \mathbb{E}\left( \sum_i \sum_t L_t^i(X_t^i, U_t^i, W_{t+1}) \right)
\]

s.t. \[ X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}) \]

\[
U_t^i = \mathbb{E}(U_t^i \mid W_0, \ldots, W_t)
\]
Couplings for stochastic problems: in space

\[
\begin{align*}
\min \mathbb{E} \left( \sum_i \sum_t L^i_t (X^i_t, U^i_t, W_{t+1}) \right) \\
\text{s.t. } X^i_{t+1} &= g^i_t (X^i_t, U^i_t, W_{t+1}) \\
U^i_t &= \mathbb{E} (U^i_t \mid W_0, \ldots, W_t) \\
\sum_i \Theta^i_t (X^i_t, U^i_t) &= 0
\end{align*}
\]
Can we decouple stochastic optimization problems?

\[
\begin{align*}
\min & \quad \mathbb{E}\left(\sum_i \sum_t L_t^i(X_t^i, U_t^i, W_{t+1})\right) \\
\text{s.t. } & \quad X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}) \\
& \quad U_t^i = \mathbb{E}(U_t^i \mid W_0, \ldots, W_t) \\
& \quad \sum_i \Theta_t^i(X_t^i, U_t^i) = 0
\end{align*}
\]
Sequential decomposition in time

\[
\min \mathbb{E}\left( \sum_i \sum_t L_t^i(X_t^i, U_t^i, W_{t+1}) \right)
\]

s.t. \[ X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}) \]

\[ U_t^i = \mathbb{E}(U_t^i | W_0, \ldots, W_t) \]

\[ \sum_i \Theta_t^i(X_t^i, U_t^i) = 0 \]

Dynamic Programming (DP)
Bellman (56)
Parallel decomposition in uncertainty/scenarios

\[
\min \mathbb{E}\left( \sum_i \sum_t L_t^i(X_t^i, U_t^i, W_{t+1}) \right)
\]

s.t. \( X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}) \)

\( U_t^i = \mathbb{E}(U_t^i \mid W_0, \ldots, W_t) \)

\[
\sum_i \Theta_t^i(X_t^i, U_t^i) = 0
\]

Progressive Hedging

Rockafellar-Wets (91)
Parallel decomposition in space/units

\[
\min \mathbb{E}\left( \sum_i \sum_t L^i_t(X^i_t, U^i_t, W_{t+1}) \right)
\]

s.t. \( X^{i+1}_t = g^i_t(X^i_t, U^i_t, W_{t+1}) \)

\( U^i_t = \mathbb{E}(U^i_t \mid W_0, \ldots, W_t) \)

\[
\sum_i \Theta^i_t(X^i_t, U^i_t) = 0
\]

Price and Resource Decompositions
Decomposition-coordination: divide and conquer

- **Temporal decomposition**
  - A state is an information summary
  - Time coordination realized through **Dynamic Programming**, by value functions (of the state)
  - Hard nonanticipativity constraints

- **Scenario decomposition**
  - Along each scenario, subproblems are deterministic (powerful algorithms)
  - Scenario coordination realized through **Progressive Hedging**, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints

- **Spatial decomposition**
  - By **prices** (multipliers of the spatial coupling constraint)
  - By **resources** (splitting the spatial coupling constraint)
Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

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Summary and research agenda
Moving from tree to fan (and scenarios)
Equivalent formulations of the nonanticipativity constraints

On a (scenario) tree, the nonanticipativity constraints

\[ \sigma(U_t) \subset \sigma(W_0, \ldots, W_t) \]

are “hardwired”

On a fan, the nonanticipativity constraints write as linear equality constraints

\[ U_t = \mathbb{E}(U_t \mid W_0, \ldots, W_t) \]
Progressive Hedging stands as a scenario decomposition method

Rockafellar-Wets (91) dualize the nonanticipativity constraints

\[ U_t = \mathbb{E}(U_t \mid W_0, \ldots, W_t) \]

- When the criterion is strongly convex, one uses a Lagrangian relaxation (algorithm "à la Uzawa") to obtain a scenario decomposition
- When the criterion is linear, Rockafellar-Wets (91) propose to use an augmented Lagrangian, and obtain the Progressive Hedging algorithm
Data: step $\rho > 0$, initial multipliers $\{\lambda_s^{(0)}\}_{s \in S}$ and mean first decision $\overline{u}^{(0)}$;

Result: optimal first decision $u$;

repeat

forall scenarios $s \in S$ do

Solve the deterministic minimization problem for scenario $s$, with a penalization $+\lambda_s^{(k)} \left( u_s^{(k+1)} - \overline{u}^{(k)} \right)$, and obtain optimal first decision $u_s^{(k+1)}$;

Update the mean first decisions

$$\overline{u}^{(k+1)} = \sum_{s \in S} \pi_s u_s^{(k+1)} ;$$

Update the multiplier by

$$\lambda_s^{(k+1)} = \lambda_s^{(k)} + \rho \left( u_s^{(k+1)} - \overline{u}^{(k+1)} \right), \quad \forall s \in S ;$$

until $u_s^{(k+1)} - \sum_{s' \in S} \pi_{s'} u_{s'}^{(k+1)} = 0$, $\forall s \in S$;
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Summary and research agenda
We consider an additive model

Consider the following minimization problem

$$\min_{u \in U_{ad} \subset U} J(u) \quad \text{subject to} \quad \Theta(u) - \theta = 0 \in \mathcal{V}$$

for which exists a decomposition of the space $U = U^1 \times \ldots \times U^N$, so that $u \in U$ writes $u = (u^1, \ldots, u^N)$ with $u^i \in U^i$, and also

- $U_{ad} = U^1_{ad} \times \ldots \times U^N_{ad}$
- $J(u) = J^1(u^1) + \ldots + J^N(u^N)$
- $\Theta(u) = \Theta^1(u^1) + \ldots + \Theta^N(u^N)$

Then the problem displays the following additive structure

$$\min_{u^1 \in U^1_{ad}, \ldots, u^N \in U^N_{ad}} \sum_{i=1}^N J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^N \Theta^i(u^i) - \theta = 0$$
Additive model — Price decomposition

\[ \min_{u \in U_{ad}} \sum_{i=1}^{N} J^i(u^i) \text{ subject to } \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0 \]

1. Form the Lagrangian of the problem

We assume that a saddle point exists, so that solving the initial problem is equivalent to

\[ \max_{\lambda \in V} \min_{u \in U_{ad}} \sum_{i=1}^{N} \left( J^i(u^i) + \langle \lambda, \Theta^i(u^i) \rangle \right) - \langle \lambda, \theta \rangle \]

2. Solve this problem by the Uzawa algorithm

\[ u^{i,(k+1)} \in \arg \min_{u^i \in U_{ad}^i} J^i(u^i) + \langle \lambda^{(k)}, \Theta^i(u^i) \rangle, \quad i = 1 \ldots, N \]

\[ \lambda^{(k+1)} = \lambda^{(k)} + \rho \left( \sum_{i=1}^{N} \Theta^i \left( u^{i,(k+1)} \right) - \theta \right) \]
Additive model — Price decomposition

\[ \lambda^{(k+1)} = \lambda^{(k)} + \rho \left( \sum \Theta_i^*(u_i^{(k+1)}) - \theta \right) \]

Coordination

Subproblem 1
\[ \min J_1(u_1) + \langle \lambda^{(k)}, \Theta_1(u_1) \rangle \]

Subproblem \(i\)
\[ \min J_i(u_i) + \langle \lambda^{(k)}, \Theta_i(u_i) \rangle \]

Subproblem \(N\)
\[ \min J_N(u_N) + \langle \lambda^{(k)}, \Theta_N(u_N) \rangle \]
Additive model — Resource allocation

\[
\min_{u \in U_{\text{ad}}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0
\]

1. Write the constraint in an equivalent manner by introducing new variables \(v = (v^1, \ldots, v^N)\) (the so-called “allocation”)

\[
\sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0 \iff \Theta^i(u^i) - v^i = 0 \quad \text{and} \quad \sum_{i=1}^{N} v^i = \theta
\]

and minimize the criterion w.r.t. \(u\) and \(v\)

\[
\min_{v \in V^N} \sum_{i=1}^{N} \left( \min_{u^i \in U^i_{\text{ad}}} J^i(u^i) \right. \quad \text{s.t.} \quad \Theta^i(u^i) - v^i = 0 \left. \right) \quad \text{s.t.} \quad \sum_{i=1}^{N} v^i = \theta
\]
Additive model — Resource allocation

\[
\min_{v \in \mathcal{V}} \sum_{i=1}^{N} \left( \min_{u^i \in \mathcal{U}_{ad}} J^i(u^i) \right. \text{ s.t. } \Theta^i(u^i) - v^i = 0 \left. \right) \text{ s.t. } \sum_{i=1}^{N} v^i = \theta
\]

\[
G^i(v^i)
\]

\[
\min_{v \in \mathcal{V}} \sum_{i=1}^{N} G^i(v^i) \quad \text{s.t. } \sum_{i=1}^{N} v^i = \theta
\]

2. Solve the last problem using a projected gradient method

\[
G^i(v^{i,(k)}) = \min_{u^i \in \mathcal{U}_{ad}} J^i(u^i) \quad \text{s.t. } \Theta^i(u^i) - v^{i,(k)} = 0 \quad \leadsto \quad \lambda^{i,(k+1)}
\]

\[
v^{i,(k+1)} = v^{i,(k)} + \rho \left( \lambda^{i,(k+1)} - \frac{1}{N} \sum_{j=1}^{N} \lambda^{j,(k+1)} \right)
\]
Additive model — Resource allocation

Subproblem $i$

Coordination

Subproblem $N$

$$v_i^{(k+1)} = v_i^{(k)} + \rho \left( \lambda_i^{(k+1)} - \frac{1}{N} \sum_j \lambda_j^{(k+1)} \right)$$

Subproblem 1

$$\text{min } J_1(u_1) \text{ s.t. } \Theta_1(u_1) - v_1^{(k)} = 0$$

Subproblem $i$

$$\text{min } J_i(u_i) \text{ s.t. } \Theta_i(u_i) - v_i^{(k)} = 0$$

Subproblem $N$

$$\text{min } J_N(u_N) \text{ s.t. } \Theta_N(u_N) - v_N^{(k)} = 0$$
Preparing Pierre Carpentier’s talk
We can also use price/resource decomposition to bound a minimization problem

\[
V_0^* = \inf_{u^1 \in U_{\text{ad}}^1, \ldots, u^N \in U_{\text{ad}}^N} \sum_{i=1}^{N} J^i(u^i) \\
\text{s.t. } (\Theta^1(u^1), \ldots, \Theta^N(u^N)) \in S
\]

- \( u^i \in U^i \) be a local decision variable
- \( J^i : U^i \to \mathbb{R}, \ i \in \llbracket 1, N \rrbracket \) be a local objective function
- \( U_{\text{ad}}^i \) be a subset of the local decision set \( U^i \)
- \( \Theta^i : U^i \to C^i \) be a local constraint mapping
- \( S \) be a subset of \( C = C^1 \times \cdots \times C^N \)

We denote by \( S^\circ \) the polar cone of \( S \)

\[
S^\circ = \{ p \in C^* \mid \langle p, r \rangle \leq 0 , \ \forall r \in S \}
\]
Price and resource local value functions

For each \( i \in \{1, N\} \),

- for any price \( p^i \in (C^i)^* \), we define the local price value

\[
V_0^i[p^i] = \inf_{u^i \in U^i_{ad}} J^i(u^i) + \langle p^i, \Theta^i(u^i) \rangle
\]

- for any resource \( r^i \in C^i \), we define the local resource value

\[
V_0^i[r^i] = \inf_{u^i \in U^i_{ad}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i
\]

Proposition (upper and lower bounds for optimal value)

- For any admissible price \( p = (p^1, \cdots, p^N) \in S^o \)
- For any admissible resource \( r = (r^1, \cdots, r^N) \in S \)

\[
\sum_{i=1}^{N} V_0^i[p^i] \leq V_0^* \leq \sum_{i=1}^{N} V_0^i[r^i]
\]
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Summary and research agenda
## Brief literature review on dynamic programming

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<td>$X$</td>
<td>$X$</td>
<td>$X$</td>
<td>$-$</td>
<td>$(\omega, U_{1:t-1})$</td>
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<td><strong>Dynamics</strong></td>
<td>$f(X, U, W)$</td>
<td>$P^u_{x,x'}$</td>
<td>$f(X, U, W)$</td>
<td>$-$</td>
<td>$X_t = (X_{t-1}, U_t)$</td>
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<td><strong>Uncertainties</strong></td>
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<td><strong>Cost</strong></td>
<td>$\sum_t$</td>
<td>$\sum_t$</td>
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<td>$\gamma(X)$</td>
<td>$\gamma(X)$ $\gamma(H)$</td>
<td>$\gamma(X)$ $\gamma(H)$</td>
<td>$\mathcal{F}_t$-meas.</td>
<td>$\gamma(x_t)$ $I_t$-meas.</td>
</tr>
<tr>
<td><strong>History</strong></td>
<td>$-$</td>
<td>$(X, U, \ldots)_t$</td>
<td>$(W, U, \ldots)_t$</td>
<td>$-$</td>
<td>$X_t$</td>
</tr>
</tbody>
</table>
We introduce the history

- The timeline is

\[ w_0 \leadsto u_0 \leadsto w_1 \leadsto u_1 \leadsto \ldots \leadsto w_{T-1} \leadsto u_{T-1} \leadsto w_T \]

- and the history is

\[
\begin{align*}
\text{history} & \quad \text{uncertainty} \quad \text{control} \quad \text{uncertainty} \\
\vec{h}_t & \quad (w_0, u_0, w_1, u_1, \ldots, u_{t-1}, w_t) \\
& \quad \in \mathcal{H}_t = \mathcal{W}_0 \times \prod_{s=0}^{t-1} (\mathcal{U}_s \times \mathcal{W}_{s+1})
\end{align*}
\]
History is the largest state

The history follows the dynamics

\[ h_{t+1} = \left( w_0, u_0, w_1, u_1, \ldots, u_{t-1}, w_t, u_t, w_{t+1} \right) \]
\[ = \left( h_t, u_t, w_{t+1} \right) \]

history \( h_t \)
control uncertainty
We formulate a sequence of minimization problems over increasing history spaces

- Once given
  - a criterion $j : \mathbb{H}_T \to \mathbb{R}$
  - a sequence of stochastic kernels $\rho_{t:t+1} : \mathbb{H}_t \to \Delta(\mathbb{W}_{t+1})$
- we define, for any history $h_t$, a minimization problem

$$V_t(h_t) = \inf_{\gamma : t : T - 1 \in \Gamma : t : T - 1} \int_{\mathbb{H}_T} \left\{ j(h'_T) \right\}_{\text{criterion}} \rho^\gamma_{t:T}(h_t, dh'_T)_{\text{controlled stochastic kernel}}$$
There is a Bellman equation involving value functions over increasing history spaces without white noise assumption

\[ V_T = j \]
\[ V_t = \mathcal{B}_{t+1:t} V_{t+1} \]

with

\[ (\mathcal{B}_{t+1:t} \varphi)(h_t) = \inf_{u_t \in \mathcal{U}_t} \int_{\mathcal{W}_{t+1}} \varphi(h_t, u_t, w_{t+1}) \rho_{t:t+1}(h_t, dw_{t+1}) \]
Preparing Jean-Philippe Chancelier’s talk
Towards state reduction by time blocks

- History $h_t$ is itself a canonical state variable, which lives in the history space
  \[ \mathcal{H}_t = \mathcal{W}_0 \times \prod_{s=0}^{t-1} (\mathcal{U}_s \times \mathcal{W}_{s+1}) \]

- However the size of this canonical state increases with $t$, which is a nasty feature for dynamic programming

- We will now
  - introduce “state” spaces $\mathbb{X}_t$
  - and then reduce the history with a mapping $\theta_r : \mathbb{H}_r \rightarrow \mathbb{X}_r$
  - to obtain a compressed “state” variable $\theta_t(h_t) = x_t \in \mathbb{X}_t$
  - but only at some specified times $0 = t_0 < t_1 < \cdots < t_N = T$

- As an application, we will handle stochastic independence between time blocks but possible dependence within time blocks
State reduction graphically

The triplet \((\theta_r, \theta_t, f_{r:t})\) is a state reduction across \((r : t)\) if

- the following diagram, for the dynamics, commutes

\[
\begin{array}{ccc}
\mathbb{H}_r \times \mathbb{H}_{r+1:t} & \xrightarrow{Id} & \mathbb{H}_t \\
\theta_r & \downarrow & \theta_t \\
\mathbb{X}_r \times \mathbb{H}_{r+1:t} & \xrightarrow{f_{r:t}} & \mathbb{X}_t
\end{array}
\]

- the following diagrams, for the stochastic kernels, commute

\[
\begin{array}{ccc}
\mathbb{H}_r \times \mathbb{H}_{r+1:s-1} & \xrightarrow{\rho_{s-1:s}} & \Delta(\mathbb{W}_s) \\
\theta_r & \downarrow & \tilde{\rho}_{s-1:s} \\
\mathbb{X}_r \times \mathbb{H}_{r+1:s-1} & \xrightarrow{\rho_{s-1:s}} & \\
& & \Delta(\mathbb{W}_s)
\end{array}
\]
Bellman operator across \((r : t)\)

\[ \mathcal{B}_{r:t} : \mathbb{L}^0_+ (\mathcal{H}_r, \mathcal{H}_r) \rightarrow \mathbb{L}^0_+ (\mathcal{H}_t, \mathcal{H}_t) \]

is defined by

\[ \mathcal{B}_{r:t} = \mathcal{B}_{t+1:t} \circ \cdots \circ \mathcal{B}_{r:r-1} , \]

where the one time step operators \(\mathcal{B}_{s:s-1}\) are

\[ (\mathcal{B}_{s:s-1} \varphi)(h_{s-1}) = \inf_{u_{s-1} \in \mathbb{U}_{s-1}} \int_{\mathbb{W}_s} \varphi(h_{s-1}, u_{s-1}, w_s) \rho_{s-1:s}(h_{s-1}, dw_s) \]
State reduction and Dynamic Programming

Denoting by \( \theta^*_r : \mathbb{L}_0^+(\mathcal{X}_r, \mathcal{X}_r) \rightarrow \mathbb{L}_0^+(\mathcal{H}_r, \mathcal{H}_r) \)
the operator defined by

\[
\theta^*_r(\tilde{\varphi}_r) = \tilde{\varphi}_r \circ \theta_r, \quad \forall \tilde{\varphi}_r \in \mathbb{L}_0^+(\mathcal{X}_r, \mathcal{X}_r),
\]

there exists a reduced Bellman operator across \((r : t)\) such that

\[
\theta^*_t \circ \tilde{B}_{r:t} = B_{r:t} \circ \theta^*_r,
\]

that is, the following diagram is commutative.

\[
\begin{array}{ccc}
\mathbb{L}_0^+(\mathcal{H}_r, \mathcal{H}_r) & \xrightarrow{\tilde{B}_{r:t}} & \mathbb{L}_0^+(\mathcal{H}_t, \mathcal{H}_t) \\
\uparrow & & \uparrow \\
\mathbb{L}_0^+(\mathcal{X}_r, \mathcal{X}_r) & \xrightarrow{\theta^*_r} & \mathbb{L}_0^+(\mathcal{H}_r, \mathcal{H}_r)
\end{array}
\]
Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

Summary and research agenda
We have sketched three main decomposition methods in multistage stochastic optimization

- **time**: Dynamic Programming
- **scenario**: Progressive Hedging
- **space**: decomposition by prices or by resources

Numerical walls are well-known

- in dynamic programming, the bottleneck is the dimension of the state
- in stochastic programming, the bottleneck is the number of stages
Here is our research agenda for stochastic decomposition

- Designing risk criteria compatible with decomposition
- Combining different decomposition methods
  - time: Dynamic Programming
  - scenario: Progressive Hedging
  - space: decomposition by prices or by resources
- to produce blends and tackle large scale energy applications
  - time blocks + prices/resources
    (talk of Jean-Philippe Chancelier)
    - dynamic programming across time blocks
      + prices/resources decomposition by time block
    - application to two time scales battery management
- time + space
  (talk of Pierre Carpentier)
  - nodal decomposition by prices or by resources
    + dynamic programming within node
  - application to large scale microgrid management