Optimal Operation and Valuation of Electricity Storages

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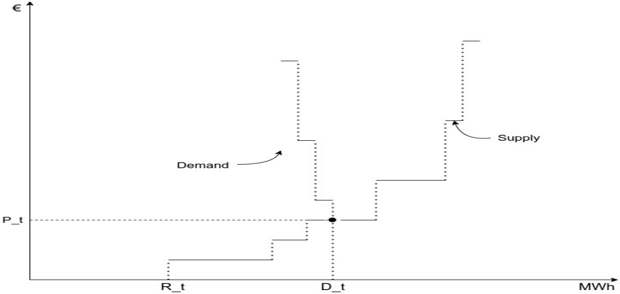
Optimal operation and valuation of electricity storages

- The growing proportion of renewable energy increases the uncertainties and seasonalities of supply and the **price of electricity**.
- This creates an incentive to **store energy**.
- We study the problem of **optimal operation** of electricity storages in the face of seasonalities and **stochastic** price developments.
- The optimization model lends itself to **indifference pricing** which is consistent with
  - the agent's views, risk preferences and existing storage/production facilities,
  - available quotes of electricity derivatives,
  - classical risk-neutral valuations in the case of **complete markets**.
- The same approach applies to optimal management and valuation of production facilities.

Optimal operation and valuation of electricity storages



- The crossing point of the supply and demand curves determines the transacted power and unit price.
- Gas prices drive the steep end of the supply curve while renewable production drives the zero-cost part of the curve. [Barlow: A diffusion model for electricity prices. Math. Finance 12 (2002)], [Coulon, Howison: Stochastic behaviour of the electricity bid stack: from fundamental drivers to power prices. J. Energy Markets (2009)], [Carmona, Coulon, Schwarz: Electricity price modeling and asset valuation: a multi-fuel structural approach, MathFinan. Econ. (2013)], [Deschatre, Féron, Gruet. *A survey of electricity spot and futures price models for risk management applications*. Energy Economics, 2021].

Optimal operation and valuation of electricity storages

Some articles closely related to our work include

- Porchet, Touzi, Warin. *Valuation of power plants by utility indifference and numerical computation*. Math. Methods Oper. Res., 2009.
- Callegaro, Campi, Giusto, Vargiolu. *Utility indifference pricing and hedging for structured contracts in energy markets*. Math. Methods Oper. Res., 2017.
- Picarelli, Vargiolu. *Optimal management of pumped hydroelectric production with state constrained optimal control*. J. Econom. Dynam. Control, 2021.
- Germain, Pham, Warin. *A level-set approach to the control of state-constrained McKean-Vlasov equations: application to renewable energy storage and portfolio selection*. Numer. Algebra Control Optim., 2023.
- Löhndorf, Wozabal. *Gas storage valuation in incomplete markets*. European J. Oper. Res., 2021.

Optimal operation of electricity storages

Given a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $(\mathcal{F}_t)_{t=0}^T$ (an increasing sequence of sub-σ-algebras of $\mathcal{F}$), consider the problem

$$\begin{aligned} \text{maximize} \quad & Eu(X_T^m) \quad \text{over } (X,U) \in \mathcal{N} \\ \text{subject to} \quad & X_0^m = w, \\ & X_0^e = 0, \\ & X_{t+1}^m \leq R_{t+1}^m(X_t^m) - S_{t+1}(U_t), \\ & X_{t+1}^e \leq R_{t+1}^e(X_t^e, U_t). \end{aligned} \tag{1}$$

- X_t^e is the amount of energy in storage, X_t^m is the amount of money invested in financial markets, U_t is the amount of energy bought at time t,
- $\mathcal{N}$ is the linear space of stochastic processes adapted to $(\mathcal{F}_t)_{t=0}^T$,
- $R_{t+1}^m(X_t^m)$ is the of wealth obtained at time $t+1$ when X_t^m units of cash is invested in the financial market at time t,
- $S_{t+1}(U_t)$ is the cost of buying U_t units of energy from the market at time t,
- $R_{t+1}^e(X_t^e, U_t)$ is the amount of energy in the storage at time $t+1$ when, at time t, the inventory is X_t^e and U_t^e more units is stored.

Numerical solution

- We rewrite the problem in the **convex stochastic control** format

$$\begin{array}{ll} \text{minimize} & E\left[\sum_{t=0}^{T} L_t(X_t,U_t)\right] \quad \text{over } (X,U)\in\mathcal{N}, \\ \text{subject to} & X_t = A_t X_{t-1} + B_t U_{t-1} + W_t \quad t=1,\dots,T, \end{array}$$

  where the **state** X and the **control** U take values in $\mathbb{R}^N$ and $\mathbb{R}^M$, respectively, A_t and B_t are $\mathcal{F}_t$-measurable random matrices, W_t is an $\mathcal{F}_t$-measurable random vector and the functions L_t are **extended real-valued** proper convex normal integrands on $\mathbb{R}^N \times \mathbb{R}^M \times \Omega$.
- This is amenable to numerical solution by **Stochastic Dual Dynamic Programming**
  - Pereira, Pinto. *Multi-stage stochastic optimization applied to energy planning*. Math. Program., 1991.
  - Dowson, Kapelevich. *Julia package for stochastic dual dynamic programming*, INFORMS Journal on Computing, 2021.
- Curin, Kettler, Kleisinger-Yu, Komaric, Krabichler, Teichmann, Wutte. *A deep learning model for gas storage optimization*. Decisions in Economics and Finance, 2021

Numerical solution

- We say that sequences of normal integrands $J_t : \mathbb{R}^N \times \Omega \to \overline{\mathbb{R}}$ and $I_t : \mathbb{R}^N \times \mathbb{R}^M \times \Omega \to \overline{\mathbb{R}}$ solve **Bellman equations** for the control problem if

$$
\begin{aligned}
I_T &= 0,\\
J_t(X_t,\omega) &= \inf_{U_t \in \mathbb{R}^M} E_t(L_t + I_t)(X_t, U_t, \omega),\\
I_{t-1}(X_{t-1}, U_{t-1}, \omega) &= J_t(X_{t-1} + A_t(\omega)X_{t-1} + B_t(\omega)U_{t-1} + W_t(\omega), \omega).
\end{aligned}
$$

- Recall that a function $f : \mathbb{R}^n \times \Omega \to \overline{\mathbb{R}}$ is a **normal integrand** if the set-valued mapping

$$\omega \mapsto \operatorname{epi} f(\cdot, \omega)$$

  is closed-valued and measurable.

- The general theory of normal integrands allows us to give general sufficient condsitions for preservation of normality in the above recursion.
- We say that a normal integrand is **lower bounded** if there exists an $m \in L^1$ such that $\inf_x f(x,\omega) \geq m(\omega)$ for almost every ω.

Numerical solution

Theorem 1

Assume that L_t *is lower bounded for each* t *and that* $(I_t, J_t)_{t=0}^T$ *solve the Bellman equations. Then the optimum value of the optimal control problem coincides with that of*

$$\begin{array}{ll} \text{minimize} & E\left[\sum_{s=0}^{t-1}(E_t L_s)(X_s, U_s) + J_t(X_t)\right] \quad \text{over } (X^t, U^t) \in \mathcal{N}^t, \\ \text{subject to} & \Delta X_s = A_s X_{s-1} + B_s U_{s-1} + W_s \quad s = 1, \dots, t \ a.s \end{array}$$

for all $t = 0, \dots, T$ *and, moreover, a pair* $(\bar{X}, \bar{U}) \in \mathcal{N}$ *solves the control problem if and only if it satisfies the system equations and*

$$\bar{U}_t \in \underset{U_t \in \mathbb{R}^M}{\operatorname{argmin}} \, E_t(L_t + I_t)(\bar{X}_t, U_t) \ a.s.$$

for all $t = 0, \dots, T$*. If the measurable mappings*

$$M_t(\omega) := \{U_t \in \mathbb{R}^M \mid (E_t(L_t + I_t))^\infty(0, U_t, \omega) \leq 0\}$$

are linear-valued for all $t = 0, \dots, T$*, then there exists an optimal control* U *with* $U_t \in M_t^\perp$ *almost surely.*

Numerical solution

Theorem 2

Assume that L_t is lower bounded for each t and that the set

$$\{(X,U) \in \mathcal{N} \mid L_t^\infty(X_t,U_t) \le 0,\ \Delta X_t = A_t X_{t-1} + B_t U_{t-1}\ \forall t\ a.s.\}$$

is linear. Then the Bellman equations have a unique solution $(J_t, I_t)_{t=0}^T$ of lower bounded convex normal integrands, and

$$M_t(\omega) := \{U_t \in \mathbb{R}^M \mid (E_t(L_t + I_t))^\infty(0, U_t, \omega) \le 0\}$$

is linear-valued for all t.

- In classical models of math finance, the above linearity condition coincides with the **no-arbitrage** condition.
- The lower boundedness condition can be relaxed to a condition that, in math finance, coincides with the "reasonable asymptotic elasticity" condition of [Kramkov and Schachermayer, 1999].

Numerical solution

Theorem 3 (Markov decision processes)

Assume that there is a Markov process $\xi = (\xi_t)_{t=0}^{T}$ *such that* L_t, A_t, B_t *and* W_t *depend on* ω *only through* ξ_t. *Then, for each* t, *the cost-to-go function* J_t *depends on* ω *only through* ξ_t.

Numerical solution

- Even in the Markovian case, the Bellman equations do not allow for analytic solutions in general.
- We will therefore first approximate the Markov process ξ by a finite-state Markov chain and then use the Stochastic Dual Dynamic Programming algorithm to solve the corresponding Bellman equations.
- To that end, we assume that ξ is $\mathbb{R}^d$-valued and that the ξ_t-conditional distribution of ξ_{t+1} has density $p(\cdot|\xi_t)$.
- Given a strictly positive probability density ϕ_{t+1} on $\mathbb{R}^d$, we have for every quasi-integrable function ψ,

$$\begin{aligned} E[\psi(\xi_{t+1}) \mid \xi_t] &= \int_{\mathbb{R}^d} \psi(\xi) p(\xi|\xi_t) d\xi \\ &= \int_{\mathbb{R}^d} \psi(\xi) \frac{p(\xi|\xi_t)}{\phi_{t+1}(\xi)} \phi_{t+1}(\xi) d\xi \\ &\approx \sum_{i=1}^{N} \psi(\xi_{t+1}^i) \frac{p(\xi_{t+1}^i|\xi_t)}{\phi_{t+1}(\xi_{t+1}^i)} w_{t+1}^i, \end{aligned}$$

where $(\xi_{t+1}^i, w_{t+1}^i)_{i=1}^N$ is a quadrature approximation of the measure P_{t+1} that has density ϕ_{t+1}.

Numerical solution

The SDDP algorithm proceeds as follows:

0. **Initialization:** Choose convex (e.g. polyhedral) lower-approximations J_t^0 of the cost-to-go functions J_t and set $k = 0$.

1. **Forward pass:** Sample a path ξ^k of the Markov process ξ and define X_t^k for $t = 0, \dots, T$ by

$$X_0^k \in \operatorname{argmin} J_k^0,$$

$$U_t^k \in \operatorname*{argmin}_{U_t \in \mathbb{R}^M} E[L_t(X_t^k, U_t) + J_{t+1}^k(X_t^k + A_{t+1}X_t^k + B_{t+1}U_t + W_{t+1}) \mid \xi_t^k],$$

$$X_{t+1}^k = X_t^k + A_{t+1}(\xi_{t+1}^k)X_t^k + B_{t+1}(\xi_{t+1}^k)U_t^k + W_{t+1}(\xi_{t+1}^k).$$

2. **Backward pass:** Let $J_{T+1}^{k+1} := 0$ and, for $t = T, \dots, 0$, compute

$$\tilde{J}_t^{k+1}(X_t^k, \xi_t^k) := \inf_{U_t \in \mathbb{R}^M} E[L_t(X_t^k, U_t) + J_{t+1}^{k+1}(X_t^k + A_{t+1}X_t^k + B_{t+1}U_t + W_{t+1}) \mid \xi_t^k$$

$$V_t^{k+1} \in \partial \tilde{J}_t^{k+1}(X_t^k, \xi_t^k),$$

$$J_t^{k+1}(X_t, \xi_t^k) := \max\{J_t^k(X_t, \xi_t^k), \tilde{J}_t^{k+1}(X_t^k, \xi_t^k) + V_t^{k+1} \cdot (X_t - X_t^k)\}.$$

Set $k := k + 1$ and go to 1.

A simple example

In the numerical illustrations below, we study the problem

$$\begin{aligned} \text{maximize} \quad & Eu(X_T^m) \quad \text{over } (X,U) \in \mathcal{N} \\ \text{subject to} \quad & X_0^m = w, \\ & X_0^e = 0, \\ & X_{t+1}^m = (1+r)X_t^m - s_{t+1}U_t \quad \forall t \ a.s., \\ & X_{t+1}^e = (1-l)X_t^e + U_t \quad \forall t \ a.s., \\ & X_t^e \in [0, C] \quad \forall t \ a.s., \\ & U_t \in [\underline{u}, \overline{u}] \quad \forall t \ a.s., \end{aligned} \tag{2}$$

where

$$u(z) = \frac{1}{\rho}[1 - \exp(-\rho z)]$$

with **risk aversion** $\rho > 0$.

A simple example

The electricity price displays seasonal variations.

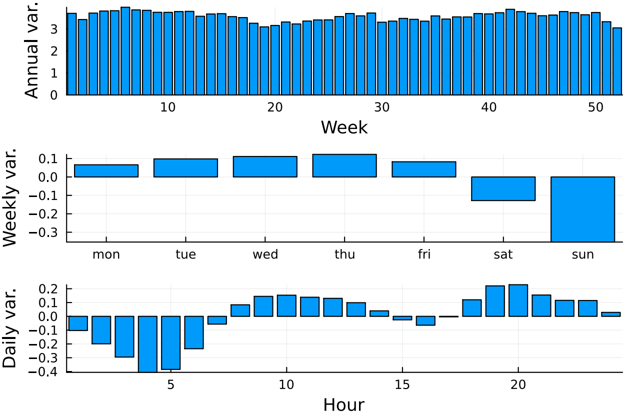
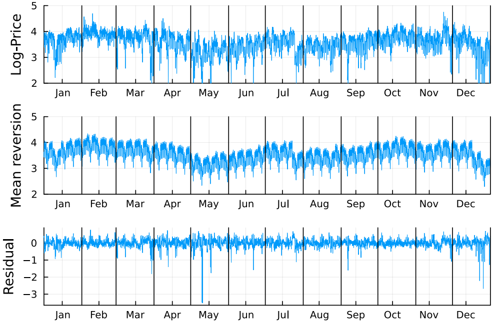


Figure: Annual, weekly and daily variations of log-prices.

A simple example



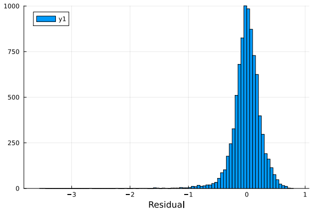


Figure: Left: price, average and residuals. Right: histogram of the residual.

We model the residual $\xi_t := \log s_t - \log \bar{s}_t$ as an Ornstein-Uhlenbeck process

$$\Delta\xi_{t+1} = -\alpha\xi_t + \sigma\epsilon_{t+1},$$

where $\alpha, \sigma > 0$ are constants and ϵ_t are iid Gaussians.

Discretized price process

- The ξ_t-conditional distribution of ξ_{t+1} is Gaussian.
- We use the conditional quadrature

$$E_t[\psi(\xi_{t+1})] \approx \sum_{i=1}^{N} \psi(\xi_{t+1}^i) \frac{p(\xi^i|\xi_t)}{\phi_{t+1}(\xi_{t+1}^i)} w_{t+1}^i$$

where ϕ_{t+1} is a Gaussian density and $(\xi_{t+1}^i, w_{t+1}^i)_{i=1}^N$ is its **Gauss-Hermite quadrature**.

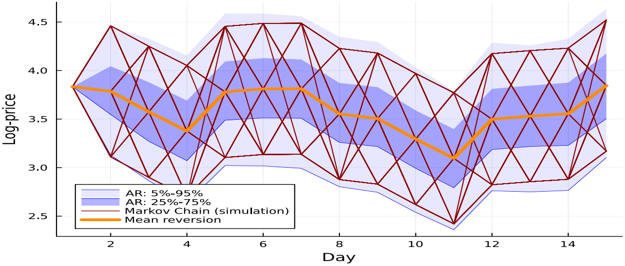


Figure: The log-price and its discretization with $N = 3$.

Convergence of the SDDP

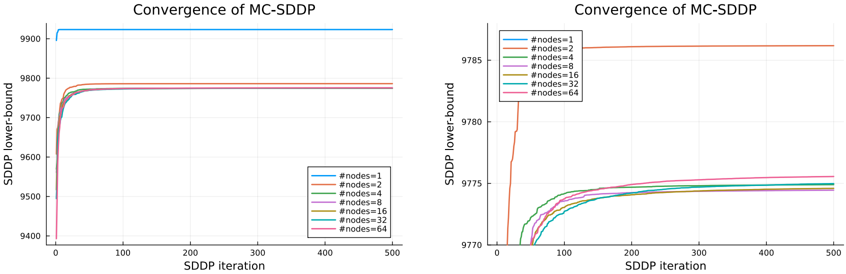


Figure: Convergence of the SDDP lower bound for varying number N of Markov states

Convergence of the SDDP

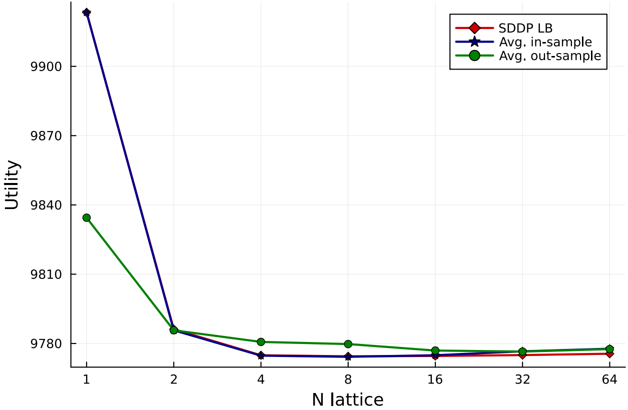


Figure: Out-of-sample evaluation of the optimized strategies obtained with varying number N of Markov states

Convergence of the SDDP

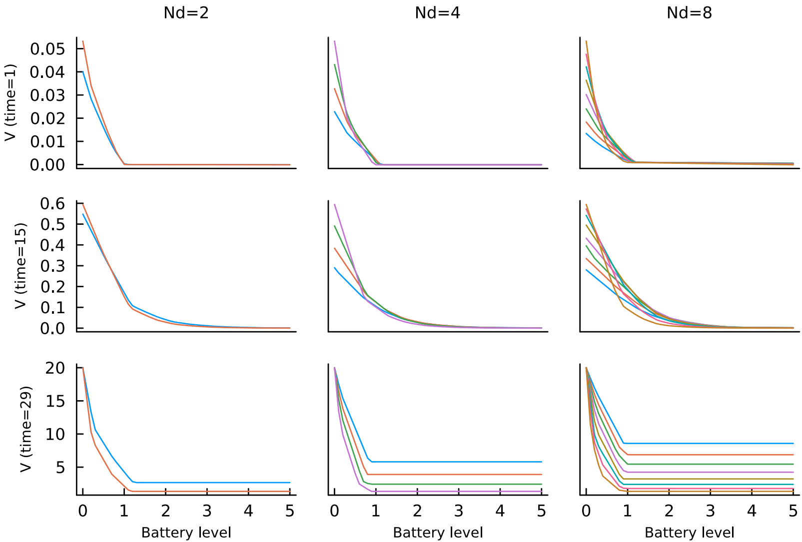


Figure: SDDP cost-to-go functions with varying number N of Markov states

Effect of the risk aversion

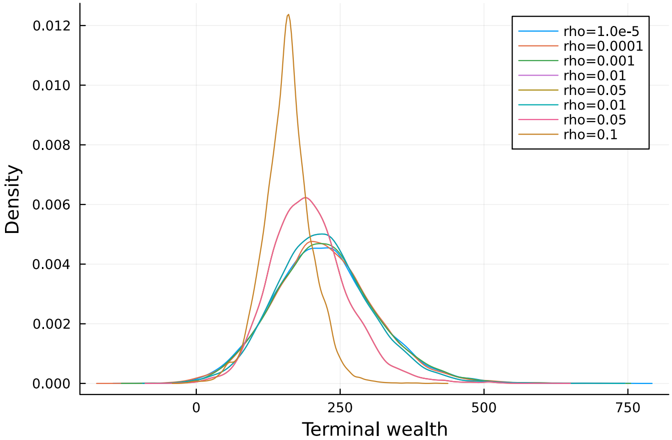


Figure: Out-of-sample distribution (obtained with 10,000 scenarios) of terminal wealth given by optimal strategies with different levels of risk aversion.

Indifference pricing

- The **indifference price** π for buying/renting the storage is the greatest price the investor can pay for it without worsening their financial position.
- Mathematically,

$$\pi = \sup\{p \in \mathbb{R} \mid \varphi(w - p) \geq \varphi_0\},$$

  where $\varphi(w)$ is the optimum value in the optimization problem with initial wealth w and φ_0 is the optimum value in a problem with initial wealth w but without the storage.
- The indifference price depends on
  - the user's views on future electricity prices and interest rates. This is described by the underlying probabilistic model,
  - the user's risk preferences described by the utility function,
  - the user's existing position.
- In complete market models, indifference prices coincide with replication costs and risk-neutral valuations.
- The above model assumes that, before the purchase of the storage, the agent has no storage nor production, but this can be generalized.

Indifference pricing

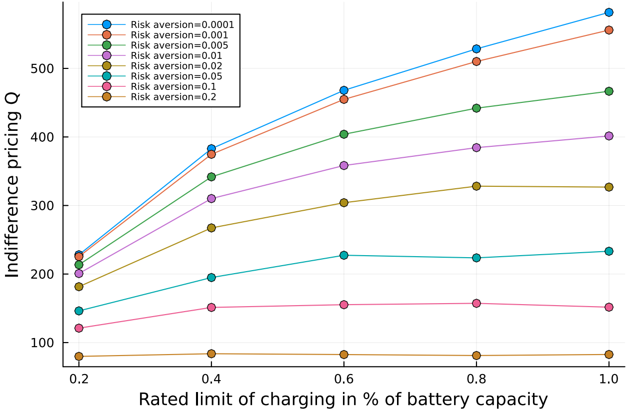


Figure: Indifference price π as a function of the maximum charging speed of the storage.

Indifference pricing

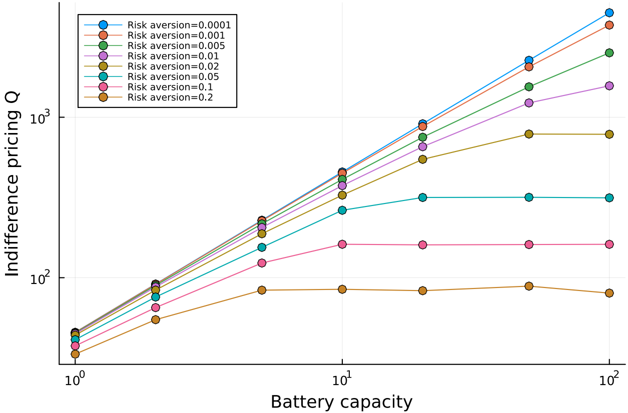


Figure: Indifference price π as a function of the storage capacity (log-log plot).

The effect of interest rates

- The UK government offers to buy offshore wind power generation at a constant unit price in order to reduce the financial risks of wind power generation developers.
- The price is set at set semi-annual auctions run by the Low Carbon Contracts Company (LCCC) through the Contracts for Difference (CfD) scheme; www.lowcarboncontracts.uk/our-schemes/contracts-for-difference
- LCCC asks for bids from developers and enters a CfD with the most generous offers (lowest offered electricity prices).
- LCCC sets a price cap it is willing to pay.
- The September 2023 auction failed to get any offers since the prices that developers were willing to offer exceeded the price cap due to the current levels of interest rates.
- High interest rate expectations make future money less valuable than today's money.
- In our model, interest rates are modeled with the functions R_t.

Calibration to option quotes

Assume that the agent has access to a set K of electricity derivatives such that

- the cost of buying $z^k \in \mathbb{R}$ units of derivative $k \in K$ costs $S^k(z^k)$ units of cash,
- derivative $k \in K$ provides c_t^k (random) units of cash at time t.

The problem becomes

$$\begin{aligned} \text{maximize} \quad & Eu(X_T^m) \quad \text{over } (X,U) \in \mathcal{N} \\ \text{subject to} \quad & X_0^m = w - C(z), \\ & X_0^e = 0, \\ & X_{t+1}^m \le R_{t+1}^m(X_t^m) - S_{t+1}(U_t) + z \cdot c_{t+1}, \\ & X_{t+1}^e \le R_{t+1}^e(X_t^e, U_t). \end{aligned} \tag{3}$$

where

$$C(z) = \sum_{k \in K} C^k(z^k) \quad \text{and} \quad z \cdot c_t = \sum_{k \in K} z^k c_t^k.$$

This can be dualized within the general Convex Stochastic Optimization theory.

Calibration to option quotes

Under the **silly** assumption that R_t^m, R_t^e, S_t and C are linear and there is no capacity constraints, the dual problem becomes

$$\begin{aligned} \text{maximize} \quad & Eu^*(q_T) + q_0 W \quad \text{over } q, w \in \mathcal{N}^1, \\ \text{subject to} \quad & E_{t-1}[q_t R_t^m] = q_{t-1}, \\ & E_{t-1}[w_t R_t^e] = w_{t-1}, \\ & q_t S_t = w_t, \\ & q_0 C = E[\sum_{t=1}^T q_t c_t]. \end{aligned}$$

The processes q and w are **stochastic discount factors** for cash and electricity, respectively. If $R_t^m \equiv R_t^e \equiv 1$ (another **silly** assumption), the constraints above mean that $q_T = dQ/dP$ where Q is a **martingale measure** for the electricity price process S and

$$E^Q[\sum_{t=1}^T c_t] = C.$$