Control Theory, Viability and Resilience for Sustainable Management

Michel De Lara
CERMICS, École des Ponts ParisTech
Université Paris-Est
France

École des Ponts ParisTech

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To make a long story short . . .

We claim that mathematical control theory offers material for an operational definition of resilience

Resilience is the capacity of a system to continually change and adapt yet remain within critical thresholds (Stockholm Resilience Centre)

Sources.

- Resilience and stability of ecological systems (Holling, 1973)
- The complexity and stability of ecosystems (Pimm, 1984)
- La résilience dans les modèles de systèmes écologiques et sociaux (Sophie Martin, 2010)

Methods. Theory provides concepts, tools and methods

- states, controls, uncertainties, dynamics
- scenarios, policies, critical thresholds
- viability kernel = viable states = resilient states

Answers. Resilience reframed within cost-efficiency theory as a minimal cost to reach a viability kernel

- the more resilient, the lower the costs to reach a viable state
- the less resilient, the farther from a viability kernel (robust or stochastic)

Steps towards an operational definition of resilience

- Identify, forge, fix
  - stages, decision steps
  - possible actions, controls, decisions, together with their restrictions
  - uncertainties, scenarios
  - states, dynamics, system
  - policies, decision rules
  - critical thresholds
- Compute
  - viability kernel = viable states for which policies exist that can keep the system within critical thresholds, despite of uncertainties
  - minimal cost to reach a viability kernel = inverse of resilience

Outline of the presentation

1. Issues in sustainable management
2. Decision models under uncertainty
3. The viability approach and resilience
4. "Self-promotion, nobody will do it for you" ;-)

Outline of the presentation

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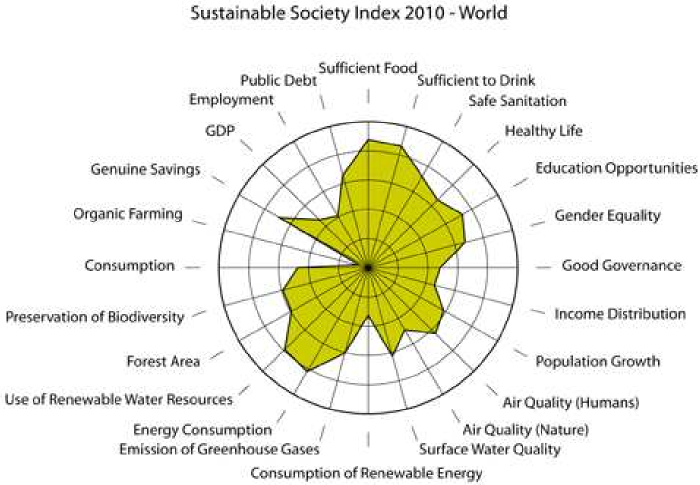
Sustainability laces bunches of indicators and critical thresholds into a Gordian knot



Sustainable management raises specific issues

- Conflicting issues
  - long term *versus* short term
  - profitability $ *versus* multiple environmental and social impacts
    ecoefficiency + climate change mitigation
    + biodiversity preservation
- Decisions
  - energy sources proportions: oil, gas, hydroelectricity, renewables, etc.
  - input quantities: energies, water, materials, etc.
  - investment in new processes, carbon emission permits buy/sell
  - biodiversity reserves design
- Uncertainties
  - combustion efficiency, pollutants diffusion, biodiversity impacts, etc.
- Dynamics
  - delayed impact of decisions on outputs
  - cumulative pollutants, ecosystem dynamics

How to manufacture management strategies and assess them?

- Invent strategies and check that they meet the objectives. Yes but
  - requires a simulation engine
  - what about uncertainties?
  - limited capacity of imagination (Mach 2 example)
- Design a method that yields proper strategies according to a proper assessment framework
  - control theory, satisficing "à la Herbert Simon" (compatible with critical thresholds)
  - cost-efficiency (compatible with critical thresholds)
  - optimality (at odds with critical thresholds because advocates smooth trade-offs)
- Modelling trade-offs between time, risk, economy, ecology, etc. appears delicate when done *a priori*, *ex ante*

Our roadmap

- Discuss critical thresholds *versus* smooth trade-offs in the climate change economic debate
- Showcase the economics standpoint of smooth trade-offs (continuity assumption)
- Showcase the assessment frameworks supposed to tackle multiple goals and risks, with exogenous critical thresholds
- Take note of the difficulty to agree on trade-offs for many issues in sustainable management: future generations, uncertainties, ecosystems
- Without giving up being methodical about trade-offs, propose a proper framework where trade-offs on exogenous critical thresholds are revealed *ex post*

Outline of the presentation

How policy-makers aggregate flows of goods or services over time, risk and within generations is crucial to policy design and choice (Nicholas Stern)

Nicholas Stern. *The Economics of Climate Change*, 2006

- Flows of goods or services over time
  - consumption $C(t)$
  - environment $E(t)$
- *How policy-makers aggregate over consequences*
  - *(i) within generations*
  - *(ii) over time*
  - *(iii) according to risk*

  will be crucial to policy design and choice

The discount rate materializes trade-offs between distant time periods

$$\sum_{t=t_0}^{+\infty}\left(\frac{1}{1+r_e}\right)^{t-t_0}\overbrace{\mathrm{L}\big(C(t),E(t)\big)}^{\text{utility}}$$

In fact, it is not an exaggeration to say that the biggest uncertainty of all in the economics of climate change is the uncertainty about which interest rate to use for discounting. In one form or another this little secret is known to insiders in the economics of climate change, but it needs to be more widely appreciated by economists at large.

(Martin Weitzman)

Expected intertemporal discounted utility is grounded in smooth trade-offs

$$\mathbb{E}\left[\sum_{t=t_0}^{+\infty}\left(\frac{1}{1+r_e}\right)^{t-t_0}\overbrace{\mathrm{L}\big(C(t),E(t)\big)}^{\text{utility}}\right]$$

Expected intertemporal discounted utility is built upon two well axiomatized theories,
where "continuity of preferences" plays a major role

- the discounted intertemporal utility
  T. Koopmans. On the concept of optimal economic growth. *Academia Scientiarium Scripta Varia*, 28:225–300, 1965
- the expected utility
  J. von Neuman and O. Morgenstern. *Theory of games and economic behaviour*. Princeton University Press, Princeton, 1947. 2nd edition

This approach is widely used; it displays time consistency

Catastrophe insurance vs. consumption smoothing

Martin L. Weitzman. A review of the Stern review on the economics of climate change. *Journal of Economic Literature*, 45(3):703–724, 2007

But I think progress begins by recognizing
that the hidden core meaning of Stern vs. Critics may be about $(\cdots)$

- *catastrophe insurance*

$$\max \mathbb{P}\left[\underbrace{C(t) \geq C^{\flat},\ E(t) \geq E^{\flat}}_{\text{indicators} \,\geq\, \text{thresholds}}, \quad \forall t = t_0, \ldots, +\infty\right]$$

- *versus consumption smoothing*

$$\max \mathbb{E}\left[\sum_{t=t_0}^{+\infty} \left(\frac{1}{1+r_e}\right)^{t-t_0} \underbrace{\mathrm{L}\big(C(t), E(t)\big)}_{\text{utility}}\right]$$

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   - Economics of risk and time vs. catastrophe insurance
   - Assigning weights is a way to deal with multiple goals
   - Assessment frameworks are supposed to tackle multiple goals and risks
2. Decision models under uncertainty
   - A few words on modelling
   - A sketch of Control Theory
   - Scenarios carry a priori / off-line information
   - An intertemporal criterion displays preferences with respect to time
   - A risk criterion displays attitudes with respect to uncertainty
3. The viability approach and resilience
   - Stochastic and robust viability in a nutshell
   - The more resilient, the lower the costs to reach a viable state
4. "Self-promotion, nobody will do it for you" ;-)

Three seems a trifle too much. . .

Comme le dit un général Français

Pour décider,
il faut être un nombre impair.
Et trois, ça me paraît beaucoup. . .

A French general saying

To decide, you must be an odd number.
And three seems a bit too much. . .

Georges Clémenceau (1841–1929)

Une commission d'enquête pour être efficace
ne doit compter que trois membres,
dont deux sont absents

Décider c'est choisir, choisir c'est pondérer et pondérer c'est donner des prix à toute chose (Marcel Boiteux)

- Décider c'est choisir
- choisir c'est pondérer
- et pondérer c'est donner des prix à toute chose,
  - matérielle ou immatérielle,
  - marchande ou non marchande

"Pondération de chacune des raretés primaires dans leur infinie diversité, bilan consolidé de tous les cheminements, les uns dans les autres imbriqués, jusqu'à remonter à chacune de ces ressources rares, cela parait a priori tout à fait inextricable"

The economic posture: defining a social optimum respecting that you and I do not have the same tastes

Martin L. Weitzman

> *An enormously important part of the "discipline" of economics is supposed to be that economists understand the difference between their own personal preferences for apples over oranges and the preferences of others for apples over oranges*

The "invisible hand", the "tâtonnement de Walras" are supposed to adjust prices so as to decentralize a Pareto optimum

- Question. How to achieve a Pareto allocation?
- An economic answer. By means of a price system (a price for any good)
- Suppose that each agent has a budget (social issue)
- There exists a price system such that
  - if every agent selects the most preferable basket of good under his/her budget constraint
  - the resulting allocation is Pareto optimal
- Thus, prices are the coefficient weights that make decentralized decisions compatible

Un vieux “truc” qui ne marche pas si mal (Marcel Boiteux)

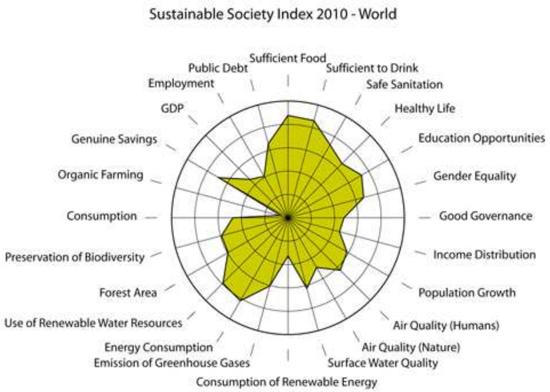
- “Et pourtant, il y a, pour ce faire, un vieux 'truc' que l'on utilise depuis des siècles et qui ne marche pas si mal.
- Cela consiste à affecter à chaque ressource élémentaire un coefficient plus ou moins élevé suivant sa rareté. . . coefficient que l'on appelle un prix.
- En multipliant par ce coefficient-prix la quantité de telle ressource rare que l'on mobilise, on obtient un coût ;
- ces coûts se cumulent tout le long des processus de fabrication pour aboutir au *prix de revient* du produit final. . .
- et la solution la meilleure, celle qui épargne au mieux les raretés élémentaires pondérées par leur importance relative, c'est celle qui coûte le moins cher !”
- “Je suis un peu confus d'avoir retenu votre attention jusqu'à maintenant pour en arriver à une telle banalité”.

Marcel Boiteux, Du Culte de l'énergie, Foi et Vie, n. 23, avril 1977, 76e année

Are prices proper weights?

- Brûler du pétrole, c'est comme brûler sa commode Louis XV (Marcel Boiteux)
- "Les prix qui règnent dans nos économies traduisent-ils correctement, et durablement, tous les aspects des raretés dont la menace pèse sur l'humanité ?" (Marcel Boiteux)
- "l'application obtuse de l'actualisation, à prix constants et sur les seules valeurs marchandes, trahit les réalités et les aspirations profondes de nos sociétés" (Marcel Boiteux)
- Debate on economic valuation of externalities

Some economists recommend objectives to be expressed in their own units, without aggregation



The “Stiglitz-Sen-Fitoussi” Commission (2009) déconseille de privilégier un indicateur synthétique unique car, quel que soit l’indicateur envisagé, l’agrégation de données disparates ne va pas de soi

A profusion of indicators compete to capture sustainability issues

En 2002, l'OCDE a dénombré 22 batteries d'indicateurs de développement durable

- 155 indicateurs de la stratégie européenne de développement durable, hiérachisés en 3 niveaux
- 800 indicateurs de la Banque Mondiale
- 99 indicateurs de la Suisse ventilés entre 24 thèmes
- 68 indicateurs de la Grande Bretagne
- 138 indicateurs des Nations unies
- les objectifs du millénaire du PNUD des Nations unies

Synthetic indicators remain numerous

- produit intérieur brut par tête (PIB)
- indice de soutenabilité environnementale du Forum Economique Mondial (ISE)
- indicateur de développement humain de l'ONU (IDH)
- empreinte écologique (EE)
- épargne nette ajustée de la Banque Mondiale
- indicateur de bien-être (IB)

No consistency emerges from such synthetic indicators

- Une étude de 2007 compare différents indicateurs en reprenant l'ensemble des performances de la plupart des pays du monde
- Le classement des différents pays apparaît alors comme fortement dépendant de l'indicateur choisi
- Le Canada
  - figure dans les huit premiers pays du monde
    en terme de PIB, d'IDH, d'ISE et d'IB
  - mais il est l'antépénultième en terme d'EE

Rapport CAS-LERNA, Préparation Grenelle de l'environnement,
La responsabilisation des entreprises, 2007

How many planets do we need?

How many planets do we need?

How many planets do we need?

???????????

The Ecological Footprint weighs in "space", whereas Economics does it in "numeraire"

As Marcel Boiteux – Honorary President of Électricité de France, and famous economist – expresses it

- to decide is to choose
- to choose is to balance
- and to balance is to give prices to all things
  - material or immaterial
  - tradable or not tradable

How Reliable an Indicator is the Ecological Footprint?

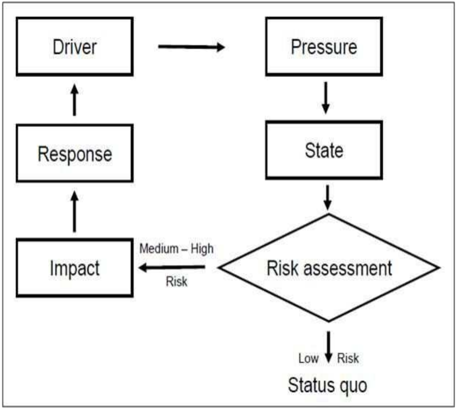
- *L'empreinte écologique : un indicateur ambigu*, Frédéric Paul Piguet, Isabelle Blanc, Tourane Corbiere-Nicollier et Suren Erkman, *Futuribles*, No 334, octobre 2007
- The concept of the ecological footprint has become well known as a composite indicator that is supposed to inform us about the space that human beings occupy in order to produce the resources they consume and the waste they create
- This is then set against the ecological capacity of the Earth (its biocapacity), and hence one can work out the environmental income that humanity has at its disposal
- The authors discuss various dead-ends that GFN reached deliberately, and the criticisms already made of the indicator, which - precisely because of its composite nature - aggregates disparate data and the proceeds to make calculations involving somewhat risky weightings
- The resulting conclusions are then open to challenge, for example when it is suggested that some countries should cut down their forests in order to increase the area for growing crops, whereas the increase of built-up areas (which also encroach on land under cultivation) is not questioned at all

Outline of the presentation

A battery of assessment frameworks have been concocted to gauge policies with respect to risk and ecological impact

- Integrated Ecosystem Assessment (IEA)
  (National Oceanic and Atmospheric Administration)
- Ecological Risk Assessment
- Ecosystem-based Management (EBM)
- Ecosystem Approach to Management
- Driver Pressure State Impact Response (DPSIR) Approach
- Management strategy evaluation (MSE)

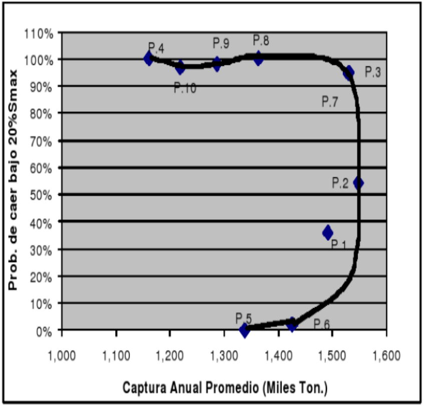
The Driver-Pressure-State-Impact-Response framework

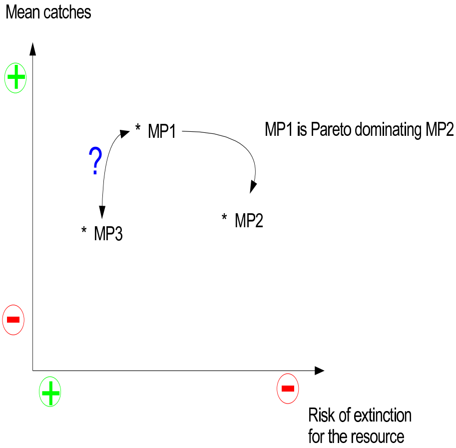


- stressor is an agent of change in the environment
- receptor
- exposure
- "effect" means the response of the receptor when it is actually exposed to the stressor
- assessment endpoint: a specific management outcome that is desired of the ecosystem

The Management Strategy Evaluation framework

"Mieux vaut être riche et bien portant que pauvre et malade"





The a priori modelling of trade-offs between time, risk, economy, ecology, etc. is a delicate task

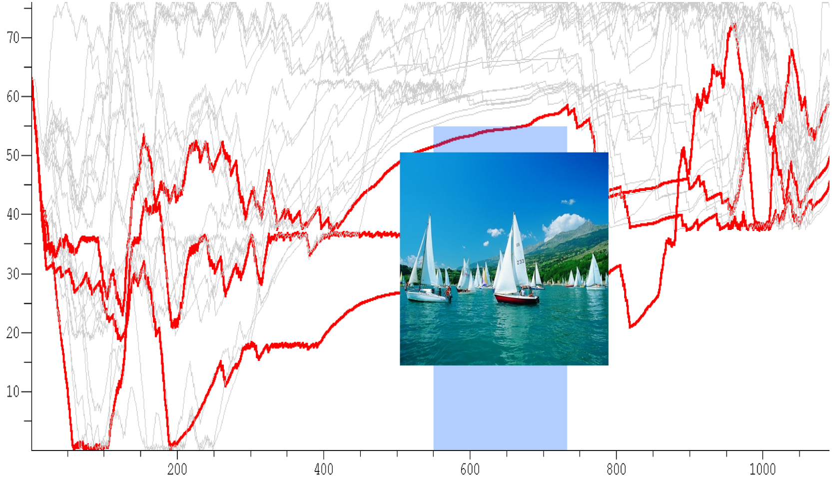
The contradictions of the Alliance of Small Island States w.r.t. shipping and aviation carbon tax

- The Alliance of Small Island States (AOSIS) is a coalition of small island and low-lying coastal countries that share similar development challenges and concerns about the environment, especially their vulnerability to the adverse effects of global climate change
- It functions primarily as an ad hoc lobby and negotiating voice for small island developing States (SIDS) within the United Nations system
- AOSIS was concerned about a proposal for a shipping and aviation tax as a way to mitigate against carbon emission, saying such a concept will discriminate against remote and far-flung members who already suffer from poor shipping and airline connections

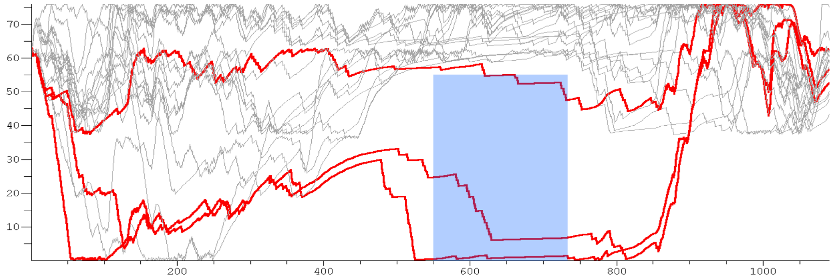
Tourism issues impose constraints upon traditional economic management of a hydro-electric dam

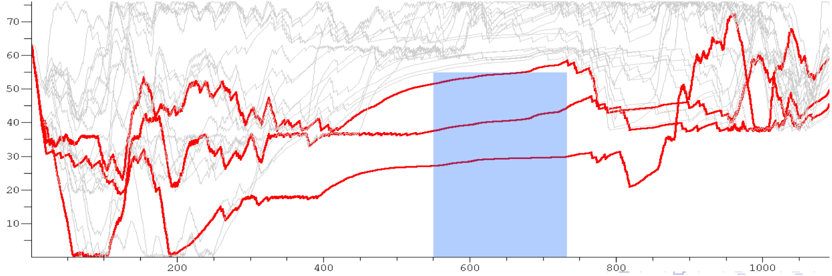
- Maximizing the revenue from turbinated water
- under a tourism constraint of having enough water in July and August

The red stock trajectories fail to meet the tourism constraint in July and August



90% of the stock trajectories meet the tourism constraint in July and August





"Please leave the toilets clean for the next person to use" ;-)

> *The notion of "stewardship" can be seen as a special form of sustainability. It points to particular aspects of the world, which should themselves be passed on in a state at least as good as that inherited from the previous generation.*

Nicholas Stern, The Economics of Climate Change, Cambridge University Press, 2006

> *If sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the very long run. It is very important to understand what that thing is: I think it has to be a generalized capacity to produce economic well-being.*

R. M. Solow. An almost practical step towards sustainability. *Resources Policy*, 19:162–172, 1993.

Our roadmap

- Showcase Control Theory as a panoply of concepts and tools to handle
  - uncertainty
  - time and dynamics
  - multiple objectives
- Prepare the ground for a "geometric" approach (acceptable sets) to handle sustainability and resilience issues
- Recover trade-offs thanks to cost-efficiency, by measuring a "cost distance" to proper sets

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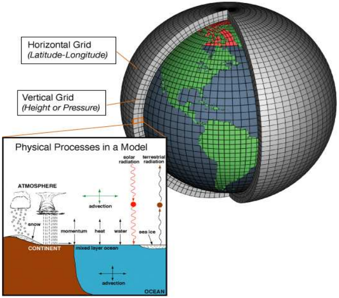
We distinguish two polar classes of models: knowledge models *versus* decision models



Knowledge models:
1/1 000 000 → 1/1 000 → 1/1 maps

Office of Oceanic and Atmospheric Research (OAR) climate model

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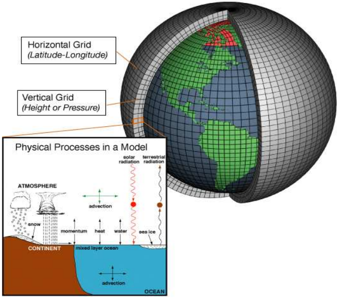

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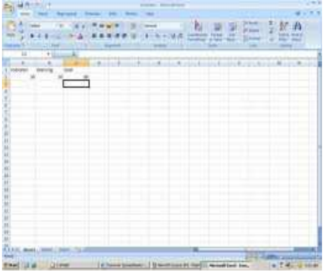


Action/decision models:
economic models are fables designed to provide insight

William Nordhaus economic-climate model

Crafting a model is a trade-off between, on the one hand, realism and complexity, and, on the other hand, mathematical tractabilility

- System: Greek *systema*, arrangement, organized whole
- Complex: Greek *complexus*, composed of parts
  - *com-* "with"
  - *plectere* "to weave, braid, twine"

This talk is *not* about crafting dynamical models

- Elaborating a dynamical model is a delicate venture
  - Peter Yodzis, *Predator-Prey Theory and Management of Multispecies Fisheries*, Ecological Applications 4:51–58, 1994

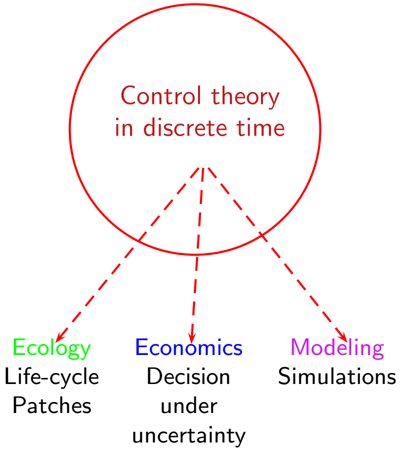
    > *In population modelling the functional forms of models are at least as important as are parameter values in expressing the underlying biology and in determining the outcome. (. . .) For instance, May et al. (1979) assumed, without comment, a particular form of predator-prey interaction; and this particular form was carried over, again without comment, by Flaaten. It turns out that this "invisible" but powerful assumption is responsible in large part for the conclusion reached by Flaaten (1988). (. . .) Flaaten's work is controversial because of his conclusion that "sea mammals should be heavily depleted to increase the surplus production of fish resources for man" (Flaaten 1988:114).*

- Our starting point will be a mathematical dynamical model that captures how sequences of decisions affect a "piece of reality"
- Then, we will use such a model to frame a decision problem

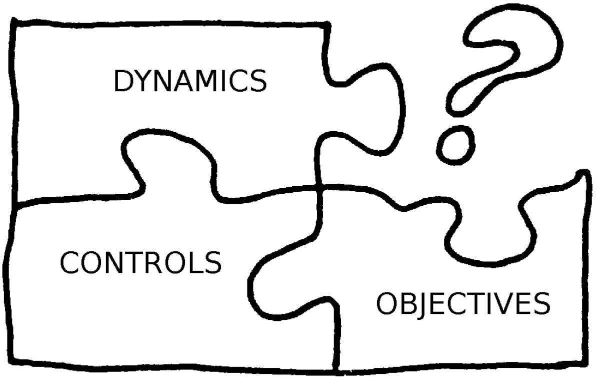
The road is long and bumpy from formulation to resolution

- Concepts, variables, categories
  mathematical modelling
- Relations, equations
  mathematical modelling
- Problem formulation
  mathematical decision theory, control, optimization
- Resolution
  - Analysis
  - Algorithm
  - Software

We showcase control theory in discrete time as a proper vehicle for problem formulation



Control theory is relevant to address the compatibility puzzle



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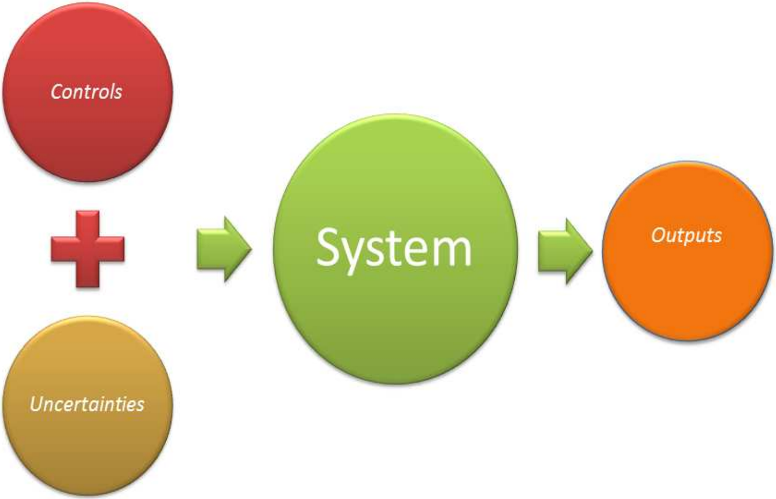
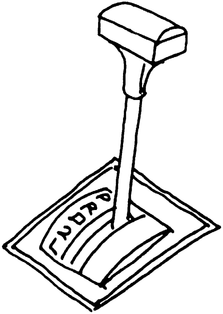
By contrast with control variables, uncertainty variables are exogenous input variables



Input control variables are in the hands of the decision-maker at successive stages

Control variables $u(t) \in \mathbb{U}$

The decision-maker can choose the values of control variables $u(t)$ at any stage within given bounds

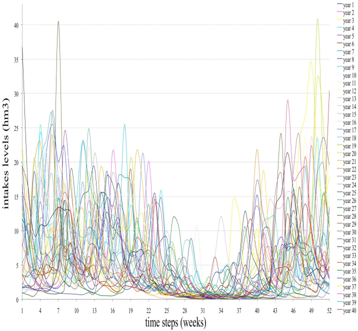


- at successive stages or time periods
  - annual catches
  - years, months:
    starting of energy units like nuclear plants
  - weeks, days, intra-day: starting of hydropower units
- within given bounds
  - fishing quotas
  - turbined capacity

Input uncertain variables are exogenous, that is, out of the control of the decision-maker

Uncertain variables $w(t) \in \mathbb{W}$ are variables

- that take more than one single value (else they are deterministic)
- and over which the decision-maker (DM) has no control whatsoever



- Stationary parameters:
  unitary cost of CO_2 emissions
- Trends or seasonal effects:
  energy consumption pathway, mean temperatures, mean prices
- Stochastic processes:
  rain inputs in a dam, energy demand, prices
- Else (set membership):
  costs of climate change damage,
  water inflows in a dam

Uncertainty variables are new input variables in a discrete-time nonlinear state-control system

A specific output is distinguished, and is labeled "state" (more on this later), when the system may be written

$$x(t+1) = \texttt{Dyn}\big(t, x(t), u(t), w(t)\big) , \quad t \in \mathbb{T} = \{t_0, t_0+1, \ldots, T-1\}$$

- time $t \in \overline{\mathbb{T}} = \{t_0, t_0+1, \ldots, T-1, T\} \subset \mathbb{N}$
  (the time period $[t, t+1[$ may be a year, a month, etc.)
- state $x(t) \in \mathbb{X} := \mathbb{R}^n$ (biomasses, abundances, etc.)
- control $u(t) \in \mathbb{U} := \mathbb{R}^p$ (catches or harvesting effort)
- uncertainty $w(t) \in \mathbb{W} := \mathbb{R}^q$
  (recruitment or mortality uncertainties, climate fluctuations or trends, etc.)
- dynamics $\texttt{Dyn}$ maps $\mathbb{T} \times \mathbb{X} \times \mathbb{U} \times \mathbb{W}$ into $\mathbb{X}$
  (biomass model, age-class model, economic model)

What have we covered so far?

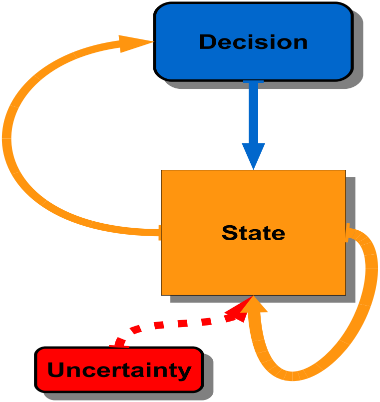
Uncertainty variables are new input variables

$$x(t+1) = \texttt{Dyn}\big(t, x(t), u(t), \underbrace{w(t)}_{\text{uncertainty}}\big)$$

- The future state $x(t+1)$ is no longer predictable
- because of the uncertain term $w(t)$,
- but the current state $x(t)$ carries information relevant for decision-making,
- and we shed light on the notion of policy

$$u(t) = \texttt{Pol}\big(t, x(t)\big)$$

"Policies" are closed-loop controls



- Deterministic control theory appeals to open-loop control, ⌚
  that is, a time-dependent sequence (planning, scheduling)

$$u : \underbrace{t \in \mathbb{T}}_{\text{time}} \mapsto \underbrace{u(t) \in \mathbb{U}}_{\text{control}}$$

- Another notion of solution is a decision rule, ⌚× 👁 a policy,
  that is, a mapping

$$\texttt{Pol} : \underbrace{(t, x) \in \mathbb{T} \times \mathbb{X}}_{\text{(time, state)}} \mapsto u = \underbrace{\texttt{Pol}(t, x) \in \mathbb{U}}_{\text{control}}$$

  which "closes the loop" between time t–state x and control u
  (and is especially relevant in presence of uncertainties)

Summary

- Control variables are defined rather unambiguously:
  the DM can select their values at any time within given sets
- The distinction between input and output variables is relative to a system:
  for two interconnected dams, the water release from the upper to the lower dam can be "seen" as an input to the lower dam or as a control variable for the two-dams system
- In various examples of natural resources management,
  we have seen so-called uncertain variables
- Uncertain variables are variables
  - which take more than one single value (else they are deterministic)
  - and over which the decision-makers have no control whatsoever
- Uncertain and control variables combine in a dynamical model

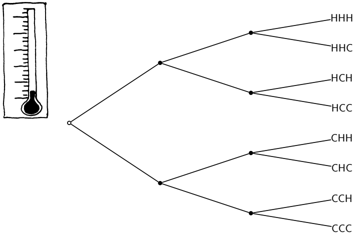
Outline of the presentation

We call scenario a temporal sequence of uncertainties

Scenarios are special cases of "states of Nature"

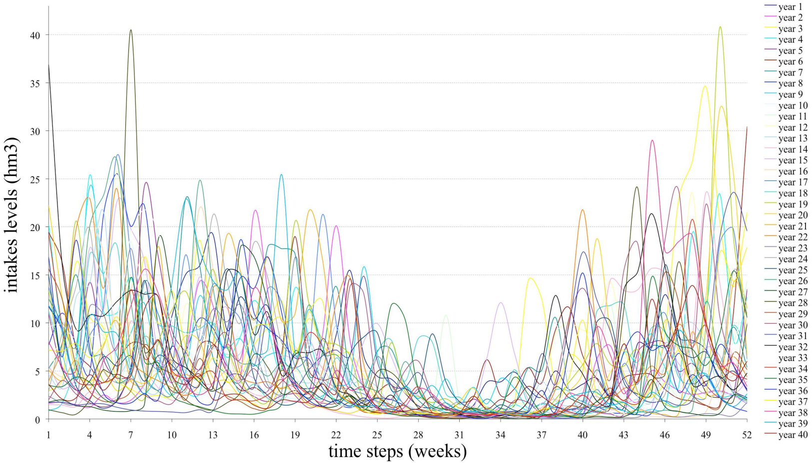
A scenario (pathway, chronicle) is a sequence of uncertainties

$$w(\cdot) := \big(w(t_0), \ldots, w(T-1)\big) \in \Omega := \mathbb{W}^{T-t_0}$$

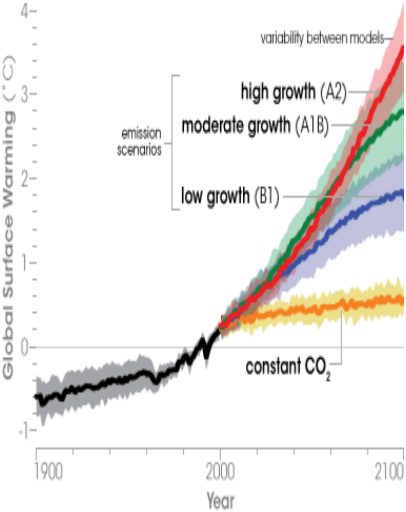


El tiempo se bifurca perpetuamente hacia innumerables futuros
(Jorge Luis Borges, *El jardín de senderos que se bifurcan*)

Water inflows historical scenarios



Beware! Scenario holds a different meaning in other scientific communities



- In practice, what modelers call a "scenario" is a mixture of
  - a sequence of uncertain variables (also called a pathway, a chronicle)
  - a policy `Pol`
  - and even a static or dynamical model
- In what follows

  scenario = pathway = chronicle

Along a given scenario, the system is deterministic

Une intelligence qui, à un instant donné, connaîtrait toutes les forces dont la nature est animée, la position respective des êtres qui la composent, si d'ailleurs elle était assez vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule les mouvements des plus grands corps de l'univers, et ceux du plus léger atome. Rien ne serait incertain pour elle, et l'avenir comme le passé seraient présents à ses yeux.

Pierre-Simon Laplace,
Essai philosophique sur les probabilités

Summary

- ▷ A scenario is a temporal sequence of uncertainties
- ▷ State feedback policies correspond to perfect observation of the state
- ▷ State and control solution maps are defined inductively along each scenario
- ▷ Outputs of a state-control system with uncertainty are now contingent upon scenarios
- ▷ What is off-line information on scenarios?

Probabilistic and set-membership approaches are ways to translate a priori / off-line information as illustrated in nuclear accidents prevention

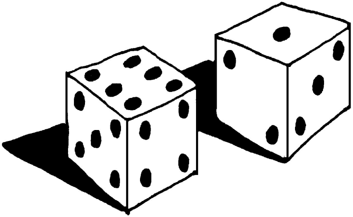
- Three Mile Island accident:
  before the fact, the core meltdown was considered as excluded
- Nuclear accidents with probability per reactor per year
  - between 10^{-6} and 10^{-4} are considered as hypothetical,
  - whereas below 10^{-6} they are not envisaged
- Fukushima nuclear plants had a 10^{-9}
  nuclear accident probability per reactor per year

Choosing a set of scenarios is excluding "things we don't know we don't know"

> *Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – the ones we don't know we don't know. And if one looks throughout the history of our country and other free countries, it is the latter category that tend to be the difficult ones.*

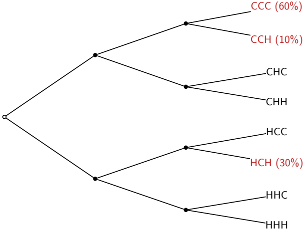
Donald Rumsfeld, former United States Secretary of Defense. From Department of Defense news briefing, February 12, 2002

In the stochastic approach, the set of scenarios is equipped with a known probability



A priori information on the scenarios may be probabilistic

- A probability distribution $\mathbb{P}$ on Ω



- In practice, one often assumes that the components $\big(w(t_0), \ldots, w(T-1)\big)$ form
  - an independent and identically distributed sequence
  - a Markov chain, a time series, etc.

Water inflows in a dam

Water inflows in a dam may be modelled as time series (ARMA, etc.)

Probabilistic assumptions and expected value

- The domain of scenarios $\Omega = \mathbb{W}^{T+1-t_0} = \mathbb{R}^q \times \cdots \times \mathbb{R}^q$ is equipped with the σ-field $\mathcal{F} = \bigotimes_{t=t_0}^{T} \mathcal{B}(\mathbb{R}^q)$ and a probability $\mathbb{P}$
- The sequences $w(\cdot) = (w(t_0), w(t_0+1), \ldots, w(T-1), w(T))$ now become the primitive random variables
- The notation $\mathbb{E}_{\mathbb{P}}$ refers to the mathematical expectation over Ω under probability $\mathbb{P}$

$$\mathbb{E}[A(w(\cdot))] = \sum_{w(\cdot)\in\Omega} \mathbb{P}\{w(\cdot)\}A(w(\cdot))$$

- The expectation operator $\mathbb{E}_{\mathbb{P}}$ enjoys linearity in the $(+,\times)$ algebra:

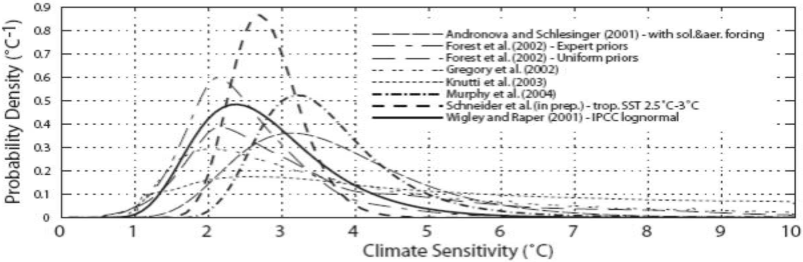
$$\mathbb{E}_{\mathbb{P}}(A+B) = \mathbb{E}_{\mathbb{P}}(A) + \mathbb{E}_{\mathbb{P}}(B)$$

- The random variables $(w(t_0), w(t_0+1), \ldots, w(T-1), w(T))$ are independent under $\mathbb{P}$ if $\mathbb{P}$ can be decompsed as a product

$$\mathbb{P} = \mu_{t_0} \otimes \cdots \otimes \mu_T$$

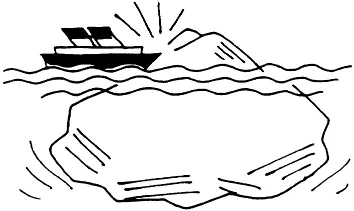
Equipping the set Ω of scenarios with a probability $\mathbb{P}$ is a delicate issue!

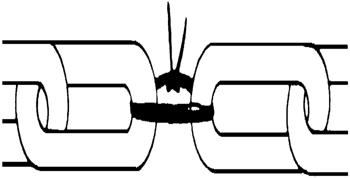
- The probabilistic distribution of the climate sensitivity parameter in climate models differs according to authors

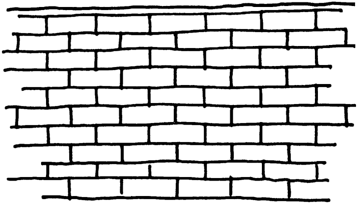


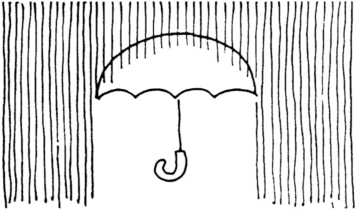
- In the multi-prior approach, the a priori information consists of different probabilities (*beliefs*, *priors*), belonging to a set $\mathcal{P}$ of admissible probabilities on Ω

In the set-membership approach, only a subset of the set of scenarios is known







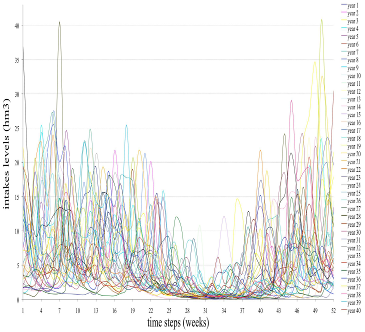


A priori information on the scenarios may be set membership

The general case

- Selected scenarios may belong to any subset $\overline{\Omega}$

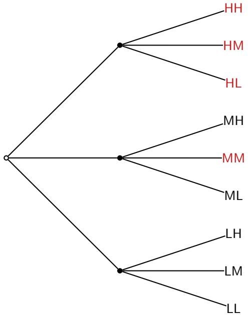
$$w(\cdot) \in \overline{\Omega} \subset \Omega$$



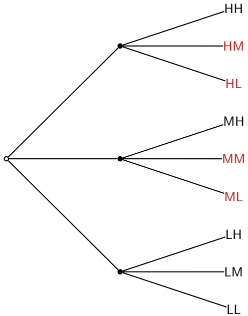
Historical water inflows scenarios in a dam

We can represent off-line information by the observed historical water inflows scenarios

Specific subsets correspond to time independence



There is no time independence because the range of values of $w(t+1)$ depends on the value of $w(t)$:
$w(t) = H \Rightarrow w(t+1) \in \{M, L\}$
$w(t) = M \Rightarrow w(t+1) \in \{M\}$



There is time independence because
$\overline{\Omega} = \{H, M\} \times \{M, L\} \subset \Omega$
is a product set

A priori information on the scenarios may be set membership

The product case

- Uncertain variables may be restricted to subsets, period by period

$$w(t) \in \mathbb{S}(t)$$

so that some scenarios are selected and the rest are excluded

$$w(\cdot) \in \mathbb{S}(t_0) \times \cdots \times \mathbb{S}(T) \subset \Omega = \mathbb{W}^{T+1-t_0}$$

Bounded water inflows in a dam

If only an upper bound on water inflows is known, we represent off-line information by

$$0 \leq a(t) \leq a^\sharp$$

A priori information on the scenarios may be softer than set membership thanks to plausibility functions

- ▷ The counterpart of a probability $\mathbb{P}$ — that weighs the likelihood of an event — is a plausibility function $\mathbb{Q}$
- ▷ Plausibility function $\mathbb{Q} : \Omega \to \mathbb{R} \cup \{-\infty\}$ can "soften" the above set membership approach
  - ▹ the higher $\mathbb{Q}\big(w(\cdot)\big)$, the more plausible the scenario $w(\cdot)$
  - ▹ totally implausible scenarios are those for which $\mathbb{Q}\big(w(\cdot)\big) = -\infty$

Historical water inflows scenarios in a dam

Attribute the value $\mathbb{Q}\big(w(\cdot)\big) = -\infty$ for all the scenarios $w(\cdot)$ which do not belong to the observed historical water inflows scenarios

The fear operator (Pierre Bernhard) is the robust counterpart of a probability

- Let $\mathbb{Q} : \Omega \to \mathbb{R} \cup \{-\infty\}$ be a a plausibility function
- The feared value of a function $A : \Omega \to \mathbb{R}$ is defined by

$$\mathbb{F}_{\mathbb{Q}}(A) := \min_{w(\cdot) \in \Omega} \big[A(w(\cdot)) - \mathbb{Q}(w(\cdot)) \big]$$

- The fear operator $\mathbb{F}_{\mathbb{Q}}$ enjoys linearity in the $(\min, +)$ algebra:

$$\mathbb{F}_{\mathbb{Q}}(\min\{A, B\}) = \min\{\mathbb{F}_{\mathbb{Q}}(A), \mathbb{F}_{\mathbb{Q}}(B)\}$$

- In the $(\min, +)$ algebra, the plausibility function $\mathbb{Q}$ plays the role of a weight, paralleling a probability distribution
- The uncertainties $(w(t_0), w(t_0 + 1), \ldots, w(T-1), w(T))$ are independent under $\mathbb{Q}$ if $\mathbb{Q}$ can be decomposed as a sum

$$\mathbb{Q} = \nu_{t_0} + \cdots + \nu_T$$

Summary

- A priori information is carried by the scenarios set, and may be
  - probabilistic
  - set membership
- This will be useful to mathematically express the objectives and the constraints in a decision problem under uncertainty

Outline of the presentation

An intertemporal criterion attaches a value to a history and performs an aggregation with respect to time, reflecting preferences across time

- The history space is

$$\mathbb{H} := \overbrace{\underbrace{\mathbb{X}^{T+1-t_0}}_{\text{state}} \times \underbrace{\mathbb{U}^{T-t_0}}_{\text{control}} \times \underbrace{\mathbb{W}^{T+1-t_0}}_{\text{uncertainty}}}^{\text{history space}}$$

- A criterion Crit is a function

$$\texttt{Crit} : \mathbb{H} \to \mathbb{R}$$

which assigns

  - a scalar value $\texttt{Crit}\big(x(\cdot), u(\cdot), w(\cdot)\big) \in \mathbb{R}$
  - to a history $\big(x(\cdot), u(\cdot), w(\cdot)\big) \in \mathbb{H}$

The additive criterion is the most common and sums payoffs over time-periods

- The traditional discounted present value is

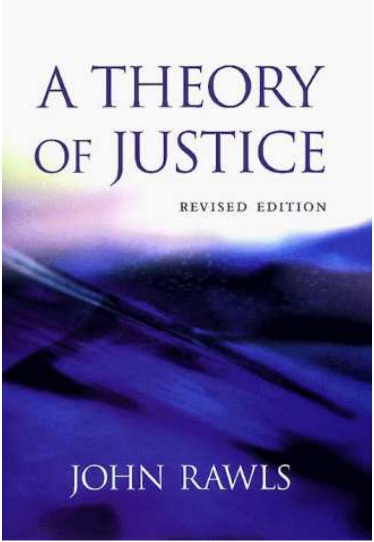
$$\sum_{t=t_0}^{+\infty} \delta^{t-t_0} \mathtt{L}\big(x(t), u(t), w(t)\big)$$

- The time-separable additive criterion includes *discounted present value, Green Golden, Chichilnisky*

$$\begin{aligned} \mathtt{Crit}\big(x(\cdot), u(\cdot), w(\cdot)\big) &= \sum_{t=t_0}^{T-1} \overbrace{\mathtt{L}\big(t, x(t), u(t), w(t)\big)}^{\text{instantaneous gain}} \\ &+ \underbrace{\mathtt{K}\big(T, x(T), w(T)\big)}_{\text{final gain}} \end{aligned}$$

- The payoffs in one time-period may be compensated by those of other time-periods

The maximin criterion focuses on the worst payoff accross time-periods



- Equity: a focus on the poorest generation
- The maximin form or Rawls criterion is

$$\texttt{Crit}\big(x(\cdot), u(\cdot), w(\cdot)\big) = \underbrace{\min_{t=t_0,\ldots,T-1} \overbrace{\texttt{L}\big(t, x(t), u(t), w(t)\big)}^{\text{generation utility}}}_{\text{worse generation utility}}$$

- The payoffs in one time-period cannot be compensated by those of other time-periods

Summary

- A criterion attaches a value to a history and performs an aggregation with respect to time, reflecting preferences across time
- How can we attach a value to a policy, so that we can rank policies?

A policy and a criterion yield a real-valued payoff

Given a policy $\texttt{Pol} \in \mathfrak{U}^{ad}$ and a scenario $w(\cdot) \in \Omega$, we obtain a payoff

$$\texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big) = \texttt{Crit}^{\texttt{Pol}}\big(t_0, x_0, w(\cdot)\big)$$

hence a mapping $\mathfrak{U}^{ad} \times \Omega \to \mathbb{R}$

Policies/Scenarios	$w^A(\cdot) \in \Omega$	$w^B(\cdot) \in \Omega$	...
$\texttt{Pol}_1 \in \mathfrak{U}^{ad}$	$\texttt{Payoff}\big(\texttt{Pol}_1, w^A(\cdot)\big)$	$\texttt{Payoff}\big(\texttt{Pol}_1, w^B(\cdot)\big)$	...
$\texttt{Pol}_2 \in \mathfrak{U}^{ad}$	$\texttt{Payoff}\big(\texttt{Pol}_2, w^A(\cdot)\big)$	$\texttt{Payoff}\big(\texttt{Pol}_2, w^B(\cdot)\big)$	...
...	...	...	...

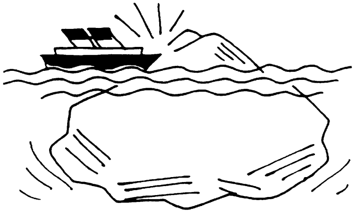
Summary

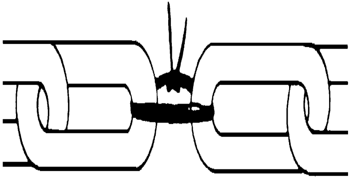
- An intertemporal criterion Crit attaches a value to a history and performs an aggregation with respect to time, reflecting preferences across time
- A policy Pol and a scenario $w(\cdot)$ yield a history, thanks to the state and control solution maps, that is evaluated by a criterion Crit (time aggregation), yielding $\texttt{Crit}^{\texttt{Pol}}\big(t_0, x_0, w(\cdot)\big)$
- A policy Pol and a criterion Crit yield a real-valued mapping $w(\cdot) \in \Omega \mapsto \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big) = \texttt{Crit}^{\texttt{Pol}}\big(t_0, x_0, w(\cdot)\big)$ over the scenarios Ω
- Therefore, comparing policies amounts to comparing mappings over the scenarios Ω
- For this purpose, we will see how to aggregate the real-valued mapping $w(\cdot) \in \Omega \mapsto \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)$ with respect to scenarios

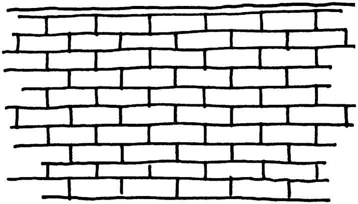
Outline of the presentation

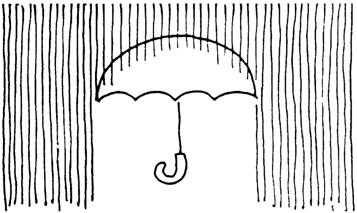
1. Issues in sustainable management
   - Sustainable management raises specific issues
   - Economics of risk and time vs. catastrophe insurance
   - Assigning weights is a way to deal with multiple goals
   - Assessment frameworks are supposed to tackle multiple goals and risks
2. Decision models under uncertainty
   - A few words on modelling
   - A sketch of Control Theory
   - Scenarios carry a priori / off-line information
   - An intertemporal criterion displays preferences with respect to time
   - A risk criterion displays attitudes with respect to uncertainty
3. The viability approach and resilience
   - Stochastic and robust viability in a nutshell
   - The more resilient, the lower the costs to reach a viable state
4. "Self-promotion, nobody will do it for you" ;-)

In the robust or pessimistic approach, Nature is supposed to be malevolent, and the DM aims at protection against all odds









In the robust or pessimistic approach, Nature is supposed to be malevolent

- In the robust approach, the DM considers the worst payoff

$$\underbrace{\min_{w(\cdot)\in\Omega} \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)}_{\text{worst payoff}}$$

- Nature is supposed to be malevolent,
  and specifically selects the worst scenario:
  the DM plays after Nature has played, and maximizes the worst payoff

$$\max_{\texttt{Pol}\in\mathcal{U}^{ad}} \min_{w(\cdot)\in\Omega} \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)$$

- Robust, pessimistic, worst-case, maximin, minimax (for costs)

Guaranteed energy production

In a dam, the minimal energy production in a given period, corresponding to the worst water inflow scenario

The robust approach can be softened with plausibility weighting

- Let $\Theta : \Omega \to \mathbb{R} \cup \{-\infty\}$ be a a plausibility function.
- The higher, the more plausible: totally implausible scenarios are those for which $\Theta\big(w(\cdot)\big) = -\infty$
- Nature is malevolent, and specifically selects the worst scenario, but weighs it according to the plausibility function Θ
- The DM plays after Nature has played, and solves

$$\max_{\texttt{Pol} \in \mathfrak{U}^{ad}} \left[\min_{w(\cdot) \in \Omega} \left(\texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big) - \underbrace{\Theta\big(w(\cdot)\big)}_{\text{plausibility}} \right) \right]$$

In the optimistic approach, Nature is supposed to benevolent

Future. That period of time in which our affairs prosper, our friends are true and our happiness is assured.

Ambrose Bierce

- Instead of maximizing the worst payoff as in a robust approach, the optimistic perspective focuses on the most favorable payoff

$$\underbrace{\max_{w(\cdot)\in\Omega} \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)}_{\text{best payoff}}$$

- Nature is supposed to benevolent, and specifically selects the best scenario: the DM plays after Nature has played, and solves

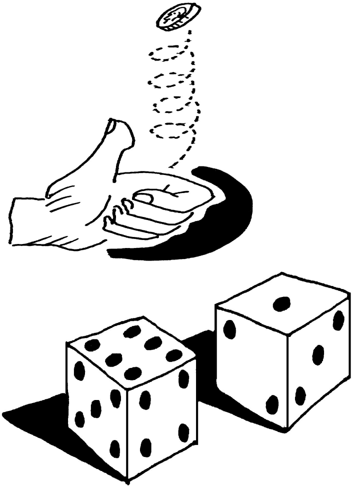
$$\max_{\texttt{Pol}\in\mathfrak{U}^{ad}} \max_{w(\cdot)\in\Omega} \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)$$

The Hurwicz criterion reflects an intermediate attitude between optimistic and pessimistic approaches

A proportion $\alpha \in [0, 1]$ graduates the level of prudence

$$\max_{\texttt{Pol} \in \mathcal{U}^{ad}} \Big\{ \alpha \overbrace{\min_{w(\cdot) \in \Omega} \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)}^{\text{pessimistic}} + (1-\alpha) \underbrace{\max_{w(\cdot) \in \Omega} \texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)}_{\text{optimistic}} \Big\}$$

In the stochastic or expected approach, Nature is supposed to play stochastically



In the stochastic or expected approach, Nature is supposed to play stochastically

- The expected payoff is

$$\overbrace{\mathbb{E}\Big[\texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)\Big]}^{\text{mean payoff}} = \sum_{w(\cdot)\in\Omega} \mathbb{P}\{w(\cdot)\}\texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)$$

- Nature is supposed to play stochastically, according to distribution $\mathbb{P}$: the DM plays after Nature has played, and solves

$$\max_{\texttt{Pol}\in\mathfrak{U}^{ad}} \mathbb{E}\Big[\texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big)\Big]$$

- The discounted expected utility is the special case

$$\mathbb{E}\Big[\sum_{t=t_0}^{+\infty} \delta^{t-t_0} \texttt{L}\big(x(t), u(t), w(t)\big)\Big]$$

The expected utility approach distorts payoffs before taking the expectation

- We consider a utility function $\mathtt{L}$ to assess the utility of the payoffs (for instance a CARA exponential utility function)
- The expected utility is

$$\underbrace{\mathbb{E}\left[\mathtt{L}\Big(\mathtt{Payoff}\big(\mathtt{Pol}, w(\cdot)\big)\Big)\right]}_{\text{expected utility}} = \sum_{w(\cdot)\in\Omega} \mathbb{P}\{w(\cdot)\}\mathtt{L}\Big(\mathtt{Payoff}\big(\mathtt{Pol}, w(\cdot)\big)\Big)$$

- The expected utility maximizer solves

$$\max_{\mathtt{Pol}\in\mathfrak{U}^{ad}} \mathbb{E}\left[\mathtt{L}\Big(\mathtt{Payoff}\big(\mathtt{Pol}, w(\cdot)\big)\Big)\right]$$

The ambiguity or multi-prior approach combines robust and expected criterion

- Different probabilities $\mathbb{P}$, termed as beliefs or priors, and belonging to a set $\mathcal{P}$ of admissible probabilities on Ω
- The multi-prior approach combines robust and expected criterion by taking the worst beliefs in terms of expected payoff

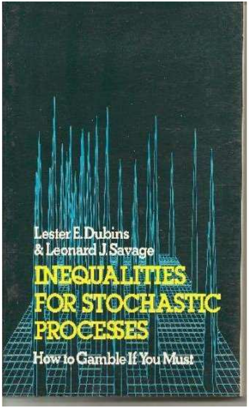
$$\max_{\texttt{Pol} \in \mathfrak{U}^{ad}} \underbrace{\min_{\mathbb{P} \in \mathcal{P}} \overbrace{\mathbb{E}^{\mathbb{P}} \Big[\texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big) \Big]}^{\text{mean payoff}}}_{\text{pessimistic over probabilities}}$$

Convex risk measures cover a wide range of risk criteria

- Different probabilities $\mathbb{P}$, termed as beliefs or priors, and belonging to a set $\mathcal{P}$ of admissible probabilities on Ω
- To each probability $\mathbb{P}$ is attached a plausibility $\Theta(\mathbb{P})$

$$\max_{\texttt{Pol} \in \mathfrak{U}^{ad}} \underbrace{\min_{\mathbb{P} \in \mathcal{P}} \overbrace{\mathbb{E}^{\mathbb{P}} \Big[\texttt{Payoff}\big(\texttt{Pol}, w(\cdot)\big) \Big]}^{\text{mean payoff}} - \overbrace{\Theta(\mathbb{P})}^{\text{plausibility}}}_{\text{pessimistic over probabilities}}$$

Non convex risk measures can lead to non diversification



How to gamble if you must, L.E. Dubbins and L.J. Savage, 1965

Imagine yourself at a casino with \$1,000. For some reason, you desperately need \$10,000 by morning; anything less is worth nothing for your purpose.

The only thing possible is to gamble away your last cent, if need be, in an attempt to reach the target sum of \$10,000.

- ▷ The question is how to play, not whether. What ought you do? How should you play?
  - ▹ Diversify, by playing 1 \$ at a time?
  - ▹ Play boldly and concentrate, by playing 10,000 \$ only one time?
- ▷ What is your decision criterion?

Savage's minimal regret criterion... "Had I known"

$$\min_{\mathtt{Pol} \in \mathfrak{U}^{ad}} \Big\{ \overbrace{\max_{w(\cdot) \in \Omega} \Big[\underbrace{\max_{\text{anticipative policies } \overline{\mathtt{Pol}}} \mathtt{Payoff}\big(\overline{\mathtt{Pol}}, w(\cdot)\big) - \mathtt{Payoff}\big(\mathtt{Pol}, w(\cdot)\big)}_{\text{regret}} \Big]}^{\text{worst regret}} \Big\}$$

- If the DM knows the future in advance, she solves $\max_{\text{anticipative policies } \overline{\mathtt{Pol}}} \mathtt{Payoff}\big(\overline{\mathtt{Pol}}, w(\cdot)\big)$, for each scenario $w(\cdot) \in \Omega$
- The regret attached to a non-anticipative policy $\mathtt{Pol} \in \mathfrak{U}^{ad}$ is the loss due to not being visionary
- The best a non-visionary DM can do with respect to regret is minimizing it

Summary

- ▷ A criterion attaches a value to a history and performs aggregation with respect to time, reflecting preferences across time
- ▷ Off-line information on scenarios allows aggregation with respect to uncertainties, reflecting risk attitudes and preferences across scenarios
- ▷ Policies are compared with respect to both time and uncertainties payoffs aggregations
- ▷ How do we compute optimal policies?

A summary table

	time compensatory	time non-compensatory
deterministic	discounted utility	Rawls, viability
risk compensatory	expected discounted utility	expected Rawls, stochastic viability
risk non-compensatory	robust discounted utility	robust Rawls, robust viability

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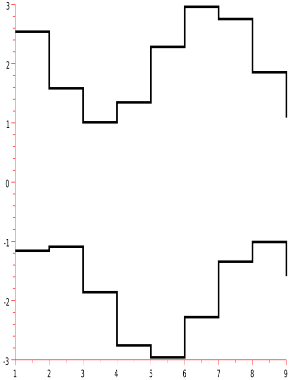
   - Scenarios carry a priori / off-line information
   - An intertemporal criterion displays preferences with respect to time
   - A risk criterion displays attitudes with respect to uncertainty
3. The viability approach and resilience
   - Stochastic and robust viability in a nutshell
   - The more resilient, the lower the costs to reach a viable state
4. "Self-promotion, nobody will do it for you" ;-)

What is resilience?



Resilience is the capacity of a system
to continually change and adapt
yet remain within critical thresholds

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Outline of the presentation

1. Issues in sustainable management
   - Sustainable management raises specific issues
   - Economics of risk and time vs. catastrophe insurance
   - Assigning weights is a way to deal with multiple goals
   - Assessment frameworks are supposed to tackle multiple goals and risks
2. Decision models under uncertainty
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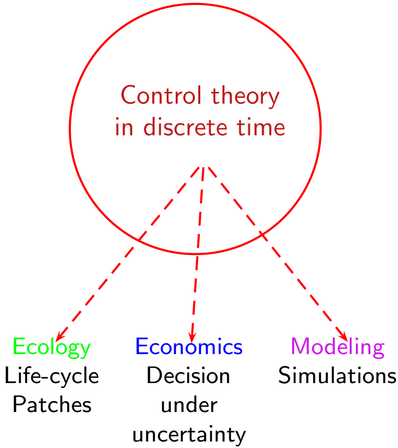
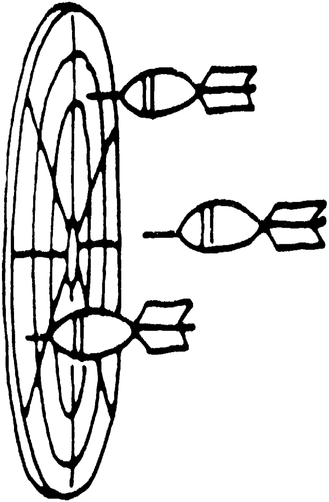
We dress natural resources management issues in the formal clothes of control theory in discrete time



- Problems are framed as
  - find controls/decisions driving a dynamical system
  - to achieve various goals
- Three main ingredients are
  - controlled dynamics
  - constraints
  - criterion to optimize

We mathematically express the objectives pursued as control and state constraints



- For a state-control system,
  we cloth objectives as constraints
- and we distinguish
  control constraints (rather easy)
  state constraints (rather difficult)
- Viability theory deals with state constraints

Constraints may be explicit on the control variable

and are rather easily handled by reducing the decision set

Examples of control constraints

- Irreversibility constraints, physical bounds 
  $0 \leq a(t) \leq 1 \ , \quad 0 \leq h(t) \leq B(t)$
- Tolerable costs $c\big(a(t), Q(t)\big) \leq c^{\sharp}$

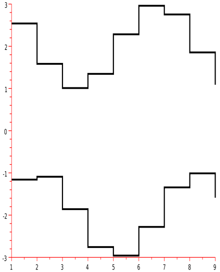
Control constraints / admissible decisions

$$\underbrace{u(t)}_{\text{control}} \in \underbrace{\mathbb{B}\big(t, x(t)\big)}_{\text{admissible set}} \ , \quad t = t_0, \ldots, T-1$$

Easy because control variables $u(t)$ are precisely those variables whose values the decision-maker can fix at any time within given bounds

Meeting constraints bearing on the state variable is delicate

due to the dynamics pipeline between controls and state



State constraints / admissible states

$$\underbrace{x(t)}_{\text{state}} \in \underbrace{\mathbb{A}(t)}_{\text{admissible set}} \quad , \quad t = t_0, \ldots, T$$

Examples ("tipping points")

- CO_2 concentration $M(t) \leq M^\sharp$
- biomass $B^\flat \leq B(t) \leq B^\sharp$

State constraints are mathematically difficult because of "inertia"

$$x(t) = \underbrace{\text{function}}_{\text{iterated dynamics}} \big(\underbrace{u(t-1), \ldots, u(t_0)}_{\text{past controls}}, x(t_0) \big)$$

Target and asymptotic state constraints are special cases

- Final state achieves some target

$$\underbrace{x(T)}_{\text{final state}} \in \underbrace{\mathbb{A}(T)}_{\text{target set}}$$

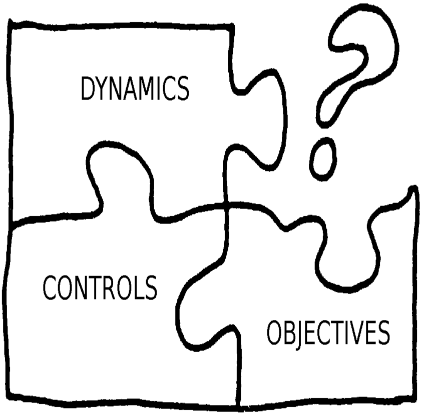
Example: CO_2 concentration

- State converges toward a target

$$\underbrace{\lim_{t \to +\infty} x(t)}_{\text{asymptotic state}} \in \underbrace{\mathbb{A}(\infty)}_{\text{target set}}$$

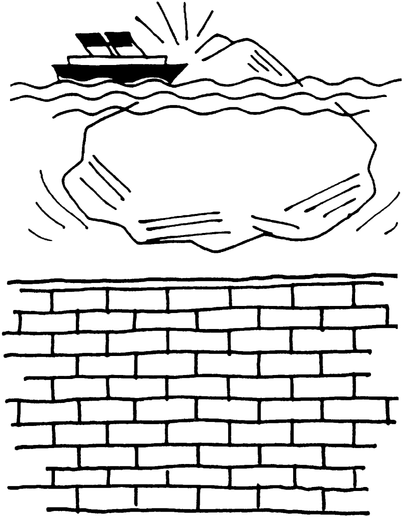
Example: convergence towards an endemic state in epidemiology

Can we solve the compatibility puzzle between dynamics and objectives by means of appropriate controls?

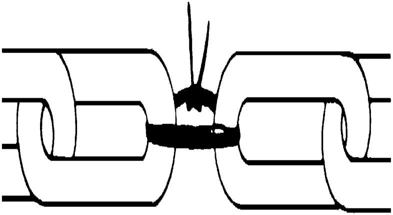


- ▷ Given a dynamics that mathematically embodies the causal impact of controls on the state
- ▷ Imposing objectives bearing on output variables (states, controls)
- ▷ Is it possible to find a control path that achieves the objectives for all times?

Crisis occurs when constraints are trespassed at least once



- ▷ An initial state is not viable if, whatever the sequence of controls, a crisis occurs
- ▷ There exists a time when one of the state or control constraints is violated

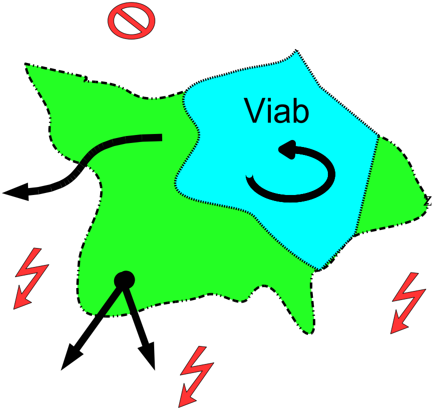


The compatibility puzzle can be solved when the initial viability kernel $\mathbb{V}\mathrm{iab}(t_0)$ is not empty

Viable initial states form the viability kernel (Jean-Pierre Aubin)

$$
\mathbb{V}\mathrm{iab}(t) := \left\{ \begin{array}{c} \text{initial} \\ \text{states} \\ x \in \mathbb{X} \end{array} \;\middle|\; \begin{array}{l} \text{there exist a control path } u(\cdot) = \\ \big(u(t), u(t+1), \ldots, u(T-1)\big) \\ \text{and a state path } x(\cdot) = \\ \big(x(t), x(t+1), \ldots, x(T)\big) \\ \text{starting from } x(t) = x \text{ at time } t \\ \text{satisfying for any time } s \in \{t, \ldots, T-1\} \\ x(s+1) = \mathtt{Dyn}\big(s, x(s), u(s)\big) \quad \textit{dynamics} \\ u(s) \in \mathbb{B}(s, x(s)) \quad \textit{control constraints} \\ x(s) \in \mathbb{A}(s) \quad \textit{state constraints} \\ \text{and } x(T) \in \mathbb{A}(T) \quad \textit{target constraints} \end{array} \right\}
$$

The viability kernel is included in the state constraint set



- The largest set is the state constraint set $\mathbb{A}$
- It includes the smaller blue viability kernel $\mathbb{V}\mathrm{iab}(t_0)$
- The green set measures the incompatibility between dynamics and constraints: good start, but inevitable crisis!

The viability program aims at turning a priori constraints, with state constraints, into a posteriori constraints, without state constraints

- A priori constraints, with state constraints

$$\begin{cases} x(t_0) \in \mathbb{X} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t)\big) \\ u(t) \in \mathbb{B}\big(t, x(t)\big) & \text{control constraints} \\ x(t) \in \mathbb{A}(t) & \text{state constraints} \end{cases}$$

- are turned into a posteriori constraints, without state constraints except for the initial state

$$\begin{cases} x(t_0) \in \mathbb{V}\mathrm{iab}(t_0) & \text{initial state constraint} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t)\big) \\ u(t) \in \mathbb{B}^{\mathsf{viab}}\big(t, x(t)\big) & \text{control constraints} \end{cases}$$

A scenario is said to be viable for a given policy if the state and control trajectories satisfy the constraints

Viable scenario under given policy

A scenario $w(\cdot) \in \Omega$ is said to be viable under policy $\mathtt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$ if the trajectories $x(\cdot)$ and $u(\cdot)$ generated by the dynamics

$$x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big) \ , \quad t = t_0, \ldots, T-1$$

with the policy

$$u(t) = \mathtt{Pol}\big(t, x(t)\big)$$

satisfy the state and control constraints

$$\underbrace{u(t) \in \mathbb{B}\big(t, x(t)\big)}_{\text{control constraints}} \text{ and } \underbrace{x(t) \in \mathbb{A}(t)}_{\text{state constraints}} \ , \quad \forall t = t_0, \ldots, T$$

The set of viable scenarios is denoted by $\Omega_{\mathtt{Pol}, t_0, x_0}$

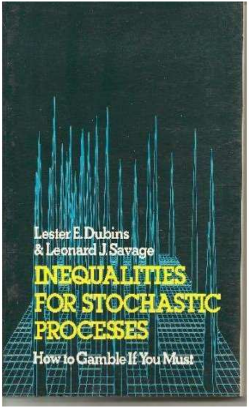
We look after policies that make the corresponding set of viable scenarios "large"

Set of viable scenarios

$$\Omega_{\texttt{Pol},t_0,x_0} := \{ w(\cdot) \in \Omega \mid \text{the state constraints } X_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)](t) \in \mathbb{A}(t) \text{ and the control constraints } U_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)] \in \mathbb{B}\big(t, x(t)\big) \text{ are satisfied for all times } t = t_0, \ldots, T \}$$

- The larger set $\Omega_{\texttt{Pol},t_0,x_0}$ of viable scenarios, the better, because the policy Pol is able to maintain the system within constraints for a large "number" of scenarios
- But "large" in what sense? Robust? Probabilistic?

Maximizing the probability of success may be an objective



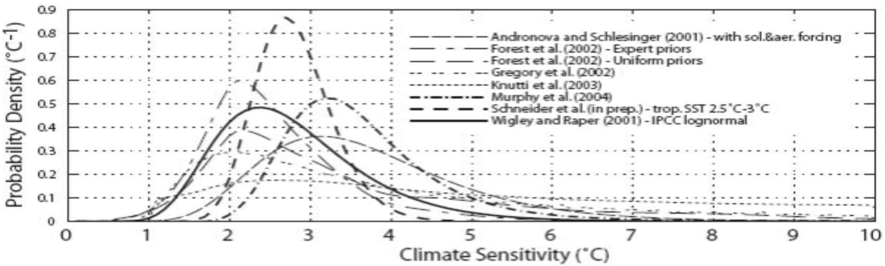
How to gamble if you must,
L.E. Dubbins and L.J. Savage,
1965

Imagine yourself at a casino with \$1,000. For some reason, you desperately need \$10,000 by morning; anything less is worth nothing for your purpose.

The only thing possible is to gamble away your last cent, if need be, in an attempt to reach the target sum of \$10,000.

- The question is how to play, not whether. What ought you do? How should you play?
  - Diversify, by playing 1 \$ at a time?
  - Play boldly and concentrate, by playing 10,000 \$ only one time?
- What is your decision criterion?

We suppose that the set Ω of scenarios is equipped with a probability $\mathbb{P}$ (though this is a delicate issue!)



In practice, one often assumes that the components $\big(w(t_0), \ldots, w(T-1)\big)$ form an independent and identically distributed sequence of random variables, or form a Markov chain, or a time series

The viability probability is the probability of satisfying constraints under a policy

Viability probability

The viability probability associated with
the initial time t_0, the initial state x_0 and the policy $\texttt{Pol}$
is the probability $\mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$ of the set $\Omega_{\texttt{Pol},t_0,x_0}$ of viable scenarios

$$\mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right] = \text{Proba}\left\{\begin{array}{l}\text{scenarios along which}\\ \text{the state } x(\cdot) \text{ and control } u(\cdot) \text{ trajectories}\\ \text{generated by dynamics } \texttt{Dyn} \text{ and policy } \texttt{Pol}\\ \text{starting from initial state } x_0 \text{ at initial time } t_0\\ \text{satisfy the constraints}\\ \text{from initial time } t_0 \text{ to horizon } T\end{array}\right\}$$

The viability probability is the probability of satisfying constraints under a policy

Viability probability

The viability probability associated with
the initial time t_0, the initial state x_0 and the policy $\texttt{Pol}$
is the probability $\mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$ of the set $\Omega_{\texttt{Pol},t_0,x_0}$ of viable scenarios

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The maximal viability probability is the upper bound for the probability of satisfying constraints

Maximal viability probability and optimal viable policy

The maximal viability probability is

$$\max_{\mathtt{Pol}} \mathbb{P}\left[\Omega_{\mathtt{Pol},t_0,x_0}\right]$$

An optimal viable policy $\mathtt{Pol}^{\star}$ satisfies

$$\mathbb{P}\left[\Omega_{\mathtt{Pol}^{\star},t_0,x_0}\right] \geq \mathbb{P}\left[\Omega_{\mathtt{Pol},t_0,x_0}\right]$$

In a sense, any optimal viable policy makes the set of viable scenarios the "largest" possible

Let us introduce the stochastic viability Bellman function

Suppose that the primitive random variables
$\big(w(t_0), w(t_0+1), \ldots, w(T-2), w(T-1)\big)$
are independent under the probability $\mathbb{P}$

Bellman function / stochastic viability value function

Define the probability-to-go as

$$V(t,x) :=$$

$$\max_{\texttt{Pol}} \mathbb{P}\Big(w(\cdot) \in \Omega \mid \overbrace{\texttt{Pol}\big(s,x(s)\big) \in \mathbb{B}\big(s,x(s)\big)}^{\text{control constraints}} \text{ and } \overbrace{x(s) \in \mathbb{A}(s)}^{\text{state constraints}} \text{ for } s \geq t \Big)$$

where $x(s+1) = \texttt{Dyn}\big(s, x(s), \texttt{Pol}\big(s,x(s)\big), w(s)\big)$ and $x(t) = x$

- ▷ The function $V(t,x)$ is called stochastic viability value function or Bellman function
- ▷ The original problem is $V(t_0, x_0)$

The dynamic programming equation is a backward equation satisfied by the stochastic viability value function

Proposition

If the primitive random variables $\big(w(t_0), w(t_0+1), \dots, w(T-2), w(T-1)\big)$ *are independent under the probability* $\mathbb{P}$*, the stochastic viability value function* $V(t,x)$ *satisfies the following backward induction, where* t *runs from* $T-1$ *down to* t_0

$$
\begin{aligned}
V(T,x) &= \mathbf{1}_{\mathbb{A}(T)}(x) \\
V(t,x) &= \mathbf{1}_{\mathbb{A}(t)}(x) \max_{u \in \mathbb{B}(t,x)} \mathbb{E}_{w(t)}\Big[V\Big(t+1, \mathtt{Dyn}\big(t,x,u,w(t)\big)\Big)\Big]
\end{aligned}
$$

Algorithm for the Bellman functions and the stochastic viable controls

initialization $V(T,x) = \mathbf{1}_{\mathbb{A}(T)}(x)$;
for $t = T, T-1, \ldots, t_0$ **do**
    forall $x \in \mathbb{X}$ **do**
        forall $u \in \mathbb{B}(t,x)$ **do**
            $\mathbb{E}_{w(t)}\Big[V\Big(t+1, \texttt{Dyn}\big(t,x,u,w(t)\big)\Big)\Big]$
        $\max_{u\in\mathbb{B}(t,x)} \mathbb{E}_{w(t)}\Big[V\Big(t+1, \texttt{Dyn}\big(t,x,u,w(t)\big)\Big)\Big]$
    $V(t,x) = \mathbf{1}_{\mathbb{A}(t)}(x) \max_{u\in\mathbb{B}(t,x)} \mathbb{E}_{w(t)}\Big[V\Big(t+1, \texttt{Dyn}\big(t,x,u,w(t)\big)\Big)\Big]$

The stochastic viable dynamic programming equation yields stochastic viable policies

For any time t and state x, let us assume that the set

$$\mathbb{B}^{\text{viab}}(t,x) := \operatorname*{argmax}_{u \in \mathbb{B}(t,x)} \Big(\mathbf{1}_{\mathbb{A}(t)}(x) \mathbb{E}_{w(t)} \Big[V\Big(t+1, \texttt{Dyn}(t,x,u,w(t))\Big) \Big] \Big)$$

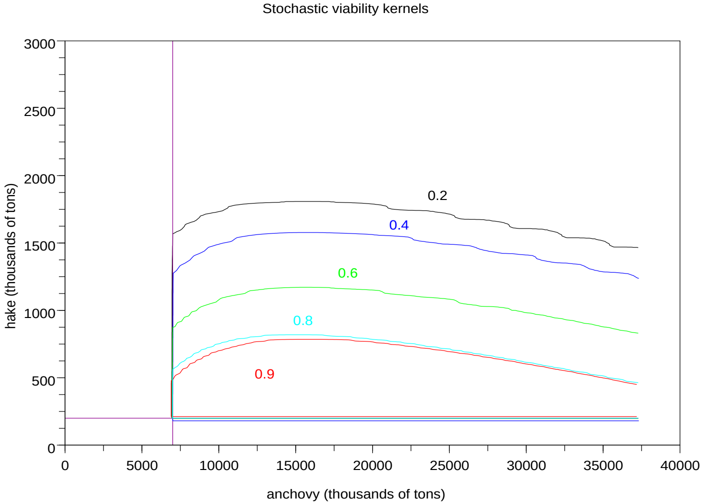
of viable controls is not empty

Proposition

Then, any (measurable) policy $\texttt{Pol}$ *such that* $\texttt{Pol}^{\star}(t,x) \in \mathbb{B}^{viab}(t,x)$ *is an optimal viable policy which achieves the maximal viability probability*

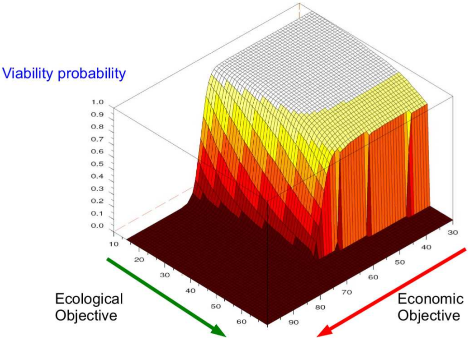
$$V(t_0,x_0) = \max_{\texttt{Pol}} \mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$$

Stochastic viability kernels $\mathbb{V}\mathrm{iab}_{\beta}(t_0)$ for a hake-anchovy fisheries model

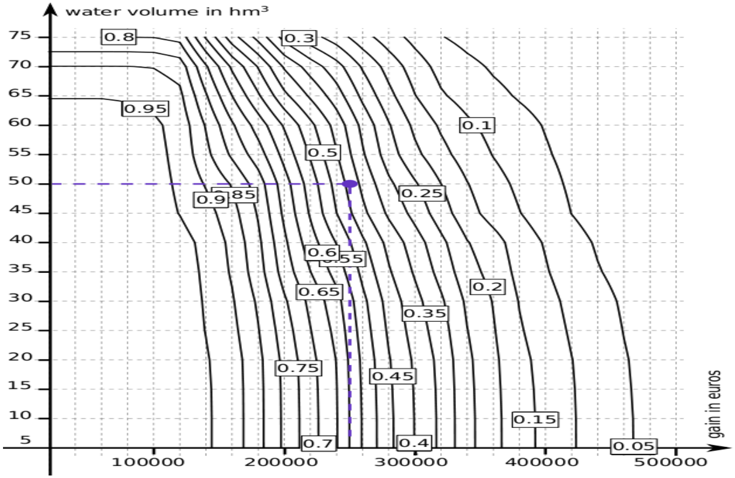


Displaying trade-offs between critical thresholds and risk

$$\mathbb{P}\left[\underbrace{C(t) \geq C^\flat,\ E(t) \geq E^\flat}_{\text{indicators} \geq \text{thresholds}},\ \forall t\right]$$



We plot iso-values for the maximal viability probability as a function of guaranteed thresholds $S^\flat$ and $P^\flat$



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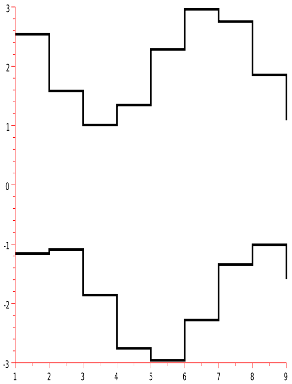
What is resilience?



Resilience is the capacity of a system
to continually change and adapt
yet remain within critical thresholds

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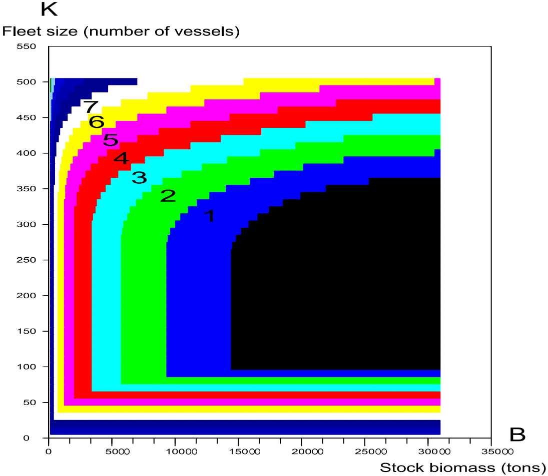
Thus, in a viable state, the system is resilient

Exposure, vulnerability, resilience?

- Acceptable set/viability constraints:
  - possible values for output variables
  - critical thresholds
- Adaptive capacity: set of viable policies
  - = policies depending on available observations and enabling the system to remain within the acceptable set for a certain number of scenarios (expressing the level of risk tolerated)
  - exist only in a viable state
- Exposure: exposure is high when
  - the current variables are close to the acceptable set boundary?
  - the minimal cost among all viable policies is high?
- Vulnerability: acceptable set/viability constraints + adaptive capacity
- Resilience:
  - the more resilient, the lower the costs to reach a viable state
  - the less resilient, the farther from a robust or stochastic viability kernel

The minimal time of crisis and recovery measures the distance to a viability kernel in terms of time units



V. Martinet, L. Doyen, and O. Thébaud. *Defining viable recovery paths toward sustainable fisheries.* Ecological Economics, 64(2):411–422, 2007.

Relaxing some constraints to try and enter into the viability kernel

In mathematical finance, risk is measured as a minimal capital requirement

A measure of risk associates with each position X

- ▷ the minimum extra capital $\rho(X)$
- ▷ required to make it "acceptable" to a regulator.
- ▷ $\rightarrow \rho(X) + X$ is acceptable.

In mathematical finance, risk is measured as a form of minimal distance (gauge) to an acceptance set

- To any (translation invariant) risk measure, we associate the acceptance set

$$\mathcal{A}_\rho = \underbrace{\{X \mid \rho(X) \leq 0\}}_{\text{acceptable prospects}} \ .$$

- To any acceptance set $\mathcal{A}$, we associate a translation invariant risk measure by

$$\rho_{\mathcal{A}}(X) = \inf\{m \in \mathbb{R} \mid m + X \in \mathcal{A}\} \ .$$

Dynamics and policies induce state-control random processes

Given a policy $\texttt{Pol}$, we define a random process

$$w(\cdot) \mapsto \big(x(\cdot), u(\cdot)\big)_{\texttt{Pol}}$$

between scenarios towards state/control trajectories

$$\overbrace{\Omega = \mathbb{W}^{T-t_0}}^{\text{uncertainty trajectories}} \to \overbrace{\mathbb{X}^{T-t_0+1}}^{\text{state trajectories}} \times \overbrace{\mathbb{U}^{T-t_0}}^{\text{control trajectories}}$$

by the closed-loop dynamics

$$\begin{aligned} x(t+1) &= \texttt{Dyn}\big(t, x(t), \texttt{Pol}\big(t, x(t)\big), w(t)\big), \quad t = t_0, \ldots, T-1 \\ u(t) &= \texttt{Pol}\big(t, x(t)\big) \end{aligned}$$

Robust viability corresponds to setting the controlled state-control random process within a product acceptable set

The acceptable set of random processes

$$\mathcal{A} = \{ \big(x(\cdot), u(\cdot) \big) \mid u(t) \in \mathbb{B}\big(t, x(t)\big) \text{ and } x(t) \in \mathbb{A}(t) , \quad \forall t = t_0, \ldots, T \}$$

has a product structure

$$\mathcal{A} = \prod_{t=t_0}^{T} \{ \big(x(t), u(t) \big) \mid u(t) \in \mathbb{B}\big(t, x(t)\big) \text{ and } x(t) \in \mathbb{A}(t) \}$$

Robust and stochastic viability correspond to controlling a random process in relation to an acceptable set of random processes

Find a policy $\mathtt{Pol}$ such that

- robust viability
  the random process $\big(x(\cdot), u(\cdot)\big)_{\mathtt{Pol}}$
  restricted to a subset $\overline{\Omega} \subset \Omega$ of the set Ω of scenarios
  belongs to the acceptable set of random processes $\mathcal{A}$
- stochastic viability
  the probability that the random process $\big(x(\cdot), u(\cdot)\big)_{\mathtt{Pol}}$
  belongs to the acceptable set of random processes $\mathcal{A}$
  is high enough: $\mathbb{P}\{\big(x(\cdot), u(\cdot)\big)_{\mathtt{Pol}} \in \mathcal{A}\} \geq p$
- extensions: more general acceptable sets of random processes? vectorial risk measures?

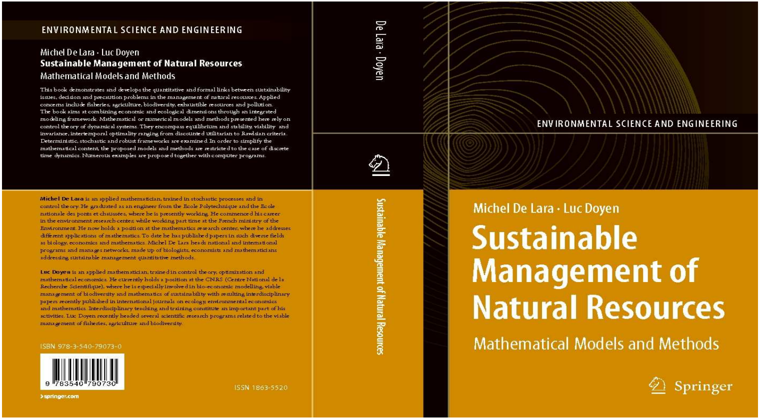
Steps towards an operational definition of resilience

- Identify, forge, fix
  - stages, decision steps
  - possible actions, controls, decisions, together with their restrictions
  - uncertainties, scenarios
  - states, dynamics, system
  - policies, decision rules
  - critical thresholds
- Compute
  - viability kernel = viable states for which policies exist that can keep the system within critical thresholds, despite of uncertainties
  - minimal cost to reach a viability kernel = inverse of resilience

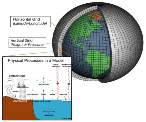
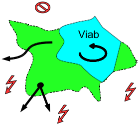
Outline of the presentation

“Nul n’est mieux servi que par soi-même”
“Self-promotion, nobody will do it for you” ;-)

M. De Lara, L. Doyen, Sustainable Management of Natural Resources. Mathematical Models and Methods, *Springer*, 2008.



THANK YOU!



Horizontal Grid
(Latitude-Longitude)
Vertical Grid
(Height or Pressure)
Physical Processes in a Model

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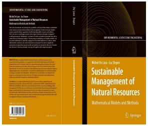
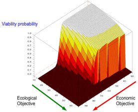
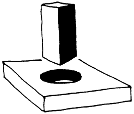
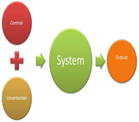
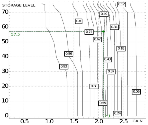
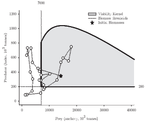
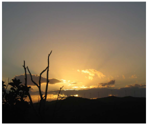
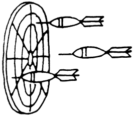
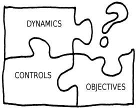

Michel De Lara · Luc Doyen
Sustainable Management of Natural Resources
Mathematical Models and Methods
Springer


Viability probability
Ecological Objective
Economic Objective


Controls
Uncertainties
System
Outputs



STORAGE LEVEL
GAIN

Viability Kernel
Biomass thresholds
Initial Biomasses




DYNAMICS
CONTROLS
OBJECTIVES