Risk and optimization for hydropower management

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PhD Defense

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Risque et optimisation pour le management d'énergies : application à la gestion de l'hydraulique (CIFRE EDF)

Dates: from October, 2010 to December, 2013

Advisors: Michel De Lara (École des Ponts ParisTech) and Pierre Carpentier (ENSTA ParisTech)

Industrial advisors: Laetitia Andrieu (EDF R&D) and Nadia Oudjane (EDF R&D)

Domains: stochastic dynamic optimization applied to hydropower planning

Hydropower

- main renewable energy produced in France
- brings both an energy reserve and a flexibility of great interest in a context of penetration of intermittent sources in the production of electricity

hydropower planning difficulties:

- uncertainties in water inflows and prices
- multiple uses of water
- number of dams

dam management under a tourist constraint:
chance constrained optimization problem

multiple dams cascade management:
large-scale optimization problem

Tourist-constrained dam hydropower management

The tourist constrained optimization problem

Reformulation of the optimality criterion

Stochastic viability approach

Dams cascade hydropower management

Managing a dams cascade: a large scale problem

Decomposition coordination methods: dual approximate dynamic programming

The three dams cascade problem

- manuscript: chapters 1, 2, 3 and 4
- C code: 2500 lines
- papers: a report, a proceeding and a submitted paper
- conferences: IFIP, Berlin (2011) – ISMP, Berlin (2012) – CLAIO, Rio (2012) – and PGMO'days, Palaiseau (2013)

ECONOMIC PURPOSE

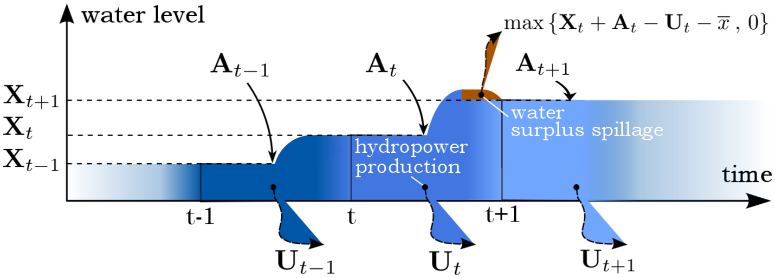
maximize cost savings

TOURIST PURPOSE

favour tourism in summer

We will develop two approaches

- optimization under probabilistic constraint
- stochastic viability

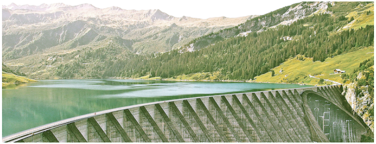


noise variables $\underbrace{\mathbf{A}_{0:T-1} = (\mathbf{A}_0, \ldots, \mathbf{A}_{T-1}), \mathbf{C}_{0:T-1}}_{\text{independent intakes and electricity prices}}$

dynamics $f_t^{\mathbf{X}}$ $\mathbf{X}_{t+1} = \min\{\mathbf{X}_t - \mathbf{U}_t + \mathbf{A}_t\,,\,\overline{x}\}$

bounds $0 \leq \mathbf{X}_t$ and $0 \leq \mathbf{U}_t \leq \overline{u}$

non anticipativity $\mathbf{U}_t \underbrace{\preceq}_{\text{measurable w.r.t.}} \sigma(\mathbf{X}_0, \mathbf{A}_{0:t}, \mathbf{C}_{0:t})$

ECONOMIC PURPOSE	**TOURIST PURPOSE**
	
maximize cost savings	favour tourism in summer

criteria maximization

$$\max_{\mathbf{X}, \mathbf{U}} \mathbb{E}\left[\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t, \mathbf{C}_t\right) + K_T\left(\mathbf{X}_T\right)\right]$$

subject to a chance constraint

$$\mathbb{P}\left[\mathbf{X}_\tau \geq x_{\mathrm{ref}},\ \forall \tau \in \mathcal{T}\right] \geq p_{\mathrm{ref}}$$

Chance-constrained maximization problem:

$$
\begin{aligned}
\max_{\mathbf{X}\,,\,\mathbf{U}} \quad & \mathbb{E}\left[\sum_{t=0}^{T-1} L_t(\mathbf{U}_t,\, \mathbf{C}_t) \;+\; K_T(\mathbf{X}_T)\right] & \\
\text{s.t.} \quad & \mathbf{X}_{t+1} = f_t^{\mathbf{X}}(\mathbf{X}_t,\, \mathbf{U}_t,\, \mathbf{A}_t)\;,\; \forall t & \text{dynamics} \\
& \mathbf{X}_0 = x_0 & \\
& 0 \leq \mathbf{U}_t \leq \min\{\mathbf{X}_t + \mathbf{A}_t\;,\; \overline{u}\}\;,\; \forall t & \text{bounds} \\
& \mathbf{U}_t \preceq \sigma\left(\mathbf{X}_0,\, \mathbf{A}_{0:t},\, \mathbf{C}_{0:t}\right)\;,\; \forall t & \text{non anticipativity} \\
& \mathbb{P}\left[\mathbf{X}_\tau \geq x_{\text{ref}}\;,\; \forall \tau \in \mathcal{T}\right] \geq p_{\text{ref}} & \text{tourist constraint}
\end{aligned}
$$

Admissible set:

$$
\mathfrak{U} = \left\{ \begin{array}{l} \mathbf{X} : \Omega \to \mathbb{R}_+^{T+1} \\ \mathbf{U} : \Omega \to \mathbb{R}_+^{T} \end{array} \;\middle|\; \begin{array}{l} \mathbf{X}_{t+1} = f_t^{\mathbf{X}}(\mathbf{X}_t,\, \mathbf{U}_t,\, \mathbf{A}_t)\;,\; \forall t \\ 0 \leq \mathbf{U}_t \leq \min\{\mathbf{X}_t + \mathbf{A}_t,\, \overline{u}\}\;,\; \forall t \\ \mathbf{U}_t \preceq \sigma\left(\mathbf{X}_0,\, \mathbf{A}_{0:t},\, \mathbf{C}_{0:t}\right)\;,\; \forall t \end{array} \right\}
$$

Chance-constrained optimization problems

introduced for the first time in

1959: Charnes and Cooper (individual chance constraint)

1965: Miller and Wagner (joint chance constraint)

meaningful to operations managers, fit well to some industrial problems

hard to handle due to theoretical and numerical difficulties

- connectedness, convexity and closedness of the induced admissible set issues
- differential calculus, stability issues

some references:

Prékopa, 2003   Henrion, 2004   Dentcheva, 2009   Nemirovski, 2012

Chance-constrained optimization problems

Theoretical results under assumptions over

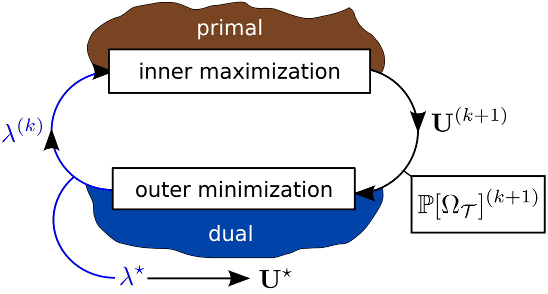
- the constraint structure: individual/joint, linear or separable w.r.t. the noise
- the noise distributions: continuous/discrete, independence, quasi/generalized concavity
- the information pattern: open/closed loop

No result applies to our case (up to our knowledge)

We dualize the chance constraint and write the maximization as a min-max problem (equivalent if a saddle point exists)

$$\min_{\lambda \in \mathbb{R}_+} \overbrace{\max_{\mathbf{X},\, \mathbf{U} \in \mathfrak{U}} \quad \mathcal{L}(\mathbf{U},\, \lambda)}^{\text{inner maximization}}$$

$$\text{where} \quad \mathcal{L}(\mathbf{U},\, \lambda) = \mathbb{E}\left[\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t\,,\, \mathbf{C}_t\right) + K_T(\mathbf{X}_T)\right] + \lambda\left(\mathbb{P}\left[\Omega_{\mathcal{T}}\right] - p_{\text{ref}}\right)$$



Inner maximization (fixed $\lambda^{(k)}$): dynamic programming

independent noise random variables ✓

additive criterion with respect to time ×

$$\max_{\mathbf{X}\,,\,\mathbf{U}\in\mathfrak{U}} \ \mathbb{E}\left[\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t,\, \mathbf{C}_t\right) + K_T(\mathbf{X}_T)\right] + \lambda^{(k)}\left(\mathbb{P}\left[\Omega_{\mathcal{T}}\right] - p_{\mathrm{ref}}\right)$$

$$\max_{\mathbf{X},\,\mathbf{U}\in\mathfrak{U}} \ \mathbb{E}\left[\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t,\, \mathbf{C}_t\right) + K_T(\mathbf{X}_T) + \lambda^{(k)}\left(\prod_{\tau\in\mathcal{T}} \mathbf{1}_{\{\mathbf{X}_\tau \geq x_{\mathrm{ref}}\}} - p_{\mathrm{ref}}\right)\right]$$

introduction of a binary state variable: we set $\boldsymbol{\pi}_0 = 1$

$$\boldsymbol{\pi}_{t+1} = f_t^{\boldsymbol{\pi}}(\mathbf{X}_t,\, \boldsymbol{\pi}_t,\, \mathbf{U}_t,\, \mathbf{A}_t) = \begin{cases} \mathbf{1}_{\{f_t^{\mathbf{X}}(\mathbf{X}_t,\,\mathbf{U}_t,\,\mathbf{A}_t) \geqslant x_{\mathrm{ref}}\}} \times \boldsymbol{\pi}_t & \text{if} \quad t+1 \in \mathcal{T} \\ \boldsymbol{\pi}_t & \text{else} \end{cases}$$

$$\rightarrow \max_{\mathbf{X},\,\boldsymbol{\pi},\,\mathbf{U}\in\mathfrak{U}} \ \mathbb{E}\left[\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t\,,\, \mathbf{C}_t\right) + K_T(\mathbf{X}_T) + \lambda^{(k)}\left(\boldsymbol{\pi}_T - p_{\mathrm{ref}}\right)\right]$$

Dynamic programming (fixed $\lambda^{(k)}$) with extended state $(\mathbf{X}, \boldsymbol{\pi})$:
we solve the following equations backward in time

$$\begin{cases} V_T(x,\, \pi) = K_T(x) \,+\, \lambda^{(k)} \, (\pi \,-\, p_{\mathrm{ref}}) \,, \\ V_t(x,\, \pi) = \mathbb{E}\left[\max_{u \in \mathfrak{U}_t(x,\, \mathbf{A}_t)} \, L_t\,(u,\, \mathbf{C}_t) + V_{t+1} \begin{pmatrix} f_t^{\mathbf{X}}(x,\, u,\, \mathbf{A}_t)\,, \\ f_t^{\boldsymbol{\pi}}(x,\, \pi,\, u,\, \mathbf{A}_t) \end{pmatrix} \right] \end{cases}$$

where

$$\mathfrak{U}_t(x,\, w) = \left\{ u \in \mathbb{R}_+^T \;\middle|\; u \leq \min\{x + w,\, \overline{u}\} \right\}$$

we obtain feedback laws

$$\chi_0^{(k+1)}, \, \ldots, \, \chi_{T-1}^{(k+1)}$$

from which we deduce the optimal control trajectories

$$\mathbf{U}_0^{(k+1)}, \, \ldots, \, \mathbf{U}_{T-1}^{(k+1)}$$

Outer minimization (known $\chi_{0:T-1}^{(k+1)}$): gradient step

- probability evaluation

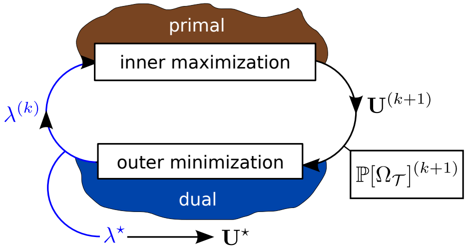
$$\begin{cases} V_T^{\boldsymbol{\pi}}(x,\,\pi) = \pi, \\ V_t^{\boldsymbol{\pi}}(x,\,\pi) = \mathbb{E}\left[V_{t+1}^{\boldsymbol{\pi}}\begin{pmatrix} f_t^{\mathbf{X}}(x,\,\chi_t^{(k+1)}(x,\,\mathbf{A}_t),\,\mathbf{A}_t), \\ f_t^{\boldsymbol{\pi}}(x,\,\pi,\,\chi_t^{(k+1)}(x,\,\mathbf{A}_t),\,\mathbf{A}_t) \end{pmatrix}\right] \end{cases}$$

we get

$$p^{(k+1)} = \mathbb{P}\left[\Omega_{\mathcal{T}}\right] = \mathbb{E}\left[\boldsymbol{\pi}_T\right] = V_0^{\boldsymbol{\pi}}(x_0,\,1)$$

- multiplier update

$$\lambda^{(k+1)} = \max\left\{\lambda^{(k)} - \rho\left(p^{(k+1)} - p_{\text{ref}}\right),\,0\right\}$$



Theorem (Everett 1963)

If the algorithm converges to a solution $\mathbf{U}^\star$ *such that the chance constraint is binding,*

$$\mathbb{P}\left[\Omega_{\mathcal{T}}\right] = \mathbb{E}\left[\boldsymbol{\pi}_T^\star\right] = p_{\text{ref}},$$

then $\mathbf{U}^\star$ *is an optimal solution*

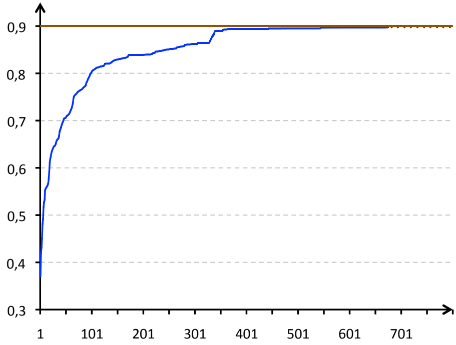
Numerical experiment

Dam problem instance

- time horizon: $\{1, \ldots, 12\}$ and $\mathcal{T} = \{7, 8\}$
- $\overline{x} = 80\,\mathrm{hm}^3$, $\overline{u} = 40\,\mathrm{hm}^3$ and $x_0 = 40\,\mathrm{hm}^3$
- $x_{\mathrm{ref}} = 50\,\mathrm{hm}^3$ and $p_{\mathrm{ref}} = 0.9$
- expectations are computed as the mean over $|\mathbf{A}_t| \times |\mathbf{C}_t| = 10 \times 20$ values that define all of the possible noise values, for each t
  $\rightarrow$ exact computations
- the state grid is $|\mathbf{X}| \times |\boldsymbol{\pi}| = 40 \times 2$, the control is discretized in 20 values and the intakes noise values are multiples of $2\,\mathrm{hm}^3$
  $\rightarrow$ no need to interpolate

Numerical results

Probability level along the iterations



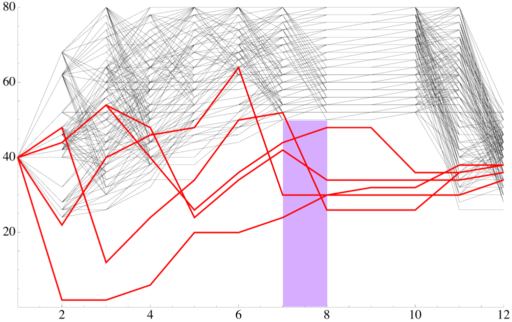
the algorithm converges to a solution

binding chance constraint: optimal solution (Everett)

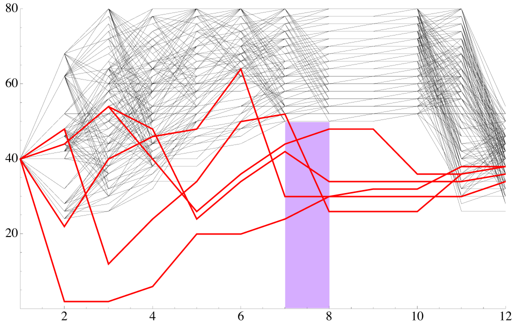
optimal multiplier value: 111 028 (1110 euros per probability %)

optimal expected cost savings: 250 136 euros

Water level trajectories with 5 “non tourist” trajectories



Water level trajectories with 5 “non tourist” trajectories



commentaries of the operations manager

- “deliberate renunciation”
- “excessive” turbined outflows
- “premature renunciation”

Tourist-constrained dam hydropower management

- The tourist constrained optimization problem
- Reformulation of the optimality criterion
- Stochastic viability approach

Dams cascade hydropower management

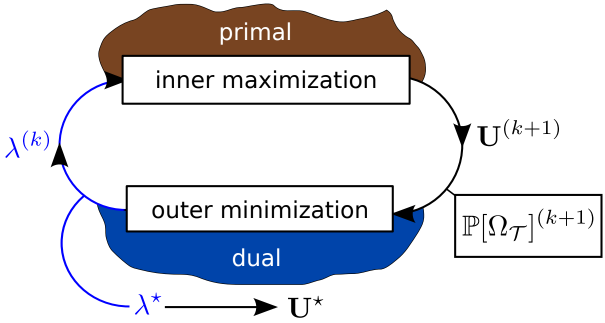
- Managing a dams cascade: a large scale problem
- Decomposition coordination methods: dual approximate dynamic programming
- The three dams cascade problem

We propose the following reformulation of the criterion

- we focus the optimization process on the tourist event realization by only giving weight to the tourist trajectories :

$$\begin{aligned} \max_{\mathbf{X}, \mathbf{U}} \quad & \mathbb{E}\left[\left(\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t, \mathbf{C}_t\right) + K_T\left(\mathbf{X}_T\right)\right) \mathbf{1}_{\Omega_{\mathcal{T}}}\right] \\ \text{s.t.} \quad & \mathbb{P}\left(\Omega_{\mathcal{T}}\right) \geq \mathrm{p}_{\mathrm{ref}} \end{aligned}$$

- we let the operations manager deal with the other trajectories


primal
inner maximization
$\lambda^{(k)}$
$\mathbf{U}^{(k+1)}$
outer minimization
$\mathbb{P}[\Omega_{\mathcal{T}}]^{(k+1)}$
dual
$\lambda^{\star}$
$\mathbf{U}^{\star}$

Inner maximization: dynamic programming

$$\max_{\mathbf{X},\,\boldsymbol{\pi},\,\boldsymbol{\sigma},\,\mathbf{U}} \mathbb{E}\left[\underbrace{\left(\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t,\,\mathbf{C}_t\right) + K_T\left(\mathbf{X}_T\right)\right)}_{\boldsymbol{\sigma}_T} \boldsymbol{\pi}_T + \lambda^{(k)}\left(\boldsymbol{\pi}_T - p_{\text{ref}}\right)\right]$$

introduction of a new state $\boldsymbol{\sigma}$ (*cumulated gain process*): the dynamic programming equations become

$$\begin{cases} V_T(x,\,\sigma,\,\pi) = \pi \times \sigma + \lambda^{(k)}\left(\pi - p_{\text{ref}}\right) \\ V_t(x,\,\sigma,\,\pi) = \mathbb{E}\left[\max\limits_{u \in \mathfrak{U}_t(x,\,\mathbf{A}_t)} V_{t+1}\left(\mathbf{X}_{t+1},\,\boldsymbol{\sigma}_{t+1},\,\boldsymbol{\pi}_{t+1}\right)\right] \end{cases}$$

Outer minimization: same gradient step method

Numerical experiment

Dam problem instance

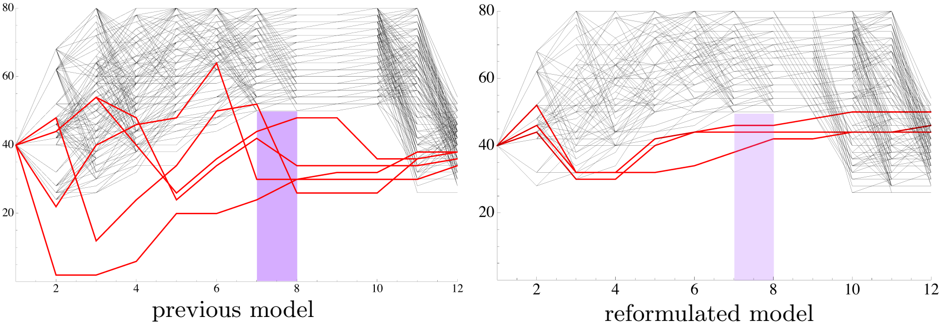
Same instance, except that $p_{\text{ref}} = 0.99$ since fixing $p_{\text{ref}} = 0.9$ makes the chance constraint inactive

Comparison to the previous model

Optimal strategy only designed for tourist trajectories:

- we apply a fixed turbined strategy to the other trajectories
- with this strategy, we compute the true economical criterion

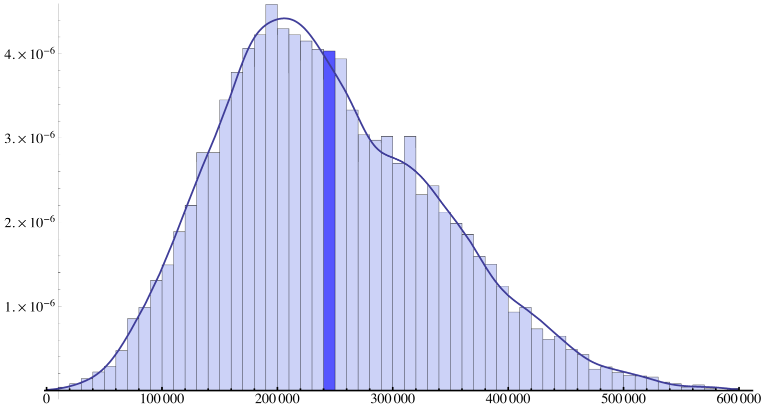
$$\mathbb{E}[\mathbf{G}] = \mathbb{E}\left[\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t, \mathbf{C}_t\right) + K_T\left(\mathbf{X}_T\right)\right]$$



	previous model	reformulated model
trajectories aspect	not OK	OK
expected cost savings (in euros)	$232,529$	$209,590$

The price to pay for “acceptable” non tourist trajectories is $\approx 10\%$

Empirical distribution of the cost savings



The gain is very dispersed

economic purpose

maximize cost savings

tourist purpose

favour tourism in summer

$$\max_{\mathbf{X},\mathbf{U}} \quad \mathbb{P}\left[\mathbf{G} \geq g_{\text{ref}} \text{ and } \mathbf{X}_\tau \geq x_{\text{ref}}\,, \quad \forall \tau \in \mathcal{T}\right]$$

$$\text{s.t.} \quad \mathbf{X}_{t+1} = f_t^{\mathbf{X}}\left(\mathbf{X}_t,\, \mathbf{U}_t,\, \mathbf{A}_t\right), \forall t \in \{0, \ldots, T-1\}$$

$$\mathbf{X}_0 = x_0\,,$$

This way, we symmetrize the economic and the tourist stakes whereas the first one was in the criterion $\mathbb{E}\left[\mathbf{G}\right]$ to maximize and the latter one was a chance constraint

Using the cumulated cost savings process $\boldsymbol{\sigma}$, the problem reads:

$$\max_{\mathbf{X}, \boldsymbol{\sigma}, \mathbf{U}} \quad \mathbb{E}\left[\prod_{\tau \in \mathcal{T}} \mathbf{1}_{\{\mathbf{X}_\tau \geq x_{\text{ref}}\}} \times \mathbf{1}_{\{\boldsymbol{\sigma}_T \geq g_{\text{ref}}\}}\right]$$

$$\text{s.t.} \quad \begin{aligned} & \mathbf{X}_{t+1} = f_t^{\mathbf{X}}\left(\mathbf{X}_t, \mathbf{U}_t, \mathbf{A}_t\right), \ \forall t \in \{0, \ldots, T-1\} \\ & \mathbf{X}_0 = x_0 \\ & \boldsymbol{\sigma}_{t+1} = f_t^{\boldsymbol{\sigma}}\left(\mathbf{X}_t, \boldsymbol{\sigma}_t, \mathbf{U}_t, \mathbf{C}_t\right), \forall t \in \{0, \ldots, T-1\} \\ & \boldsymbol{\sigma}_0 = 0 \end{aligned}$$

Note that the criterion is multiplicative w.r.t. time

Theorem: multiplicative dynamic programming

Solving the dynamic programming equations

$$V_T(x, \sigma) = \mathbf{1}_{\{\sigma \geq g_{\mathrm{ref}}\}} ,$$

$$\forall t \in \mathcal{T}, \quad V_t(x, \sigma) =$$

$$\mathbb{E}\left[\max_{u \in \mathfrak{U}_t(x, \mathbf{A}_t)} \mathbf{1}_{\{x \geq x_{\mathrm{ref}}\}} \times V_{t+1}\left(f_t^{\mathbf{X}}(x, u, \mathbf{A}_t), f_t^{\boldsymbol{\sigma}}(x, \sigma, u, \mathbf{C}_t)\right)\right]$$

$$\forall t \notin \mathcal{T} \cup \{T\}, \quad V_t(x, \sigma) =$$

$$\mathbb{E}\left[\max_{u \in \mathfrak{U}_t(x, \mathbf{A}_t)} V_{t+1}\left(f_t^{\mathbf{X}}(x, u, \mathbf{A}_t), f_t^{\boldsymbol{\sigma}}(x, \sigma, u, \mathbf{C}_t)\right)\right]$$

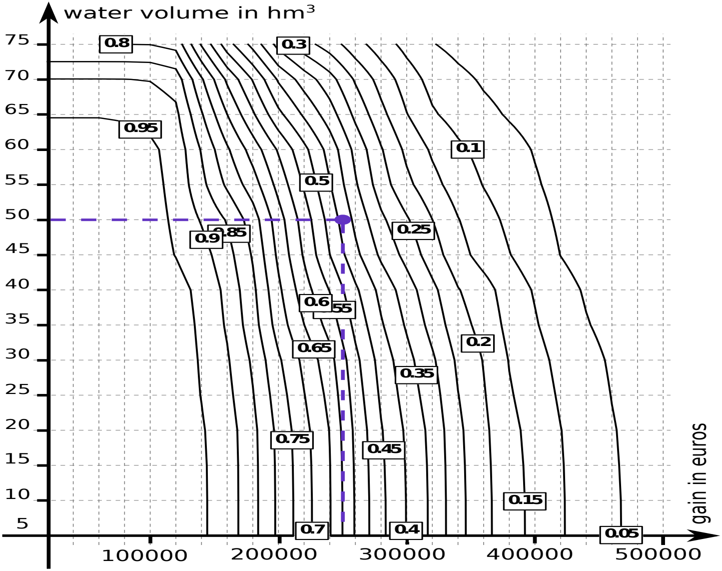
gives the solution of the stochastic viability problem

We now imbed the multiplicative dynamic programming algorithm in a loop where the thresholds $(x_{\text{ref}}, g_{\text{ref}})$ vary to compute the isovalues of the maximal viability probability as function of the guaranteed gain and stock g_{ref} and x_{ref}

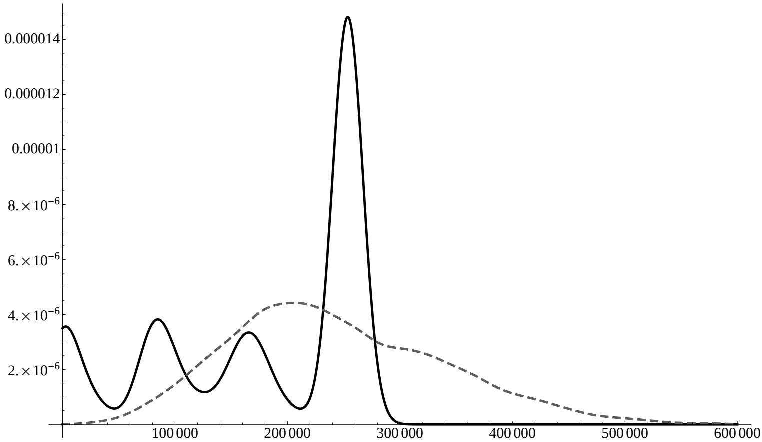
---

for every gain value g_{ref} **do**
  for every storage level x_{ref} **do**
    solve: the dynamic programming equations
    save: $\phi^*(x_{\text{ref}}, g_{\text{ref}}) = V_0(x_0, \sigma_0)$
  end for
end for

Isovalues of the viability probability

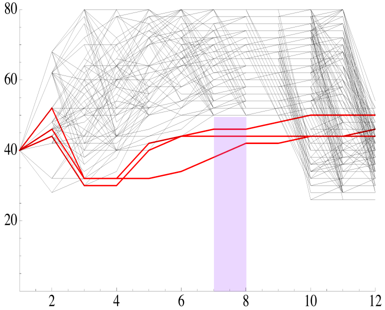


Empirical gain distributions (previous gain is dotted)

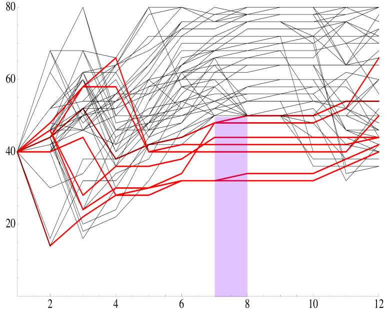


$g_{\text{ref}} = 250,136$ and $x_{\text{ref}} = 50$

Storage level trajectories



reformulated model



stochastic viability model

Conclusion

- tourist-constrained dam hydropower management relevant for operations managers
- extension to n level-constraints (Chapter 3)
- complementary approach (stochastic viability)

Perspectives

- extension to dependent probability distributions or to continuous probability distributions
- extension to dams cascade hydropower management

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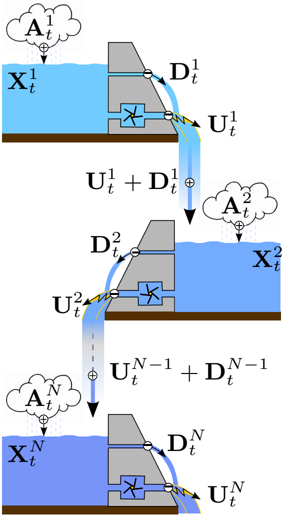
- manuscript: chapters 5, 6 and 7
- C code: 4500 lines
- papers: one proceeding, a paper under writing
- conferences: ICSP, Bergame (2013) and PGMO'days, Palaiseau (2013)

Optimal management of a N dams cascade hydroelectric production by means of a Discrete Time Stochastic Optimal Control Problem



Optimal management of a N dams cascade hydroelectric production by means of a Discrete Time Stochastic Optimal Control Problem

state	$\mathbf{X}_t^i$: storage level
noise	$\mathbf{A}_t^i$: exogeneous inflows
control	$\mathbf{U}_t^i$: turbinated water
	$\mathbf{D}_t^i$: spilled water surplus

$\rightarrow$ N state and N control variables

Dynamic Programming:
untractable as soon as N > 4

Methods to deal with large-scale optimization problems

- **Stochastic Programming**
  model the problem using the scenario tree
- **Dynamic Programming**
  - **Aggregation Methods**
  - **Approximate Dynamic Programming**
  - **Stochastic Dual Dynamic Programming**
    Bellman function approximation by cuts
- **Decomposition/Coordination Methods**

Decomposition coordination methods: main ideas

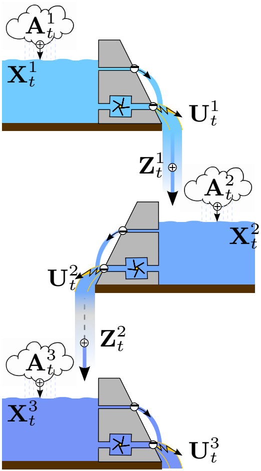
1. decompose a large scale problem into smaller subproblems susceptible to be solved by efficient algorithms
2. coordinate the subproblems to forge the initial problem solution

How to decompose the problem:

1. identify the coupling dimensions of the problem: *time*, *space*, *uncertainty*
2. dualize the coupling constraints linked to the dimension over which the problem is to be decomposed
3. split the problem into the resulting subproblems

The optimization problem we are interested in:

$$\begin{aligned} \max_{\mathbf{X},\, \mathbf{Q} \in \mathfrak{U}} \quad & \mathbb{E}\left[\sum_{i=1}^{N} \sum_{t=0}^{T} G_t^i\big(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i\big) \right] \\ \text{s.t.} \quad & \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{Q}_t^i, \mathbf{W}_t^i) \;,\; \forall (t\;,\; i) \\ & \mathbf{Q}_t^i \preceq \mathcal{F}_t \;,\; \forall (t\;,\; i) \\ & \sum_{i=1}^{N} \Theta_t^i(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i) = 0 \;,\; \forall t \end{aligned}$$



$$\max_{\mathbf{X},\,\mathbf{U}\in\mathfrak{U}} \ \mathbb{E}\left[\sum_{i=1}^{3}\sum_{t=0}^{T-1} L_t\left(\mathbf{U}_t^i,\,\mathbf{Z}_t^i\right) + K_T^i\left(\mathbf{X}_T^i\right)\right]$$

f_t^i :

$$\mathbf{X}_{t+1}^i = \min\left\{\mathbf{X}_t^i + \mathbf{A}_t^i - \mathbf{U}_t^i + \mathbf{Z}_t^i,\,\overline{x}^i\right\}$$

$$\mathcal{F}_t = \sigma\left(\mathbf{A}_{0:t}^{1:N}\right)$$

g_t^i :

$$\mathbf{Z}_t^{i+1} = \max\left\{\mathbf{X}_t^i + \mathbf{A}_t^i + \mathbf{Z}_t^i - \overline{x}^i,\,\mathbf{U}_t^i\right\}$$

$$\sum_{i=1}^{3}\Theta_t^i(\mathbf{X}_t^i,\,\mathbf{U}_t^i,\,\mathbf{W}_t^i) = 0 :$$

$$\begin{cases}\mathbf{Z}_t^2 - g_t^1\left(\mathbf{X}_t^1,\,\mathbf{U}_t^1,\,\mathbf{A}_t^1,\,\mathbf{Z}_t^1\right) = 0\\ \mathbf{Z}_t^3 - g_t^2\left(\mathbf{X}_t^2,\,\mathbf{U}_t^2,\,\mathbf{A}_t^2,\,\mathbf{Z}_t^2\right) = 0\end{cases}$$

The optimization problem we are interested in:

$$\begin{aligned} \max_{\mathbf{X},\, \mathbf{Q} \in \mathfrak{U}} \quad & \mathbb{E}\left[\sum_{i=1}^{N} \sum_{t=0}^{T} G_t^i\big(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i\big) \right] \\ \text{s.t.} \quad & \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{Q}_t^i, \mathbf{W}_t^i) \,,\ \forall (t\,,\, i) \\ & \mathbf{Q}_t^i \preceq \mathcal{F}_t \,,\ \forall (t\,,\, i) \\ & \sum_{i=1}^{N} \Theta_t^i(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i) = 0 \,,\ \forall t \end{aligned}$$

$(\boldsymbol{\lambda}_t)_{t\in\{0,\dots,T\}}$: $\mathcal{F}_t$ -adapted processes of the coupling constraints multipliers. By dualization:

$$\max_{\substack{\mathbf{X},\, \mathbf{Q} \in \mathfrak{U} \\ \mathbf{Q}_t \preceq \mathcal{F}_t}} \; \min_{\boldsymbol{\lambda}} \; \mathbb{E}\left[\sum_{i=1}^{N}\sum_{t=0}^{T} G_t^i\big(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i\big) + \langle \boldsymbol{\lambda}_t,\, \Theta_t^i(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i)\rangle\right]$$

$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{Q}_t^i, \mathbf{W}_t^i) \;,\; \forall (t \,,\, i)$$

Assuming the existence of a saddle point, we can exchange the min and max operators:

$$\min_{\boldsymbol{\lambda}} \sum_{i=1}^{N} \max_{\substack{\mathbf{X}^i,\, \mathbf{Q}^i \in \mathfrak{U}^i \\ \mathbf{Q}_t^i \preceq \mathcal{F}_t}} \; \mathbb{E}\left[\sum_{t=0}^{T} G_t^i\big(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i\big) + \langle \boldsymbol{\lambda}_t,\, \Theta_t^i(\mathbf{X}_t^i,\, \mathbf{Q}_t^i,\, \mathbf{W}_t^i)\rangle\right]$$

$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{Q}_t^i, \mathbf{W}_t^i) \;,\; \forall t$$

Uzawa algorithm: at step k and for a given $(\boldsymbol{\lambda})^{(k)}$,

1. we solve N problems $(\mathcal{P}_i)$ that are

$$\max_{\substack{\mathbf{X}^i,\, \mathbf{Q}^i \\ \mathbf{Q}^i_t \preceq \mathcal{F}_t}} \quad \mathbb{E}\left[\sum_{t=0}^{T} G^i_t\big(\mathbf{X}^i_t,\, \mathbf{Q}^i_t,\, \mathbf{W}^i_t\big) + \langle \boldsymbol{\lambda}_t,\, \Theta^i_t(\mathbf{X}^i_t,\, \mathbf{Q}^i_t,\, \mathbf{W}^i_t)\rangle\right]$$

$$\text{s.t.} \quad \mathbf{X}^i_{t+1} = f^i_t(\mathbf{X}^i_t, \mathbf{Q}^i_t, \mathbf{W}^i_t)\ ,\ \forall t$$

2. we update the multipliers by a gradient method

$$(\boldsymbol{\lambda}_t)^{(k+1)} = (\boldsymbol{\lambda}_t)^{(k)} + \rho \sum_{j=1}^{N} \Theta^i_t\left(\big(\mathbf{X}^i_t,\, \mathbf{Q}^i_t,\, \mathbf{W}^i_t\big)^{(k+1)}\right)\ ,\ \forall t$$

The subproblems $(\mathcal{P}_i)$:

- are small size standard SOC problems
- involve state variables that follow Markovian dynamics

their solutions should be computable by Dynamic Programming

But:

- the noise processes in $(\mathcal{P}_i)$ are $\mathbf{W}$ and $(\boldsymbol{\lambda})^{(k+1)}$
- $(\boldsymbol{\lambda})^{(k+1)}$ has no reason to be white or Markovian

we cannot solve $(\mathcal{P}_i)$ by dynamic programming with the state $\mathbf{X}^i$

The idea of DADP: replacing the multipliers by their conditional expectations w.r.t. chosen information variables $\mathbf{Y}_t$, namely

$$\mathbb{E}\left[(\boldsymbol{\lambda}_t)^{(k)} \middle| \mathbf{Y}_t\right]$$

$\rightarrow$ We transfer the measurability problem of $(\boldsymbol{\lambda})^{(k)}$ to the measurability issue of a chosen additional variable $(\mathbf{Y}_t)$

Equivalent to replace the space coupling constraints by (Girardeau, 2010)

$$\mathbb{E}\left[\sum_{i=1}^{N} \Theta_t^i(\mathbf{X}_t^i, \mathbf{Q}_t^i, \mathbf{W}_t^i) \middle| \mathbf{Y}_t\right] = 0, \quad \forall i$$

The choice of the information variable:

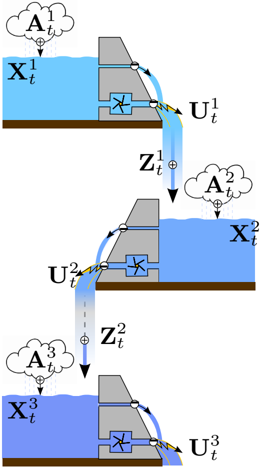
- is in the hands of the user
- can have a great impact on the efficiency of the DADP algorithm

In practice, $\mathbf{Y}_t$ is a short-memory process. Possible choices are:

(1) $\mathbf{Y}_t \equiv$ cste: we deal with the constraint in expectation

(2) $\mathbf{Y}_t = \varphi_t(\mathbf{W}_t)$: we incorporate a noise

(3) $\mathbf{Y}_{t+1} = \tilde{f}_t(\mathbf{Y}_t, \mathbf{W}_t)$: we incorporate a new state variable in the problem



Information variables

(1) $\mathbf{Y}_t \equiv$ cste: we deal with the constraint in expectation

(2) $\mathbf{Y}_t = (\mathbf{A}_t^1, \mathbf{A}_t^2)$: we incorporate the upstream exogeneous inflows

(3) $\mathbf{Y}_{t+1} = \tilde{f}_t^1(\mathbf{Y}_t, \mathbf{A}_t^1)$: we mimic the first dam storage level

(1) $\mathbf{Y}_t \equiv$ cste: we deal with the constraint in expectation

The DP equation for $(\mathcal{P}_i)$ reads:

$$V_T^i(x) = \mathbb{E}\left[\max_q \; G_T^i\left(x,\, q,\, \mathbf{W}_T\right) + \left\langle \mathbb{E}[(\boldsymbol{\lambda}_T)^{(k)}],\, \Theta_T^i(x,\, q,\, \mathbf{W}_T^i)\right\rangle\right]$$

$$V_t^i(x) = \mathbb{E}\left[\max_q \; \left\{ \begin{array}{l} G_t^i\left(x,\, q,\, \mathbf{W}_t\right) + V_{t+1}^i\left(f_t^i(x,\, q,\, \mathbf{W}_t)\right) \\ + \left\langle \mathbb{E}[(\boldsymbol{\lambda}_t)^{(k)}],\, \Theta_t^i(x,\, q,\, \mathbf{W}_t^i)\right\rangle \end{array} \right\}\right]$$

no additional state variable

(2) $\mathbf{Y}_t = \varphi_t(\mathbf{W}_t)$: we incorporate a noise

The DP equation for $(\mathcal{P}_i)$ reads:

$$V_T^i(x) = \mathbb{E}\left[\max_q \left\{ \begin{array}{l} G_T^i\left(x,\, q,\, \mathbf{W}_T^i\right) \\ + \left\langle \mathbb{E}[(\boldsymbol{\lambda}_T)^{(k)} | \varphi_T(\mathbf{W}_T)],\, \Theta_T^i(x,\, q,\, \mathbf{W}_T^i) \right\rangle \end{array} \right\}\right]$$

$$V_t^i(x) = \mathbb{E}\left[\max_q \left\{ \begin{array}{l} G_t^i\left(x,\, q,\, \mathbf{W}_t^i\right) + V_{t+1}^i\left(f_t^i(x,\, q,\, \mathbf{W}_t^i)\right) \\ + \left\langle \mathbb{E}[(\boldsymbol{\lambda}_t)^{(k)} | \varphi_t(\mathbf{W}_t)],\, \Theta_t^i(x,\, q,\, \mathbf{W}_t^i) \right\rangle \end{array} \right\}\right]$$

no additional state variable

(3) $\mathbf{Y}_{t+1} = \tilde{f}_t(\mathbf{Y}_t, \mathbf{W}_t)$: we add a non controlled variable to the state

The DP equation for $(\mathcal{P}_i)$ reads:

$$V_T^i(x, y) = \mathbb{E}\left[\max_q \left\{ \begin{array}{l} G_T^i\left(x, q, \mathbf{W}_T^i\right) \\ + \left\langle \mathbb{E}[(\boldsymbol{\lambda}_T)^{(k)} | \mathbf{Y}_T = y], \Theta_T^i(x, q, \mathbf{W}_T^i)\right\rangle \end{array}\right\}\right]$$

$$V_t^i(x, y) = \mathbb{E}\left[\max_q \left\{ \begin{array}{l} G_t^i\left(x, q, \mathbf{W}_t^i\right) \\ + V_{t+1}^i\left(f_t^i(x, q, \mathbf{W}_t^i), \tilde{f}_t(y, \mathbf{W}_t)\right) \\ + \left\langle \mathbb{E}[(\boldsymbol{\lambda}_t)^{(k)} | \mathbf{Y}_t = y], \Theta_t^i(x, q, \mathbf{W}_t^i)\right\rangle \end{array}\right\}\right]$$

additional state variable

Update of the conditional expectation of the multipliers w.r.t. $\mathbf{Y}_t$.

- save the strategies computed at i for the fixed $(\boldsymbol{\lambda}_t)^{(k)}$
- use these strategies to simulate the trajectories $(X_t^i, U_t^i, W_t, Y_t^i)_l^{(k+1)}$ over given scenarios
- estimate the conditional expectation

$$\mathbb{E}\left[\sum_{i=1}^{N} \Theta_t^i(X_t^i, U_t^i, W_t^i) \middle| \mathbf{Y}_t\right]$$

- update the multipliers conditional expectations by a gradient method

At this point, the algorithm solves

$$\max_{\mathbf{X},\,\mathbf{Q}} \mathbb{E}\left[\sum_{i=1}^{N}\sum_{t=0}^{T} G_t^i\big(\mathbf{X}_t^i,\,\mathbf{Q}_t^i,\,\mathbf{W}_t^i\big)\right] \text{ s.t. } \mathbb{E}\left[\sum_{i=1}^{N}\Theta_t^i(\mathbf{X}_t^i,\,\mathbf{Q}_t^i,\,\mathbf{W}_t^i)\middle|\mathbf{Y}_t\right] = 0$$

which is different from the initial problem

$$\max_{\mathbf{X},\,\mathbf{Q}} \mathbb{E}\left[\sum_{i=1}^{N}\sum_{t=0}^{T} G_t^i\big(\mathbf{X}_t^i,\,\mathbf{Q}_t^i,\,\mathbf{W}_t^i\big)\right] \text{ s.t. } \sum_{i=1}^{N}\Theta_t^i(\mathbf{X}_t^i,\,\mathbf{Q}_t^i,\,\mathbf{W}_t^i) = 0$$

- We use heuristics to compute a feasible strategy

$$\text{Bellman function approximation: } \quad V \approx \sum_{i=1}^{N} V^i$$

We solve the three dams cascade problem by DADP

- can be solved exactly by dynamic programming
  → "accurate" choice of the information variables
  → DADP efficiency evaluation
- extendable to $N > 3$ dams cascade problems
  → first step to solve large-scale dams cascades

Tourist-constrained dam hydropower management

The tourist constrained optimization problem

Reformulation of the optimality criterion

Stochastic viability approach

Dams cascade hydropower management

Managing a dams cascade: a large scale problem

Decomposition coordination methods: dual approximate dynamic programming

The three dams cascade problem

Numerical experiments

Dams cascade instance

horizon: $T = 12$

state:

$$\mathbf{X}_t^i \in \{0, 2, \ldots, 80\}, \forall (i, t)$$

control:

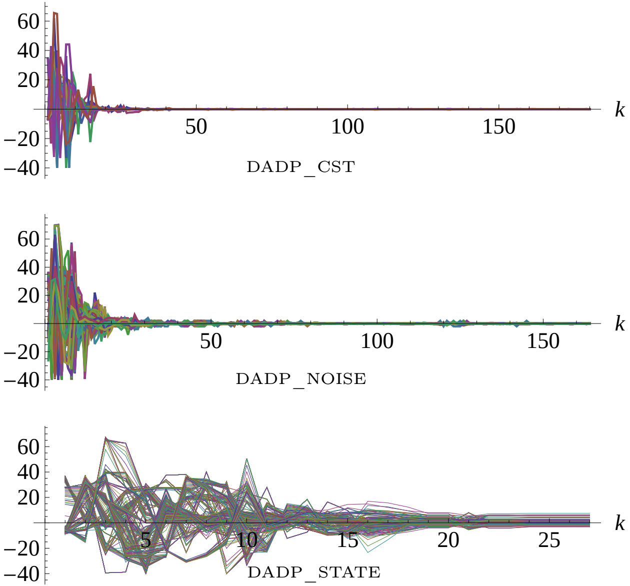
$$\mathbf{U}_t^i \in \{0, 8, \ldots, 40\}, \forall (i, t)$$
$$\mathbf{Z}_t^2 \in \{0, 2, \ldots, 40\} \text{ and } \mathbf{Z}_t^3 \in \{0, 2, \ldots, 80\}, \forall t$$

noise:

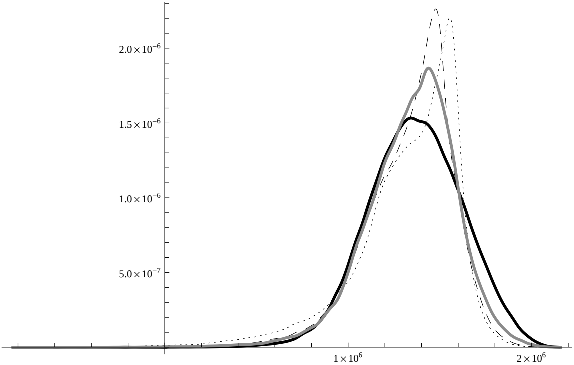
$$\mathbf{W}_t^i \in \{0, 2, \ldots, 32\}, \forall (i, t)$$

100,000 scenarios to compute conditional expectations

Deviation from coupling constraints respect along the iterations



Empirical cost savings distributions



Means:

- DP (black): $1,365,770$ euros
- DADP_CST (dashed): $1,334,900$ euros (-2.3%)
- DADP_NOISE (line-dotted): $1,320,740$ euros (-3.3%)
- DADP_STATE (gray): $1,344,180$ euros (-1.6%)

Conclusion

- encouraging results
  - numerical convergence of the algorithm
  - satisfactory numerical results
- more information does not imply better results (heuristics)
- first use of a dynamic information variable in DADP

Perspectives

- try other methods to compute conditionnal expectations
- realistic dams cascade problems
- theoretical studies (convergence proof, epiconvergence, control of errors)
- comparison with other methods
- extention to other topologies (Y, smart grids)