MONOTONICITY PROPERTIES FOR THE VIABLE CONTROL OF DISCRETE TIME SYSTEMS

Michel De Lara[1], Luc Doyen[2],
Thérèse Guilbaud[3], Marie-Joëlle Rochet[3]

June 12, 2007

[1]CERMICS, École des ponts, ParisTech, France
[2]CNRS, CERESP, Muséum National d'Histoire Naturelle, Paris, France
[3]Département EMH (Ecologie et Modèles pour l'Halieutique), IFREMER, Nantes, France

Credits

- ACI Écologie quantitative MOOREA (Méthodes et outils d'optimisation pour la recherche en écologie appliquée)
- M. De Lara., L. Doyen, T. Guilbaud, M.-J. Rochet, *Is a management framework based on spawning stock biomass indicators sustainable? A viability approach*, *ICES J. Mar. Sci.*, 64(4):761–767, 2007. "high science content", "very relevant"
- M. De Lara., L. Doyen, T. Guilbaud, M.-J. Rochet, *Monotonicity properties for the viable control of discrete time systems*, *Systems and Control Letters*, Volume 56, Number 4, 2006, Pages 296-302.

Outline of the presentation

DISCRETE TIME

VIABILITY ISSUES

Discrete time nonlinear control system

$$\begin{cases} x(t+1) = g(x(t), u(t)), \quad t = t_0, t_0+1, \dots \\ x(t_0) \quad \text{given,} \end{cases}$$

where

- the *state variable* $x(t)$ belongs to the finite dimensional state space $\mathbb{X} = \mathbb{R}^{n_{\mathbb{X}}}$;
- the *control variable* $u(t)$ is an element of the *control set* $\mathbb{U} = \mathbb{R}^{n_{\mathbb{U}}}$;
- the *dynamics* g maps $\mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$.

Harvested fish population age structured model

- *Time* index t in years
- *State* variable $N = (N_a)_{a=1,\dots,A} \in \mathbb{X} = \mathbb{R}_+^A$, the *abundances* at ages (age class index $a \in \{1, \dots, A\}$, with $A = 3$ for anchovy and $A = 8$ for hake)
- *Control* variable $\lambda \in \mathbb{U} = \mathbb{R}_+$, *multiplier* of the *exploitation pattern* $F_1, \dots, F_A$
- *Dynamics* g given by:

Dynamics

$$\left\{\begin{array}{lll} N_1(t+1) & = & \varphi(SSB(N(t))), \\ N_a(t+1) & = & e^{-(M_{a-1}+\lambda(t)F_{a-1})}N_{a-1}(t), \quad a=2,\ldots,A-1 \\ N_A(t+1) & = & e^{-(M_{A-1}+\lambda(t)F_{A-1})}N_{A-1}(t)+\pi e^{-(M_A+\lambda(t)F_A)} \end{array}\right.$$

If we neglect the survivors after age A then $\pi = 0$, else $\pi = 1$ and the last age class is a *plus group*.

Dynamics

$$\begin{cases} g_1(N,\lambda) &= \varphi(SSB(N)), \\ g_a(N,\lambda) &= e^{-(M_{a-1}+\lambda F_{a-1})} N_{a-1}, \quad a = 2,\dots,A-1 \\ g_A(N,\lambda) &= e^{-(M_{A-1}+\lambda F_{A-1})} N_{A-1} + \pi e^{-(M_A+\lambda F_A)} N_A \end{cases}$$

- $SSB(N) = \sum_{a=1}^{A} \gamma_a w_a N_a$ is the **spawning stock biomass**, with γ_a *proportion of matures* at age a, w_a *weight* at age a,
- φ describes a **stock-recruitment relationship**,
- M_a is the *mortality* at age a,
- F_a is the *exploitation pattern-at-age* a;
- λ is the *exploitation pattern multiplier*.

Desirable configurations

We introduce a subset

$$\mathbb{D} \subset \mathbb{X} \times \mathbb{U} = \quad \text{“states”} \quad \times \quad \text{“controls”}$$

termed the **desirable configurations set**.

We aim at finding at least one trajectory such that

$$(x(t), u(t)) \in \mathbb{D}, \quad t = t_0, t_0 + 1, \ldots$$

Current fisheries management advice

Indicators and their associated **reference points** are key elements of current fisheries management advice, in the **International Council for the Exploration of the Sea** (**ICES**) precautionary approach.

The **Study Group for long term advice** is

- keeping (or restoring) **spawning stock biomass** SSB **indicator above a threshold reference point** B_{lim};
- restricting fishing effort so that **mean fishing mortality** F **indicator is below a threshold reference point** F_{lim}.

The ICES indicators and reference points

- **Spawning stock biomass**,

$$SSB(N) := \sum_{a=1}^{A} \gamma_a w_a N_a$$

with reference threshold $SSB(N) \geq B_{\lim}$.

- **Mean fishing mortality** over a pre-determined age range from a_r to A_r, that is,

$$F(\lambda) := \frac{\lambda}{A_r - a_r + 1} \sum_{a=a_r}^{a=A_r} F_a$$

with reference threshold $F(\lambda) \leq F_{\lim}$.

Examples of desirable configurations sets

We consider *sustainable management within* ICES *bounds* involving biomass and fishing mortality indicators. It corresponds to the following ICES **desirable reference configuration set**

$$\mathbb{D}_{\text{lim}} := \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid SSB(N) \geq B_{\text{lim}} \text{ and } F(\lambda) \leq F_{\text{lim}}\}.$$

- maintain biomass of reproductors above a viable level
- restrict fishing effort

Viability kernel

Definition

The following set of states

$$\mathbb{V}(g,\mathbb{D}) := \left\{ x(0) \in \mathbb{X} \;\middle|\; \begin{array}{l} \exists\ (u(0), u(1), \dots) \text{ and } (x(0), x(1), \dots) \\ \text{satisfying } x(t+1) = g(x(t), u(t)) \\ \text{and } (x(t), u(t)) \in \mathbb{D}, \quad t \geq 0 \end{array} \right\}$$

is called the **viability kernel** associated with the dynamics g in the desirable set $\mathbb{D}$.

Starting from $x(0) \in \mathbb{V}(g,\mathbb{D})$, there exists a sequence of states and of controls which both satisfy the dynamics g and belong to the desirable set $\mathbb{D}$.

Viability kernel

Definition

The **state constraints set** associated with $\mathbb{D}$ is obtained by projecting the desirable set $\mathbb{D}$ onto the state space $\mathbb{X}$:

$$\mathbb{V}^0 := \mathrm{Proj}_{\mathbb{X}}(\mathbb{D}) = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}\,,\,(x,u) \in \mathbb{D}\}.$$

By definition, we have

$$\mathbb{V}(g,\mathbb{D}) \subset \mathbb{V}^0$$

but, in general, the inclusion is strict. The so called **comfortable case** is when $\mathbb{V}(g,\mathbb{D}) = \mathbb{V}^0$.

Strong invariance

Definition

A subset $\mathbb{V}$ of the state space $\mathbb{X}$ is said to be **strongly invariant** for the dynamics g in the desirable set $\mathbb{D}$ if

$$\forall x \in \mathbb{V}\,, \quad \forall u \in \mathbb{U}\,, \quad (x,u) \in \mathbb{D} \Longrightarrow g(x,u) \in \mathbb{V}.$$

That is, if one starts from $\mathbb{V}$, any desirable control may transfer the state in $\mathbb{V}$ into $\mathbb{V}$.
This is generally a too demanding requirement.

Weak invariance

Definition

A subset $\mathbb{V}$ is said to be **weakly invariant** for the dynamics g in the desirable set $\mathbb{D}$, or a **viability domain** of g in $\mathbb{D}$, if

$$\forall x \in \mathbb{V}\,, \quad \exists u \in \mathbb{U}\,, \quad (x, u) \in \mathbb{D} \text{ and } g(x, u) \in \mathbb{V}.$$

That is, if one starts from $\mathbb{V}$, a suitable control may transfer the state in $\mathbb{V}$ and the system into a desirable configuration.

Desirable equilibrium

Definition

A **desirable equilibrium** is an equilibrium of the system that belongs to $\mathbb{D}$, that is a pair $(\bar{x}, \bar{u}) \in \mathbb{X} \times \mathbb{U}$ such that

$$(\bar{x}, \bar{u}) \in \mathbb{D} \text{ and } \bar{x} = g(\bar{x}, \bar{u}).$$

Any desirable equilibrium $\{\bar{x}\}$ is a viability domain of g in $\mathbb{D}$.

Viability kernel and viability domains

It turns out that the viability kernel is the largest viability domain.

Theorem

The viability kernel $\mathbb{V}(g, \mathbb{D})$ is the union of all viability domains, or the largest viability domain:

$$\mathbb{V}(g, \mathbb{D}) = \bigcup \Big\{ \mathbb{V} \subset \mathbb{V}^0,\ \mathbb{V} \text{ viability domain for } g \text{ in } \mathbb{D} \Big\}.$$

Viable controls

Definition

When $\mathbb{V}$ is a viability domain, the following set of **viable controls** is not empty:

$$\mathbb{U}_{\mathbb{V}}(x) := \{u \in \mathbb{U} \mid (x,u) \in \mathbb{D} \text{ and } g(x,u) \in \mathbb{V}\}.$$

A **viable policy** is a mapping $\Psi : \mathbb{X} \to \mathbb{U}$ which associates with each state $x \in \mathbb{V}$ a control $u = \Psi(x)$ satisfying $\Psi(x) \in \mathbb{U}_{\mathbb{V}}(x)$.

Starting from $x(t_0) \in \mathbb{V}$ and applying a viable policy $u(t) = \Psi(x(t))$ yields a trajectory satisfying

$$(x(t), u(t)) \in \mathbb{D}, \quad t = t_0, t_0 + 1, \ldots$$

APPROXIMATIONS

OF THE VIABILITY KERNEL

Lower approximation of the viability kernel

A major interest of the property that the viability kernel is the union of all viability domains lies in the following fact:

any viability domain for the dynamics g in the desirable set $\mathbb{D}$ provides a **lower approximation** of the viability kernel.

Upper approximations of the viability kernel

Definition

An *upper approximation* $\mathbb{V}_k$ of the viability kernel is given by the so called **viability kernel until time** k **associated with** g **in** $\mathbb{D}$:

$$\mathbb{V}_k := \left\{ x(0) \in \mathbb{X} \;\middle|\; \begin{array}{l} \exists\ (u(0), u(1), \ldots, u(k)) \text{ and } (x(0), x(1), \ldots, x(k)) \\ \quad \text{satisfying } x(t+1) = g(x(t), u(t)), \\ \qquad\quad \text{for } t = 0, \ldots, k-1 \\ \text{and } (x(t), u(t)) \in \mathbb{D}, \text{ for } t = 0, \ldots, k \end{array} \right\}$$

We have

$$\mathbb{V}(g, \mathbb{D}) \subset \mathbb{V}_{k+1} \subset \mathbb{V}_k \subset \mathbb{V}_0 = \mathbb{V}^0 , \quad \forall k \in \mathbb{N} .$$

Upper approximation algorithm

It may be seen by induction that the decreasing sequence of viability kernels until time k satisfies the following equation:

$$\begin{aligned} \mathbb{V}_0 &= \mathbb{V}^0 \\ \mathbb{V}_{k+1} &= \{x \in \mathbb{V}_k \mid \exists u \in \mathbb{U}\,,\ g(x,u) \in \mathbb{V}_k \text{ and } (x,u) \in \mathbb{D}\}\,. \end{aligned}$$

Such an algorithm provides an *upper approximation* of the viability kernel as follows:

$$\mathbb{V}(g,\mathbb{D}) \subset \bigcap_{k \in \mathbb{N}} \mathbb{V}_k = \lim_{k \to +\infty} \downarrow \mathbb{V}_k\,.$$

MONOTONICITY PROPERTIES

The lattices $\mathbb{R}^{n_\mathbb{X}}$ and $\mathbb{R}^{n_\mathbb{U}}$

Let us assume that the state space $\mathbb{X}$ and the control space $\mathbb{U}$ are $\mathbb{X} \subset \mathbb{R}^{n_\mathbb{X}}$ and $\mathbb{U} \subset \mathbb{R}^{n_\mathbb{U}}$ supplied with the componentwise order: $x' \geq x$ if and only if each component of x' is greater than or equal to the corresponding component of x:

$$x' \geq x \iff x'_i \geq x_i, \ i = 1, \ldots, n.$$

The maximum $x \bigvee x'$ of (x, x') is

$$x \bigvee x' := (x_1 \bigvee x'_1, \ldots, x_n \bigvee x'_n)$$

Set monotonicity

Definition

We say that a set $S \subset \mathbb{X}$ is **increasing** (or an **upper set**) if it satisfies the following property:

$$\forall x \in S\,, \quad x' \geq x \Rightarrow x' \in S\,.$$

We say that $K \subset \mathbb{X} \times \mathbb{U}$ is **increasing** (or a **lower set**) if it satisfies the following property:

$$\forall (x, u) \in K\,, \quad x' \geq x \Rightarrow (x', u) \in K\,.$$

Dynamics monotonicity

Definition

We say that $g : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is **increasing with respect to the state** if it satisfies

$$\forall (x,u) \in \mathbb{X} \times \mathbb{U}\,, \quad x' \geq x \Rightarrow g(x',u) \geq g(x,u),$$

and is **decreasing with respect to the control** if

$$\forall (x,u) \in \mathbb{X} \times \mathbb{U}\,, \quad u' \geq u \Rightarrow g(x,u') \leq g(x,u).$$

Maximal and saturated dynamics

Definition

The **maximal dynamics** $\check{g}$ is defined by

$$\forall x \in \mathbb{V}^0 , \quad \check{g}(x) := \bigvee_{u \in \mathbb{U}, (x,u) \in \mathbb{D}} g(x,u) \, .$$

Since the dynamics g has several components, $\bigvee_{u \in \mathbb{U}, (x,u) \in \mathbb{D}} g(x,u)$ is generally not achieved by a common $\bar{u}$. This is why we introduce the notion of function "saturated at x".

We say that the maximal dynamics $\check{g}$ is **saturated at** $x \in \mathbb{V}^0$ if there exists $u \in \mathbb{U}$ such that $(x,u) \in \mathbb{D}$ and $\check{g}(x) = g(x,u)$.

VIABILITY RESULTS

UNDER

MONOTONICITY PROPERTIES

Proposition

Assume that

1. *the desirable set $\mathbb{D}$ is increasing;*
2. *the dynamics g is increasing with respect to the state.*

Then the associated viability kernel $\mathbb{V}(g,\mathbb{D})$ is an increasing set, as well as all the sets $\mathbb{V}_k$, $k \in \mathbb{N}$ given by

$$\left\{ \begin{array}{rcl} \mathbb{V}_0 & = & \mathbb{V}^0 \\ \mathbb{V}_{k+1} & = & \{x \in \mathbb{V}_k \mid \exists u \in \mathbb{U}, \quad g(x,u) \in \mathbb{V}_k \\ & & \qquad\qquad \textit{and} \quad (x,u) \in \mathbb{D}\}\,. \end{array} \right.$$

A first lower approximation of the viability kernel

Proposition

Assume that

1. *the desirable set $\mathbb{D}$ is increasing;*
2. *the dynamics g is increasing with respect to the state;*
3. *there exists a desirable equilibrium $(\bar{x}, \bar{u})$.*

Then

- *the orthant $\left\{x \in \mathbb{X} \mid x \geq \bar{x}\right\}$ is a viability domain for g in $\mathbb{D}$;*
- *consequently $\left\{x \in \mathbb{X} \mid x \geq \bar{x}\right\} \subset \mathbb{V}(g, \mathbb{D})$.*

A first upper approximation of the viability kernel

Proposition

Assume that

1. *the desirable set* $\mathbb{D}$ *is increasing;*
2. *the dynamics* g *is increasing with respect to the state.*

Then the domain $\{x \in \mathbb{X} \mid x \leq \tilde{x}\}$ *is strongly invariant, whenever* $\tilde{x} \in \mathbb{V}^0$ *satisfies* $\breve{g}(\tilde{x}) \leq \tilde{x}$ *(in particular when* $\tilde{x}$ *is a fixed point of the maximal dynamics* $\breve{g}$*).*

Proposition

Assume that the desirable set $\mathbb{D}$ is increasing and that the dynamics g is increasing with respect to the state. Assume also that the maximal dynamics $\check{g}$ is continuous, and that $\mathbb{V}^0$ is bounded from below.
Define $\mathbb{M}$ as the set of those elements which are larger than at least one fixed point of $\check{g}$ in the closure $\overline{\mathbb{V}^0}$ of the state constraints set:

$$\mathbb{M} := \{x \in \mathbb{X} \mid \exists x' \in \overline{\mathbb{V}^0}, \quad \check{g}(x') = x', \quad x \geq x'\}.$$

Then

$$\mathbb{V}(g, \mathbb{D}) \subset \mathbb{V}^0 \backslash \{x \in \mathbb{V}^0 \mid \check{g}(x) \leq x \text{ or } x \notin \mathbb{M}\}.$$

A second lower approximation of the viability kernel

Proposition

If $\mathbb{V}$ is a viability domain of g in $\mathbb{D}$, then

$$\widetilde{\mathbb{V}} = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U},\ (x,u) \in \mathbb{D} \ \textit{and}\ g(x,u) \in \mathbb{V}\}$$

is a viability domain which contains $\mathbb{V}$. As a consequence

1. *the induction $\widetilde{\mathbb{V}}_0 = \mathbb{V}$ and*

$$\widetilde{\mathbb{V}}_{k+1} = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U},\ (x,u) \in \mathbb{D} \ \textit{and}\ g(x,u) \in \widetilde{\mathbb{V}}_k\}$$

   generates an increasing sequence of viability domains;

2. *and its limit is included in the viability kernel:*

$$\bigcup_{k \in \mathbb{N}} \widetilde{\mathbb{V}}_k = \lim_{k \to +\infty} \uparrow \widetilde{\mathbb{V}}_k \subset \mathbb{V}(g, \mathbb{D}).$$

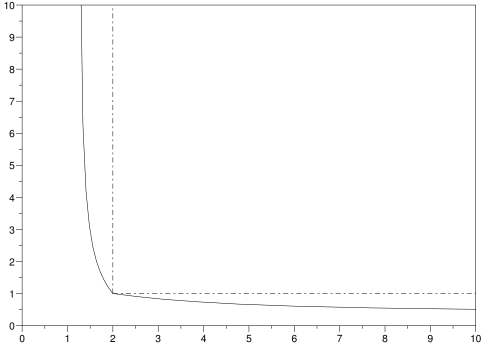

Figure: Enlargement of a viable orthant in the plan (x^1, x^2).

A second upper approximation of the viability kernel

Proposition

Assume that the desirable set $\mathbb{D}$ is increasing and that the dynamics g is increasing with respect to the state. Assume also that the maximal dynamics $\check{g}$ is saturated at all $x \in \mathbb{V}^0$. Then

1. *the decreasing sequence of viability kernels until time k satisfies the induction*

$$\mathbb{V}_0 = \mathbb{V}^0 \text{ and } \mathbb{V}_{k+1} = \mathbb{V}_k \bigcap \check{g}^{-1}(\mathbb{V}_k)\,, \quad \forall k \in \mathbb{N}\,,$$

2. *the decreasing sequence $(\mathbb{V}_k)_{k\in\mathbb{N}}$ converges to $\mathbb{V}(g,\mathbb{D})$:*

$$\mathbb{V}(g,\mathbb{D}) = \bigcap_{k\in\mathbb{N}} \mathbb{V}_k = \lim_{k\to+\infty} \downarrow \mathbb{V}_k\,.$$

A third lower approximation of the viability kernel

Proposition

Assume that

1. *the desirable set* $\mathbb{D}$ *is increasing;*
2. *the dynamics* g *is bounded below*

$$\forall (x,u) \in \mathbb{X} \times \mathbb{U}\,, \quad g^{\flat}(x,u) \leq g(x,u)$$

   by a dynamics $g^{\flat} : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ *which is increasing with respect to the state.*

Then, $\mathbb{V}(g^{\flat}, \mathbb{D})$ *is a viability domain associated with* g *in* $\mathbb{D}$*, and thus*

$$\mathbb{V}(g^{\flat}, \mathbb{D}) \subset \mathbb{V}(g, \mathbb{D}).$$

ARE THE ICES

FISHING QUOTAS RECOMMANDATIONS

"SUSTAINABLE"?

Comments on ICES precautionary approach

The precautionary approach (PA) may be sketched as follows:

- the condition $SSB(N) \geq B_{\lim}$ is checked;
- if valid, the following usual advice is given:

$$\lambda_{UA}(N) = \max\{\lambda \in \mathbb{R}_+ \mid SSB(g(N,\lambda)) \geq B_{\lim} \text{ and } F(\lambda) \leq F_{\lim}\}$$

The problem is that... **nothing ensures the existence of $\lambda \geq 0$ such that $SSB(g(N,\lambda)) \geq B_{\lim}$ and $F(\lambda) \leq F_{\lim}$.**

The existence of a fishing mortality multiplier for any stock vector N such that $SSB(N) \geq B_{\lim}$ is **tantamount to non-emptyness of a set of viable controls**. This justifies the following definitions.

Defining "sustainability"

Let us define the PA state set

$$\mathbb{V}_{\mathrm{lim}} := \{ N \in \mathbb{R}_+^A \mid SSB(N) \geq B_{\mathrm{lim}} \} .$$

We shall say that **the precautionary approach is sustainable** if the PA state set $\mathbb{V}_{\mathrm{lim}}$ is a viability domain for dynamics g in the desirable set

$$\mathbb{D}_{\mathrm{lim}} = \{ (N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid SSB(N) \geq B_{\mathrm{lim}} \text{ and } F(\lambda) \leq F_{\mathrm{lim}} \}.$$

Indeed, starting from $N(t_0) \in \mathbb{V}_{\mathrm{lim}}$ and applying the usual policy λ_{UA} yields a trajectory satisfying

$$SSB(N(t)) \geq B_{\mathrm{lim}} \text{ and } F(\lambda(t)) \leq F_{\mathrm{lim}} , \quad \forall t = t_0, t_0 + 1, \ldots$$

Testing "sustainability"

Proposition

The PA *is sustainable if and only if*

$$\min_{B \in [B_{\lim}, +\infty[} [\Theta B + \gamma_1 w_1 \varphi(B)] \geq B_{\lim}$$

that is, if and only if the lowest possible sum of survivors (weighted by growth and maturation) and newly recruited spawning biomass is above $B_{\lim}$.

$$\Theta = \min \left(\min_{a=1,\ldots,A-1,\ \gamma_a w_a \neq 0} \left[\frac{\gamma_{a+1} w_{a+1}}{\gamma_a w_a} e^{-M_a} \right], \pi e^{-M_A} \right)$$

The proof relies upon *monotonicity properties*.

The answer depends upon... the stock-recruitment relationship φ.
Notice that condition

$$\min_{B \in [B_{\lim}, +\infty[} [\Theta B + \gamma_1 w_1 \varphi(B)] \geq B_{\lim}$$

does not depend on the stock-recruitment relationship φ between 0 and $B_{\lim}$.
It does not depend on $F_{\lim}$ either.

Testing "sustainability" with constant recruitment

A constant recruitment is generally used for fishing advice, so the following simplified condition can be used.
If we suppose that

- the natural mortality is independent of age, that is $M_a = M$,
- the proportion γ_a of mature individuals and the weight w_a at-age are increasing with age a,
- the stock-recruitment is a constant R,

the PA is sustainable if and only if

$$R \geq \underline{R} \quad \text{where} \quad \underline{R} := \frac{1 - \pi e^{-M}}{\gamma_1 w_1} B_{\lim} \ ,$$

making thus of $\underline{R}$ a minimum recruitment required to preserve $B_{\lim}$.

Bay of Biscay anchovy

S/R Relationship	Constant	Constant	Constant (2002)	Constant (2004)	Linear	Ricker
Condition to check	$R_{\text{mean}} \geq \underline{R}$	$R_{\text{gm}} \geq \underline{R}$	$R_{\text{min}} \geq \underline{R}$	$R_{\text{min}} \geq \underline{R}$	$\gamma_1 w_1 r \geq 1$	$\min\limits_{B \geq B_{\text{lim}}} [\cdots] \geq$
Left hand side	14 016 $\times 10^6$	7 109 $\times 10^6$	3 964 $\times 10^6$	696 $\times 10^6$	0.84	0
Right hand side	1 312 $\times 10^6$	1 312 $\times 10^6$	1 312 $\times 10^6$	1 312 $\times 10^6$	1	21 000
Sustainable	yes	yes	yes	no	no	no

Table: Bay of Biscay anchovy: sustainability of advice based on the spawning stock biomass indicator for various stock recruitment relationships. The answer is given in the last row of the table. The second row contains an expression whose value is given in the third line. It has to be compared to the threshold in the fourth row.

Northern stock of hake

For hake, ICES precautionary approach is never sustainable because the proportion of mature individuals at age 1 is zero, $\gamma_1 = 0$, so condition

$$\min_{B \in [B_{\lim}, +\infty[} [\Theta B + \gamma_1 w_1 \varphi(B)] \geq B_{\lim}$$

is never satisfied, whatever the value of $B_{\lim}$. Indeed, $\Theta \leq \pi e^{-M_A} < 1$ since $\pi \in \{0, 1\}$ and $M_A > 0$.

Thus, **a viability domain based upon the only indicator** ***SSB*** **proves unsufficient**.

A confusion due to the double role of SSB indicator

The *SSB* indicator is used for two different purposes:

- for designing *short term advice*: when $SSB(N(t)) \geq B_{\lim}$ is checked, compute usual advice $\lambda_{UA}(N(t)) = \max\{\lambda \in \mathbb{R}_+ \mid SSB(g(N(t),\lambda)) \geq B_{\lim} \text{ and } F(\lambda) \leq F_{\lim}\}$;
- for delineating a domain to which states and controls should belong *year after year*:

$$SSB(N(t)) \geq B_{\lim} \text{ and } F(\lambda(t)) \leq F_{\lim}\,, \quad \forall t = t_0, t_0+1, \ldots$$

There is no reason why yearly objectives described by means of the single *SSB* indicator should be achieved by means of advice based upon this single indicator.

HOW TO DESIGN "SUSTAINABLE" RECOMMANDATIONS

Sustainable management

Definition

We say that **sustainable management is possible within** ICES **bounds** if the viability kernel $\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}})$ associated with dynamics g in the acceptable set

$$\mathbb{D}_{\mathrm{lim}} = \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid SSB(N) \geq B_{\mathrm{lim}} \text{ and } F(\lambda) \leq F_{\mathrm{lim}}\}$$

is not empty.

Proposition

$$\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}}) = \left\{ N \in \mathbb{R}_+^A \middle| SSB\left(g^{(n)}(N, 0)\right) \geq B_{\mathrm{lim}} \quad \forall n \right\},$$

with $g^{(n)}(\cdot, 0) = g(\cdot, 0) \circ g(\cdot, 0) \circ \cdots \circ g(\cdot, 0)$, the n-time

Constant recruitment and no plus-group

Proposition

With a constant recruitment R and no plus-group ($\pi = 0$), the set $\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}})$ is described by $A+1$ affine constraints:

$$\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}}) = \Big\{ N \in \mathbb{R}_+^A \mid \quad R\, spr(0) \geq B_{\mathrm{lim}} \quad and \quad SSB\left(g^{(i)}(N, 0)\right) \geq B_{\mathrm{lim}} \quad \forall i = 0, \ldots, A \Big\} .$$

Constant recruitment and no plus-group

Proposition

Sustainability is characterized as follows:

$$\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}}) \neq \emptyset \quad \Leftrightarrow \quad R\, spr(0) \geq B_{\mathrm{lim}} .$$

The well known *spawners per recruit* indicator appears naturally. We denote by $\mathrm{spr}(\lambda)$ the equilibrium *spawners per recruit* obtained with the fishing mortality multiplier λ. By definition,

$$\mathrm{spr}(\lambda) = \frac{SSB(\overline{N})}{\varphi(SSB(\overline{N}))} \quad \text{where} \quad \overline{N} = g(\overline{N}, \lambda).$$

Bay of Biscay anchovy

S/R Relationship	Constant	Constant	Constant (2002)	Constant (2004)
Condition	$R_{\text{mean}}\,\text{spr}(0) \geq B_{\text{lim}}$	$R_{\text{gm}}\,\text{spr}(0) \geq B_{\text{lim}}$	$R_{\text{min}}\,\text{spr}(0) \geq B_{\text{lim}}$	$R_{\text{min}}\,\text{spr}(0) \geq B_{\text{lim}}$
Left hand side Right hand side	194.1×10^6 21×10^6	98.5×10^6 21×10^6	54.9×10^6 21×10^6	9.6×10^6 21×10^6
sustainable management	yes	yes	yes	no

Table: Bay of Biscay anchovy: Sustainable management for some stock recruitment constant relationships.

For stock recruitment constant R such that

$$1\ 312 \times 10^6 \leq R \leq 1\ 516 \times 10^6$$

sustainable management is possible but . . .
not following ICES advice! New indicators, others than SSB.

Uncertainty on stock-recruitment relationship

Recruitment involves complex biological and environmental processes that fluctuate in time, and are difficult to integrate into a population model.

Typical examples are

- constant: $\varphi(B) = R$;
- linear: $\varphi(B) = rB$;
- Beverton-Holt: $\varphi(B) = \frac{B}{\alpha+\beta B}$;
- Ricker: $\varphi(B) = \alpha B \mathrm{e}^{-\beta B}$.

Coping with uncertain dynamics in a precautionary way

For a stock-recruitment relationship

$$\varphi^{\flat} \leq \varphi$$

and the associated dynamics

$$g^{\flat} \leq g$$

we have

$$\mathrm{Viab}(g^{\flat}, \mathbb{D}_{\mathsf{lim}}) \subset \mathrm{Viab}(g, \mathbb{D}_{\mathsf{lim}}) \,.$$

Hence, by choosing a constant mimimum recruitment, we obtain a lower approximation for all majorizing dynamics.

CONCLUSIONS

Claims about sustainable management

Although **sustainable management** is claimed to be a guide for decision making, there is great **confusion** between

- **operational objectives (advice)**
- and **perpetual objectives** (not explicitely stated).

Viability concepts and methods have helped

- giving a framework for **setting decision making**;
- **testing** "sutainability" of current fishing advice and practices;
- **proposing** viable policies;

giving thus coherence to claims and practices.

Perspectives

The approach developed

- relies upon monotonicity properties of viability domains with respect to the dynamics and to the desirable set;
- may be extended to multiple species *without ecological interactions but with technical interactions*;
- may include explicit economic requirements such as mimimum yield

$$\mathbb{D}_{\text{yield}} := \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid Y(N, \lambda) \geq y_{\min}\}.$$

Perspectives

Mathématiques et décision pour le développement durable
RTP M3D
Réseau thématique pluridisciplinaire CNRS

- département Environnement et développement durable
- département Sciences humaines et sociales
- département Mathématiques, physique, planète et univers