Viable control of a dengue epidemiological model

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Outline of the presentation

Dengue control issues in Cali

Viable control of dengue epidemiological models

Robust viable control of a dengue epidemiological model

Conclusions

Outline of the presentation

Dengue control issues in Cali

Viable control of dengue epidemiological models

Robust viable control of a dengue epidemiological model

Conclusions

World panorama of dengue

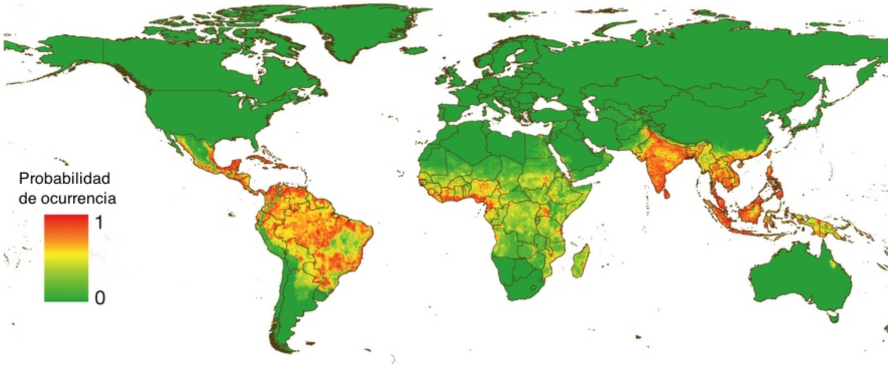

Figure: Global map of the incidence of dengue. Source: World Health Organization

Dengue in Cali, Colombia

Cali is a tropical urban environment of Colombia

Local authorities

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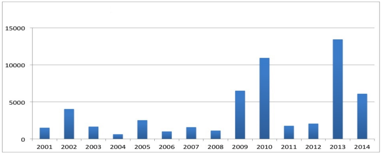


Figure: Reported cases of dengue in Cali 2001 to 2014. Source: Data from Secretaría Muncipal de Salud de Cali

“Canal Endémico” stands as the reference to control dengue

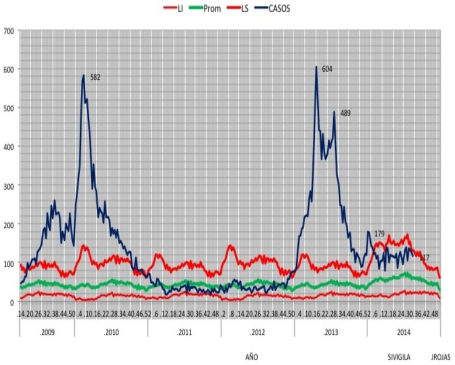


Figure: Cases of dengue between 2009 and 2014.
Source: Secretaría Municipal de Salud de Cali.

Program "Dengue Control" of SMS

Control mosquito breeding sites

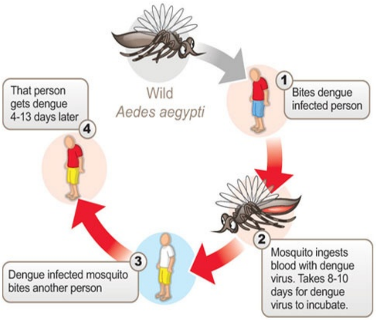
What is coming ahead

- Viable control of dengue
- Robust viability analysis of dengue

Outline of the presentation

Outline of the presentation

Dengue is transmitted by the mosquito vector



Ross-Macdonald epidemic model

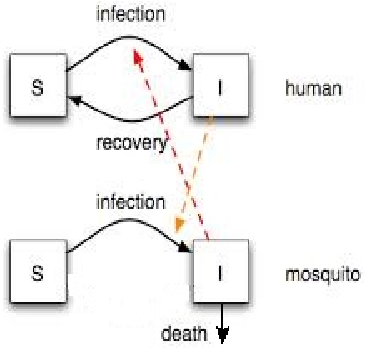


Figure: Dengue transmission cycle. (http://www.eliminatedengue.com/our-research/dengue-fever)

Ross-Macdonald epidemic model

Denote by m and h the proportions of infected mosquitoes and humans, respectively

$$\text{mosquitos} \Rightarrow \frac{dm}{dt} = \alpha\, p_m h(1-m) - \delta m$$

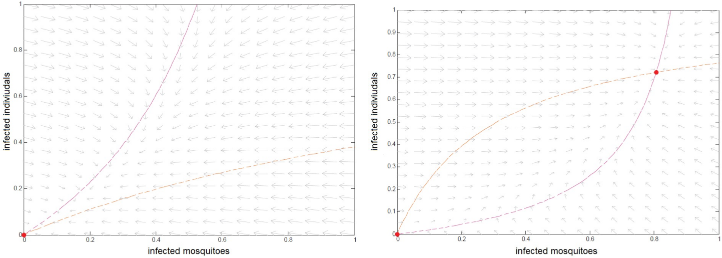
$$\text{humans} \Rightarrow \frac{dh}{dt} = \alpha\, p_h \xi\, m(1-h) - \gamma h$$

Parameter	Description
ξ	number of mosquito females per person
α	per capita rate of mosquito bites on humans
p_m	probability of infection of a susceptible mosquito by biting an infected human
p_h	probability of infection of a susceptible human by the bite of an infected mosquito
δ	per capita rate death of mosquitos
γ	rate at which humans recover from infection

Table: Parameters of the Ross-Macdonald model.

Most mathematical analysis focus on asymptotical properties without control (or stationary ones)

Asymptotic analysis relies upon the *basic reproductive number* $\mathscr{R}_0 = \dfrac{\alpha^2 p_h p_m \xi}{\gamma\delta}$



(a) A unique equilibrium point ($\mathscr{R}_0 < 1$)

(b) Two equilibrium points ($\mathscr{R}_0 > 1$)

Outline of the presentation

Formulation of the viability problem for Ross-Macdonald Model

- The dynamics of the system is given by

$$\begin{aligned} \text{infected mosquito proportion} \quad \frac{dm}{dt} &= A_m h(t)(1-m(t)) - u(t)m(t) \\ \text{infected human proportion} \quad \frac{dh}{dt} &= A_h m(t)(1-h(t)) - \gamma h(t) \end{aligned}$$

- Determine, if it exists, a piecewise continuous function (fumigation policy rates) $u(\cdot)$,

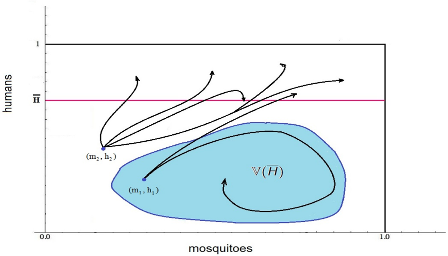
$$u(\cdot) : t \mapsto u(t) \,, \;\; \underline{u} \leq u(t) \leq \overline{u} \,, \;\; \forall t \geq 0 \,,$$

such that the following so-called viability constraint is satisfied:

$$h(t) \leq \overline{H} \,, \;\; \forall t \geq 0$$

The viability kernel

$$\mathbb{V}(\overline{H},\overline{u}) = \left\{ (m_0, h_0) \;\middle|\; \begin{array}{l} \textit{there exists } u(\cdot) \textit{ with } \underline{u} \le u(t) \le \overline{u} \\ \textit{such that the trajectory state } (m(t), h(t)) \textit{ of} \\ \dfrac{dm}{dt} = A_m h(t)(1-m(t)) - u(t)m(t) \\ \dfrac{dh}{dt} = A_h m(t)(1-h(t)) - \gamma h(t) \\ \textit{starting from } (m_0, h_0) \textit{ satisfies } \quad h(t) \le \overline{H}, \ \forall t \ge 0 \end{array} \right\}$$



Outline of the presentation

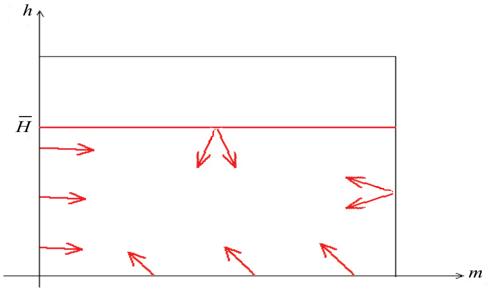
Theorem (Characterization of the viability kernel)

(C) *Comfortable case: if*

$$\frac{A_h}{A_h+\gamma} \leq \overline{H}$$

the viability kernel is

$$\mathbb{V}(\overline{H},\overline{u}) = \mathbb{V}^0(\overline{H}) = \{(m,h)|0 \leq m \leq 1, 0 \leq h \leq \overline{H}\} = [0,1]\times[0,\overline{H}]$$



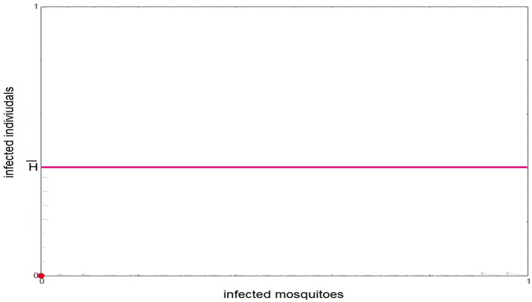
Theorem (Characterization of the viability kernel)

(D) *Desperate case: if*

$$A_m(A_h + \gamma)\overline{H} + \gamma\overline{u} < A_m A_h \,,$$

the viability kernel is

$$\mathbb{V}(\overline{H}, \overline{u}) = \{(0,0)\}$$



Theorem (Characterization of the viability kernel)

(V) *Viable case: If*

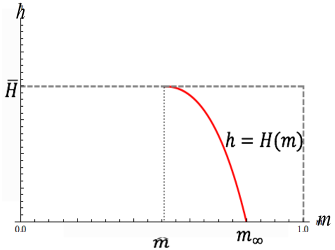
$$\overline{H} < \frac{A_h}{A_h+\gamma} \quad \text{and} \quad A_m(A_h+\gamma)\overline{H}+\gamma\overline{u} > A_m A_h \ ,$$

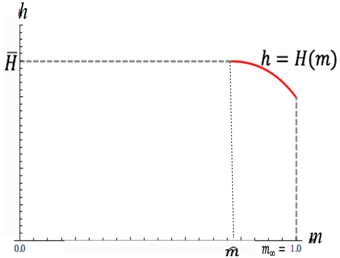
the viability kernel is

$$\mathbb{V}(\overline{H},\overline{u}) = ([0,\overline{M}]\times[0,\overline{H}]) \bigcup \left\{(m,h) \middle| \overline{M} \le m \le M_\infty \ , \ \ h \le \mathcal{H}(m)\right\}$$

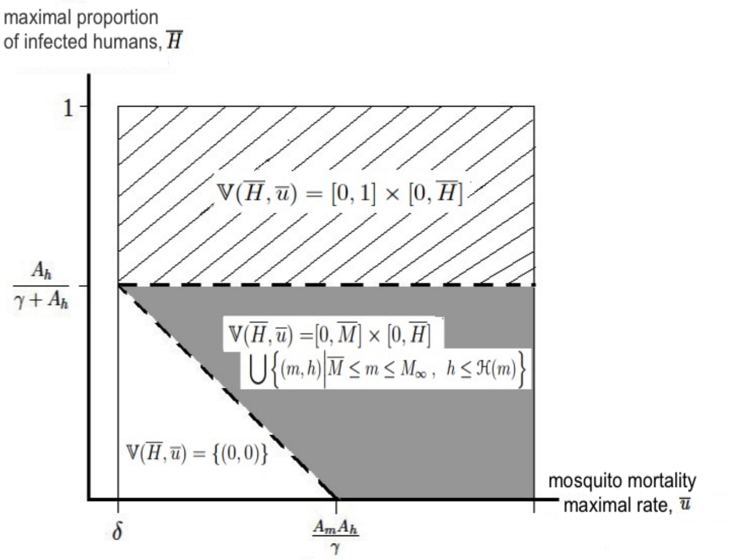
where $\overline{M} = \gamma\overline{H}/A_h(1-\overline{H})$ *and* $\mathcal{H} : [\overline{M}, M_\infty] \to [0,\overline{H}]$ *is solution of*

$$-g_m(m,\mathcal{H}(m),\overline{u})\mathcal{H}'(m) + g_h(m,\mathcal{H}(m)) = 0 \ , \ \ \mathcal{H}(\overline{M}) = \overline{H}$$

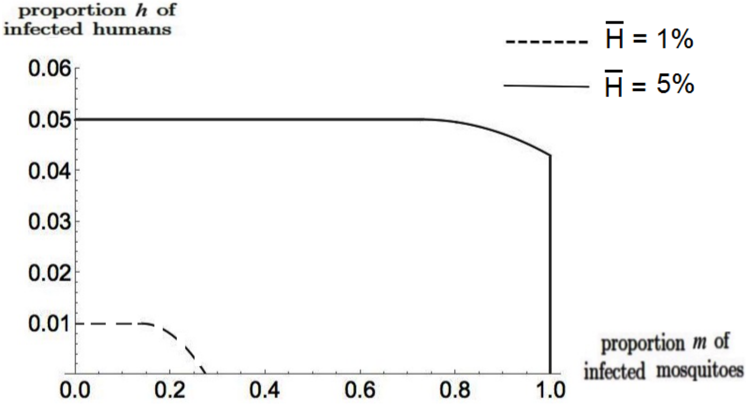




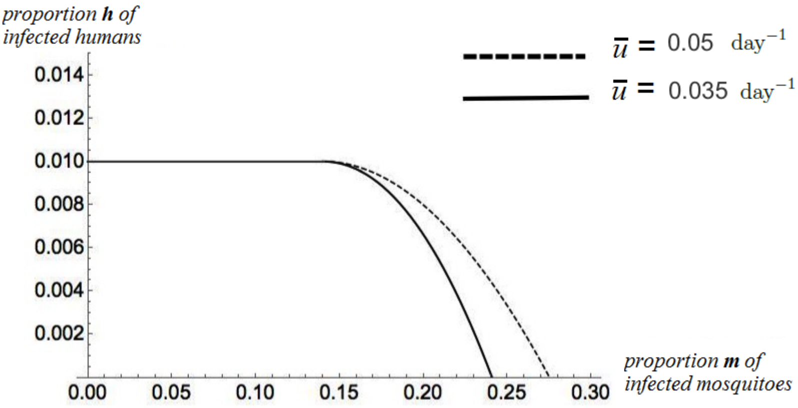
Three cases for the viability kernel



Sensitivity of $\mathbb{V}(\overline{H},\overline{u})$ with respect to the infection cap $\overline{H}$ on the proportion of infected humans

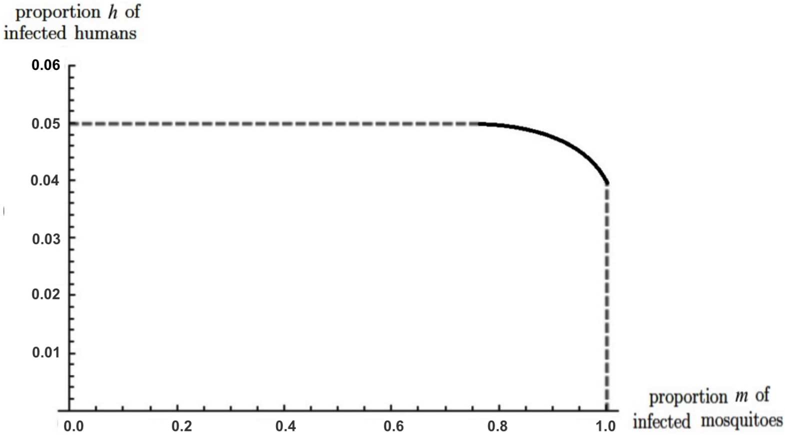


Sensitivity of $\mathbb{V}(\overline{H}, \overline{u})$ with respect to the mosquito mortality maximal rate $\overline{u}$

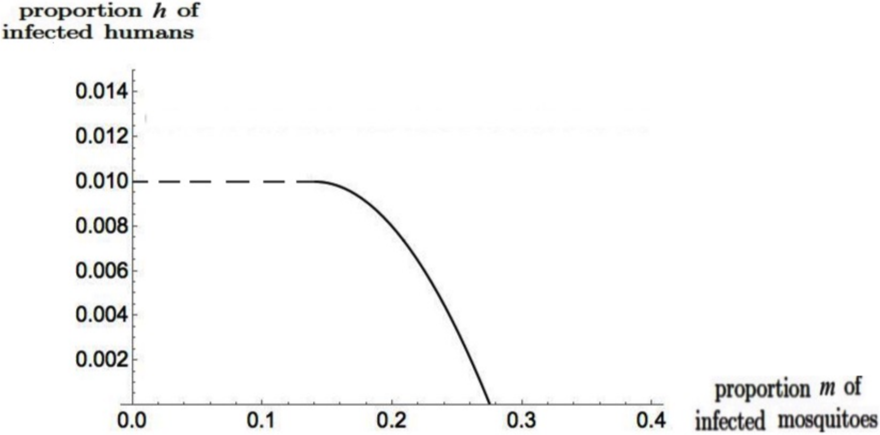


Outline of the presentation

Case $\overline{u} = 0.04\ \text{day}^{-1}$ and $\overline{H} = 5\%$



Case $\overline{u} = 0.05$ day^{-1} and $\overline{H} = 1\%$



Possible design for a viable policy

- Monitoring without fumigation
  When the proportion of infected humans is below the infection cap $\overline{H} = 1\%$ and when the proportion of infected mosquitoes is below the proportion $\overline{M} = 14\%$,
  do not fumigate
- Monitoring with (maximal) fumigation
  When the proportion of infected mosquitoes
  is between the proportions $\overline{M} = 14\%$ and $M_\infty = 27\%$,
  fumigate with maximal capacity
- Alert
  When the proportion of infected mosquitoes is above $M_\infty = 27\%$,
  additional measures should be taken
  to prevent a high peak of infected humans

Conclusion on viability analysis

- Comfortable case
  - whatever state $(m_0, h_0) \in [0,1] \times [0,H]$ belongs to the viability kernel
  - no control is needed to satisfy the viability constraint
  - all trajectories satisfy the viability constraint
- Desperate case
  - the viability kernel reduces to the point $(0,0)$
  - the unique trajectory that satisfies viability constraint is $m(t) \equiv 0$ and $h(t) \equiv 0$ for all $t \geq 0$
- Viable case
  - the viability kernel is

$$\mathbb{V}(\overline{H}, \overline{u}) = \left([0,\overline{M}] \times [0,\overline{H}]\right) \bigcup \left\{ (m,h) \middle| \overline{M} \leq m \leq M_\infty \,, \quad h \leq \mathcal{H}(m) \right\}$$

  - viable controls increase fumigation at the viability kernel upper frontier

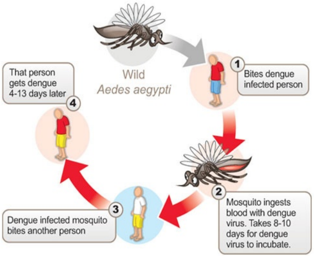
What is coming ahead

- Till now
  - continuous time model
  - deterministic model
  - deterministic viability kernel and viable controls
- And now
  - discrete time model
  - dynamic model with uncertainties
  - robust viability kernel (and viable policies)

Outline of the presentation

Outline of the presentation

Sources of uncertainty abound



Uncertainties are captured by $\begin{cases} \text{mosquitoes transmission rate} & A_M(t) \\ \text{human transmission rate} & A_H(t) \end{cases}$

in the forthcoming model

New variables

- Time
  - Discrete-time $t = 0, 1, \dots, T$ with interval $[t, t+1[$ representing one day
- State variables
  - $M(t)$ denotes the proportion of infected mosquitoes during the interval $[t, t+1[$
  - $H(t)$ denotes the proportion of infected humans during the interval $[t, t+1[$
- Control variable
  - $U(t)$ denotes the mosquito mortality due to fumigation during the interval $[t, t+1[$

Discrete-time dynamic control model with uncertainties

- Let us denote by $\Phi(M, H, u, A_M, A_H)$ the solution, at time $s = 1$, of the deterministic differential system with initial condition $\big(m(0), h(0)\big) = (M, H)$
- We obtain the following sampled and controlled Ross–Macdonald model

$$\big(M(t+1), H(t+1)\big) = \Phi\big(M(t), H(t), u(t), A_M(t), A_H(t)\big)$$

- The control constraints capture limited fumigation resources during a day

$$\underline{U} \leq U(t) \leq \overline{U}\ , \ \ \forall t = 0, \ldots, T-1$$

Viability problem statement

- We impose that the viability constraint

$$H(t) \leq \overline{H}\,, \quad \forall t = 0, \dots, T$$

- holds true whatever the scenario (sequence of uncertainties)

$$(A_M(\cdot), A_H(\cdot)) = \Big((A_M(0), A_H(0)), \dots, (A_M(T-1), A_H(T-1)) \Big)$$

belonging to a subset $\Omega \subset (\mathbb{R}^2)^T$

In the robust framework, we need a new definition of solution

- A policy $\mathfrak{U}$ is defined as a sequence of mappings

$$\mathfrak{U} = \{\mathfrak{U}_t\} = 0, \ldots, T-1, \quad \text{with} \quad \mathfrak{U}_t : [0,1]^2 \to \mathbb{R}$$

  where each $\mathfrak{U}_t$ maps state (M, H) towards control U
- A strategy induces a sequence of controls by

$$U(t) = \mathfrak{U}_t\big(M(t), H(t)\big)$$

- A policy $\mathfrak{U}$ is said to be admissible if it satisfies the control constraints

$$\mathfrak{U}_t : [0,1]^2 \to [\underline{U}, \overline{U}]$$

Outline of the presentation

Robust viability problem statement

The robust viability kernel is the set of initial conditions $\big(M(0), H(0)\big)$ from which at least one policy $\mathfrak{U}$ produces infected mosquitoes and infected humans trajectories by the dynamics

$$\big(M(t+1), H(t+1)\big) = \Phi\big(M(t), H(t), u(t), A_M(t), A_H(t)\big)$$

with input controls

$$U(t) = \mathfrak{U}_t\big(M(t), H(t)\big)$$

so that

$$H(t) \leq \overline{H}\,, \quad \forall t = 0, \ldots, T$$

for all the scenarios

$$\Big(\Big(A_M(0), A_H(0)\Big), \ldots, \Big(A_M(T-1), A_H(T-1)\Big)\Big) \in \Omega \subset (\mathbb{R}^2)^T$$

We make a tough assumption on the set of scenarios

- An uncertainty scenario is a time sequence of uncertainty couples

$$\big(A_M(\cdot), A_H(\cdot)\big) = \Big(\big(A_M(0), A_H(0)\big), \dots, \big(A_M(T-1), A_H(T-1)\big)\Big)$$

- We make the strong independence assumption that

$$\big(A_M(t)(\cdot), A_H(\cdot)\big) \in \Omega = \mathbb{S}_0 \times \mathbb{S}_1 \times \cdots \times \mathbb{S}_{T-1}$$

- Therefore, from one time t to the next $t+1$, uncertainties can be drastically different since $\big(A_M(t), A_H(t)\big)$ is not related to $\big(A_M(t+1), A_H(t+1)\big)$
- Such an assumption makes it possible to write a dynamic programming equation with (M, H) as state variable
- For the sake of simplicity, we take

$$\mathbb{S}_0 = \mathbb{S}_1 = \cdots = \mathbb{S}_{T-1} = \mathbb{S}$$

Numerical resolution of the dynamic programming equation

initialization $\mathrm{V}_T(M,H) = 1_{[0,1]\times[0,\overline{H}]}(M,H)$;

for $t = T, T-1, \ldots, 0$ **do**

  forall $(M,H) \in [0,1] \times [0,\overline{H}]$ **do**

    forall $U \in [\underline{U}, \overline{U}]$ **do**

      forall $(A_M, A_H) \in \mathbb{S}$ **do**

        $\mathrm{V}_{t+1}\big(\Phi(M,H,U,A_M,A_H)\big)$

      $\min_{(A_M,A_H)\in\mathbb{S}} \mathrm{V}_{t+1}\big(\Phi(M,H,U,A_M,A_H)\big)$

    $\max_{U\in[\underline{U},\overline{U}]} \min_{(A_M,A_H)\in\mathbb{S}} \mathrm{V}_{t+1}\big(\Phi(M,H,U,A_M,A_H)\big)$

  $\mathrm{V}_t\big(t,M,H\big) = 1_{[0,1]\times[0,\overline{H}]}(M,H) \times \mathrm{V}_{t+1}\big(\Phi(M,H,U,A_M,A_H)\big)$

Outline of the presentation

Uncertainty sets

We consider three nested sets of uncertainties

$$\mathbb{S}_L \subset \mathbb{S}_M \subset \mathbb{S}_H \subset \mathbb{R}^2_+$$

L) deterministic case

$$\mathbb{S}_L = \left\{\widehat{A_M}\right\} \times \left\{\widehat{A_H}\right\}$$

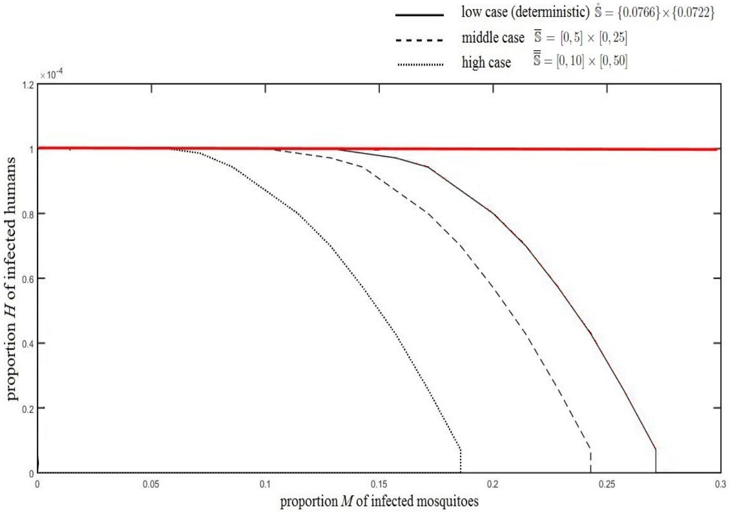
M) medium case

$$\mathbb{S}_M = \left[\underline{A_M}, \overline{A_M}\right] \times \left[\underline{A_H}, \overline{A_H}\right]$$

H) high case

$$\mathbb{S}_H = \left[\underline{\underline{A_M}}, \overline{\overline{A_M}}\right] \times \left[\underline{\underline{A_H}}, \overline{\overline{A_H}}\right]$$

Robust viability kernels shrink when uncertainties expand



Conclusion on robust viability analysis

The numerical results show that the viability kernel without uncertainties is highly sensitive to the variability of parameters such as

- biting rate
- probability of infection to mosquitoes and humans
- proportion of female mosquitoes per person

Maybe we should focus the effort on reducing these three sources of uncertainty

Outline of the presentation

General conclusions

- Analysis of strategies of dengue control
  - not only preoccupied by asymptotics ($\mathscr{R}_0$ like in most of the literature)
  - but focusing on transients (viability)
- Obtention of theoretical results
- Insight into possible viable policies by means of numerical applications
- Analysis of the impact of uncertainties thanks to the robust viability kernel
- Proposal of practical strategies
  - measure the proportion of infected mosquitoes
    (at least above a cut-off value) to cap the infected human at the peak
  - pay attention to three specific sources of uncertainty

THANKS FOR YOUR ATTENTION