# The Reduced Basis method for aeroacoustic simulations solved by integral equations

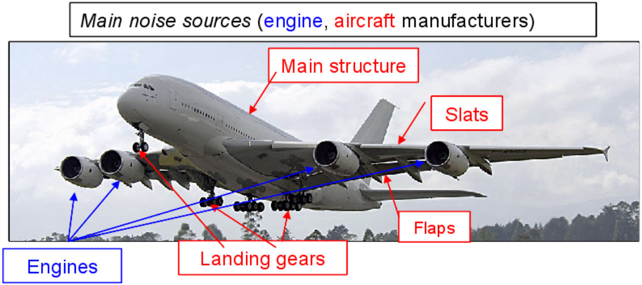
Fabien Casenave

joint work with Alexandre Ern and Tony Lelièvre

April 16, 2014

# Motivations

## Aeroacoustic in civil aviation




Numerical methods for computing acoustic scattering in civil aviation contexts

# Motivations

## Parametric problems

Different PDE's depending on physical assumptions, different parameters to take into account (position and frequency of the acoustic source, absorption coefficients, flow)

## Large scale computing

Uncertainty propagation, optimization, or real-time computation : need to solve parametric problem for a large number of parameter values

## Goal

Numerical procedure for fast computations of an approximation to the solution of a large class of parametric problem, with fast and accurate quantification of the approximation error

# Outline

## Two aeroacoustic problems solved by integral equations

- Acoustic scattering by impedant objects in the air at rest
- Acoustic scattering in moving air

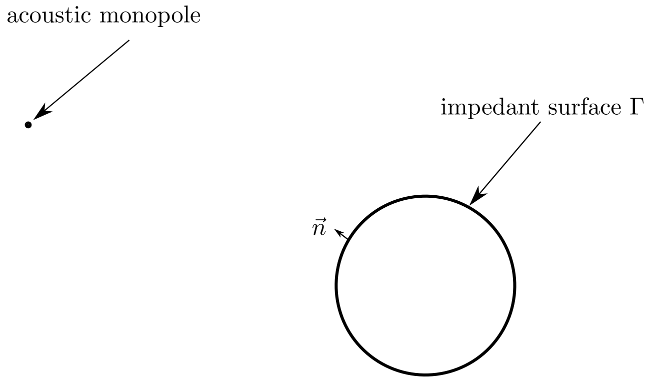
## Some contributions to the Reduced Basis method

- (A quick reminder on) The reduced basis method
- An accurate and online-efficient evaluation of the error bound
- A nonintrusive technique for the RB method

## Some numerical illustrations

# Outline

# Acoustic scattering by impedant objects in the air at rest



## Variational formulation

Find $\chi, \lambda : \Gamma \to \mathbb{C}$ s.t. for all $\chi^t, \lambda^t$

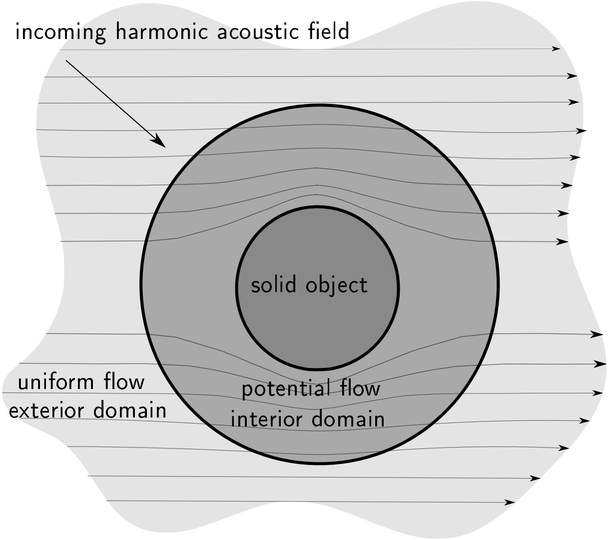
$$\begin{cases} \left( N\chi - \frac{ik}{2\mu}\chi, \chi^t \right) + \left( \tilde{D}\lambda, \chi^t \right) = \left( \gamma_1 p_{\mathsf{inc}}, \chi^t \right), \\ \left( D\chi, \lambda^t \right) - \left( S\lambda + \frac{i\mu}{2k}\lambda, \lambda^t \right) = -\left( \gamma_0 p_{\mathsf{inc}}, \lambda^t \right), \end{cases}$$

- $\mu$ is the impedant coefficient, quantifies the absorbed and scattered parts of the acoustic pressure field
- $N$, $D$, $\tilde{D}$ and $S$ are integral operators involving the Green kernel associated to the Helmholtz equation

$$G(x, y) = \frac{\exp(ik|x-y|)}{4\pi|x-y|}$$

- $p_{\mathrm{inc}}(\boldsymbol{x}) = A\frac{\exp(ik|x-x_S|)}{4\pi|x-x_S|}$ for a monopole source of amplitude $A$ located at $x_S$
- $k = \frac{2\pi f}{c}$, with $f$ the frequency of the source

# Acoustic scattering in moving air



# Aeroacoustics problem

## Characteristics

- acoustics in a mean flow not at rest
- unbounded domain of propagation

## Tools

- Linearization of the Euler equations $\rightarrow$ convected Helmholtz equation
- Prandtl–Glauert transformation [Glauert (1928), Amiet and Sears (1970)] $\rightarrow$ transforms the uniformly convected Helmholtz eq. into the classical Helmholtz eq. in the exterior domain
- Coupled BEM-FEM [McDonald and Wexler (1972), Zienkiewicz, Kelly and Bettess (1977), Johnson and Nédélec (1980)]
- Stabilization with respect to resonant frequencies [Buffa and Hiptmair (2005), Hiptmair and Meury (2006)]

# Variational formulation

$$\begin{cases} \mathcal{V}(f,f^t) + \left(N(\gamma_0 f), \gamma_0 f^t\right)_{\Gamma_\infty} + \left(\left(\tilde{D} - \frac{1}{2}I\right)(\lambda), \gamma_0 f^t\right)_{\Gamma_\infty} = \left(\gamma_1 f_{\mathrm{inc}}, \gamma_0 f^t\right)_{\Gamma_\infty} \\ \left(\left(D - \frac{1}{2}I\right)(\gamma_0 f), \lambda^t\right)_{\Gamma_\infty} - \left(S(\lambda), \lambda^t\right)_{\Gamma_\infty} + i\left(p, \lambda^t\right)_{\Gamma_\infty} = -\left(\gamma_0 f_{\mathrm{inc}}, \lambda^t\right)_{\Gamma_\infty} \\ \left(N(\gamma_0 f), p^t\right)_{\Gamma_\infty} + \left(\left(\tilde{D} + \frac{1}{2}I\right)(\lambda), p^t\right)_{\Gamma_\infty} - \left(p, p^t\right)_{H^1(\Gamma_\infty)} = \left(\gamma_1 f_{\mathrm{inc}}, p^t\right)_{\Gamma_\infty} \end{cases}$$

- $\mathcal{V}(f,f^t) = \int_{\Omega^-} r\Xi\nabla\overline{f}\cdot\nabla f^t - \int_{\Omega^-} rk^2\beta\overline{f}f^t + i\int_{\Omega^-} rk\boldsymbol{V}\cdot\left(\overline{f}\nabla f^t - f^t\nabla\overline{f}\right)$
- $N$, $D$, $\tilde{D}$ and $S$ are integral operators
- $f_{\mathrm{inc}}(\boldsymbol{x}) = A\dfrac{\exp(ik|x-x_S|)}{4\pi|x-x_S|}$ for a monopole source of amplitude $A$ located at $x_S$
- $k = \dfrac{2\pi f}{c}$, with $f$ the frequency of the source

# Well-posed formulation

## Fredholm alternative

- uniqueness : Rellich lemma and unique continuation property, with "weakly regular" coefficients [Garofalo et Lin (1987)]
- isolate a coercive part of the sesquilinear form (including stabilization)
- compact perturbation

## Finite dimension approximation

- tetrahedral mesh, meshsize $h$
- finite elements P1 for $f$ and $p$, and finite elements P0 for $\lambda$
- inf-sup stable discrete formulation (for $h$ small enough)

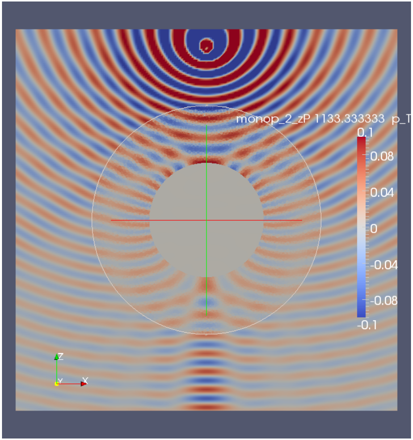
## Conclusion

- Direct use of BEM codes for Helmhotz
- Numerical method valid for all frequencies of the source

## Well-posed variational formulation

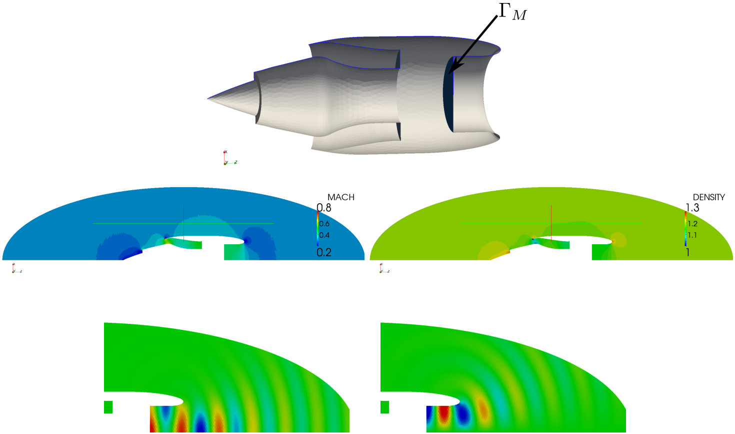
[F.C., A. Ern and G. Sylvand (2014)]

# Numerical illustration




Computation by Airbus (350k dof, FMM : 1h30 sur 32 proc)

# Acoustic scattering by a turbojet




Computation by Airbus (4.7M ddl, FMM : 1h30 sur 160 proc)

# Outline

Two aeroacoustic problems solved by integral equations

- Acoustic scattering by impedant objects in the air at rest
- Acoustic scattering in moving air

Some contributions to the Reduced Basis method

- (A quick reminder on) The reduced basis method
- An accurate and online-efficient evaluation of the error bound
- A nonintrusive technique for the RB method

Some numerical illustrations

# The reduced basis method

## Parametrized problem - idealized case

$$a_\mu(u_\mu, v) = b(v) \qquad \forall v \in \mathcal{V}$$

where

- $\mu \in \mathcal{P}$ is the parameter
- $a_\mu$ is a continuous and coercive bilinear form (uniformly in $\mu$)
- $b$ is a continuous linear form
- $\mathcal{V}$ is a $N$-dimensional linear space, approx. of a Hilbert space $H$, $N \gg 1$

## Goal

Compute an approximation $\hat{u}_\mu$ of $u_\mu$ and (sharp) upper bound of $|\hat{u}_\mu - u_\mu|$ in complexity independent of $N$

## Some early references

[Machiels, Maday, Oliveira, Patera and Rovas (2000)]
[Prud'homme, Rovas, Veroy, Machiels, Maday, Patera and Turinici (2002)]

# Principles

## Offline stage

- some parameter values $(\mu_i)_{1\leq i\leq \hat{N}}$ are chosen in $\mathcal{P}_{\mathrm{trial}} \subset \mathcal{P}$ by a greedy algorithm, $\hat{N} \ll N$
- the full solutions $u_{\mu_i}$ are computed for all $1 \leq i \leq \hat{N}$
- approximation space $\hat{\mathcal{V}}_{\hat{N}} = \mathrm{Vect}\{u_{\mu_1}, \cdots, u_{\mu_{\hat{N}}}\}$

## Online problem

Galerkin procedure on $\hat{\mathcal{V}}_{\hat{N}}$

$$a_\mu(\hat{u}_\mu, u_{\mu_j}) = b(u_{\mu_j}) \qquad \forall j \in \{1, ..., \hat{N}\}$$

The decomposition of $\hat{u}_\mu$ in $\hat{\mathcal{V}}_{\hat{N}}$ is denoted

$$\hat{u}_\mu = \sum_{i=1}^{\hat{N}} \gamma_i(\mu) u_{\mu_i}$$

# A posteriori error bound

## Residual

Let $G_\mu : H \to H$ such that

$$\langle G_\mu u, v \rangle_H = a_\mu(u, v) - b(v)$$

where $\langle \cdot, \cdot \rangle_H$ is the inner product of $H$

## A posteriori error bound

$$\mathcal{E}(\mu) = \frac{\|G_\mu \hat{u}_\mu\|_H}{\beta} \geq \|\hat{u}_\mu - u_\mu\|_H$$

where $\beta$ is the coercivity constant of $a_\mu$ and $\|u\|_H = \sqrt{\langle u, u \rangle_H}$

## Goal-oriented

Need to introduce a dual problem

# Online-efficiency

## Affine dependence

$a_\mu(\cdot,\cdot)$ has an affine dependence in $\mu$ if there exists $\mu \mapsto \alpha_k(\mu)$ and $a_k(\cdot,\cdot)$, $1 \leq k \leq d$, such that $a_\mu(\cdot,\cdot) = \sum_{k=1}^{d} \alpha_k(\mu) a_k(\cdot,\cdot)$

## The key assumption

Affine dependence allows to construct the online problem and compute the error bound in complexity independent of $N$, since

$$a_\mu(u_{\mu_i}, u_{\mu_j}) = \sum_{k=1}^{d} \alpha_k(\mu) a_k(u_{\mu_i}, u_{\mu_j})$$

$$\mathcal{E}(\mu) = \frac{1}{\beta} \left( \langle Jb, Jb \rangle_H - 2 \sum_{k=1}^{d} \sum_{i=1}^{\hat{N}} \alpha_k(\mu) \gamma_i(\mu) \langle Jb, Ja_k(u_{\mu_i}, \cdot) \rangle_H \right.$$
$$\left. + \sum_{k=1}^{d} \sum_{l=1}^{d} \sum_{i=1}^{\hat{N}} \sum_{j=1}^{\hat{N}} \alpha_k(\mu) \gamma_i(\mu) \alpha_l(\mu) \gamma_j(\mu) \left\langle Ja_k(u_{\mu_i}, \cdot), Ja_p(u_{\mu_j}, \cdot) \right\rangle_H \right)^{\frac{1}{2}}$$

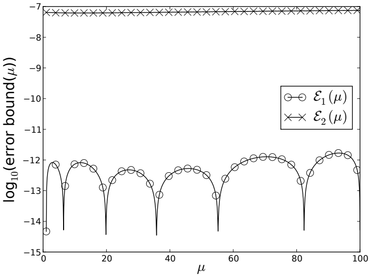
where the only $N$-dependent quantities are $\mu$-independent, and therefore can be precomputed during the offline stage

## 1 - the formula for the error bound is very sensitive to round-off errors

Denote

- $\mathcal{E}_1(\mu) = \dfrac{\|G_\mu \hat{u}_\mu\|_H}{\beta}$
- $\mathcal{E}_2(\mu) =$ online-efficient formula

$\mathcal{E}_1(\mu) = \mathcal{E}_2(\mu)$ in exact arithmetics




### Simple example

$$-u'' + \mu u = 1$$

on $]0,1[$ with $u(0) = u(1) = 0$

## 2 - the RB method is intrusive

The precomputations in $a_\mu(u_{\mu_i}, u_{\mu_j}) = \sum_{k=1}^{d} \alpha_k(\mu) a_k(u_{\mu_i}, u_{\mu_j})$ suppose that we can modify the elementary assembly routines of the code

# New formula for computing the error bound

## Rewriting of $\mathcal{E}_2$

Define

- $x(\mu) \in \mathbb{R}^{d\hat{N}}$, with components $\alpha_k(\mu)\gamma_i(\mu)$, $1 \leq k \leq d$, $1 \leq i \leq \hat{N}$
- $\sigma = 1 + d\hat{N} + \frac{1}{2} d\hat{N}(d\hat{N} + 1)$
- $X(\mu) \in \mathbb{R}^{\sigma}$, with components $(1, x_I(\mu), x_I(\mu)x_J(\mu))$, $1 \leq I \leq J \leq d\hat{N}$

$(\beta\mathcal{E}(\mu))^2$ is a linear form in $X(\mu)$ : $\exists q \in \mathbb{R}^{\sigma}$, independent of $\mu$, s.t.

$$(\beta\mathcal{E}(\mu))^2 = \sum_{p=1}^{\sigma} q_p X_p(\mu)$$

## Separated representation of $X_p(\mu)$

Empirical Interpolation Method (EIM) [Barrault, Maday, Nguyen, Patera (2004)]

## Empirical Interpolation Method for $X_p(\mu)$

- Initialization $k = 1$
  - Compute $p_1 := \underset{p \in \{1,\dots,s\}}{\operatorname{argmax}} \|X_p(\cdot)\|_{\ell^\infty(\mathcal{P}_{\mathbf{trial}})}$
  - Compute $\mu_1 := \underset{\mu \in \mathcal{P}_{\mathbf{trial}}}{\operatorname{argmax}} |X_{p_1}(\mu)|$ and set $\mathcal{P}_{\mathrm{inter}} = \{\mu_1\}$
  - Set $q_1(\cdot) := \dfrac{X_{p_1}(\cdot)}{X_{p_1}(\mu_1)}$ and $B^1_{11} := 1$
- While $k < \hat{\sigma}$
  - Compute $p_{k+1} := \underset{p \in \{1,\dots,s\}}{\operatorname{argmax}} \|(\delta^k X)_p(\cdot)\|_{\ell^\infty(\mathcal{P}_{\mathbf{trial}})}$
  - Compute $\mu_{k+1} := \underset{\mu \in \mathcal{P}_{\mathbf{trial}}}{\operatorname{argmax}} |(\delta^k X)_{p_{k+1}}(\mu)|$ and set $\mathcal{P}_{\mathrm{inter}} := \mathcal{P}_{\mathrm{inter}} \cup \{\mu_{k+1}\}$
  - Set $q_{k+1}(\cdot) := \dfrac{(\delta^k X)_{p_{k+1}}(\cdot)}{(\delta^k X)_{p_{k+1}}(\mu_{k+1})}$ and $B^{k+1}_{ij} := q_j(\mu_i)$, $1 \le i, j \le k+1$
  - $k \leftarrow k + 1$

where $\delta^k := \mathrm{Id} - I^k$ and

$$I^k X(\mu) := \sum_{r=1}^{k} \lambda^k_r(\mu) X(\mu_r),$$

with $\lambda^k(\mu) \in \mathbb{C}^k$ solves $B^k \lambda^k(\mu) = q^k(\mu)$

## Change of base

Since $\underset{1\le m\le k}{\mathrm{Vect}}(q_m(\cdot)) = \underset{1\le m\le k}{\mathrm{Vect}}(X_{p_m}(\cdot))$, there exists a matrix $\Gamma \in \mathbb{R}^{k\times k}$ such that, for all $1 \le l \le k$,

$$\sum_{m=1}^{k} \Gamma_{lm} q_m(\mu) = X_{p_m}(\mu), \qquad \forall \mu \in \mathcal{P}$$

$\Gamma$ is constructed recursively :

- $k = 1$ :

$$\Gamma_{1,1} = X_{p_1}(\mu_1),$$

- $k \to k+1$ :

$$\begin{aligned} \Gamma_{k+1,k+1} &= (\delta_k X)_{p_{k+1}}(\mu_{k+1}), & \\ \Gamma_{l,k+1} &= 0, & \forall 1 \le l \le k, \\ \Gamma_{k+1,l} &= \kappa_l, & \forall 1 \le l \le k, \end{aligned}$$

where $\kappa$ is such that $\sum_{m=1}^{k} B_{lm}\kappa_m = X_{p_{k+1}}(\mu_l)$, for all $1 \le l \le k$

## New formula for computing the error bound

### Separated representation of $X_p(\mu)$

$$X_p(\mu) \approx (I^{\hat{\sigma}} X)_p(\mu) = \sum_{i=1}^{\hat{\sigma}} \sum_{j=1}^{\hat{\sigma}} \Delta_{ij} X_{p_j}(\mu) X_p(\mu_i) \qquad \hat{\sigma} \leq \sigma$$

where $\Delta := B^{-t} \Delta^{-1}$

### New online estimator

Using the separated representation of $X(\mu)$ and exchanging summation order

$$\begin{aligned}(\beta \mathcal{E}(\mu))^2 &\approx \sum_{p=1}^{\sigma} q_p \left\{ \sum_{i=1}^{\hat{\sigma}} \sum_{j=1}^{\hat{\sigma}} \Delta_{ij} X_{p_j}(\mu) X_p(\mu_i) \right\} \\ &= \sum_{i=1}^{\hat{\sigma}} \underbrace{\left\{ \sum_{j=1}^{\hat{\sigma}} \Delta_{ij} X_{p_j}(\mu) \right\}}_{\lambda_i(\mu)} \underbrace{\left\{ \sum_{p=1}^{\sigma} q_p X_p(\mu_i) \right\}}_{(\beta \mathcal{E}(\mu_i))^2}\end{aligned}$$

leading to

$$\mathcal{E}_3(\mu) := \frac{1}{\beta} \left( \sum_{i=1}^{\hat{\sigma}} \lambda_i(\mu) \left( \beta \mathcal{E}_1(\mu_i) \right)^2 \right)^{\frac{1}{2}}$$

# New formula for computing the error bound

## An online-efficient formula

The accurate evaluations $\mathcal{E}_1(\mu_i)$, $1 \leq i \leq \hat{\sigma}$, are precomputed during the offline stage and $\lambda_i(\mu)$ are computed in complexity independent of $N$

## Summary

| | $\mathcal{E}_1(\mu)$ | $\mathcal{E}_2(\mu)$ | $\mathcal{E}_3(\mu)$ |
|---|---|---|---|
| Accuracy scales like | $\epsilon$ | $\sqrt{\epsilon}$ | $\epsilon$ |
| Online efficiency | No | Yes | Yes |

where $\epsilon$ is the machine precision

## Usefulness

- bad stability constant (can be the case for the Helmholtz equation)
- nonlinear problems. Brezzi-Rappaz-Raviart theory : no error bound is possible until a very tight tolerance is reached ("overkill"). $\mathcal{E}_3$ used in [M. Yano (2013)] for Boussinesq certified RB at Grashof numbers of engineering interest.

## References

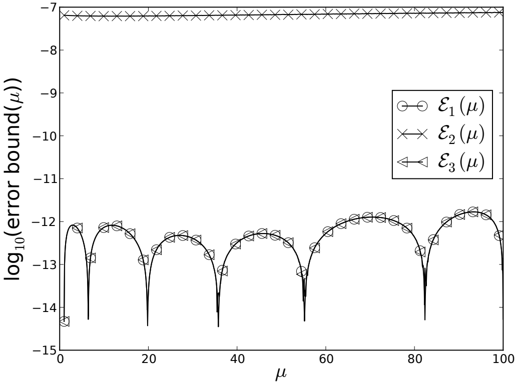

[F.C. (2012)]
[F.C., A. Ern and T. Lelièvre (2013)]

## Back to the simple problem

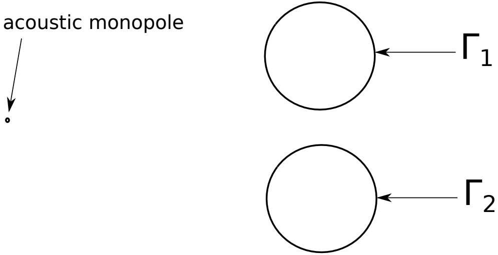
$$-u'' + \mu u = 1$$

on $]0,1[$ with $u(0) = u(1) = 0$

- $d = 2,\ \hat{N} = 7$
  $\Longrightarrow \sigma = 120$
- $\hat{\sigma} = 23$

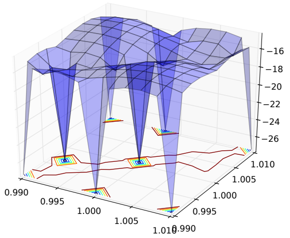


# Acoustics problem

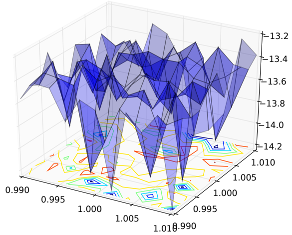



$$\begin{cases} \left(N\chi - \frac{ik}{2\mu}\chi, \chi^t\right) + \left(\tilde{D}\lambda, \chi^t\right) = \left(\gamma_1 p_{\text{inc}}, \chi^t\right), \\ \left(D\chi, \lambda^t\right) - \left(S\lambda + \frac{i\mu}{2k}\lambda, \lambda^t\right) = -\left(\gamma_0 p_{\text{inc}}, \lambda^t\right), \end{cases}$$

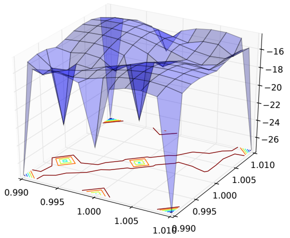
# Acoustics problem




$\mathcal{E}_1$




$\mathcal{E}_2$




$\mathcal{E}_3$

Two scattering balls, s.t.

- $d = 5$, $\hat{N} = 8$
  $\Longrightarrow \sigma = 1681$ (complex)
- $\hat{\sigma} = 60$

# Nonintrusivity of reduced problem construction

## Example of affine decomposition

$$A_{\mu,ij} = \underbrace{\int_\Omega \nabla\varphi_i \cdot \nabla\varphi_j dx}_{:=A_{0\,ij}} + \mu \underbrace{\int_\Omega \varphi_i\varphi_j dx}_{:=A_{1\,ij}}, \qquad 1 \le i,j \le N$$

Computing $A_0$ and $A_1$ requires entering the assembly routines of the code

- not feasable for large industrial codes
- nonintrusivity : use only full matrix $A_\mu$ at user-selected $\mu$'s

## Simple fix

Select two values of the parameter and observe that

$$A_\mu = \frac{\mu_2 - \mu}{\mu_2 - \mu_1} A_{\mu_1} + \frac{\mu - \mu_1}{\mu_2 - \mu_1} A_{\mu_2}$$

## Goal 1

Extend this idea for more complex parameter dependencies

# From affine dependence to nonintrusivity

## General affine dependence

$$A_{\mu,ij} = \sum_{s=1}^{\varsigma} g_s(\mu) \int_\Omega \psi_{s,ij}(x)dx$$

(previous example : $\varsigma = 2$, $g_1(\mu) = 1$, $\psi_{1,ij}(x) = \nabla\varphi_i(x) \cdot \nabla\varphi_j(x)$, $g_2(\mu) = \mu$, $\psi_{2,ij}(x) = \varphi_i(x)\varphi_j(x)$)

## Key idea

Separate $\mu$ and $s$ in $g_s(\mu)$ using EIM (of rank $d \leq \varsigma$)

$$g_s(\mu) \approx (I_d g)(\mu, s) = \sum_{k=1}^{d} \underbrace{\left\{ \sum_{l=1}^{d} \Delta_{kl} g_{s_l}(\mu) \right\}}_{:=\beta_k(\mu)} g_s(\mu_k)$$

## Nonintrusive approximation

Exchange summation order

$$A_{\mu,ij} \approx \sum_{s=1}^{\varsigma} (I_d g)(\mu, s) \int_\Omega \psi_{s,ij}(x)dx = \sum_{k=1}^{d} \beta_k(\mu) A_{\mu_k,ij}$$

# Nonaffine parameter dependencies

## General case

$$A_{\mu,ij} = \sum_{s=1}^{\varsigma} \int_\Omega g_s(\mu, x)\psi_{s,ij}(x)dx$$

## Goal 2

Achieve nonintrusivity in this general setting

## Step 1 - classical

Use $\varsigma$ EIM's (same rank $d$ for simplicity) to separate $(\mu, x)$ in the $g_s$'s :
Approximate $A_{\mu,ij} \approx I_d^g A_{\mu,ij}$ with

$$I_d^g A_{\mu,ij} = \sum_{s=1}^{\varsigma} \sum_{k=1}^{d} \underbrace{\left\{ \sum_{l=1}^{d} \Delta_{s,kl} g_s(\mu, x_{s,m}) \right\}}_{:=z_{sk}(\mu)} \underbrace{\int_\Omega g_s(\mu_{s,k}, x)\psi_{s,ij}(x)dx}_{:=Q_{sk,ij}}$$

# Nonaffine parameter dependencies

## Regroup indices

$s$ and $k$ to write ($d^p = \varsigma d$)

$$I_d^g A_{\mu,ij} = \sum_{p=1}^{d^p} z_p(\mu) Q_{p,ij}$$

## Step 2 - key idea

Separate $(\mu, p)$ in $z_p(\mu)$ using a second EIM (rank $d^z \leq d^p$)

$$z_p(\mu) \approx \sum_{k=1}^{d^z} \sum_{l=1}^{d^z} \Delta_{kl}^{(z)} z_{p_l^{(\mu)}}(\mu) z_p(\mu_k^{(\mu)})$$

## Nonintrusive formula

Exchanging summation order

$$A_{\mu,ij} \approx I_d^g A_{\mu,ij} = \sum_{p=1}^{d^p} \left\{ \sum_{k=1}^{d^z} \sum_{l=1}^{d^z} \Delta_{kl}^{(z)} z_{p_l^{(\mu)}}(\mu) z_p(\mu_k^{(z)}) \right\} Q_{p,ij}$$

$$= \sum_{k=1}^{d^z} \underbrace{\left\{ \sum_{l=1}^{d^z} \Delta_{kl}^{(z)} z_{p_l^{(z)}}(\mu) \right\}}_{:=\beta_k^{(z)}(\mu)} \underbrace{\left\{ \sum_{p=1}^{d^p} z_p(\mu_k^{(z)}) Q_{p,ij} \right\}}_{=I_d^g A_{\mu_k^{(z)},ij} \approx A_{\mu_k^{(z)},ij}}$$

$$\approx \sum_{k=1}^{d^z} \beta_k^{(z)}(\mu) A_{\mu_k^{(z)},ij}$$

# "Nonintrusive" Reduced Basis

Not black-box

Coefficients of the reduced matrix

$$\hat{A}_{\mu,ij} = \sum_{k=1}^{d^z} \beta_k^{(z)}(\mu) \hat{U}_i^t A_{\mu_k^{(z)}} \hat{U}_j$$

Nonintrusive in the sense that we only need

- the function $(\mu, V) \mapsto A_\mu V$ (quite a mild assumption). Any optimized matrix-vector product available in the industrial code can be directly used
- to know the terms of the variational formulation

Reference

[F.C., A. Ern and T. Lelièvre], submitted

# Outline

Two aeroacoustic problems solved by integral equations

- Acoustic scattering by impedant objects in the air at rest
- Acoustic scattering in moving air

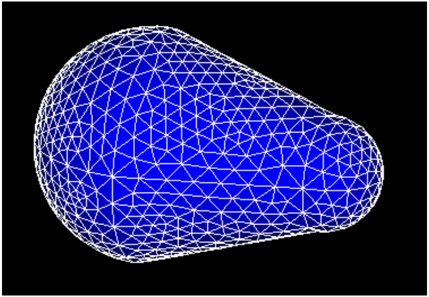
Some contributions to the Reduced Basis method

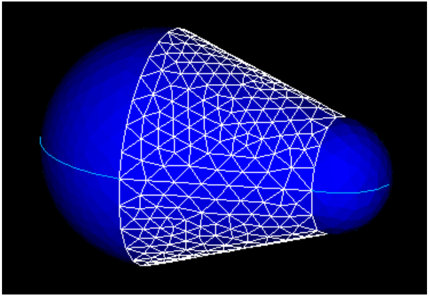
- (A quick reminder on) The reduced basis method
- An accurate and online-efficient evaluation of the error bound
- A nonintrusive technique for the RB method

Some numerical illustrations

# Acoustic scattering by impedant objects in the air at rest

## Mesh, 2240 dof





## Parametric problem

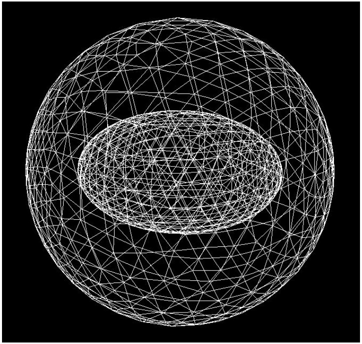
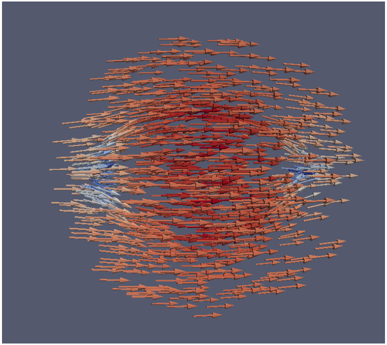
- parameters : frequency of the source (plane wave), 3 impedant coefficients
- quantity of interest : far-field acoustic scattered field on the axis of symmetry

## Reduced basis

- approximation formula for the matrix : 20 terms, for the right-hand side of the direct and dual problems : 13 terms
- 20 basis vectors (max of the error bound $10^{-6}$)
- speed-up factor $> 10^4$ ( 2.8 ms vs 30 s)

# Acoustic scattering in moving air

## Mesh, 1711 dof

## Parametric problem

- parameters : frequency of the source (monopole), uniform perturbation of the flow (3 components)
- quantity of interest : acoustic potential at a point

Frequency-dependent terms

Flow-dependent terms

$$\begin{cases} \mathcal{V}(f,f^t) + \left(N(\gamma_0 f), \gamma_0 f^t\right)_{\Gamma_\infty} + \left(\left(\tilde{D} - \frac{1}{2}I\right)(\lambda), \gamma_0 f^t\right)_{\Gamma_\infty} = \left(\gamma_1 f_{\mathrm{inc}}, \gamma_0 f^t\right)_{\Gamma_\infty} \\ \left(\left(D - \frac{1}{2}I\right)(\gamma_0 f), \lambda^t\right)_{\Gamma_\infty} - \left(S(\lambda), \lambda^t\right)_{\Gamma_\infty} + i\left(p, \lambda^t\right)_{\Gamma_\infty} = -\left(\gamma_0 f_{\mathrm{inc}}, \lambda^t\right)_{\Gamma_\infty} \\ \left(N(\gamma_0 f), p^t\right)_{\Gamma_\infty} + \left(\left(\tilde{D} + \frac{1}{2}I\right)(\lambda), p^t\right)_{\Gamma_\infty} - \left(p, p^t\right)_{H^1(\Gamma_\infty)} = \left(\gamma_1 f_{\mathrm{inc}}, p^t\right)_{\Gamma_\infty} \end{cases}$$

with $\mathcal{V}(f,f^t) = \int_{\Omega^-} r\Xi\boldsymbol{\nabla}\overline{f}\cdot\boldsymbol{\nabla}f^t - \int_{\Omega^-} rk^2\beta\overline{f}f^t + i\int_{\Omega^-} rk\boldsymbol{V}\cdot\left(\overline{f}\boldsymbol{\nabla}f^t - f^t\boldsymbol{\nabla}\overline{f}\right)$

## Reduced basis

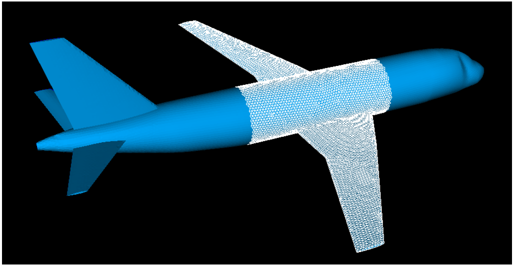
- approximation formula for the matrix : 25 terms, for the right-hand side of the direct and dual problems : 18 terms
- 20 basis vectors (max of the error bound $10^{-7}$)
- speed-up factor $> 5 \times 10^3$ ( 2.8 ms vs 14 s)

# Towards industrial applications

## "Scalable" implementation

- never save a matrix on hard-drive
- fast matrix-vector products (parallel FMM)
- parallel exploration of $\mathcal{P}_{\mathrm{trial}}$

## Mesh, 60866 dof



parameters : frequency of the source (monopole), 3 impedant coefficients

# Towards industrial applications

## Reduced basis

- approximation formula for the matrix : 50 terms, for the right-hand side of the direct problem : 60 terms
- 30 basis vectors
- offline stage : $\approx$ 2 days, exploration of $\mathcal{P}_{\mathrm{trial}} \approx 1$ hour at the 30th loop
- speed-up factor $\approx 1.6 \times 10^5$ ( 15 ms vs 40 min)
- all computations of this laptop with 4 CPUs and 4 GB of RAM

# Towards industrial applications

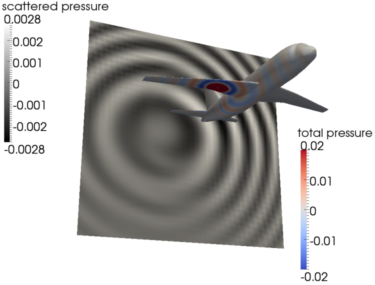



Figure: Left : direct solution, right : difference between direct and RB solutions

## Real-time online computation

$\Longrightarrow$ demonstration