A mathematical and numerical framework for ultrasonically-induced Lorentz force electrical impedance tomography

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Mots-clés : electrical impedance tomography, hybrid imaging, ultrasonically-induced Lorentz force, optimal control, transport equation, viscosity-type regularization

Ultrasonic vibration of a tissue in the presence of a static magnetic field induces an electrical current by the Lorentz force. This current can be detected by electrodes placed around the tissue; it is proportional to the velocity of the ultrasonic pulse, but depends nonlinearly on the conductivity distribution. The imaging problem is to reconstruct the conductivity distribution from measurements of the induced current. I will explain how this nonlinear inverse problem is solved by making use of a virtual potential to relate explicitly the current measurements to the conductivity distribution and the velocity of the ultrasonic pulse and how one can reduce the problem to imaging the conductivity from an internal electric current density. I will compare two numerical methods for solving such a problem. First an optimal control method and then a new direct reconstruction scheme that we proposed, involving a transport partial differential equation. I will show how solving such an equation with a viscosity-type regularization yields the true conductivity distribution as the regularization parameters approaches zero. This work is the result of a collaboration between Habib Ammari, Pol Grasland-Mongrain, Pierre Millien, Laurent Seppecher and Kin-Keun Seo, see [2].

Références

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