Stochastic Optimization - 2 hours

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Name: ____________________

1 Linear quadratic control (11 points)

We consider the following problem

$$\min_{\boldsymbol{x}_1,\boldsymbol{x}_2} \quad \mathbb{E}\left[\frac{1}{2}(\boldsymbol{x}_1-\boldsymbol{\xi}_1)^2+\frac{1}{2}(\boldsymbol{x}_2-\boldsymbol{\xi}_2)^2+\frac{1}{2}(\boldsymbol{x}_2-\boldsymbol{x}_1)^2\right] \tag{1a}$$

$$s.t. \quad \sigma(\boldsymbol{x}_1)\subset\sigma(\boldsymbol{\xi}_1) \tag{1b}$$

$$\sigma(\boldsymbol{x}_2)\subset\sigma(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2) \tag{1c}$$

$$\tag{1d}$$

where $\boldsymbol{\xi}_i$ are centered random variable (i.e. with $\mathbb{E}[\boldsymbol{\xi}_i]=0$) with finite covariance matrix Σ. This means that, $var(\boldsymbol{\xi}_i)=\Sigma_{i,i}$ and $cov(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)=\Sigma_{1,2}$. When possible the result are to be given in function of the coefficient of Σ.

1. (1 point) Assuming that $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ are deterministic, find the optimal solution to Problem (1) in function of x_0, ξ_1, ξ_2 and k, and show that the optimal value is of the form

$$V^{det}(\xi_1,\xi_2)=\kappa(\xi_2-\xi_1)^2,$$

κ to be determined.

2. (2 points) Using Question 1 propose a natural open-loop solution to Problem (1) and compute the associated upper bound. Can you also give a lower bound ?

3. (1 point) Using Question 1 give a lower bound of Problem (1).

4. (4 points) Assume that $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ are independent. Solve Problem (1) giving both the value and optimal strategy.

5. (2 points) For generic $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2$ prove that the policy obtained in the previous question is ε-optimal for Problem (1), where ε is to be given in function of Σ.

2 A unit commitment problem (11 points)

We consider an energy production company which has a set of production unit $\mathcal{I}$. Each unit $i \in \mathcal{I}$ is either on ($x^i = 1$) or off ($x^i = 0$) for the day (decided the day before). If it is on, its production u_t^i at time $t \in [\![1, 24]\!]$ should be in $[\underline{u}^i, \bar{u}^i]$. Turning on a unit has a cost c^i (for the day), while the production cost per hour is e^i.

Let's denote $\boldsymbol{z}_t$ the production of the company sold on the market, and $\boldsymbol{\varepsilon}_t \geq 0$ the lost production, i.e. $\sum_{i \in \mathcal{I}} \boldsymbol{u}_t^i = \boldsymbol{z}_t + \boldsymbol{\varepsilon}_t$. For each hour there is a demand $\boldsymbol{d}_t$ such that $\boldsymbol{z}_t \in [0.8\boldsymbol{d}_t, 1.2\boldsymbol{d}_t]$ almost surely. The company is paid $\boldsymbol{p}_t \boldsymbol{z}_t$ at time t.

$\boldsymbol{d}_t$ and $\boldsymbol{p}_t$ are revealed at the beginning of hour t. u_t^i is decided once they are revealed.

The company aims at minimizing the expected cost.

1. (3 points) Write the problem has a multistage stochastic program and give the information structure. Justify the choice of expectation in the objective.

2. (2 points) Justify that this problem can be reduced to a two stage program. Specify the decomposition in first stage and second stage program.

3. (1 point) Give a simple necessary and sufficient condition under which this problem has finite value. Give a necessary and sufficient condition for the decomposition to present relatively complete recourse.

4. (2 points) Assuming that you have a sample of $S = 1000$ scenarios of $(\boldsymbol{d}_t, \boldsymbol{p}_t)_{t \in [\![1,24]\!]}$. Write the extensive formulation SAA approximation of the above two-stage program as a MILP. Precise the number and type of first stage and second stage variables.

5. (3 points) Is the SAA problem better addressed by Progressive Hedging or L-Shaped method ? Justify your answer. Write the master and slave problems.