ECOLE NATIONALE
DES PONTS ET CHAUSSEES

LCPC

Vibration of granular materials

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UR Navier

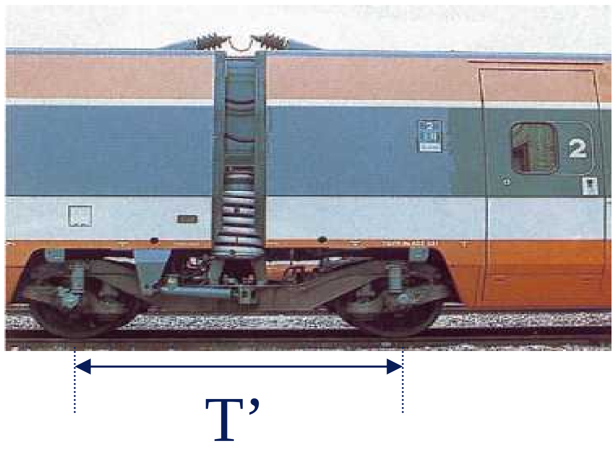
21 November 2007

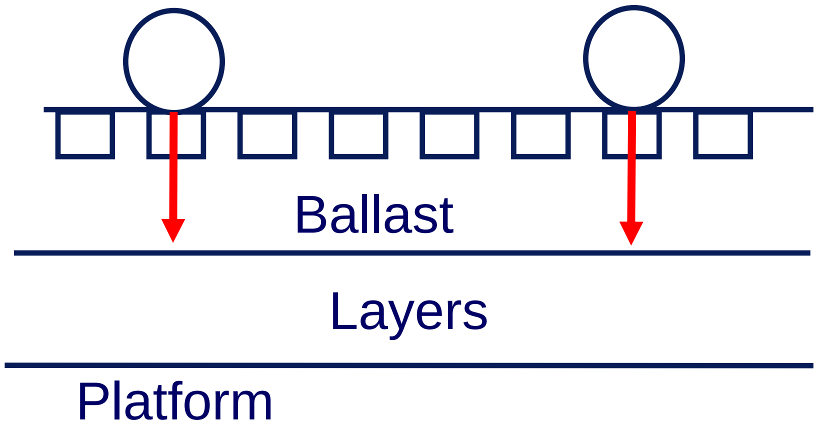
Outline

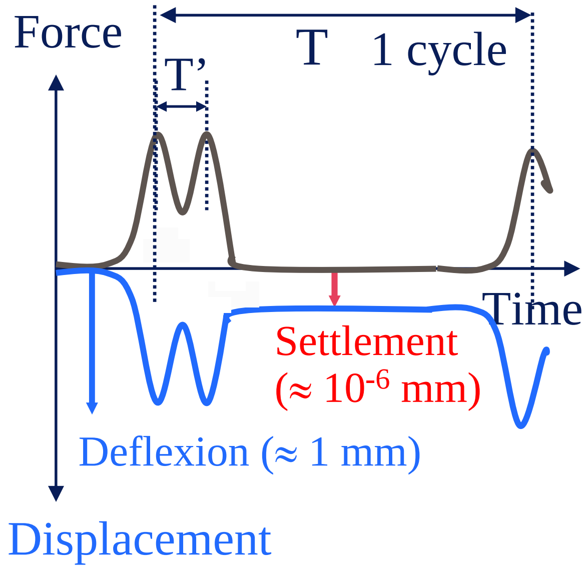
1. Different behaviors under vibration
2. Discrete element model
3. Long term settlement
4. Continuous model

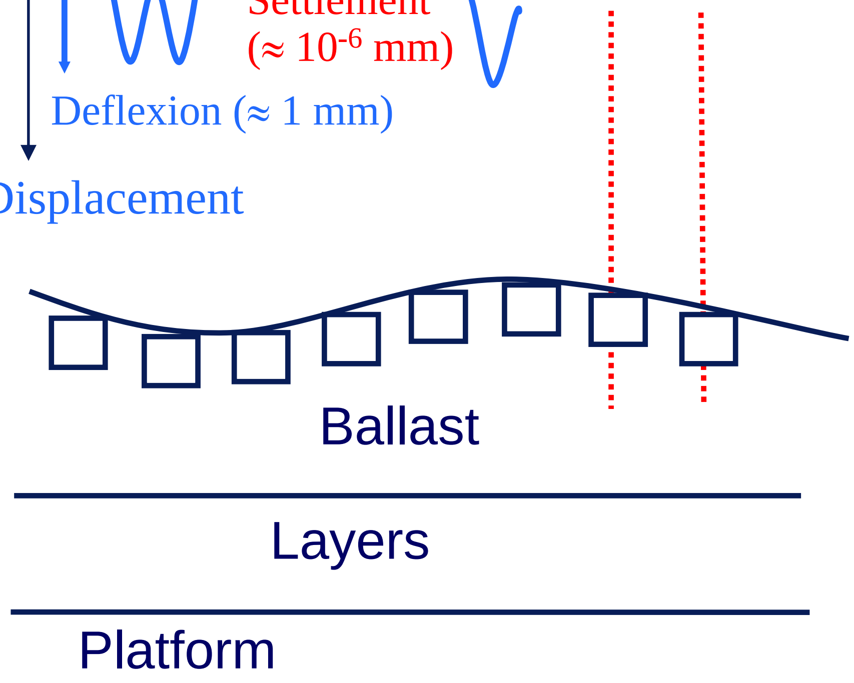
1. Different behaviors under vibration

Problem of settlement of railway tracks

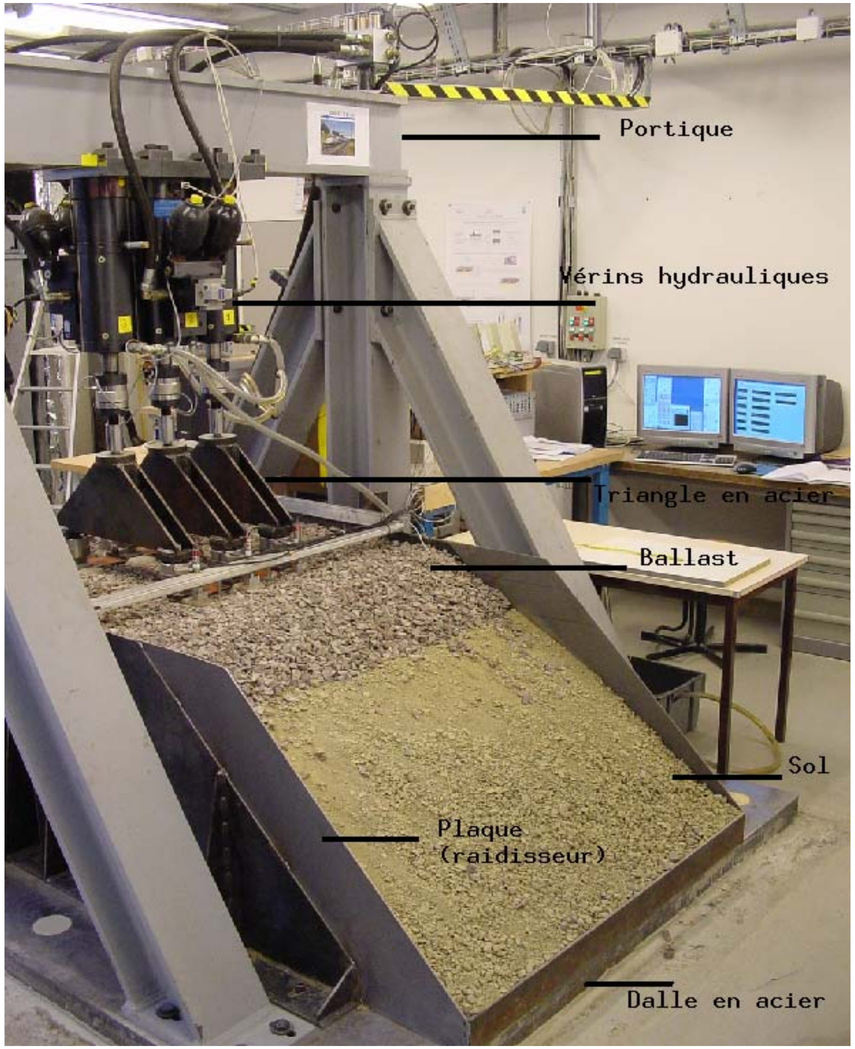




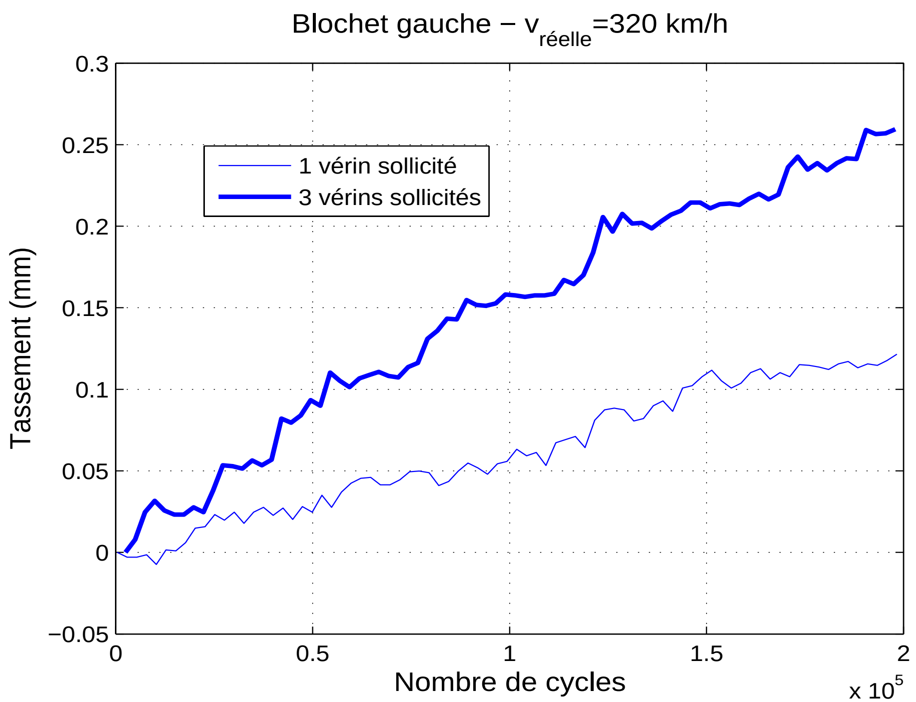


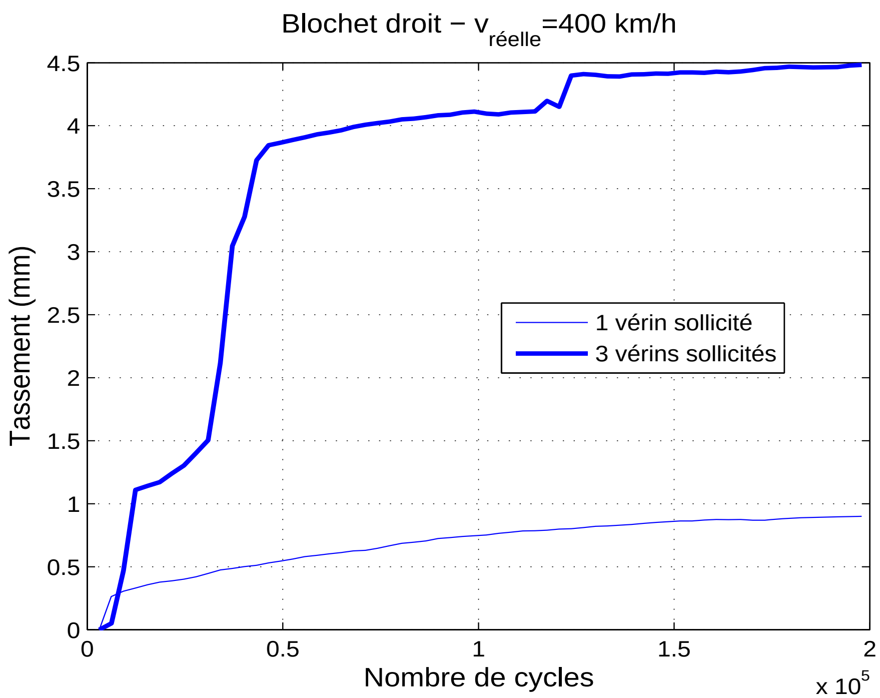


Experimental device

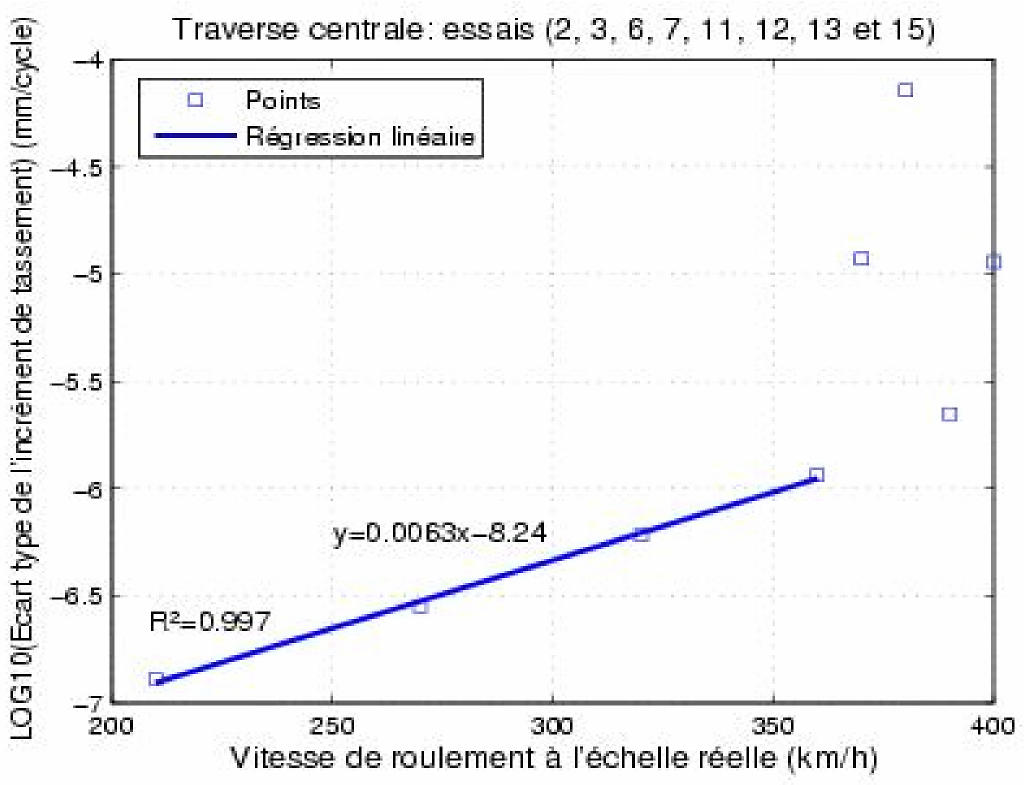


Settlement at 320 and 400 km/h

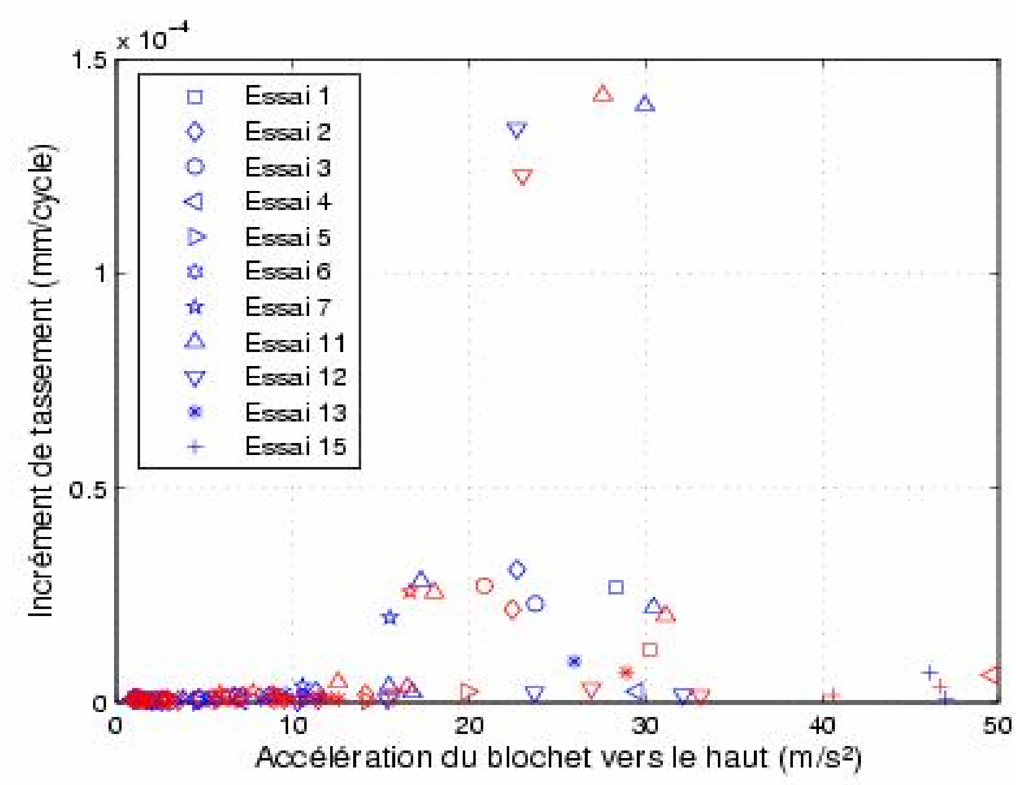




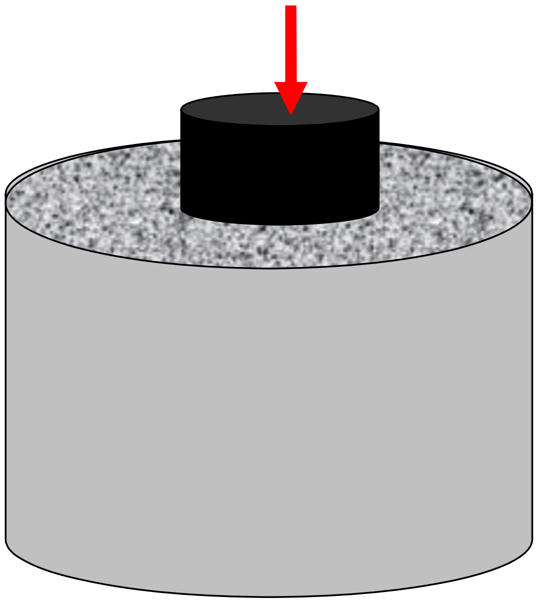
Threshold of speed at 360 km/h



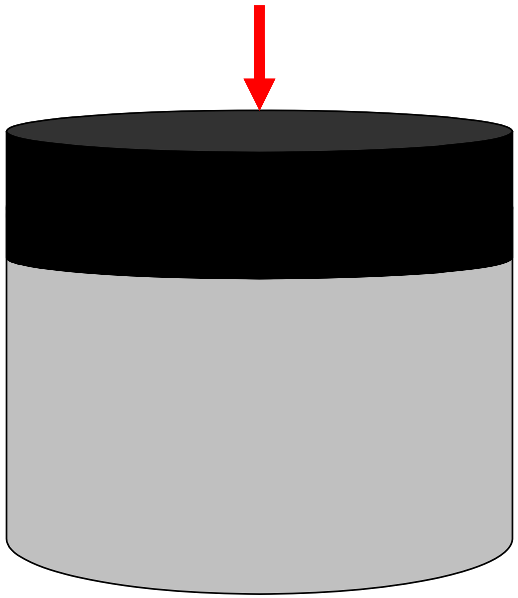
Threshold of acceleration at 10 m/s²



Experimental device

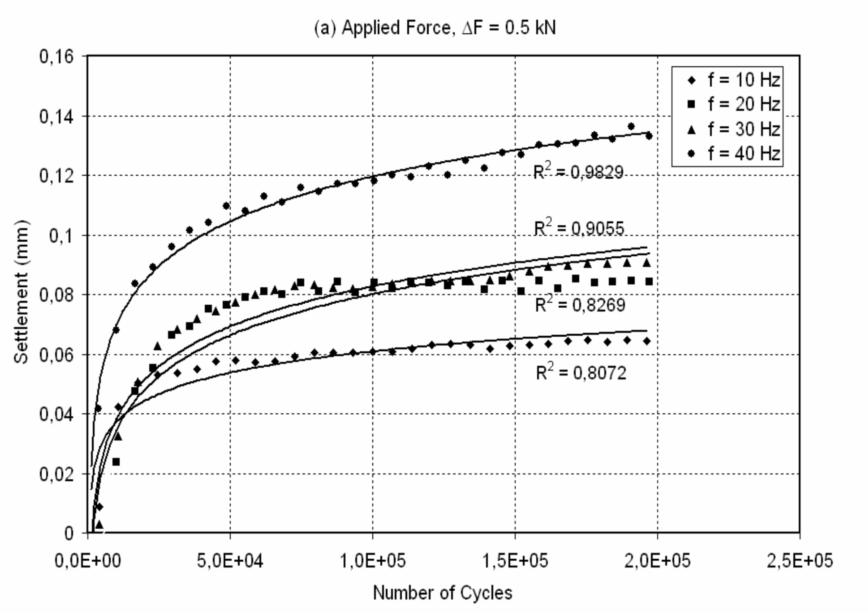


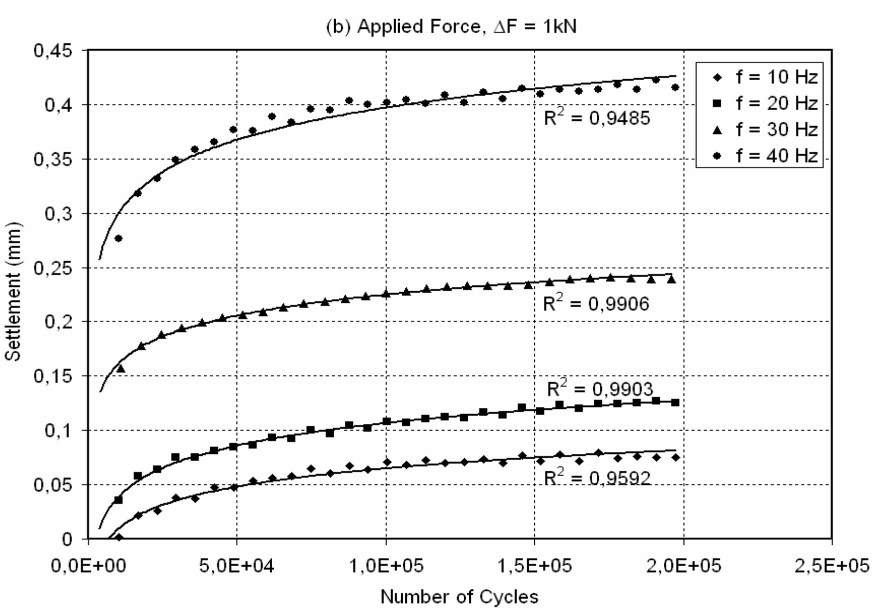
Partially confined

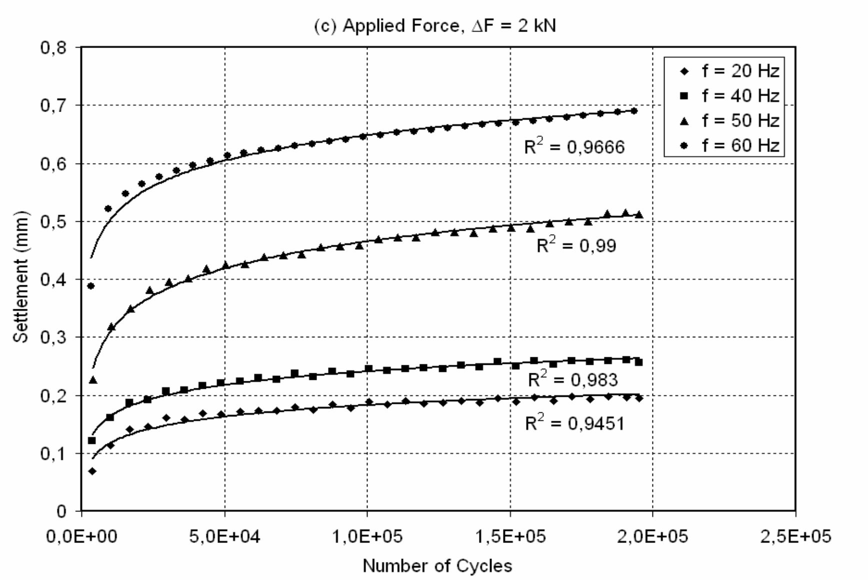


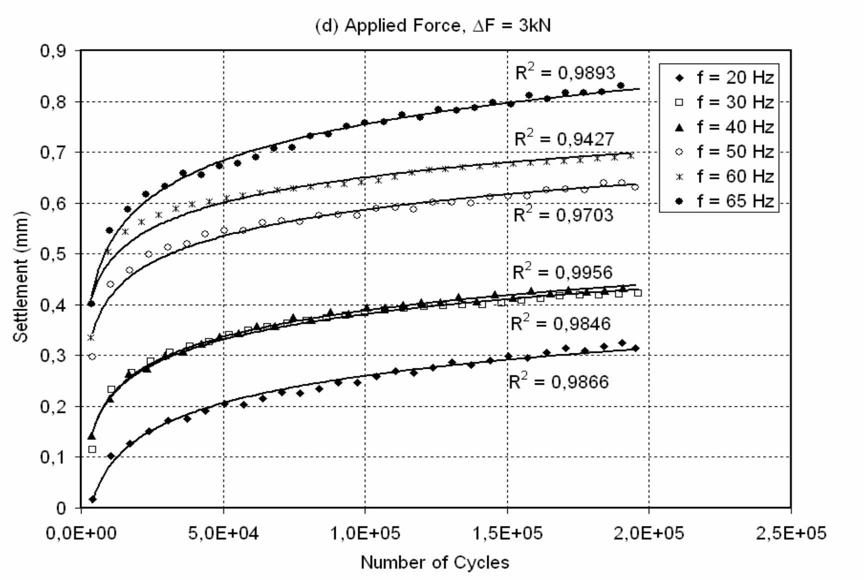
Fully confined

Fully confined samples

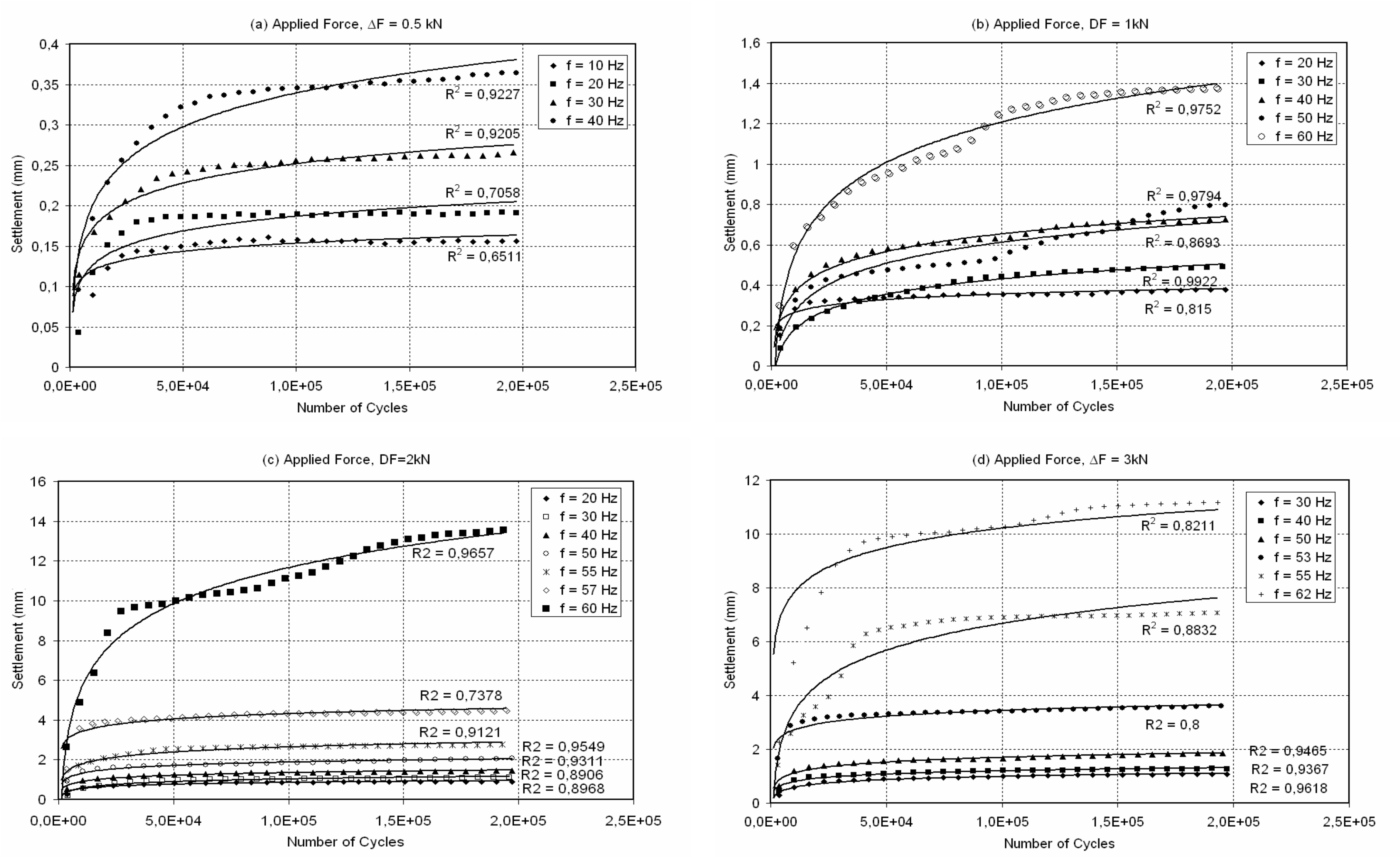




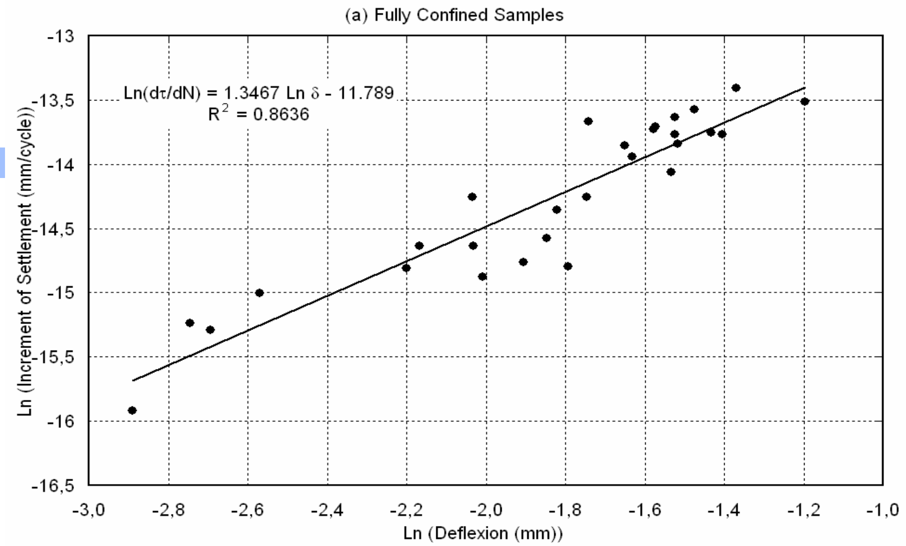


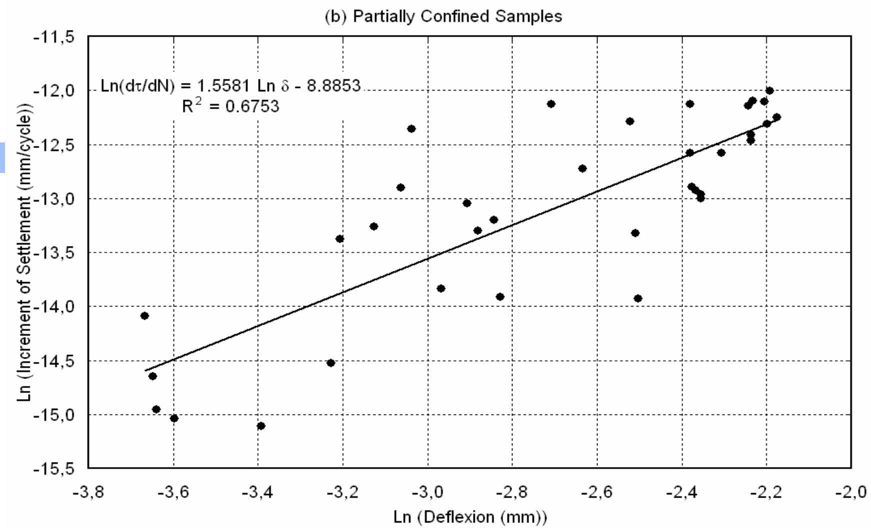


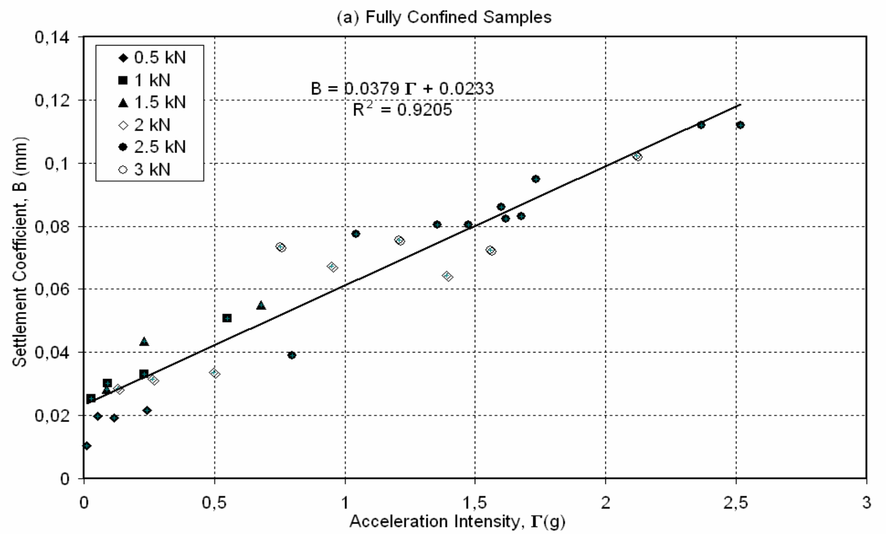
Partially confined samples

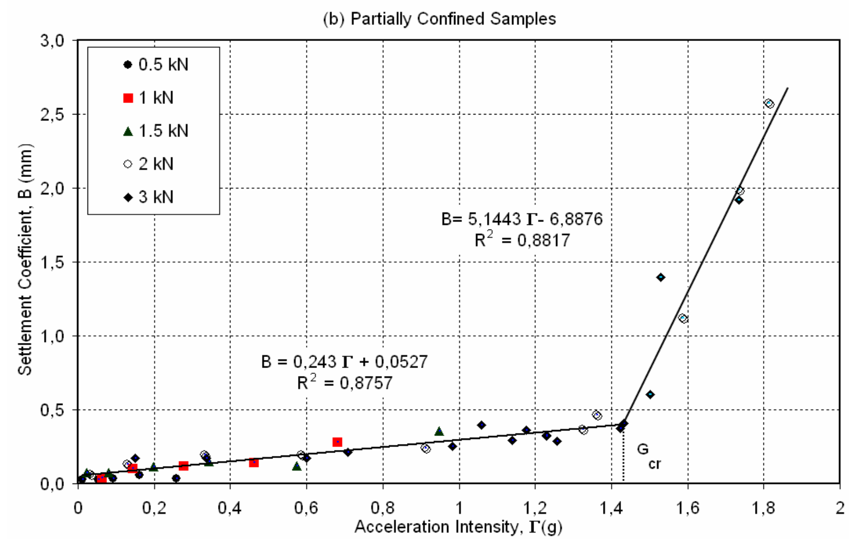


Most important parameters

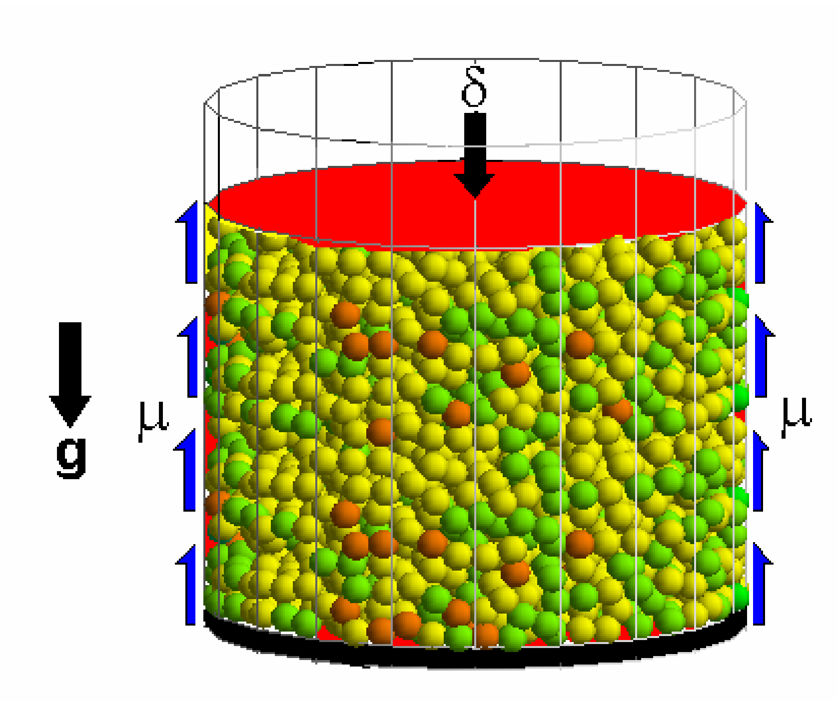


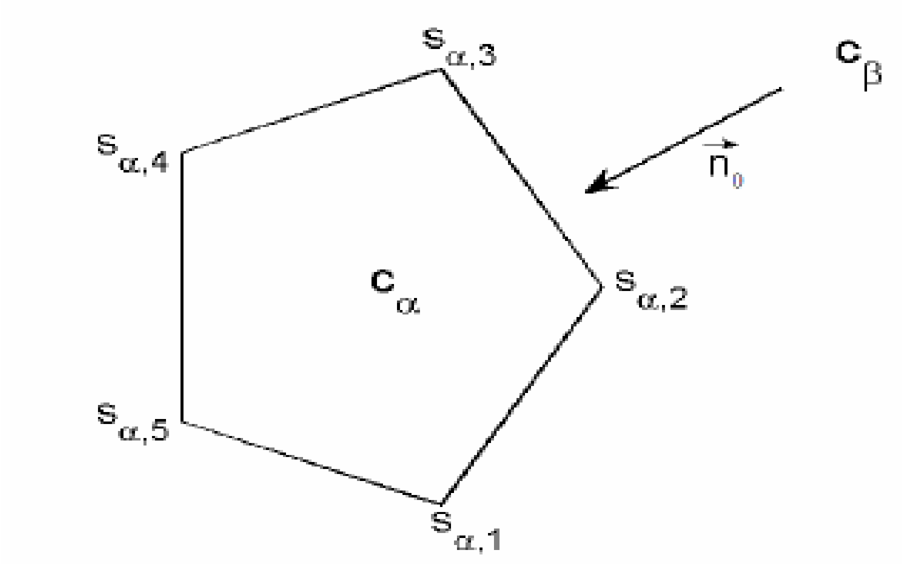


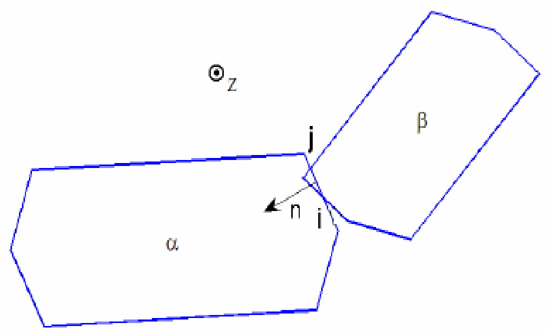




2. Discrete element model







Equations

Dynamic equations

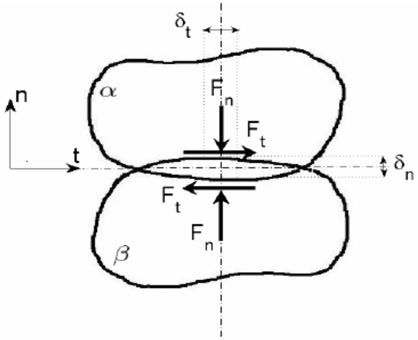
$$m^i \frac{d^2 r^i}{dt^2} = m^i g + \sum_c F^{ic}$$

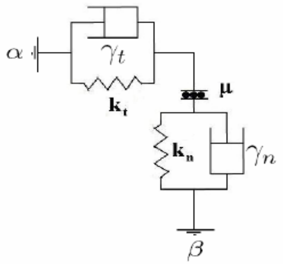
$$I^i \frac{d^2 \omega^i}{dt^2} = R^i \sum_c F_t^{ic}$$

Contact forces for spherical particules

$$F_n^e = k_n \delta_n = \frac{4}{3} E^* \sqrt{R^*} \delta_n^{3/2}$$

$$F_t^e = k_t \delta_t = 8 G^* \sqrt{R^*} \delta_n^{1/2} \delta_t$$



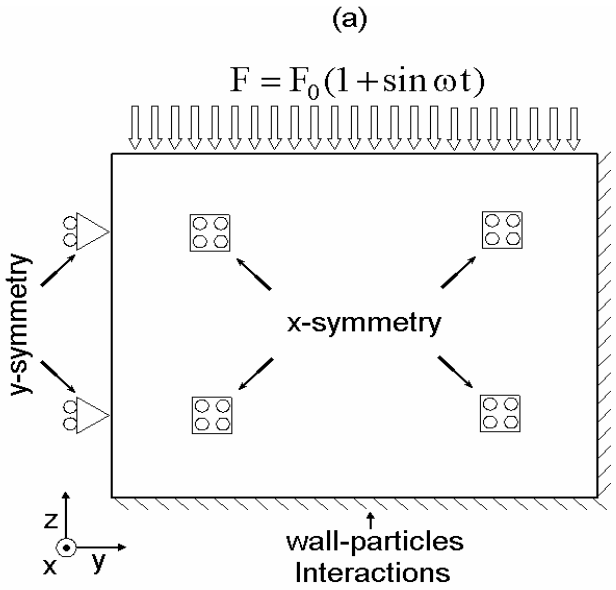


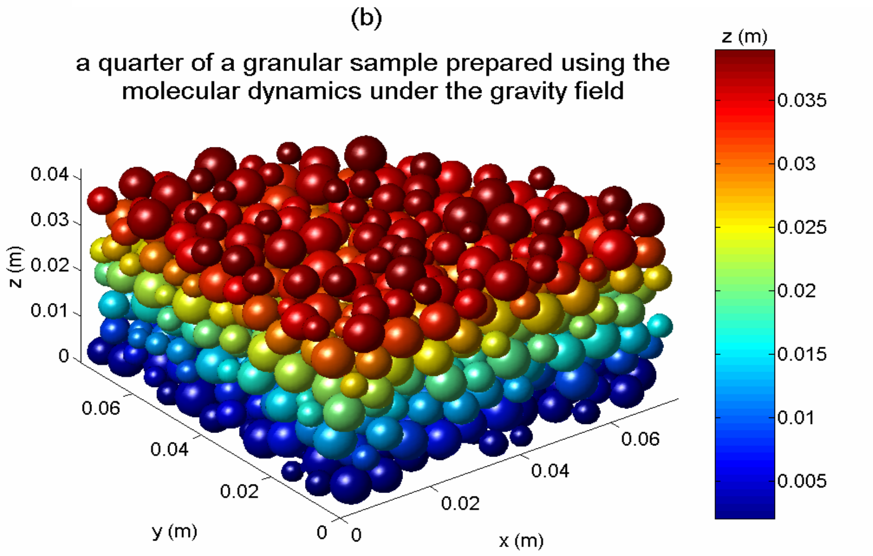
Dissipative terms

$$F_n = F_n^e - \frac{m}{2} \gamma_n v_n$$

$$F_t = \begin{cases} F_t^* - \frac{m}{2} \gamma_t v_t & si \ \|F_t^*\| \leq \mu F_n \\ sign(F_t^*) \mu F_n - \frac{m}{2} \gamma_t v_t & si \ \|F_t^*\| > \mu F_n \end{cases}$$

Load and boundary conditions

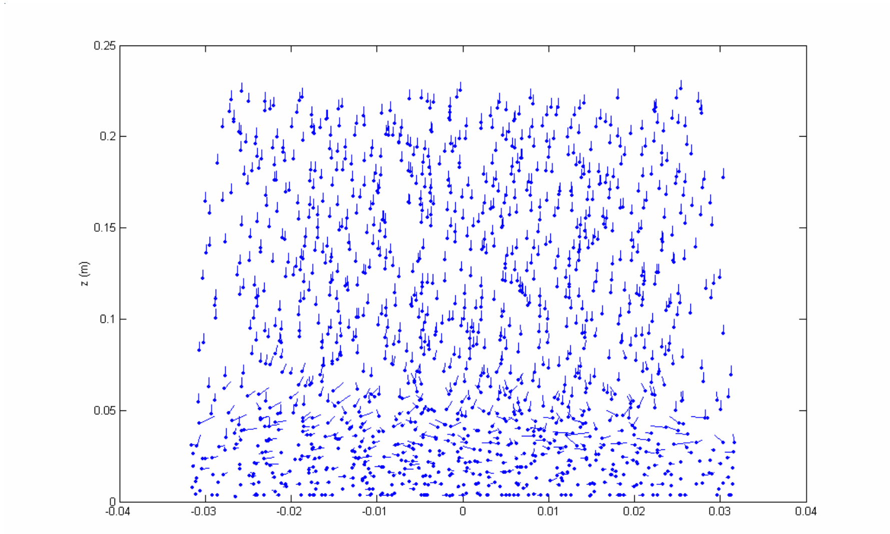


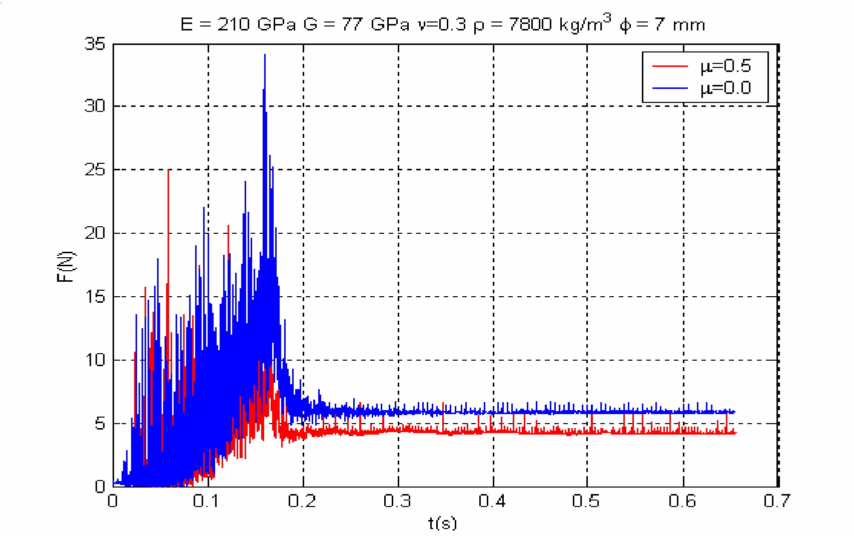


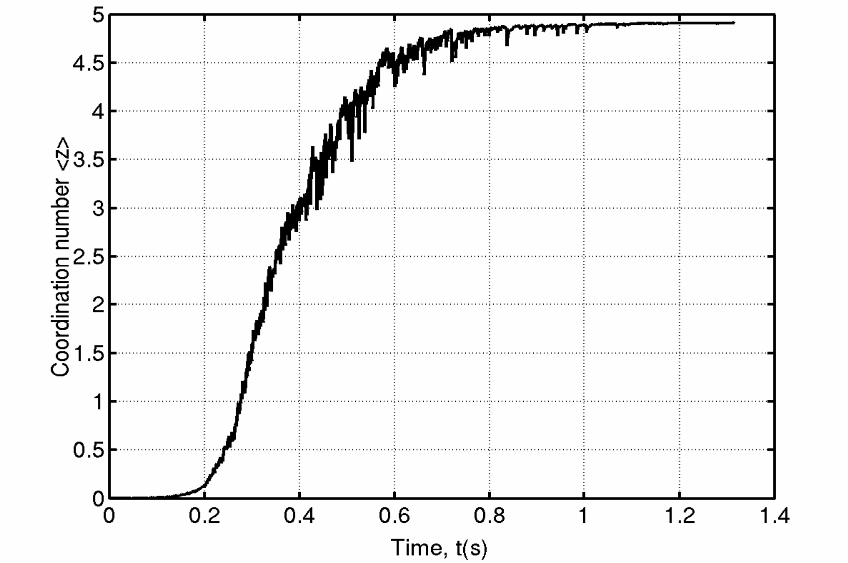
Dimensions			Properties		
Initial density	ϕ_i	0.5615	Particle density	ρ_p	$2710\ kg/m^3$
Mean particle radius	$\bar{r}$	3.2 mm	Particle Young Modulus	E	46.9 GPa
Radius standard deviation	σ_r	$5.65 10^{-4}$	Particle Poisson's ratio	ν	0.25
Radius of the cylindrical container	R	75 mm	Internal friction coefficient	μ	0.5
Sample's Length	H	60 mm	Restitution coefficients	e_n, e_t	0.5

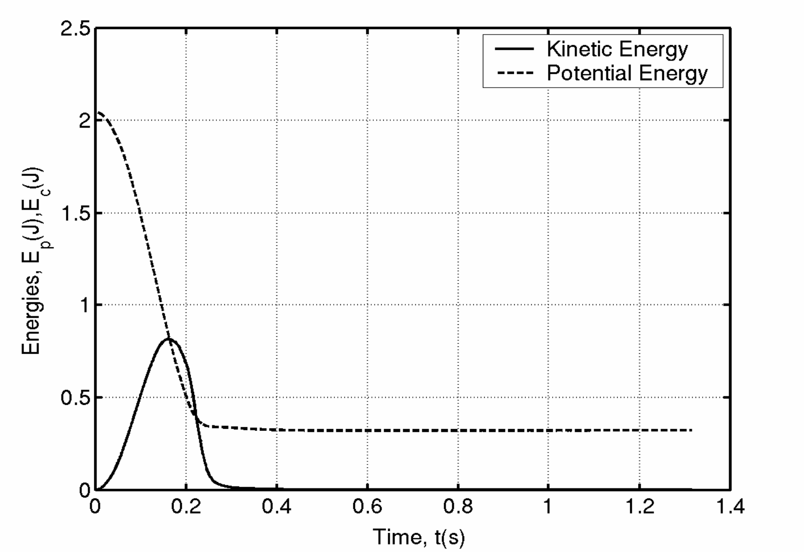
Table 1. System parameters and material properties used for simulation.

Preparation of samples

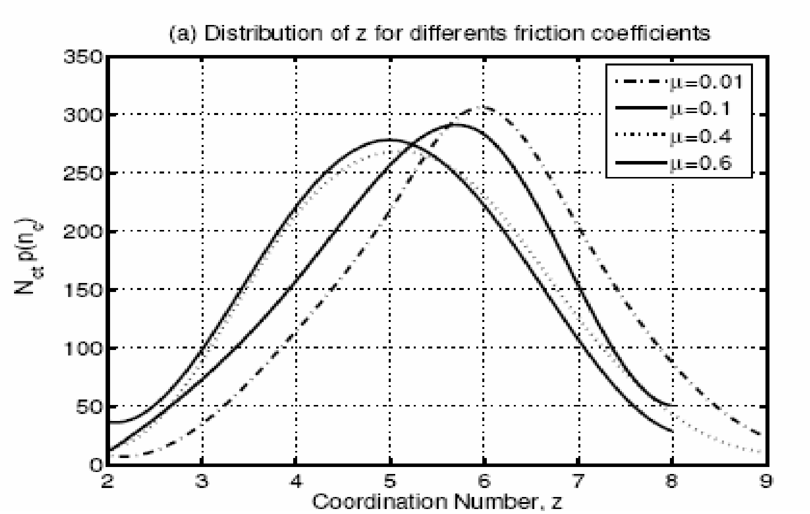


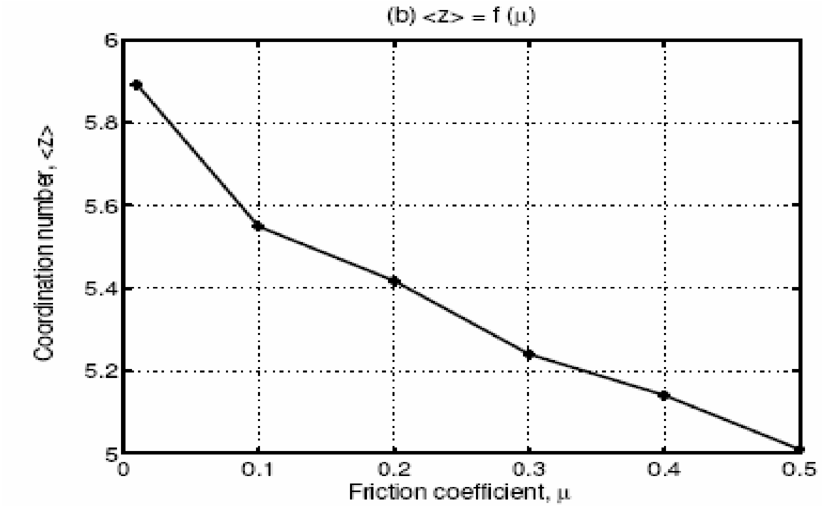


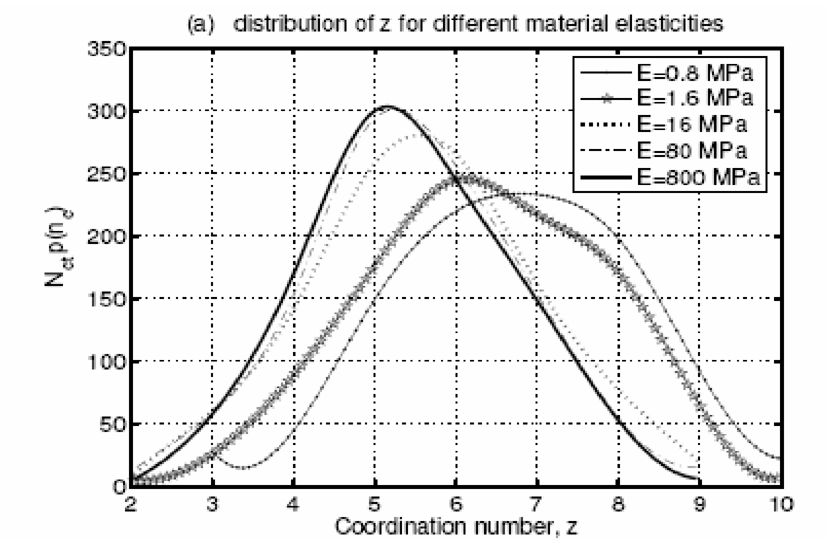


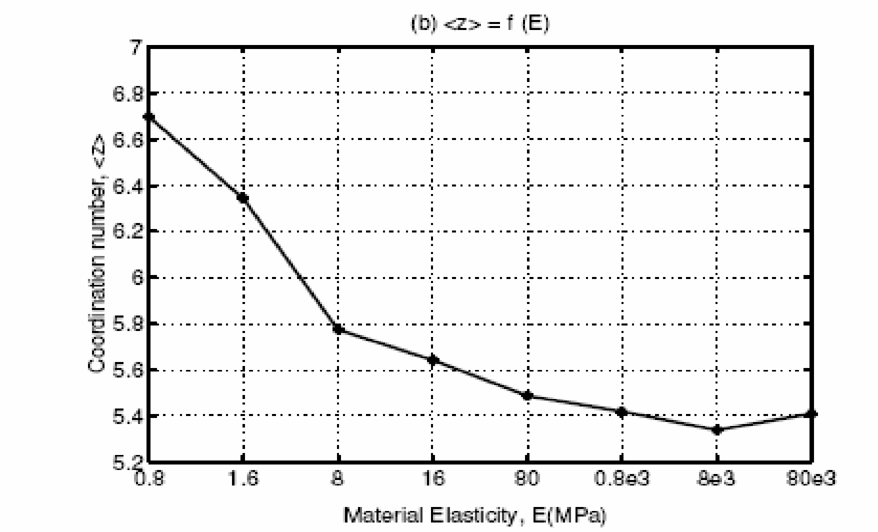


Influence of different parameters



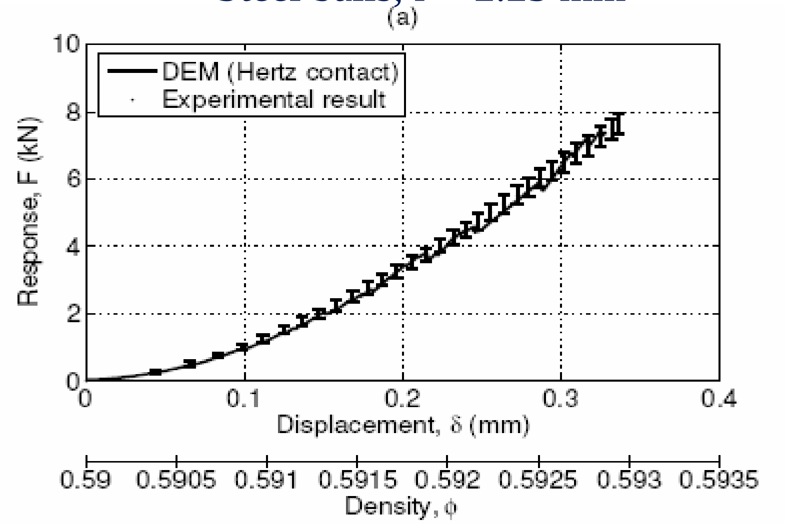




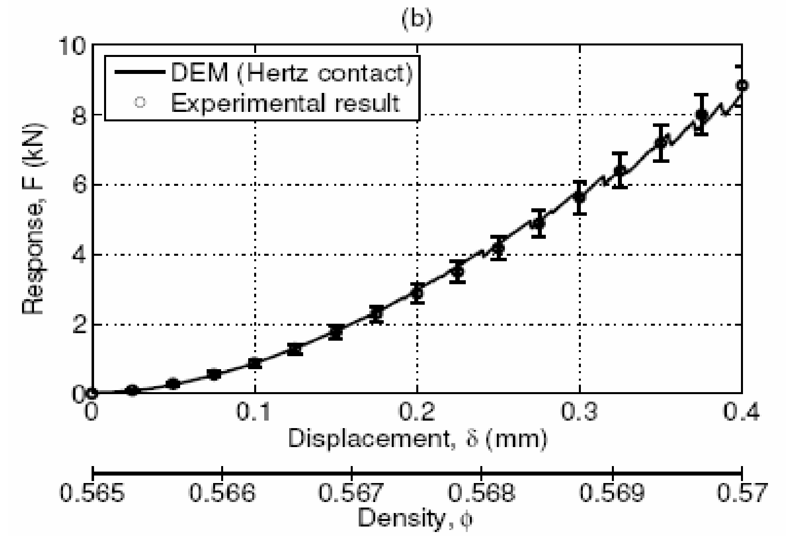


Comparison with experiments for static loads

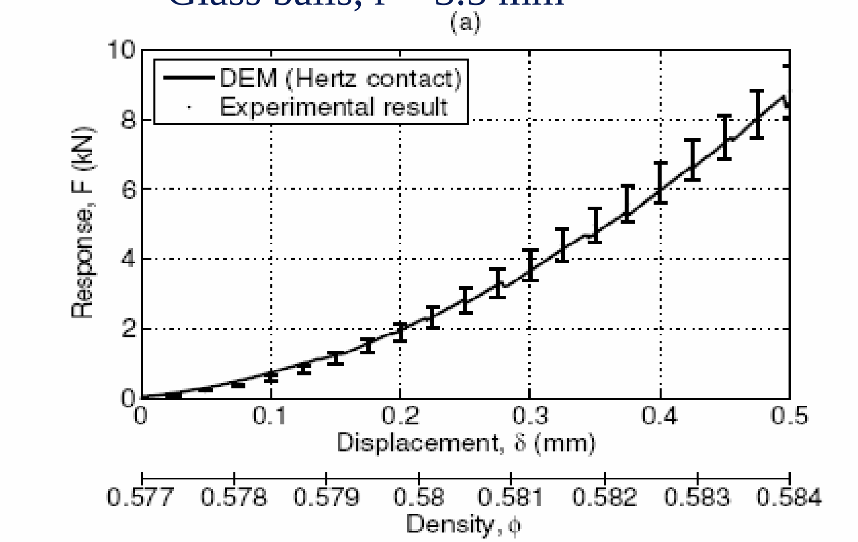
Steel balls, r = 2.25 mm

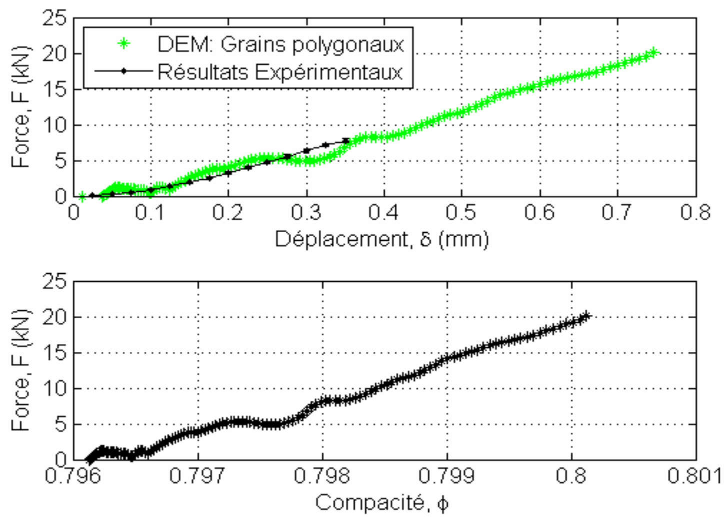


Steel balls, r = 4.75 mm

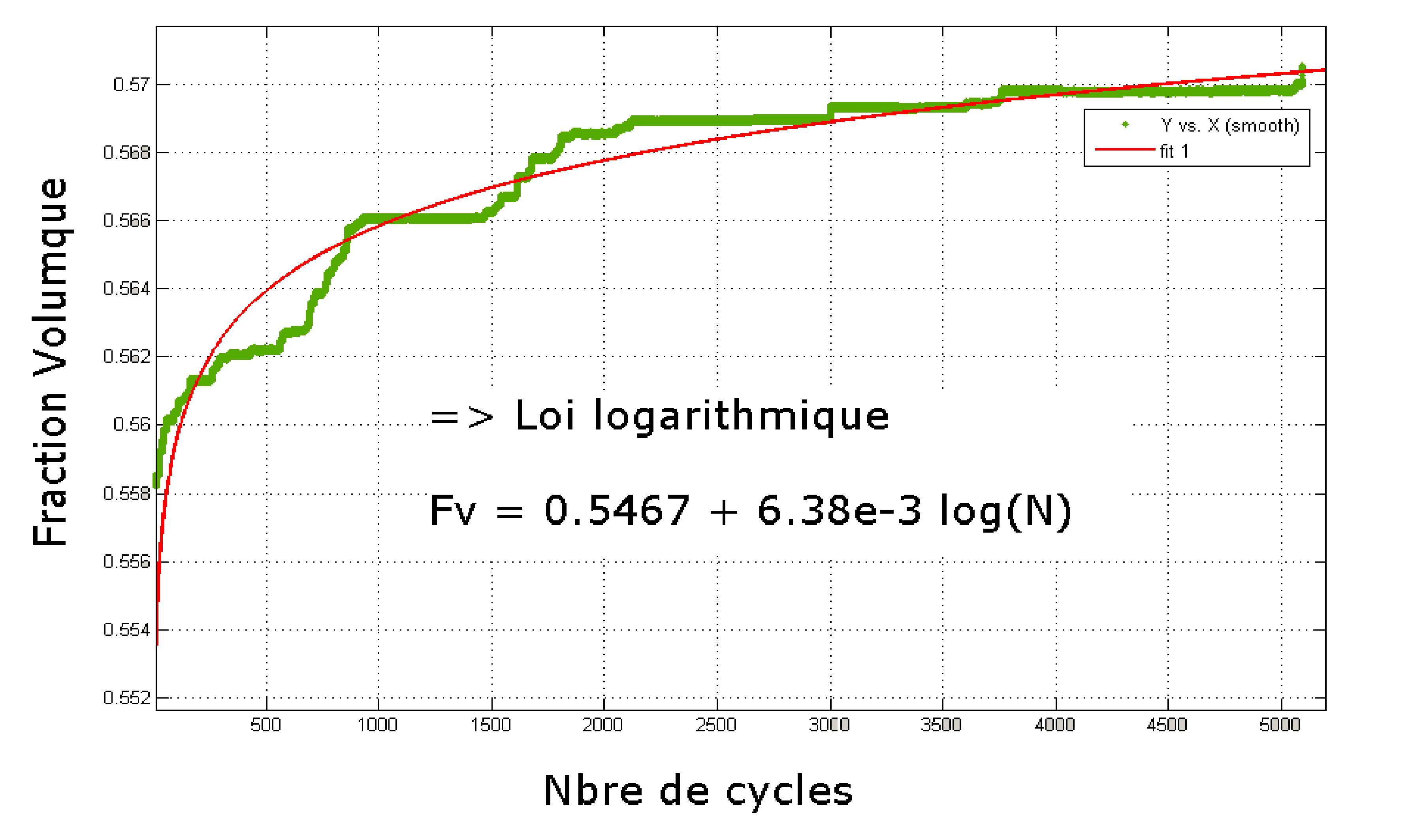


Glass balls, r = 3.5 mm

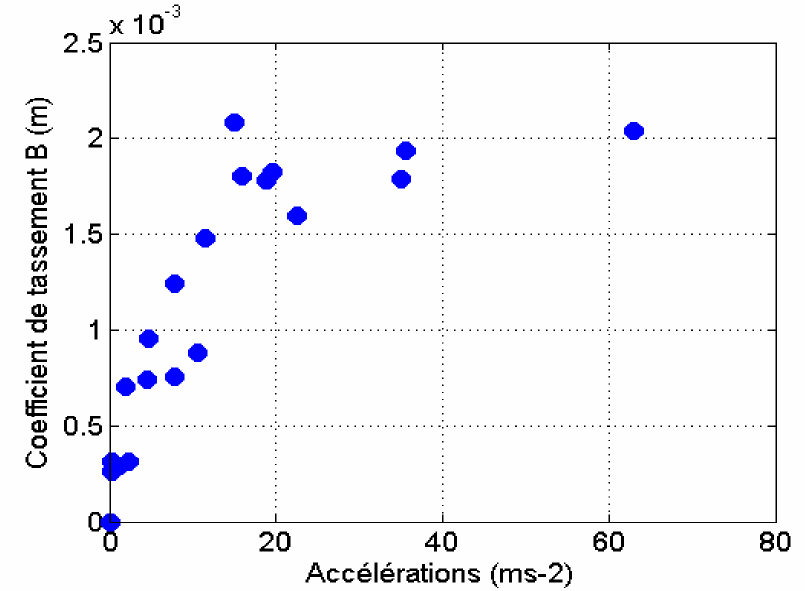




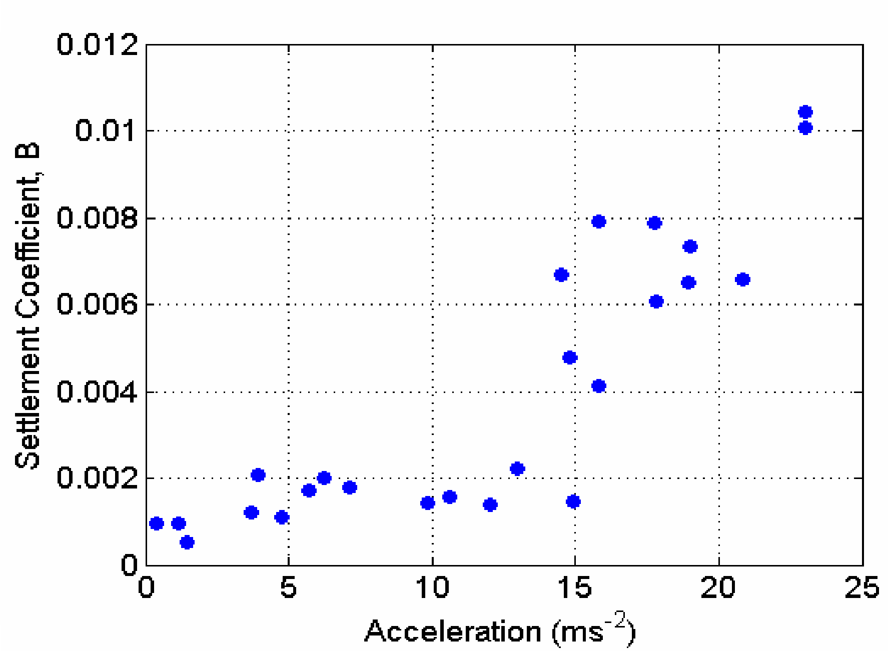
Density versus number of cycles



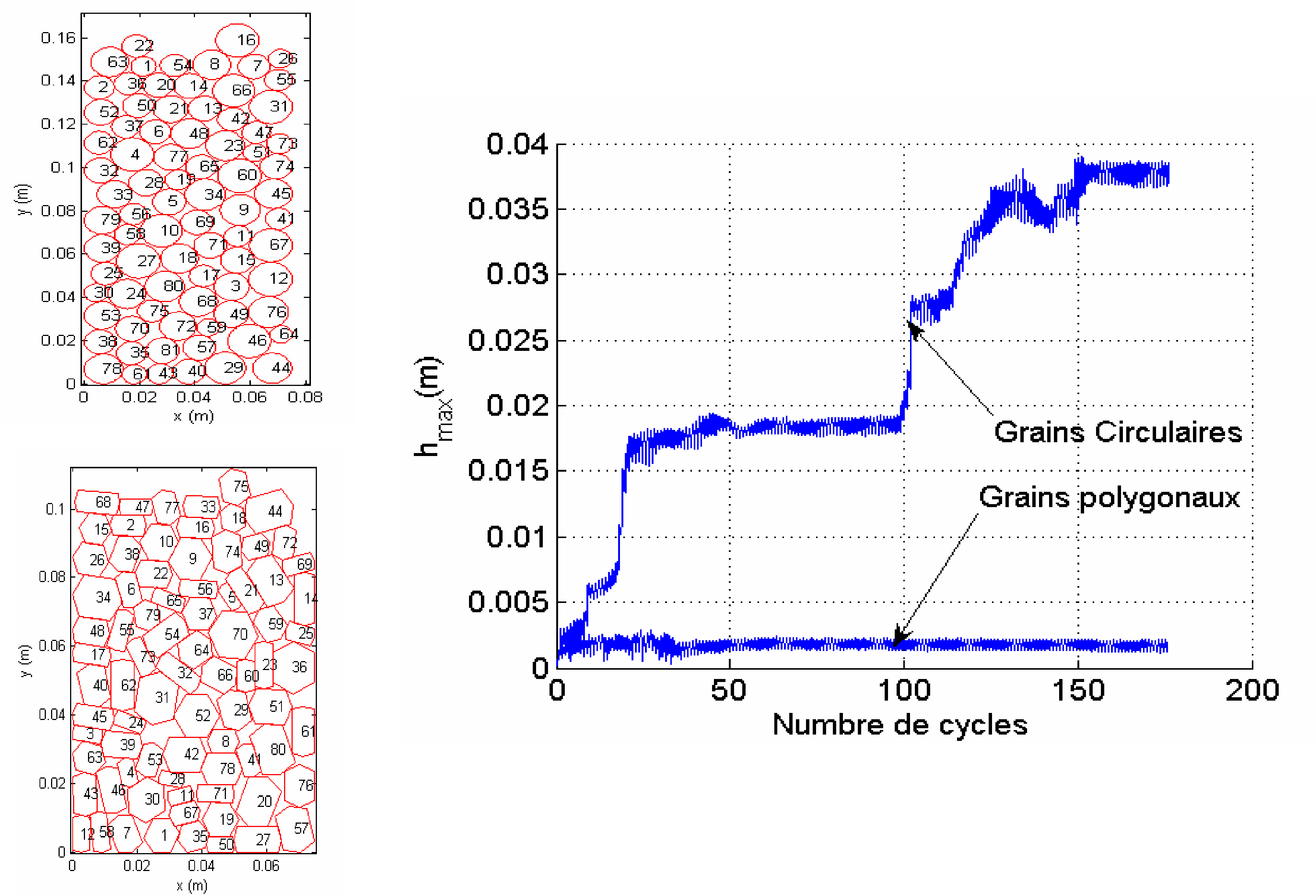
fully confined



partially confined

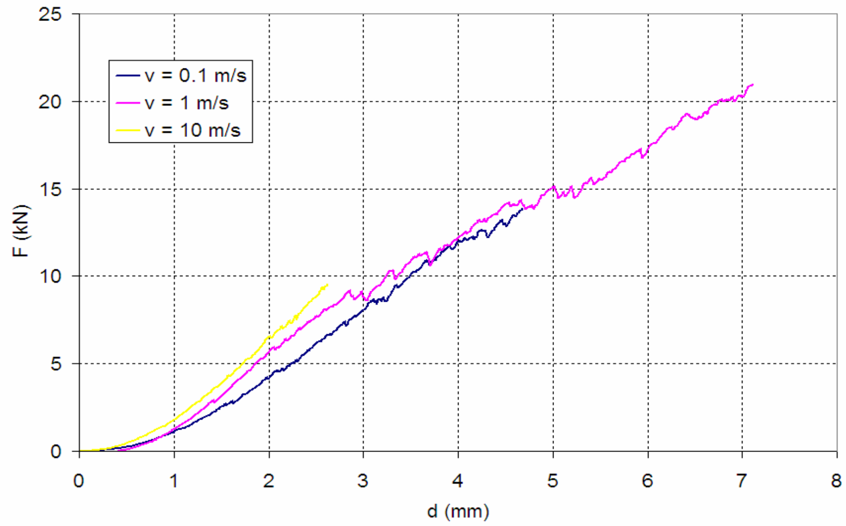


- The settlement increases with the acceleration
- Movement of grains in the partially confined case
- Critic acceleration at ~ 1.5g, in the partially confined case

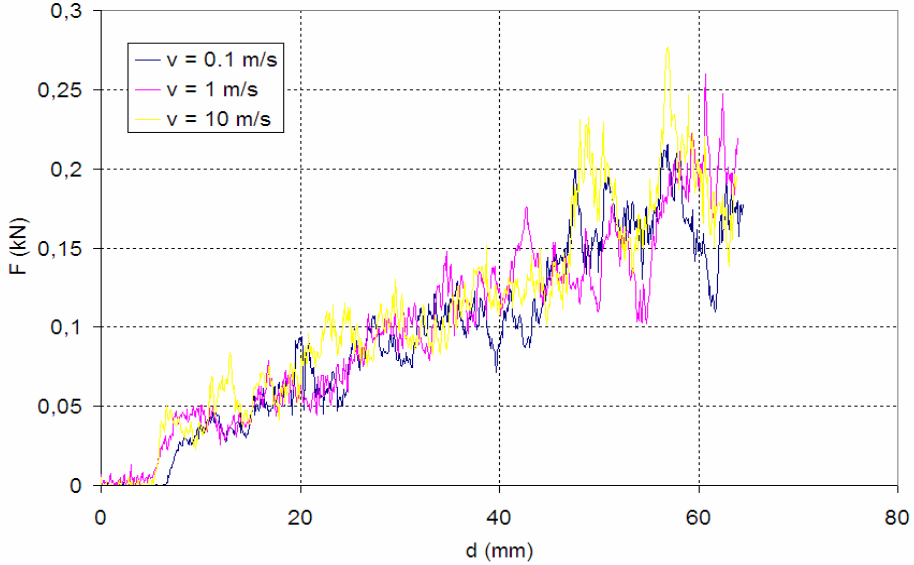


y (m)
x (m)
Grains Circulaires
Grains polygonaux
h_{max}(m)
Numbre de cycles

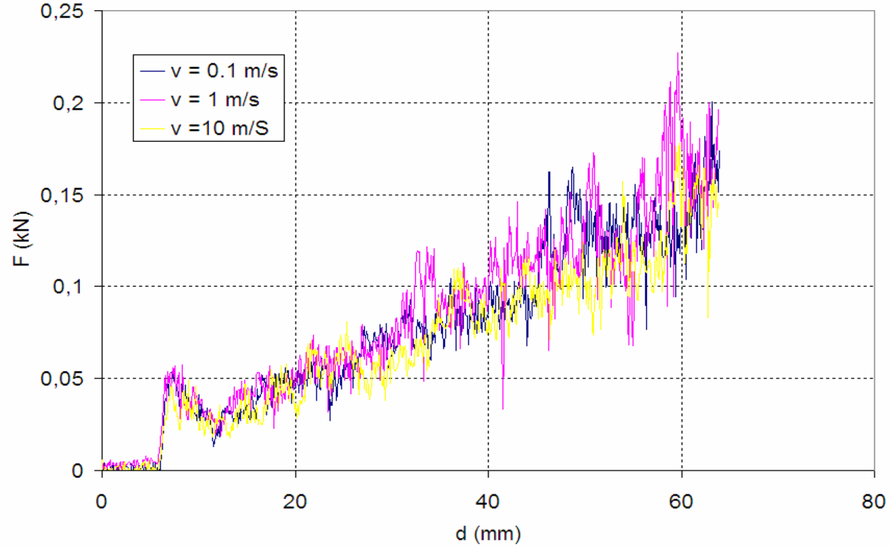
Ballast grains

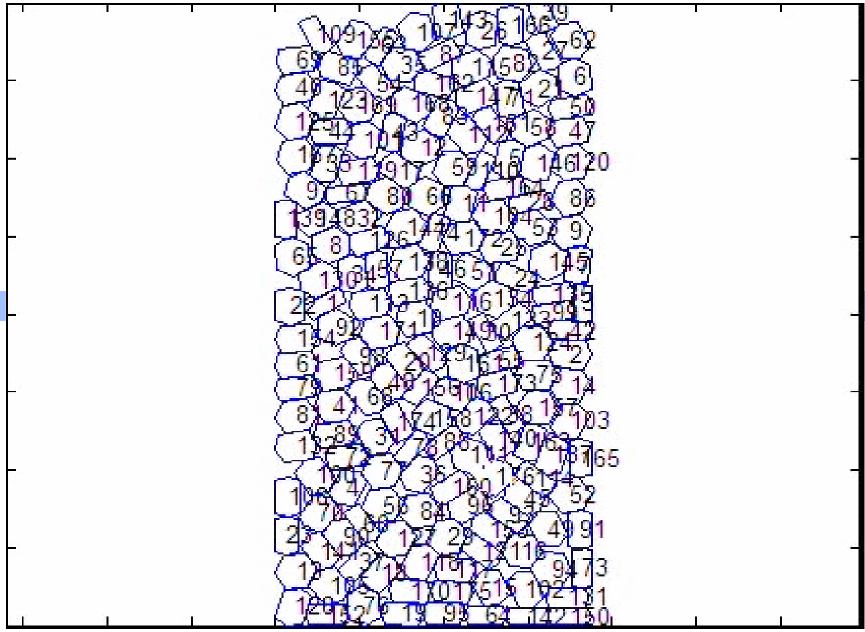


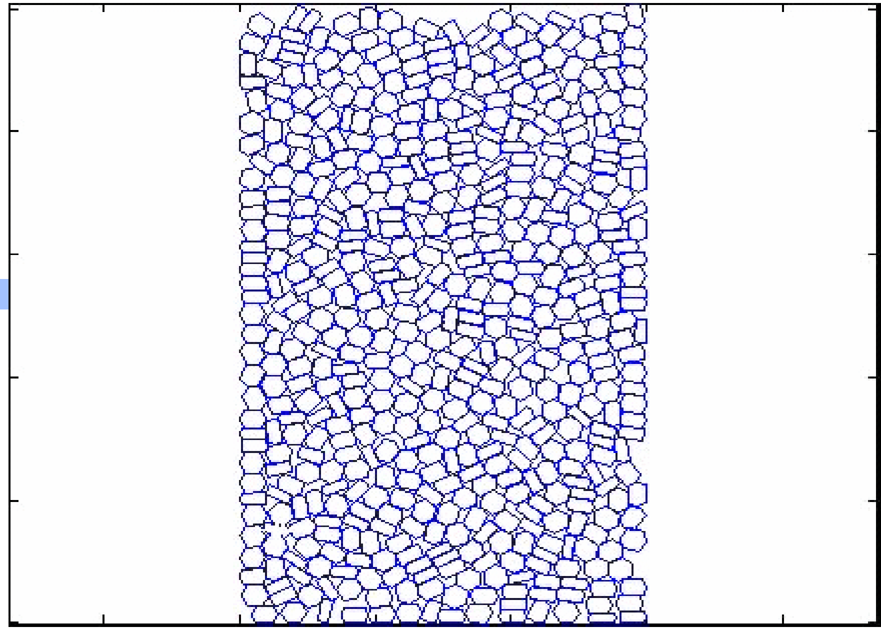
Steel balls

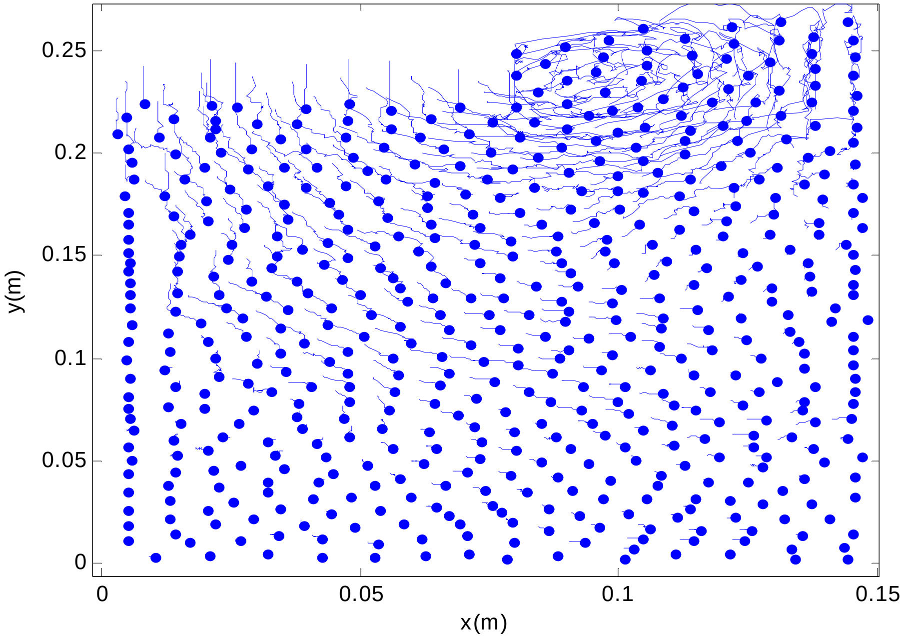


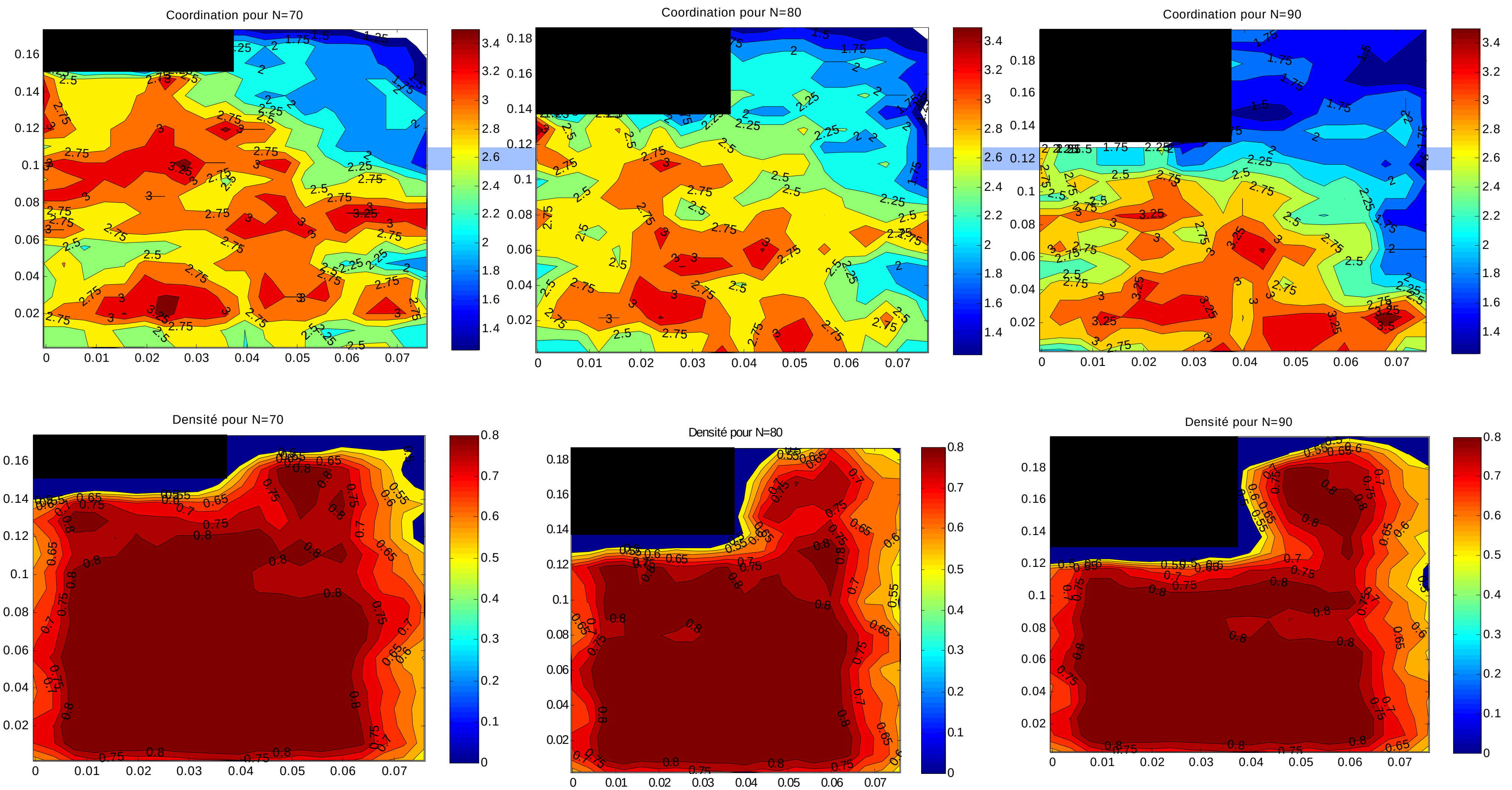
Glass balls








0.25
0.2
0.15
0.1
0.05
0
y(m)
0
0.05
0.1
0.15
x(m)



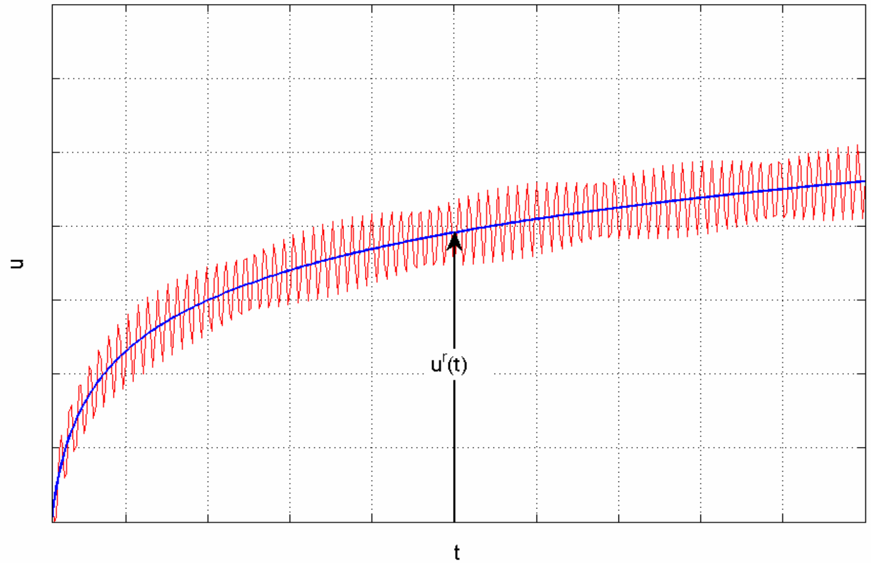
Coordination pour N=70
Coordination pour N=80
Coordination pour N=90
Densité pour N=70
Densité pour N=80
Densité pour N=90

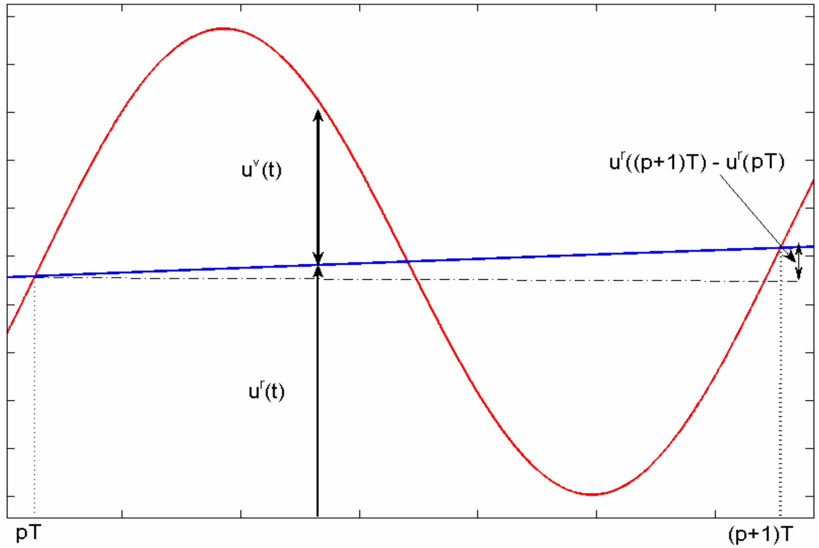
3. Computation of long term settlements

Definition of residual displacement

$$u^r_\beta(T) = \lim_{\tau \to +\infty} \left(u_\beta(T+\tau) - u_\beta(0)\right)$$

For a cyclic loading this quantity can be defined by





$$\begin{cases} \forall t \in [0,T], \forall \beta \in \mathcal{S} \\ u_\beta(t) = u^r_\beta(t) + u^v_\beta(t) \end{cases} \Rightarrow \quad u_\beta(t+T) = u_\beta(t) + \left[u^r_\beta(t+T) - u^r_\beta(t)\right] + \xi_\beta$$

The parameters of the model are obtained by minimizing the quantity

$$\mathcal{R} = \sum_{p=1}^{k} \left(\xi_\beta^p\right)^2 = \sum_{p=1}^{k} \left(u_\beta^r(t+pT) - \tilde{u}_\beta^r(t+pT)\right)^2$$

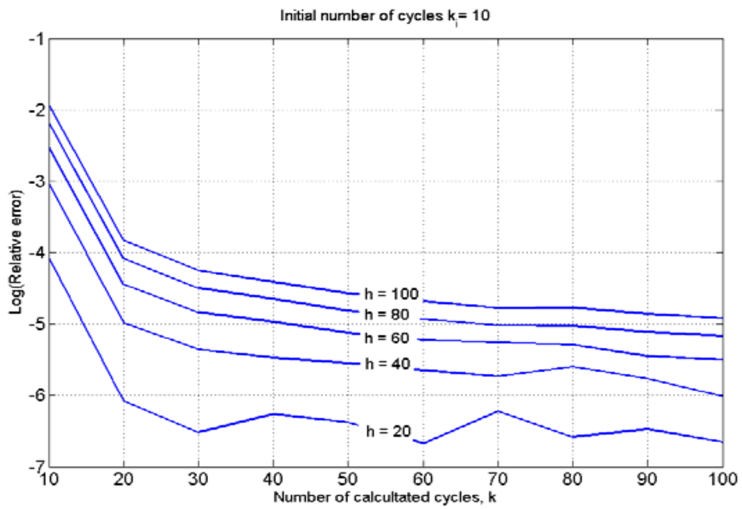
After k cycles of calculation using the molecular dynamic procedure, a linear estimate of the residual displacements is obtained by

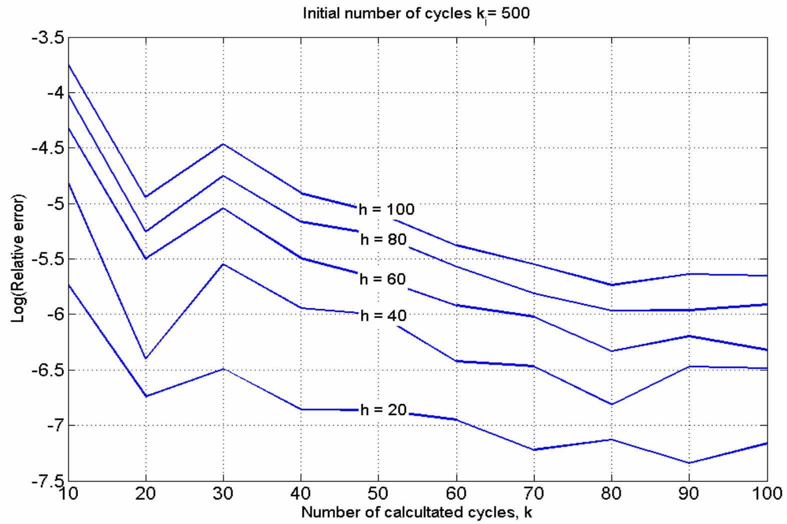
$$\hat{u}_\beta(t+(k+h)T) = u_\beta(t+kT) + \frac{h}{k}\sum_{p=1}^{k} u_\beta^r(t+pT) - u_\beta^r(t+(p-1)T)$$

The logarithmic estimate is obtained by

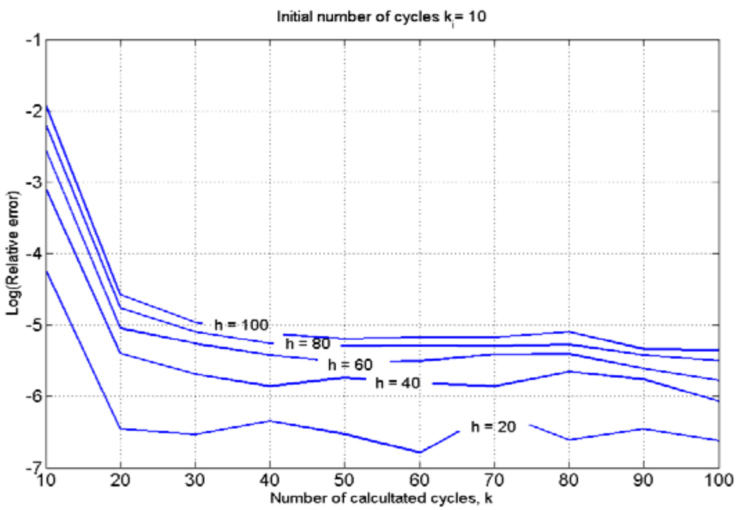
$$\hat{u}_\beta(t+(k+h)T) = u_\beta(t+kT) + a_\beta Ln(t+(k+h)T) + b_\beta(t+(k+h)T)$$

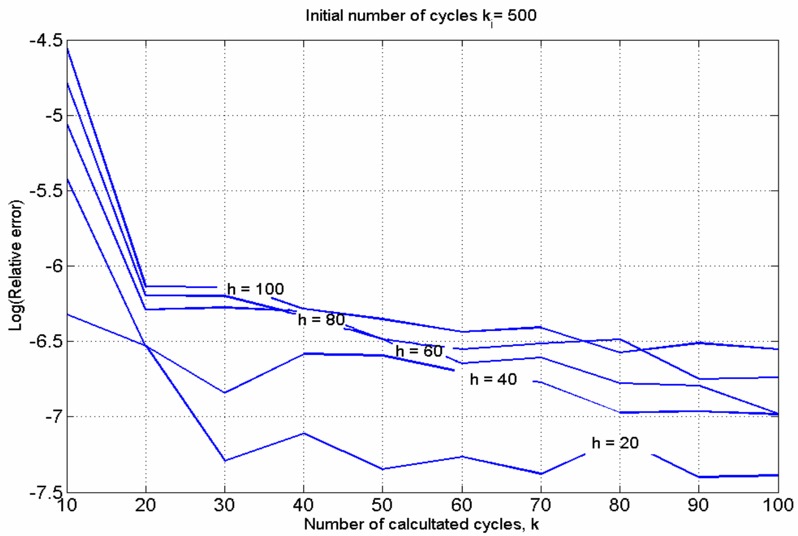
For linear estimations, the error is given by





For logarithmic estimations, the error is given by

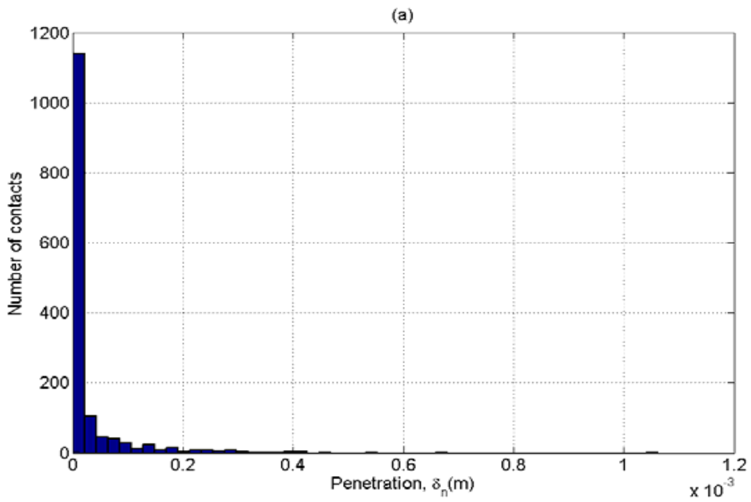


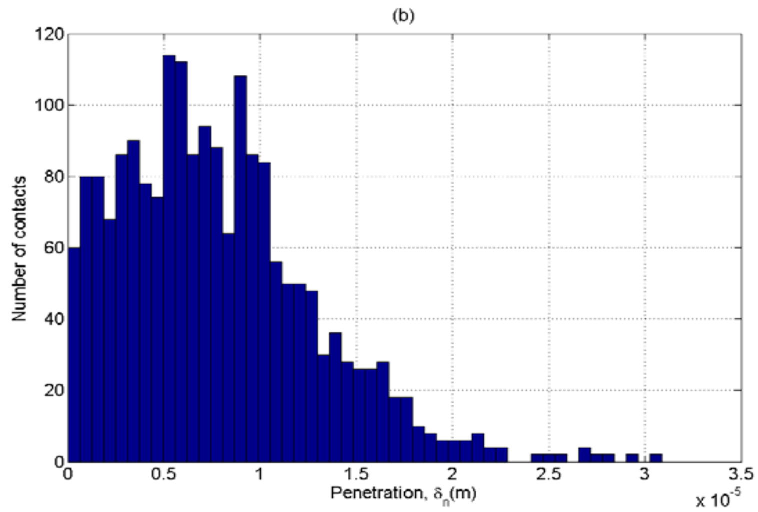


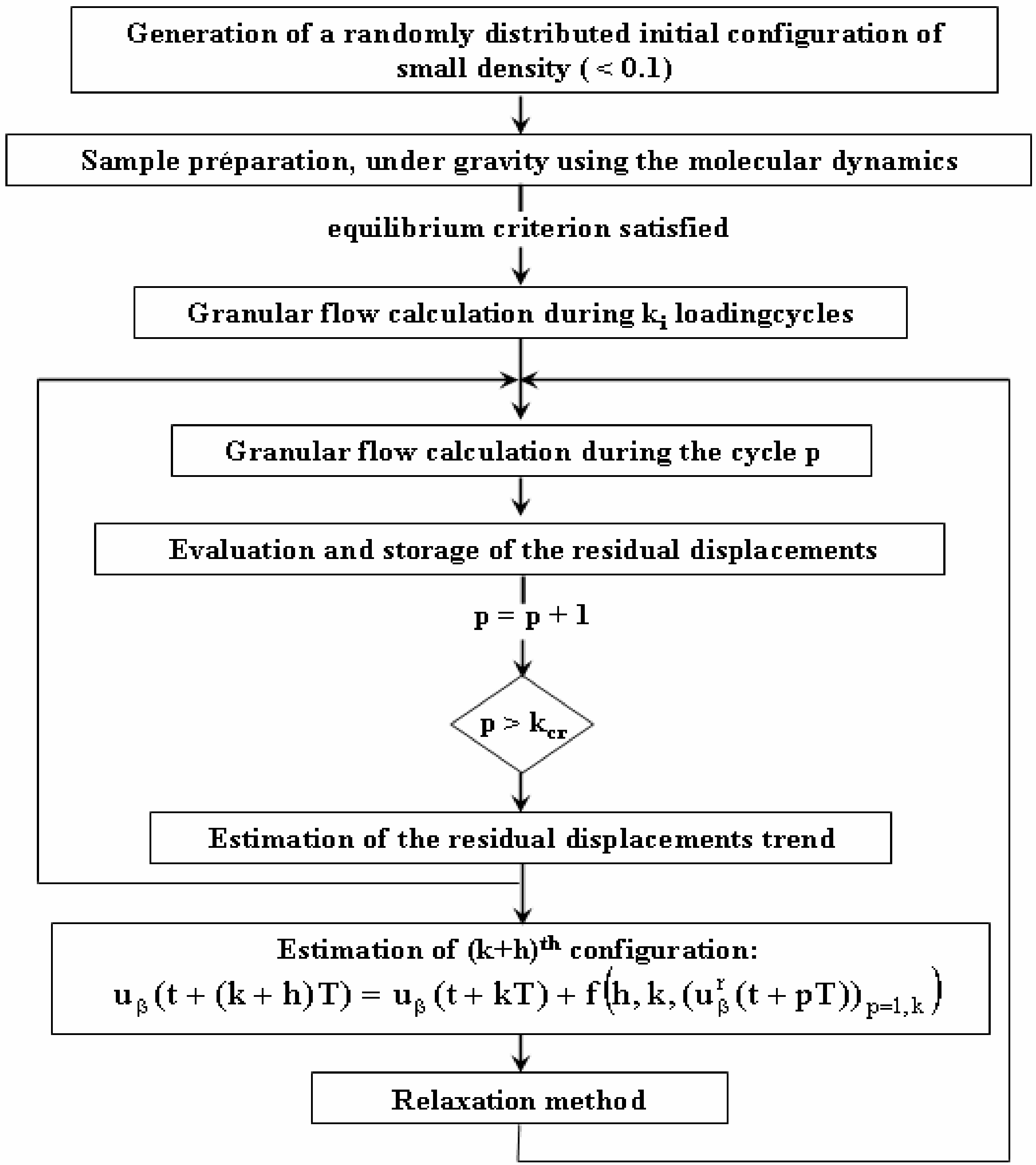
The function to minimize is

$$\mathcal{F}(u) = \sum_{\beta \in \overline{\mathcal{S}}^f} F^f_\beta u_\beta + \sum_{\beta \in \mathcal{S}} m_\beta g u_\beta + \sum_{\beta \in \mathcal{S}} \sum_{\alpha \neq \beta \in \mathcal{S}} \psi_{\alpha\beta}(u_\beta)$$

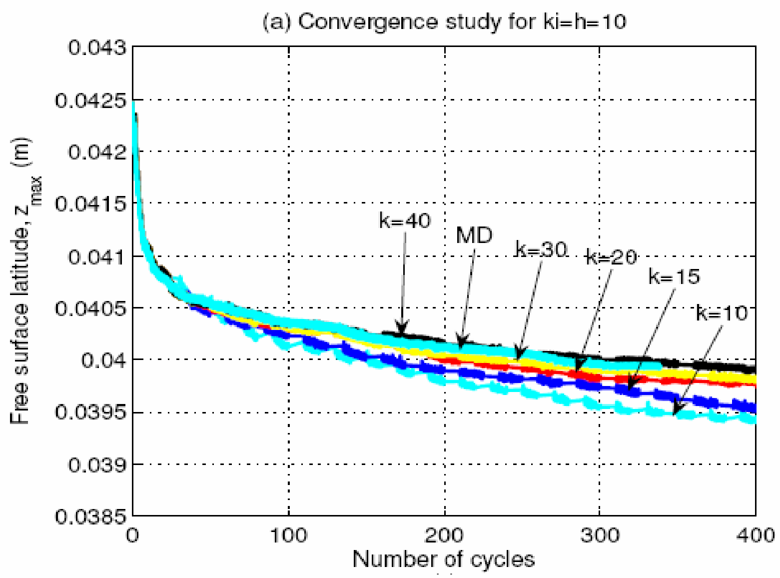
Example: obtained by a DM computation and an extrapolation with (ki = 20, k = 20, and h= 20)



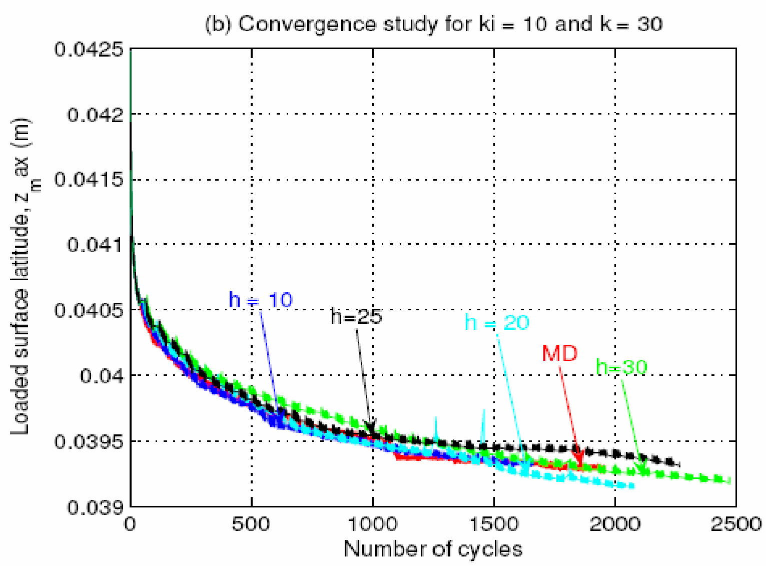



Generation of a randomly distributed initial configuration of small density (< 0.1)
Sample préparation, under gravity using the molecular dynamics
equilibrium criterion satisfied
Granular flow calculation during k_i loadingcycles
Granular flow calculation during the cycle p
Evaluation and storage of the residual displacements
p = p + 1
$p > k_{cr}$
Estimation of the residual displacements trend
Estimation of $(k+h)^{th}$ configuration:
$u_\beta(t+(k+h)T) = u_\beta(t+kT) + f\left(h, k, (u^r_\beta(t+pT))_{p=1,k}\right)$
Relaxation method

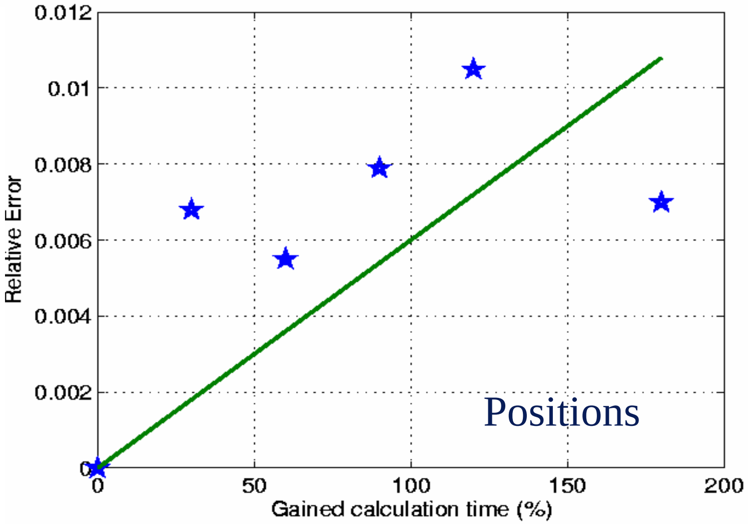
Examples



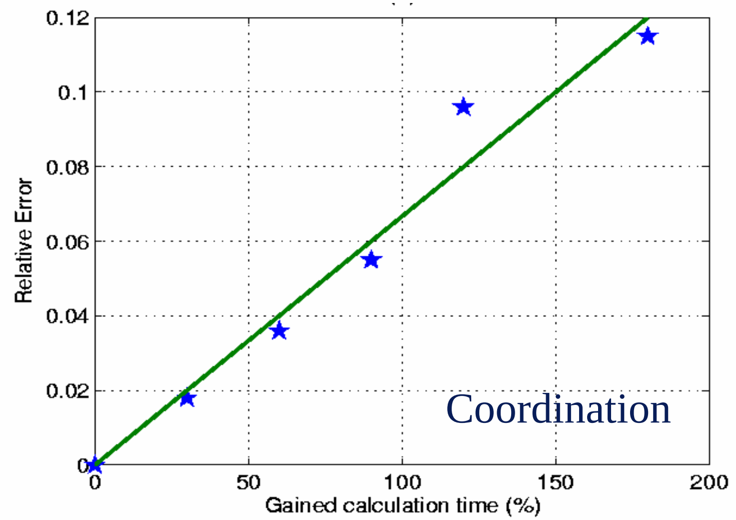
(a) Convergence study for ki=h=10
Free surface latitude, z_{max} (m)
Number of cycles
k=40
MD
k=30
k=20
k=15
k=10



(b) Convergence study for ki = 10 and k = 30
Loaded surface latitude, z_{max} (m)
Number of cycles
h = 10
h=25
h = 20
MD
h=30

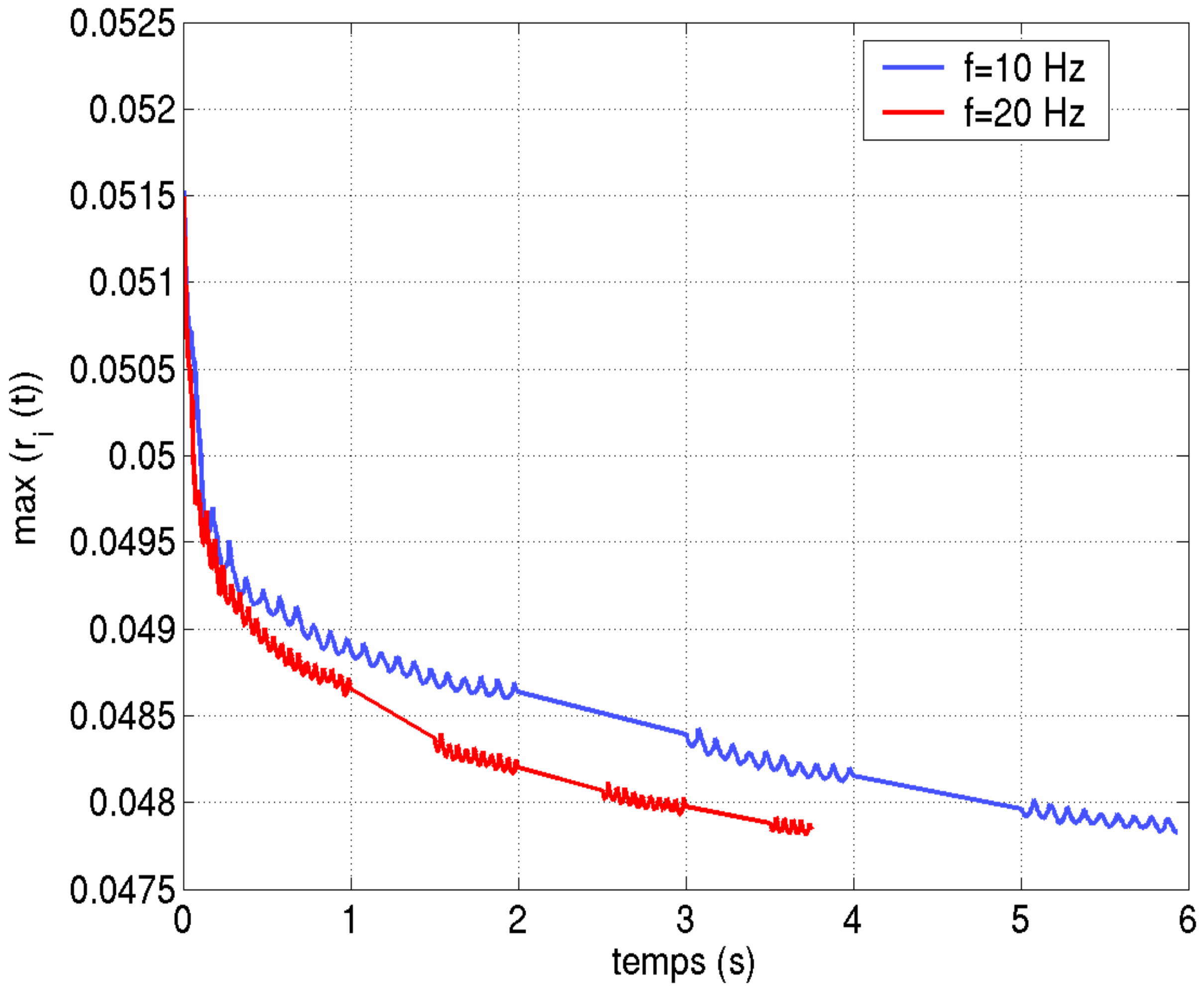


Relative Error
Gained calculation time (%)
Positions



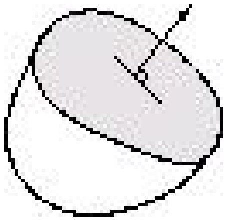
Relative Error
Gained calculation time (%)
Coordination

Numerical simulation of settlement for cyclic loads



4. Continuous model of granular media

- **Unilateral constitutive law in 3D :**
  - Isotropic and homogeneous medium
  - Small deformation
  - Stress tensor negative definite



$$\sigma_{ij} n_i n_j \leq 0 \qquad \sigma(\varepsilon) = \frac{\partial W}{\partial \varepsilon}$$

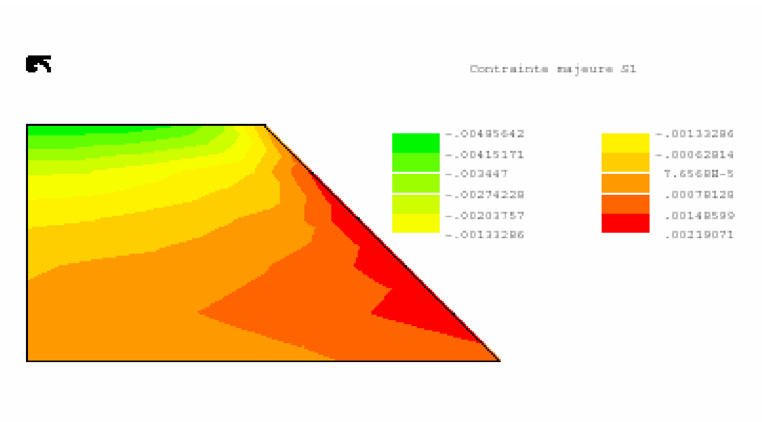
- **Linear continuous piecewise law**

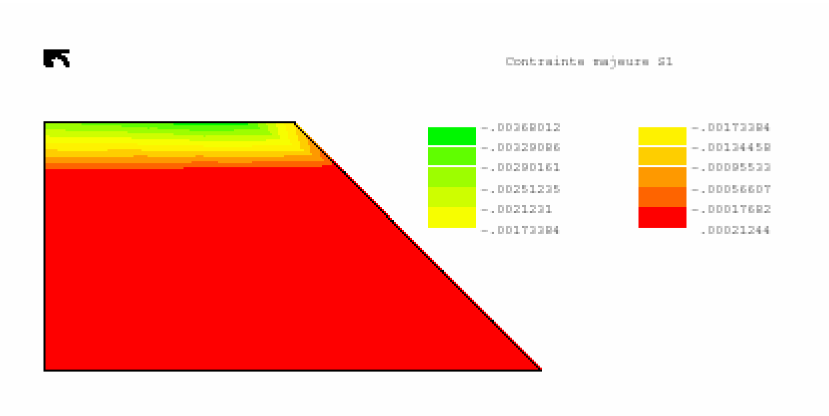
Model of ballast with unilateral behavior

Example : Pressure on an embankment made of ballast

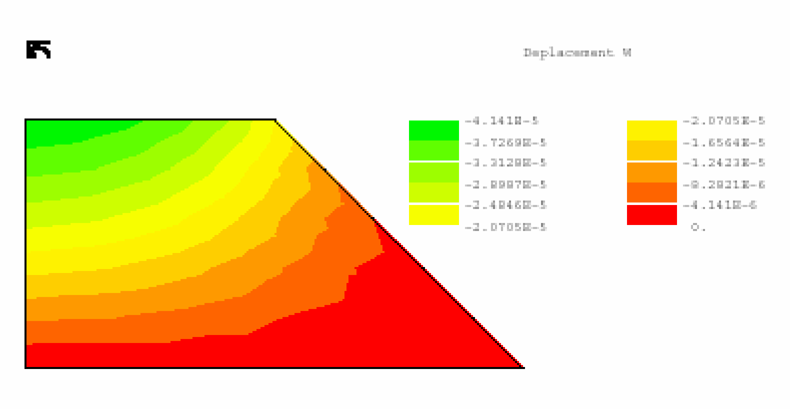
élastique linéaire ***unilatéral***

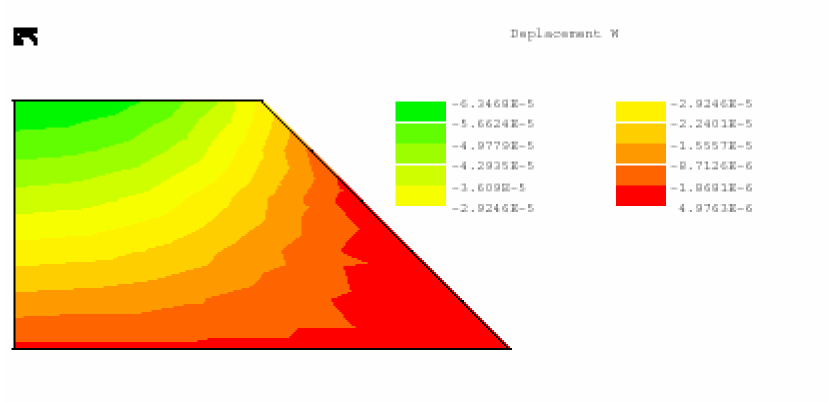
Contrainte principale min.



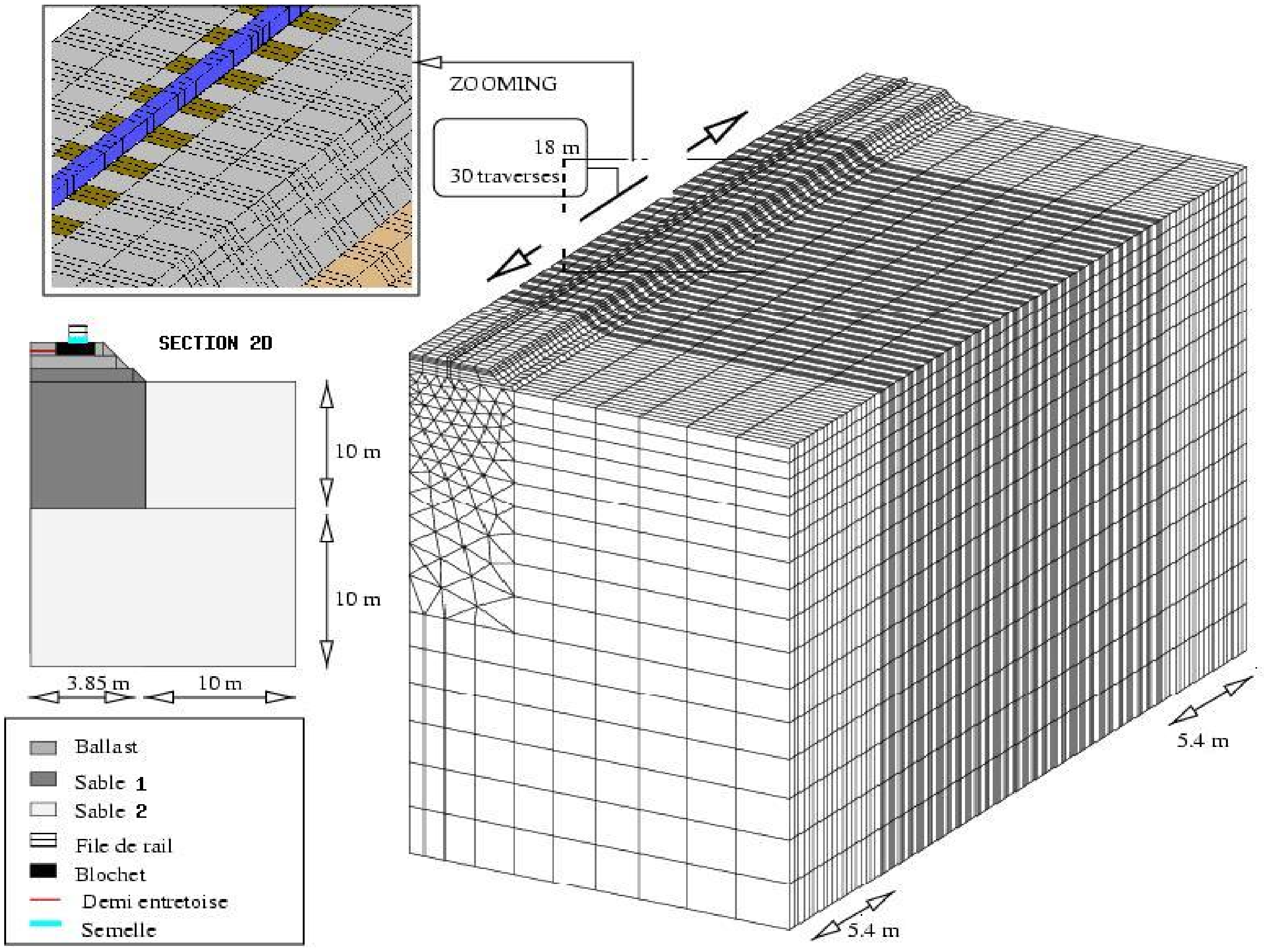


déplacement vertical





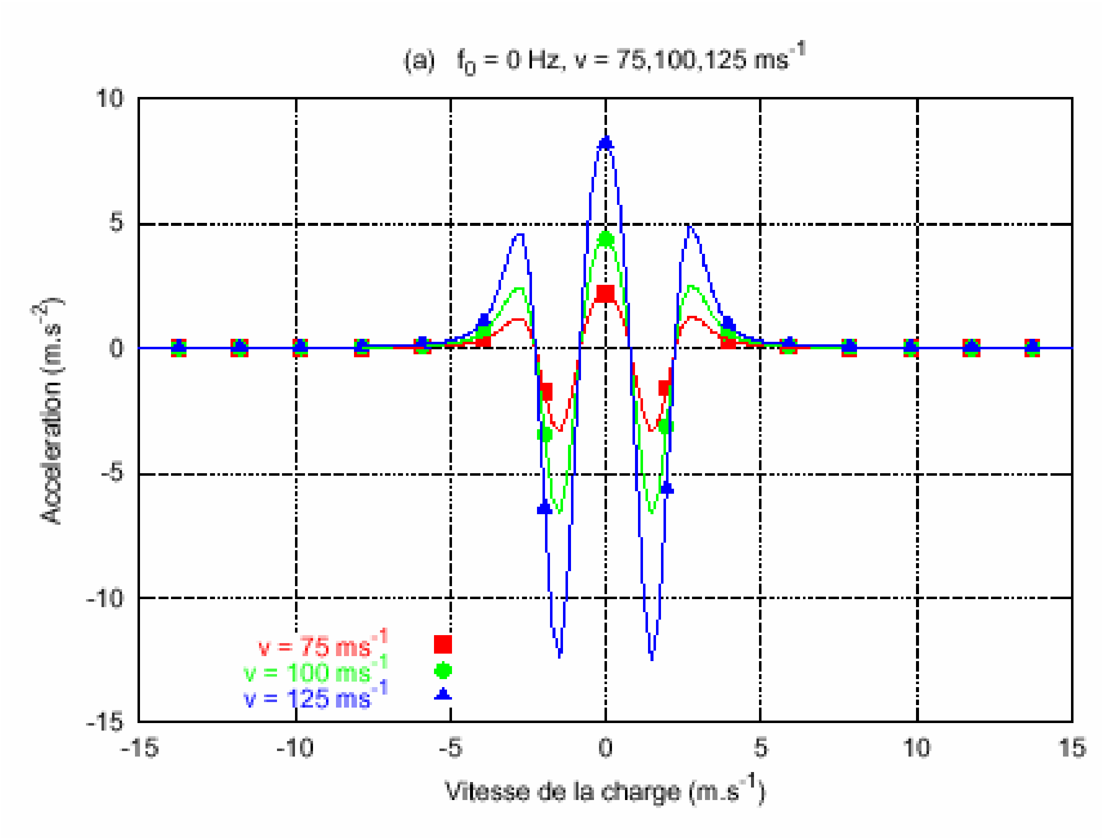
Finite element model of a railway track



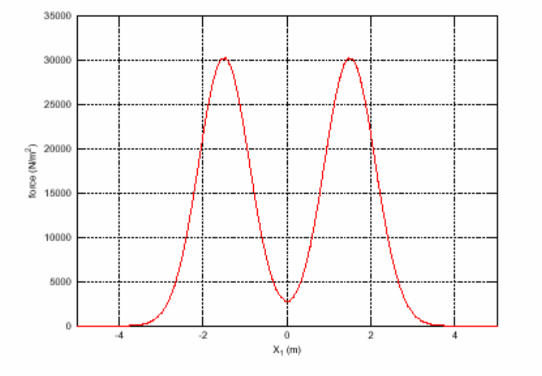
Vertical acceleration

Two layers model

- *ballast : 30 cm*
- *soil : infinite half-space*



Force signal by a bogie with two axles



Conclusion

- Influence of the shape of the grains
  - Ball versus polygons
  - Dynamic versus static behavior
- Problem a large number of cyclic loads
  - Long term procedure
- Mechanisms of settlement
- Coupling granular with continuous media for structural computations