AN INTRODUCTION TO THE NUMERICAL MODELING OF FUSION PLASMAS

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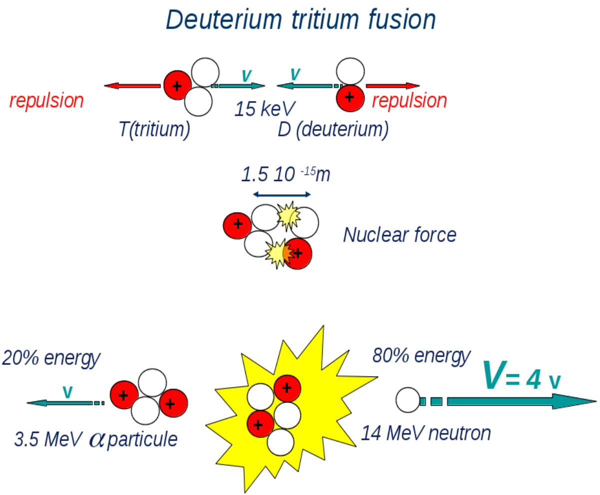
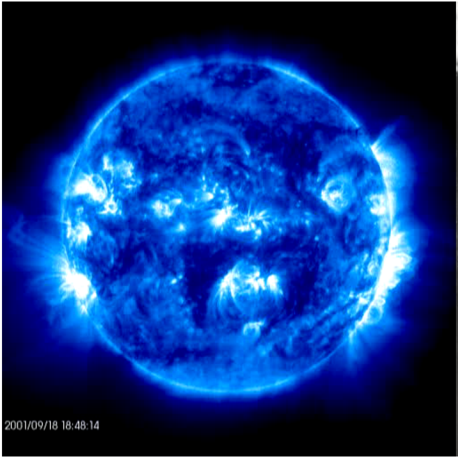
Outline

Outline

Deuterium-Tritium Reaction



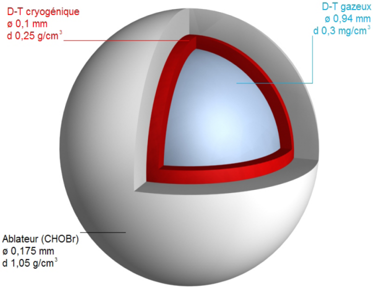
Nuclear reactions



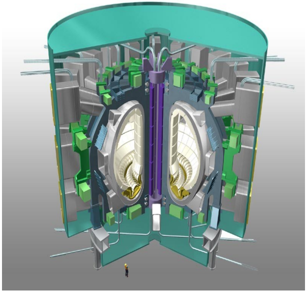
Fusion on earth ???

Essentially two studied technologies :

Inertial confinement

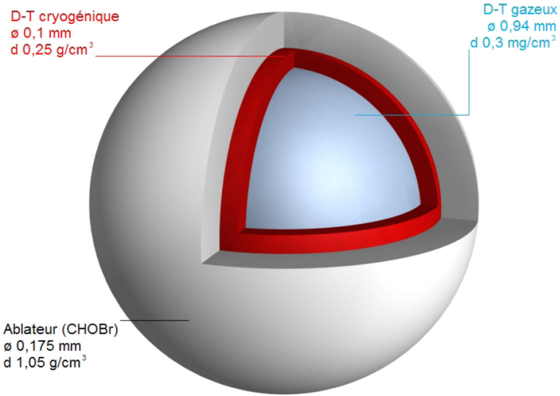


Magnetic confinement



Inertial confinement technology

relies on fast and violent heating of fusion targets.



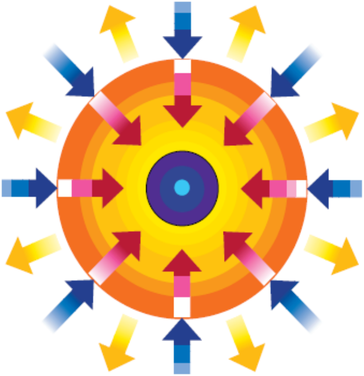
Target Ablator heating

Ablator is illuminated by powerful - X rays and reaches a plasma state

Ablator expansion & DT Compression

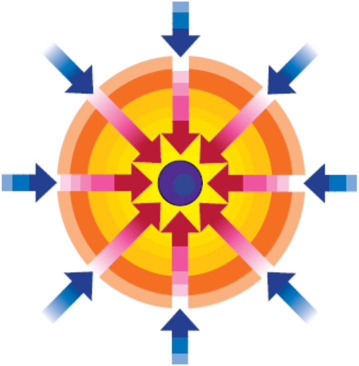
Ablator is accelerated outwards

By rocket effect DT is accelerated inwards (v $\sim$ 300 km/s)



Deuterium - Tritium Compression

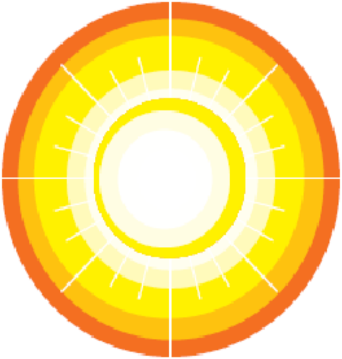
DT ice is compressed up to 1000 times its initial density
(20 time lead density)



Hot spot Ignition

Central gas is heated up to several 10^6 of degrees
When $\rho R \sim 0.3 g/cm^2$ the gas is self heated by alpha particles

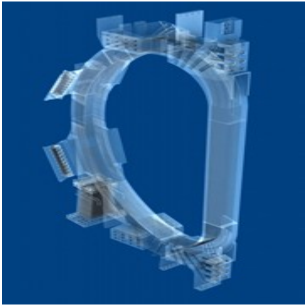
Fusion reactions start and propagate into DT ice
Note : 1 mg D-T $\rightarrow$ 340 MJ : Fusion is equivalent to combustion of 10 kg coal



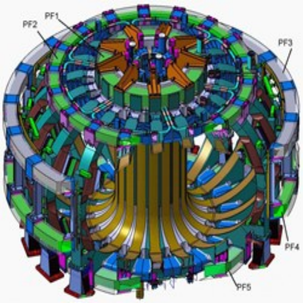
The other way to fusion : magnetic confinement

Objective : confine hot D-T plasma by strong magnetic fields

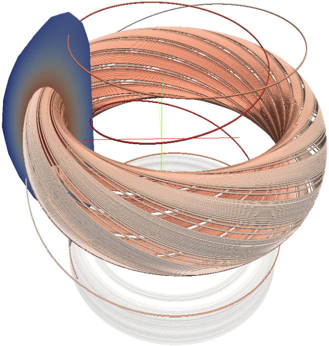
Toroidal coils



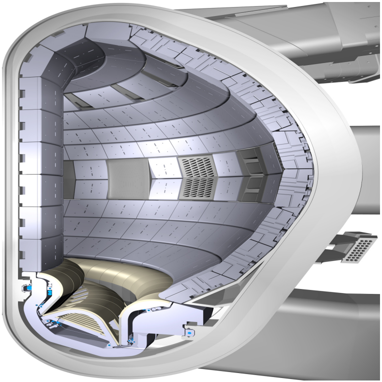
Poloidal coils



Tokamaks :



magnetic field lines



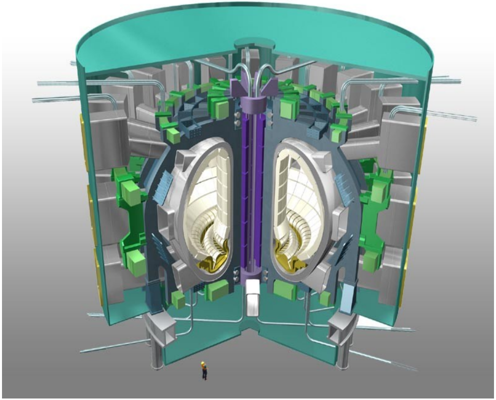
vacuum chamber

Tokamaks design in the 1950s by Igor Tamm and Andrei Sakharov

ITER Tokamak

ITER (International Thermonuclear Experimental Reactor) in construction in Cadarache, construction begins in 2010, first plasma in 2020, first fusion plasma in 2027

- 840 m^3
- 150M ^{O}K
- 13 Tesla = 200 000 x earth magnetic field



Numerical simulations

Necessary for

- Equilibrium computation : steady state and control of the machines
- prediction of the possible occurrence of instabilities
  - magnetic instabilities
  - hydrodynamic instabilities
- determination of the value of key parameters e.g transport coefficients due to turbulence
- understand and explain physical phenomena

Very large number of different numerical models

Outline

Kinetic models

Vlasov or Boltzmann eq for each particle (ions, electrons, neutral)

$$\frac{\partial f_s}{\partial t} + \text{div}_x(\mathbf{v} f_s) + \text{div}_v(\mathbf{F} f_s) = \mathbf{C}_s = \sum C_{ss'}$$

Force field $\mathbf{F} = \frac{e_s}{m_s}[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$ (note $\text{div}_v \mathbf{F} = 0$)

$\mathbf{E}, \mathbf{B}$ given by Maxwell equations:

$$\frac{\partial \mathbf{B}}{\partial t} + \text{ curl } \mathbf{E} = 0 \qquad \Upsilon = \sum e_s \int f_s(\mathbf{x}, \mathbf{v}, t) dv^3$$

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \text{ curl } \mathbf{B} = \mu_0 \mathbf{J} \qquad \mathbf{J} = \sum e_s \int f_s \mathbf{v}(\mathbf{x}, \mathbf{v}, t) dv^3$$

$$\varepsilon_0 \text{ div } \mathbf{E} = \Upsilon \qquad \text{div } \mathbf{B} = 0$$

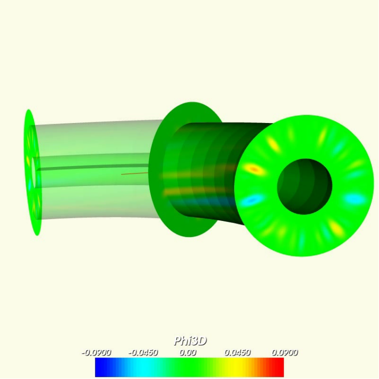
Kinetic models

- Self-consistent and closed model
- extremely heavy from computational point of view
  - 6 D model : 3 space dimensions, 3 velocity dimensions
  - covers huge range of time and space scales
- some simplifications possible : Ampere law, Quasi-Neutral assumption, Electrostatic assumption, gyrokinetic theory
- Used for some very specific tasks

Kinetic models : one example of applications

Anomalous transport in tokamaks

Radial diffusion of mass and temperature exceed by order of magnitude "laminar" values
due to micro-turbulence and micro-instabilities :
Estimate the value of turbulent transport coefficients by direct simulation
Development of gyrokinetic codes (e.g GYSELA code of CEA)



Outline

From kinetic to fluid

Idea of fluid models :
instead of computing the whole distribution function f_s
compute only a small number of its moments, i.e : Fluid variables are moments of the distribution function f_s

- fluid density

$$n_s(\mathbf{x}, t) = \int f_s(\mathbf{x}, \mathbf{v}, t) dv^3$$

- fluid velocity

$$n_s(\mathbf{x}, t)\mathbf{u}_s(\mathbf{x}, t) = \int f_s(\mathbf{x}, \mathbf{v}, t)\mathbf{v} dv^3$$

From kinetic to fluid

- Pressure tensor

$$P_s(\mathbf{x}, t) = \mathrm{m}_s \int f_s(\mathbf{x}, \mathbf{v}, t)(\mathbf{v} - \mathbf{u}_s) \otimes (\mathbf{v} - \mathbf{u}_s) dv^3$$

- scalar pressure = 1/3 trace of pressure tensor

$$p_s(\mathbf{x}, t) = \frac{\mathrm{m}_s}{3} \int f_s(\mathbf{x}, \mathbf{v}, t)|(\mathbf{v} - \mathbf{u}_s)|^2 dv^3$$

- temperature

$$T_s(\mathbf{x}, t) = \frac{p_s(\mathbf{x}, t)}{n_s(\mathbf{x}, t)}$$

- Energy flux

$$\mathbf{Q}_s(\mathbf{x}, t) = \frac{\mathrm{m}_s}{2} \int f_s(\mathbf{x}, \mathbf{v}, t)|\mathbf{v}|^2 \mathbf{v} dv^3$$

More complex fluid models, e.g 14 Moment closure by D. Levermore

From kinetic to fluid

Mass conservation

First velocity moment of the Boltzmann (Vlasov) equation

$$\int [\frac{\partial f_s}{\partial t} + \mathrm{div}_x(\mathbf{v} f_s) + \mathrm{div}_v(\mathbf{F} f_s) = C_s]$$

gives

$$\int f_s dv^3 \stackrel{\text{def}}{=} n_s \qquad \int \mathbf{v} f_s dv^3 \stackrel{\text{def}}{=} n_s \mathbf{u}_s$$

$$\int \mathrm{div}_v(\mathbf{F} f_s) = 0 \quad \int C_s dv^3 = 0$$

$$\frac{\partial n_s}{\partial t} + \mathrm{div}_x(n_s \mathbf{u}_s) = 0$$

Fluid models

proceeding in the same way for the other moments :

$m_s\mathbf{v}\times$ Boltzmann and integrating, gives momentum (velocity) equation

$\frac{1}{2}m_s|\mathbf{v}|^2\times$ Boltzmann and integrating, gives energy (temperature) equation

Fluid models

$s \in ion,\ electron,\ neutral\ particle$

$$\frac{\partial n_s}{\partial t} + \mathrm{div}(n_s \mathbf{u}_s) = 0$$

$$\mathrm{m}_s(\frac{\partial n_s \mathbf{u}_s}{\partial t} + \mathrm{div}(n \mathbf{u}_s \otimes \mathbf{u}_s)) + \nabla p_s + \mathrm{div}_x \Pi_s - n_s e_s(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) = \sum_{s' \neq s} R_{ss'}$$

$$\frac{\partial}{\partial t}(\mathrm{m}_s \frac{n_s}{2}|\mathbf{u}_s|^2 + \frac{3}{2} p_s) + \mathrm{div}\mathbf{Q} - e_s \mathbf{u}_s.\mathbf{E} = Q_s$$

The 2-fluid model

- Much simpler than kinetic model (3D)
- Clear mathematical structure (1 compressible Navier-Stokes system per species coupled by force terms)
- But needs closure assumptions
- Questionable in tokamaks
- very large disparity in length and time scales
- still costly e.g for a ion-electron model 10 dof per mesh point
- not really used but good starting point for subsequent approximations

From 2-fluid to one Fluid models

Normalization units

time τ_{obs}	
velocity u_*	
lenght $L_* = u_* \tau_{obs}$	
ion gyro-period $\tau_{c_i} = m_i / Z_i eB$	
electron gyro-period $\tau_{c_e} = m_e / eB$	
plasma period $\tau_{pe} = (\varepsilon_0 m_e / n_e e^2)^{1/2}$	$\tau_{c_e} / \tau_{pe} \sim 1$
collision $\tau_{coll} \propto (\Lambda / \ln\Lambda) \tau_{pe}$	

Non dimensional parameters

$$\varepsilon_i = \tau_{c_i} / \tau_{obs} \quad M_i = u_* / \nu_{T_i}, \quad \Lambda = \frac{4\pi}{3} n \lambda_D^3, \quad \mu = m_e / m_i$$

Relevant asymptotic regimes $\varepsilon_i \to 0 \quad \Lambda \to \infty \quad \mu = m_e / m_i \to 0$ etc

Give a huge set of one fluid models

Outline

MHD : violent instabilities affecting the whole plasma region :

$$x_* = e.ga \quad \text{minor radius}$$

fast event : speed of MHD waves : Alfen velocity

$$v_A = \frac{B}{\sqrt{\mu_0 \rho}}$$

$$u_* = \mathcal{O}(\nu_{T_i}) \sim \mathcal{O}(v_A)$$

MHD asymptotic limit

$$\varepsilon_i = \frac{\tau_{c_i}}{\tau} \to 0, \quad M_i = \mathcal{O}(1)$$

Zero order model in ε_i ($\varepsilon_i = 0$)

$$\frac{\partial}{\partial t}\rho + div(\rho\mathbf{u}) = 0$$

$$\frac{\partial}{\partial t}\rho\mathbf{u} + div(\rho\mathbf{u} \otimes \mathbf{u}) + \nabla p - (\mathbf{J} \times \mathbf{B}) = 0$$

$$E + \mathbf{u} \times \mathbf{B} = 0$$

need the current $\mathbf{J}$ → Ampere's law $\mathbf{J} = \text{ curl } \mathbf{B}$
need the magnetic field $\mathbf{B}$ → Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} + \text{ curl } \mathbf{E} = 0$$

⇒ autonomous system for a One Fluid model

$$\frac{\partial}{\partial t}\rho + div(\rho\mathbf{u}) = 0$$

$$\frac{\partial}{\partial t}\rho\mathbf{u} + div(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla p + \frac{1}{\mu_0}(\mathbf{B}\times \text{curl } \mathbf{B}) = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} - \text{ curl } (\mathbf{u}\times\mathbf{B}) = 0$$

$+$ energy equation

One example of hydrodynamic instabilities in magnetized plasmas

2 scalar variables : ion density n; ion parallel velocity $u_{||}$

$$\mathbf{u} = u_{||}\mathbf{b} + \mathbf{u}_{\perp}$$

$$\frac{\partial}{\partial t}\rho + div(\rho \mathbf{u}) = 0$$

$$\frac{\partial}{\partial t}\rho u_{||} + \mathrm{div}\rho u_{||}\mathbf{u} + \nabla_{||}p = \rho \mathbf{u}.\frac{D\mathbf{b}}{Dt}$$

with (this is the ideal Ohms law solved for the $\perp$ velocity)

$$\mathbf{u}_{\perp} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

B given, **E** computed by the adiabatic assumption

Hydrodynamic instabilities

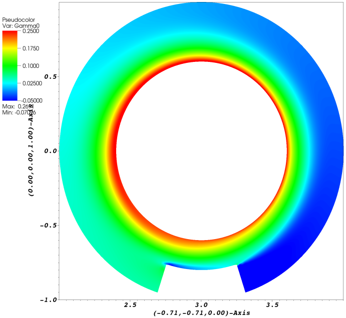
In the core plasma (white region) the plasma rotate counterclockwise

In the edge region, the plasma touches the limiter

Due to electric charging of the wall (Bohm's BC) the plasma enters the wall

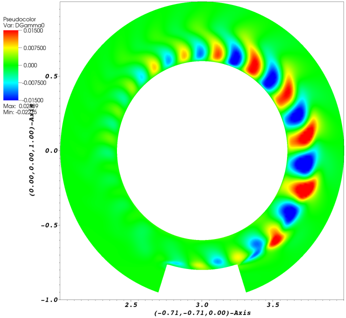
$\rightarrow$ intense shear near the wall

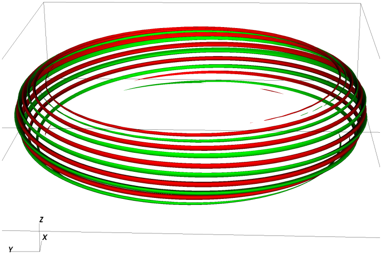
Development of hydrodynamical instabilities (Kelvin-Helmholtz like) ?



momentum plot at t=0

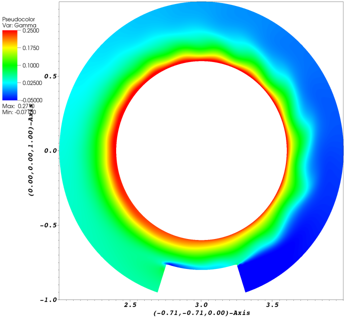
Hydrodynamic instabilities





Hydrodynamic instabilities

Oscillation of the plasma boundary



Conclusions

- large number of unsolved phenomena requiring the development of
  - numerical models
  - numerical methods
  - parallel algorithms
- short overview of numerical simulations for fusion plasmas
- See you next year in Cemracs2014