Numerical Simulation of Rhone's Glacier between 1874 and 2100

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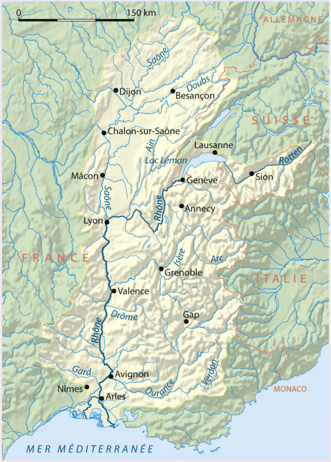
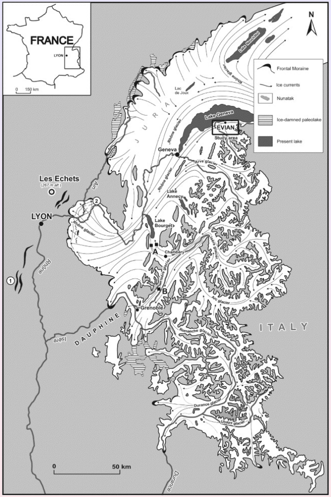
Rhône's river



Source : wikipedia

Rhône's river 25 000 years ago

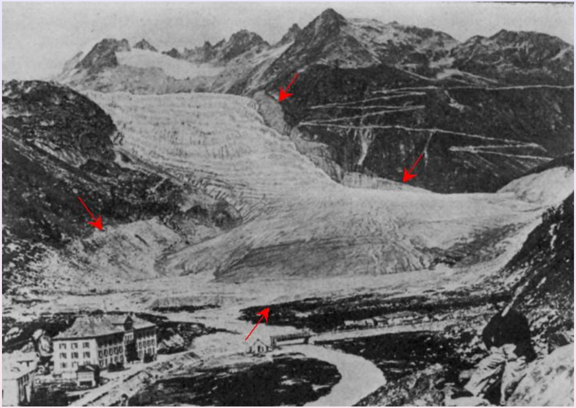


Source : wikipedia

Rhône's glacier in 1850

Source : http://www.unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1870



Source : http://www.unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1900

Source : http://www.unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1914

Source : http://www.unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1925

Source : http://www.unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1985

Source : http://www.unifr.ch/geosciences/geographie/glaciers

Rhône's glacier : comparison at 2000 m

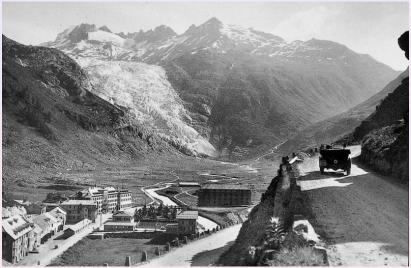
Rhône's glacier in 1860 (M. Funk's reconstruction)

Rhône's glacier in 1970 (M. Funk's reconstruction)

Rhône's glacier in 2050 (M. Funk's prediction)

Numerical validation : 1900

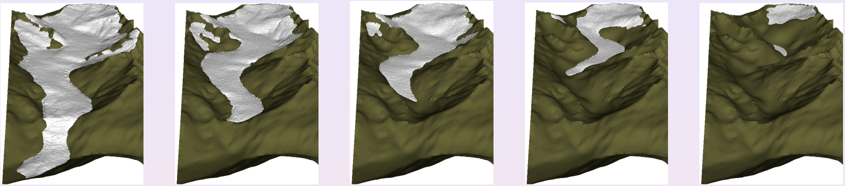
Numerical validation : 1932



Numerical validation : 1960

Numerical validation : 1985

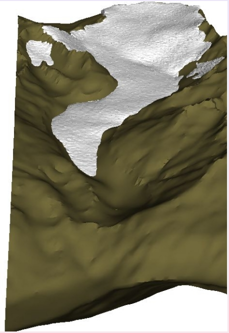
Numerical prediction from 2007 to 2100



1874 2008 2050 2075 2100

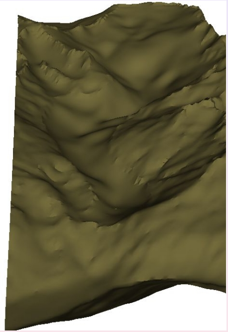
- Median climatic scenario (occh.ch)
  - Temperature trend : $+3.8^o C$
  - Precipitation trend : $-6m/year$
  - Data from M. Huss and D. Farinotti, ETHZ.
  - Animation

Rhône's glacier in 2100 (3 scenario)



Scenario 1 : cold-wet

Scenario 2 : median



Scenario 3 : warm-dry

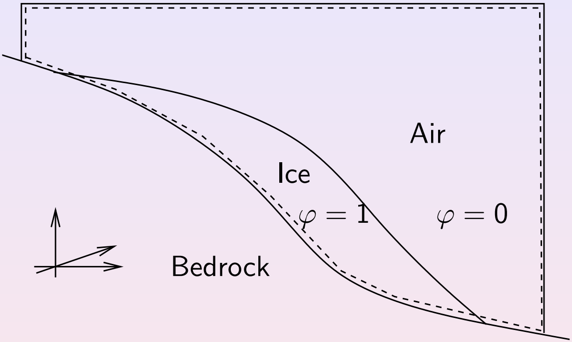
Outline

- Starting point : 3D fluid flows with complex free surfaces.
- Model.
- Numerical method.
- Ongoing work : crevasse formation.

Starting Point : 3D fluid flows with complex free surfaces

- Newtonian flows : V. Maronnier, M. Picasso, J. Rappaz (JCP 1999, IJNMF 2003).
  - Broken dam
  - Mould filling
- Newtonian flows and compressible gas and surface tension : A. Caboussat, M. Picasso, J. Rappaz (JCP 2005).
  - Mould filling
- Viscoelastic flows : A. Bonito, M. Picasso, M. Laso (JCP 2006).
  - Jet buckling.
  - Fingering instabilities : experiment, G. McKinley, MIT
  - Fingering instabilities : simulation

The model



- The computational domain is the region inside the dotted line.
- The ice region is defined by the volume fraction of ice φ.

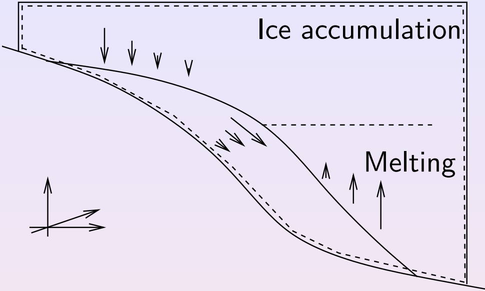
Ice flows in glaciers

Trift glacier, one picture a day in 2003 Animation

The model (Volume of Fluid)

Climatic input : b.

Unknowns :
velocity u and pressure p,
volume fraction of ice φ.



$$\rho \frac{\partial u}{\partial t} + \rho (u \cdot \nabla) u - \text{div} \left(2\mu\epsilon(u) \right) + \nabla p = \rho g,$$

$$\text{div } u = 0,$$

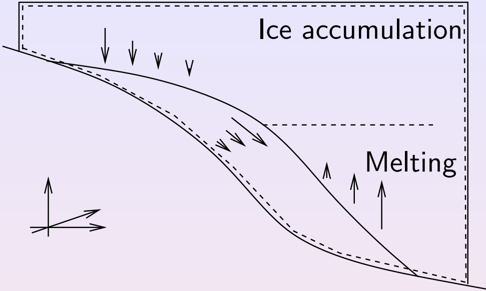
$$\frac{\partial \varphi}{\partial t} + u \cdot \nabla \varphi = b \delta_\Gamma,$$

on the ice/air interface Γ : $(2\mu\epsilon(u) - pI)n = 0$,

on the bedrock : no-slip or sliding,

Glen's law for the viscosity.

The model (Volume of Fluid)



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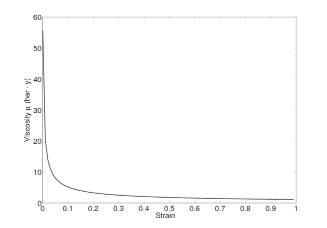
Glen's flow law

- Given $\epsilon(u)$ find μ such that

$$\frac{1}{2\mu} = A\left(\sigma_0^{n-1} + \left(2\mu\sqrt{\frac{1}{2}\epsilon(u):\epsilon(u)}\right)^{n-1}\right).$$

- Viscosity with respect to $s = \sqrt{\frac{1}{2}\epsilon(u):\epsilon(u)}$



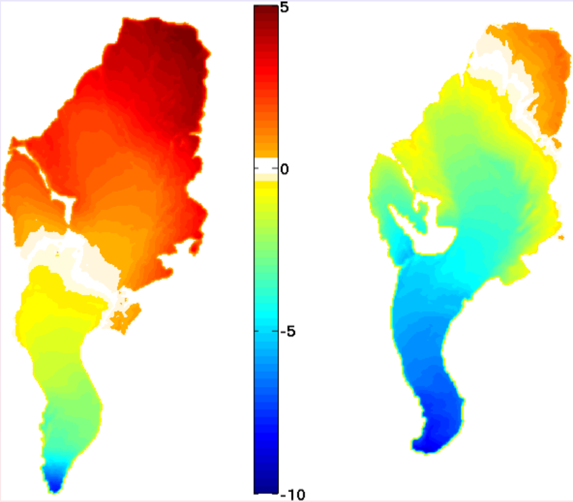
- The nonlinear Stokes problem has a solution in $W^{1,1+1/n}$ (minimum of a strictly convex functional), Colinge Rappaz (M2AN 1999), Barrett Liu (NM 1994).

Sliding condition on the bedrock

$$u \cdot n = 0 \quad \text{and} \quad (2\mu\varepsilon(u)n) \cdot t_i = -\alpha u \cdot t_i \quad i = 1, 2,$$

$$\alpha = \frac{1}{c^{\frac{1}{3}}} \frac{1}{\left(\sqrt{u^2 + v^2 + w^2} + s_0\right)^{\frac{2}{3}}},$$

Climatic input : b (m of ice per year)



Cold year 1913 Warm year 2003

Climatic input : b (m of ice per year)

- Based on 150 years of measurements.
- $b = P - M$.
- P : solid precipitations (snow)

$$P(x,y,z,t) = P_{ws}(t)\left(1 + \frac{dP}{dz}(z - z_{ws})\right) C_{prec} DIST(x,y,z).$$

- M : ice melting

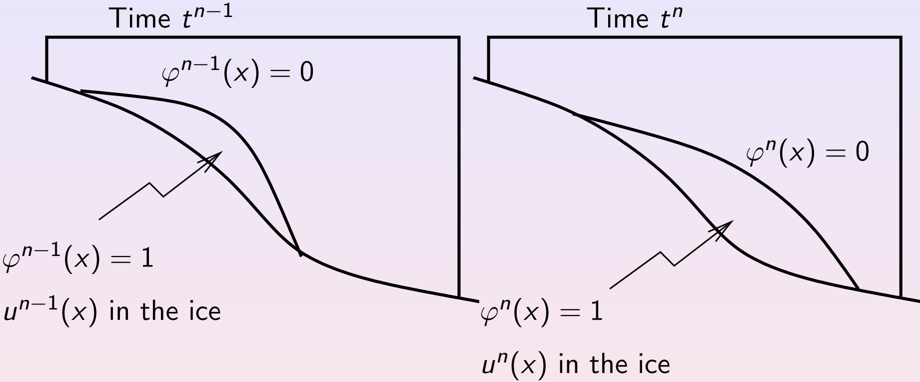
$$M(x,y,z,t) = \begin{cases} (F_M + r_{ice/snow} I(x,y,z) T(z,t) & \text{if } T(z,t) > 0, \\ 0 & \text{else.} \end{cases}$$

- $T(z,t) = T_{ws}(t) - 0.006(z - z_{ws})$.
- The coefficients F_M, r_{ice}, r_{snow}, C_{prec}, dP/dz are tuned so that

$$\int_{1874}^{2007} \int_{\text{ice}} (b - b_{meas})^2 dV dt$$

is minimum.

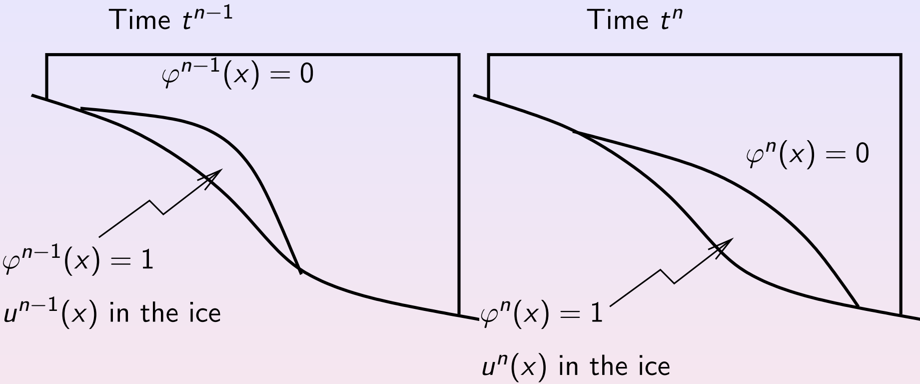
Time discretization : a decoupling scheme



- Shape computation : solve between $t = t^{n-1}$ and $t = t^n$

$$\frac{\partial \varphi}{\partial t} + u^{n-1} \cdot \nabla \varphi = b\delta_\Gamma.$$

Time discretization : a decoupling scheme



- Velocity computation : solve

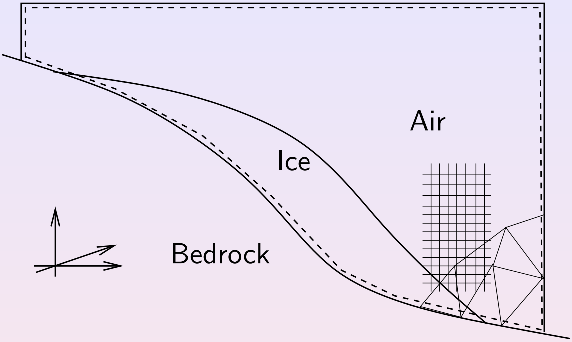
$$-\text{div}\left(2\mu\epsilon(u)\right) + \nabla p = \rho g,$$

$$\text{div } u = 0,$$

$$\text{on the ice/air interface } \Gamma : (2\mu\epsilon(u) - pI)n = 0,$$

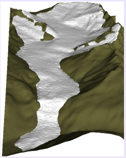
on the bedrock : no-slip or sliding.

Space discretization : structured cells and finite elements

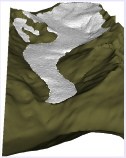


- Shape computation (transport + ice accumulation/melting) : small structured cells
- Velocity computation (nonlinear Stokes) : unstructured finite elements
- To avoid numerical diffusion : $\frac{\text{FE spacing}}{\text{cells spacing}} \simeq 5$.

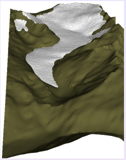
The 3D finite element mesh



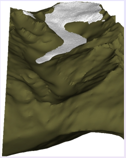
1874



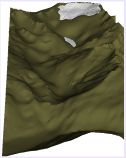
2008



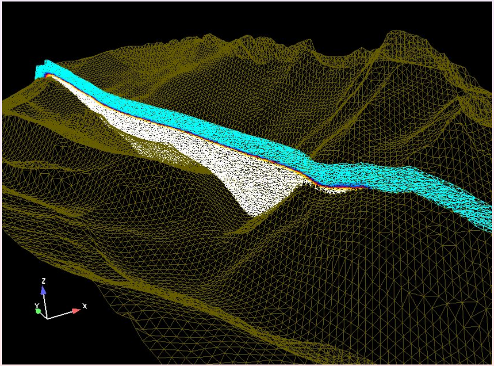
2050



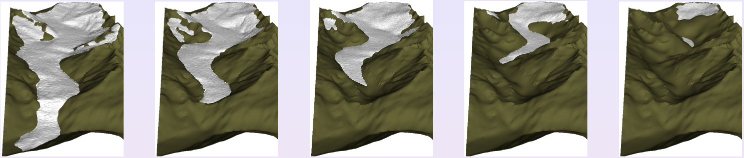
2075



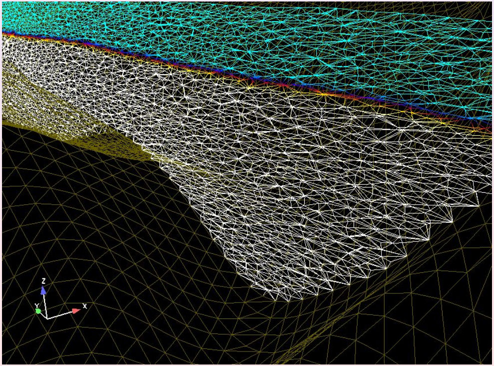
2100



The 3D finite element mesh



1874 2008 2050 2075 2100



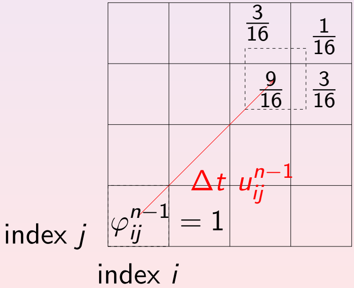
Shape computation : transport + ice accumulation/melting (1/3)

- Solve between $t = t^{n-1}$ and $t = t^n$

$$\frac{\partial \varphi}{\partial t} + u^{n-1} \cdot \nabla \varphi = b\delta_\Gamma.$$

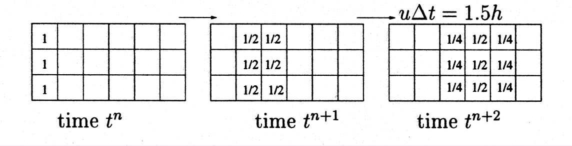
- Forward characteristics method :

$$\varphi^n(x + \Delta t\ u^{n-1}(x)) = \varphi^{n-1}(x).$$

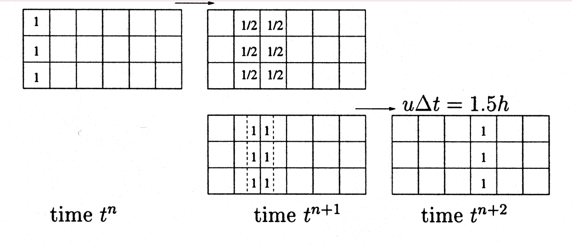


Shape computation : transport + ice accumulation/melting (2/3)

- SLIC Postprocessing (Simple Line Interface Calculation), see for instance Scardovelli Zaleski Ann. Rev. Fluid Mech. 1999.
- Without SLIC.



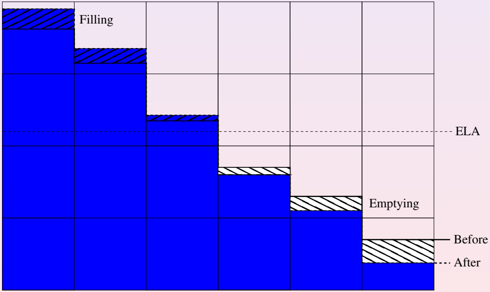
- With SLIC.



Shape computation : transport + ice accumulation/melting (3/3)

- Solve between $t = t^{n-1}$ and $t = t^n$

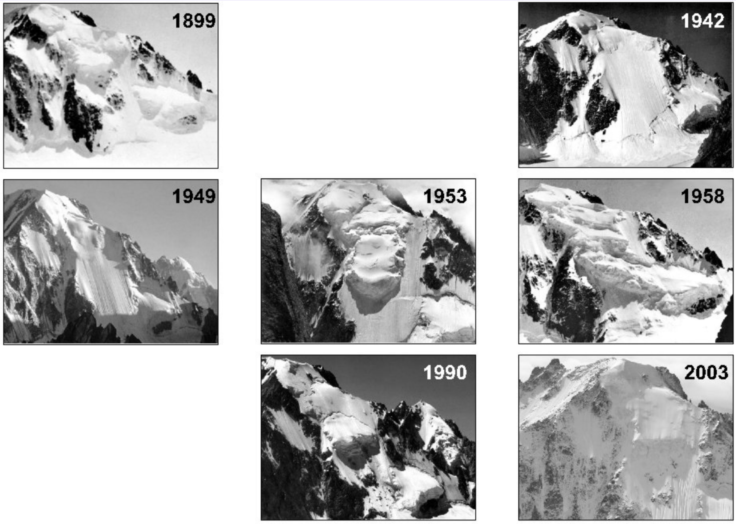
$$\frac{\partial \varphi}{\partial t} + u^{n-1} \cdot \nabla \varphi = b\delta_{\Gamma}.$$



Conclusions - Perspectives - Ongoing work

- Given the climatic input b, we can simulate the motion of a glacier.
- Optimization problem : observed moraines $\longrightarrow$ climatic input b.
- Crevasse formation : rupture of hanging glaciers, calving glaciers $\longrightarrow$ mechanics of damage.

Rupture of hanging glaciers : Argentière



Calving glaciers : Rhone Glacier

Damage modelling

- Microcracks can be modeled at macroscopic level by introducing a state variable $D(x,t)$ called damage : A. Pralong and M. Funk, Annals of Glaciology, 2003.
- Isotropic damage : D is a scalar field, $D=0$ no damage, $D=1$ full damage.
- Momentum equation :

$$\rho\frac{\partial u}{\partial t}+\rho(u\cdot\nabla)u-\text{div}\,(2\mu(1-D)\epsilon(u))+\nabla p=\rho g,$$

- Damage nonlinear transport equation :

$$\frac{\partial D}{\partial t}+u\cdot\nabla D=f(D,\epsilon(u),p).$$

- Simulation of crevasse formation : Eiger Gruben.