



Luxembourg National
Research Fund



Deutsche
Forschungsgemeinschaft



Machine Learning of Molecular Quantum Chemical Space *Opportunities and Challenges*

Alexandre Tkatchenko

Physics and Materials Science (PhyMS), University of Luxembourg



UNIVERSITÉ DU
LUXEMBOURG

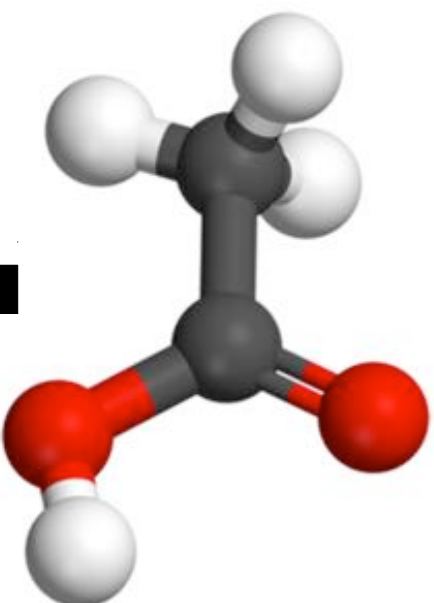
alexandre.tkatchenko@uni.lu

CECAM@Sanofi, December 5, 2018



BERLIN BIG
DATA CENTER

Quantum physics/chemistry today



$$\begin{array}{l} \text{DFT} \\ \text{MP2} \\ \text{CCSD(T)} \\ \dots \end{array} \hat{H}(R_1, Z_1, \dots, R_N, Z_N) \tilde{\Psi} = E \tilde{\Psi}$$



Properties: Energy, polarizability, HOMO, LUMO, ...

Dynamics: Thermal properties, spectroscopy, ...

Quantum physics/chemistry tomorrow?

ML

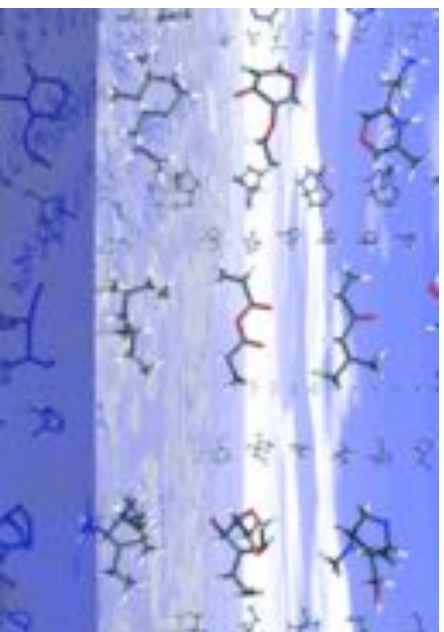
Insights:

- Structure of chemical space

- Reactivity trends, aromaticity, “new” chemistry

- Molecular design through multi-property optimization

- ...

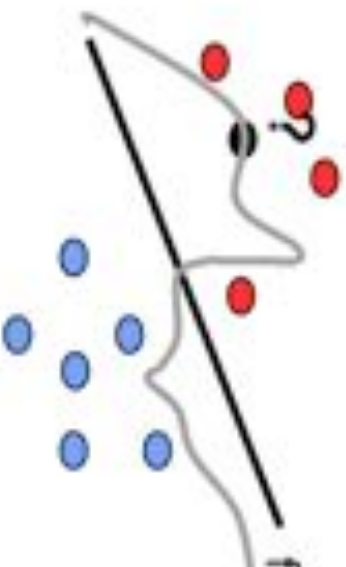


Training data:
molecular properties



Machine Learning in a nutshell

Typical scenario: learning from data



- given data set **X** and labels **Y** (generated by some joint probability distribution $p(x,y)$)

- **LEARN/INFERR** underlying **unknown** mapping

$$Y = f(X)$$

fit

Example: ~~understand~~ chemical compound space, distinguish brain states ...

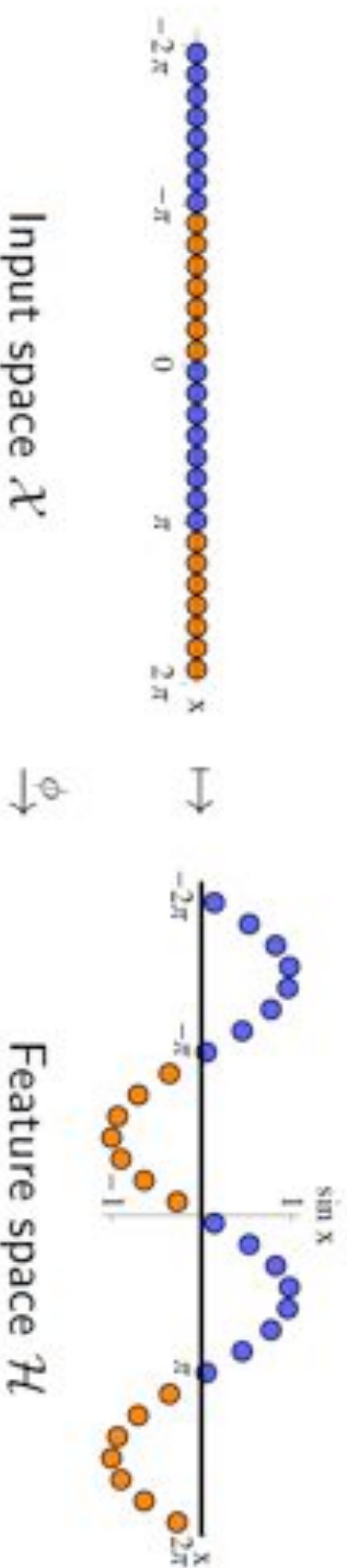
BUT: how to do this optimally with good performance on **unseen** data?

Most popular techniques **kernel methods** and (deep) **neural networks**.

Kernel Learning

Idea:

- Transform samples into higher-dimensional space
- *Implicitly* compute inner products there
- Rewrite linear algorithm to use only inner products



$$k : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}, \quad k(x, z) = \langle \phi(x), \phi(z) \rangle$$

Regularized Kernel Ridge Regression

- Regularized form of ordinary regression
- Regularization prevents over-fitting by penalizing large coefficients
- Use of kernels for non-linearity

Solution has form

$$f(\mathbf{x}) = \sum_{i=1}^n \alpha_i k(\mathbf{x}_i, \mathbf{x})$$

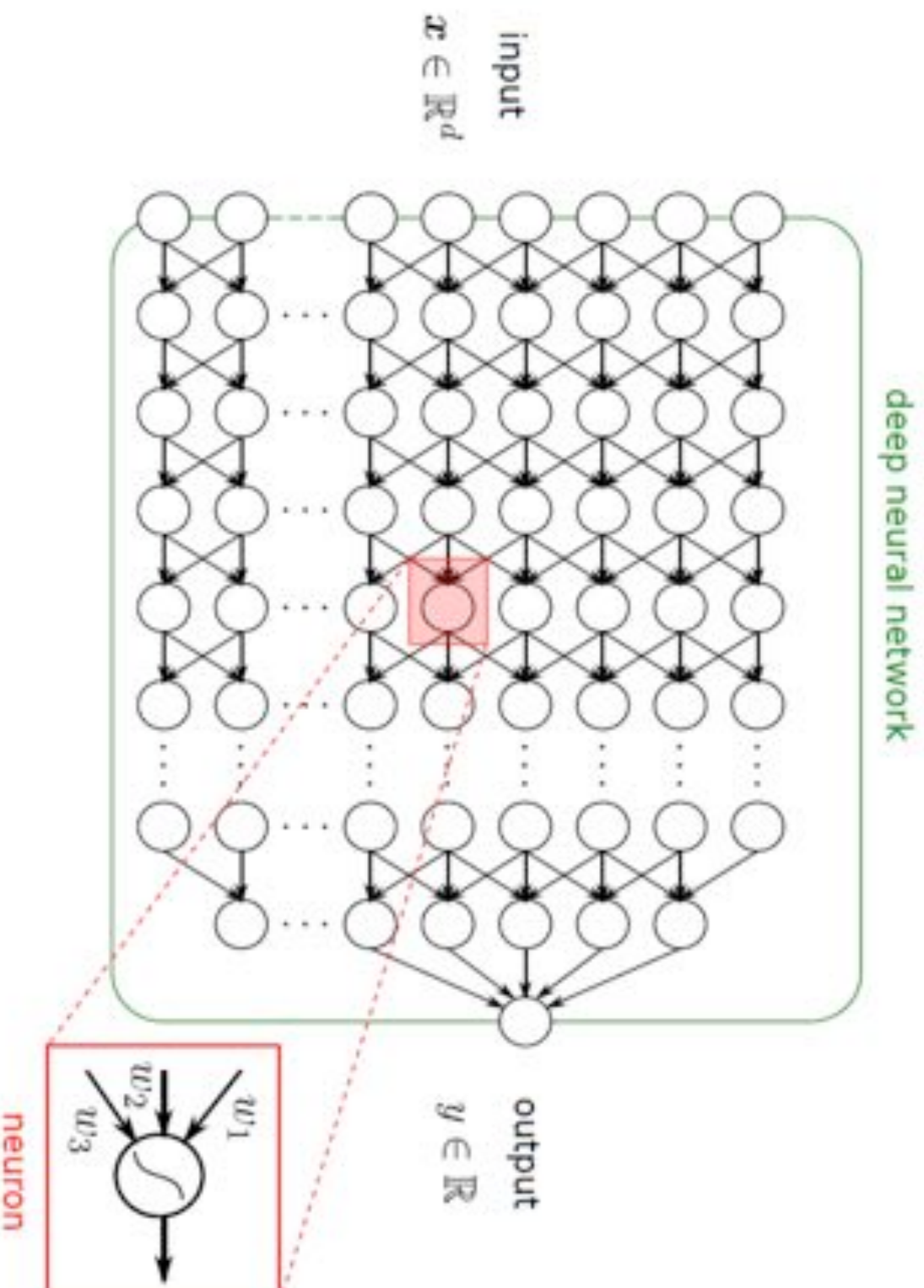
Coefficients α are obtained by solving

$$\sum_{i=1}^n (f(\mathbf{x}_i) - y_i)^2 + \lambda \alpha^T \mathbf{K} \alpha,$$

which has solution

$$\alpha = (\mathbf{K} + \lambda \mathbf{I})^{-1} \mathbf{y}.$$

Neural Networks



- ▶ Neuron applies a nonlinear function to its input.
- ▶ Examples of functions: hyperbolic tangent, rectification.

Big Data for Molecules and Materials



nomad-coe.eu



e-cam2020.eu



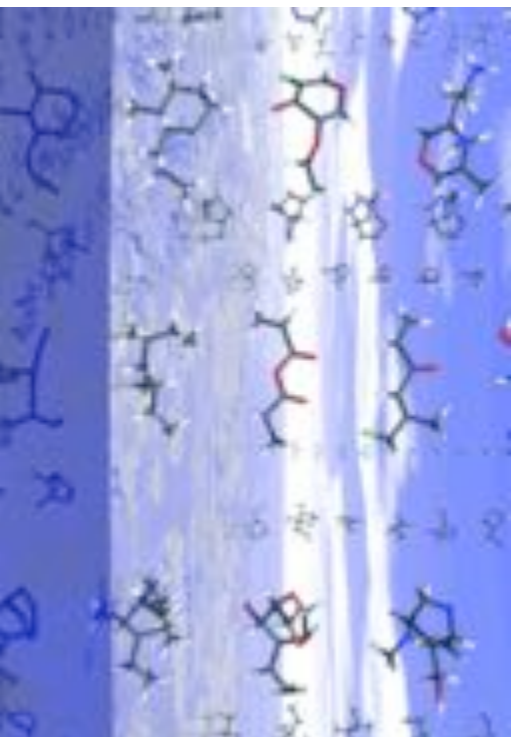
max-centre.eu



NATIONAL CENTRE OF COMPETENCE IN RESEARCH

ncr-marvel.ch

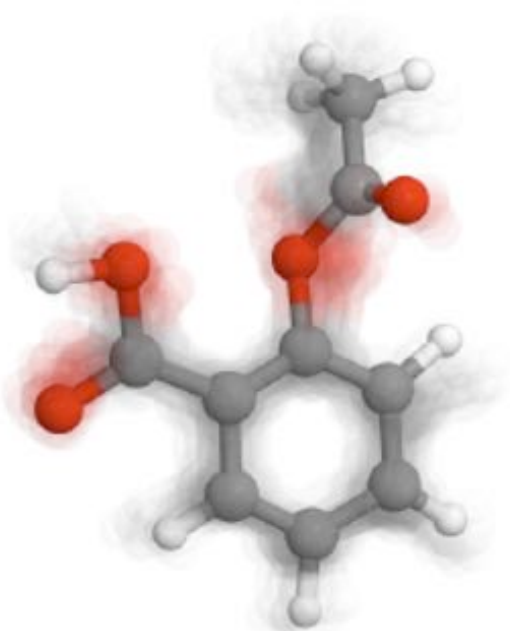
Molecular Data in this Talk



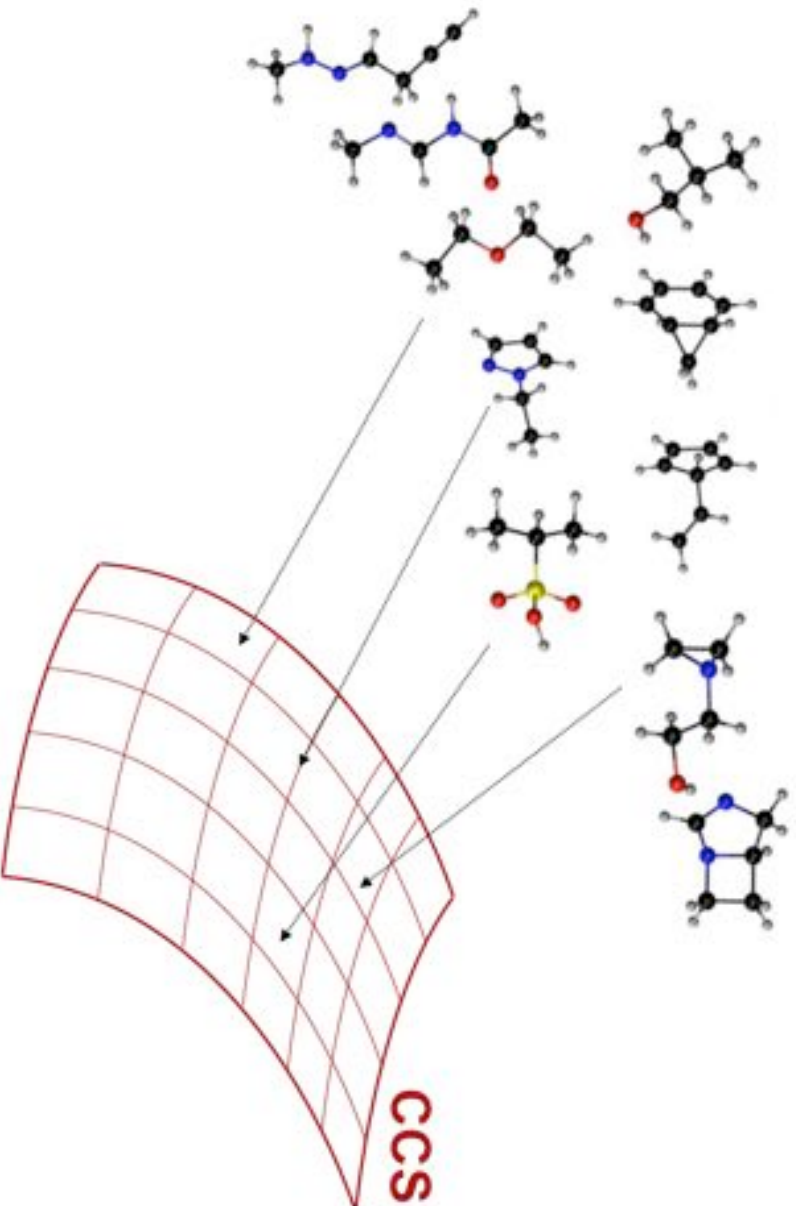
GDB mol graphs: **J. L. Reymond** (U. Bern)
<http://gdb.unibe.ch/downloads/>

QM7/QM9 datasets: Hybrid DFT
calculations by **von Lilienfeld's** group
(Sci. Data 2014) and my group (PRL 2012).

MD17/ISO17 datasets: Molecular
dynamics trajectories from my group
(DFT and CCSD(T) levels)



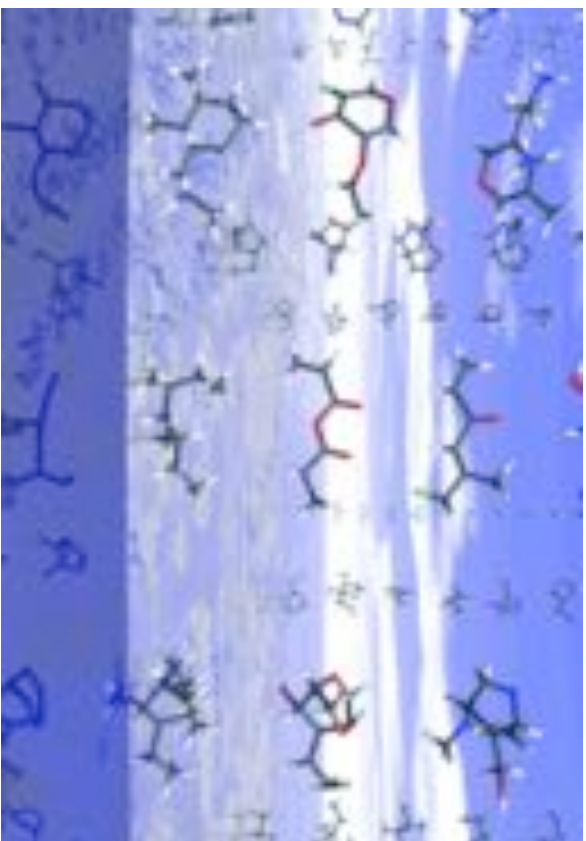
Molecular big data



- Graph theory: combinatorial explosion
- At least 10^{60} small drug candidate molecules
- Finding needles in a haystack

$\{\mathbf{R}_i, \mathbf{Z}_i\}$ maps to $\{P_1, P_2, P_3, P_4, \dots\}$

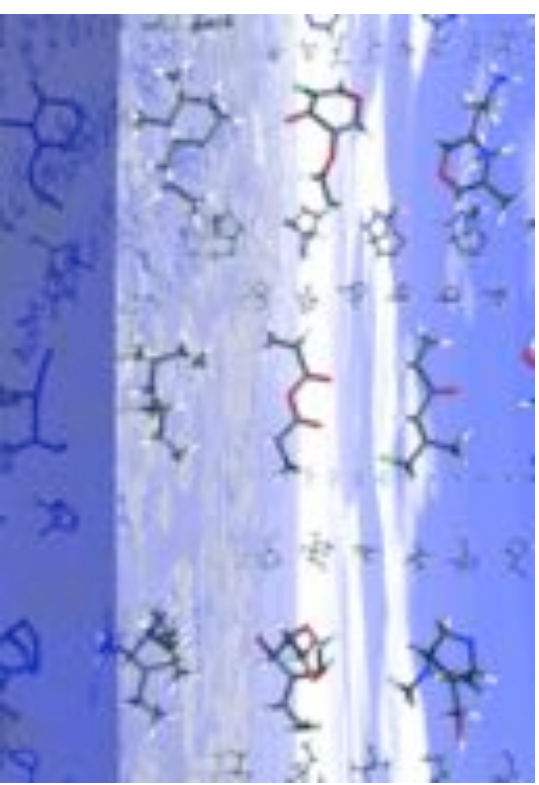
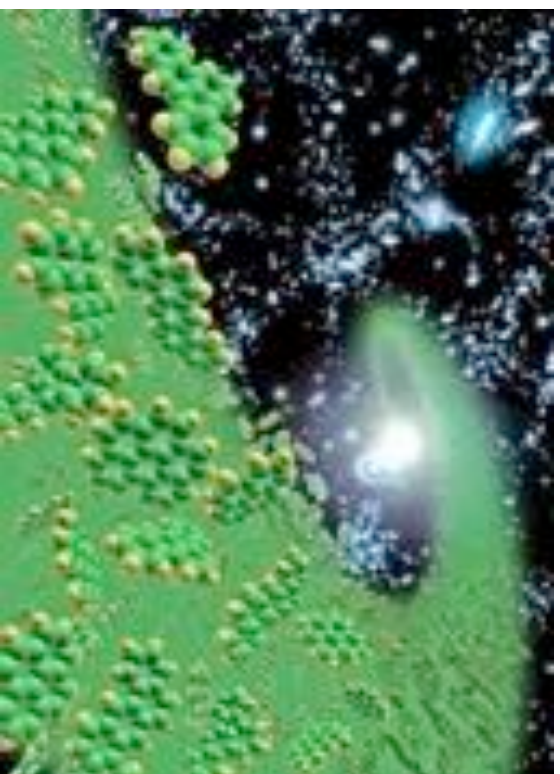
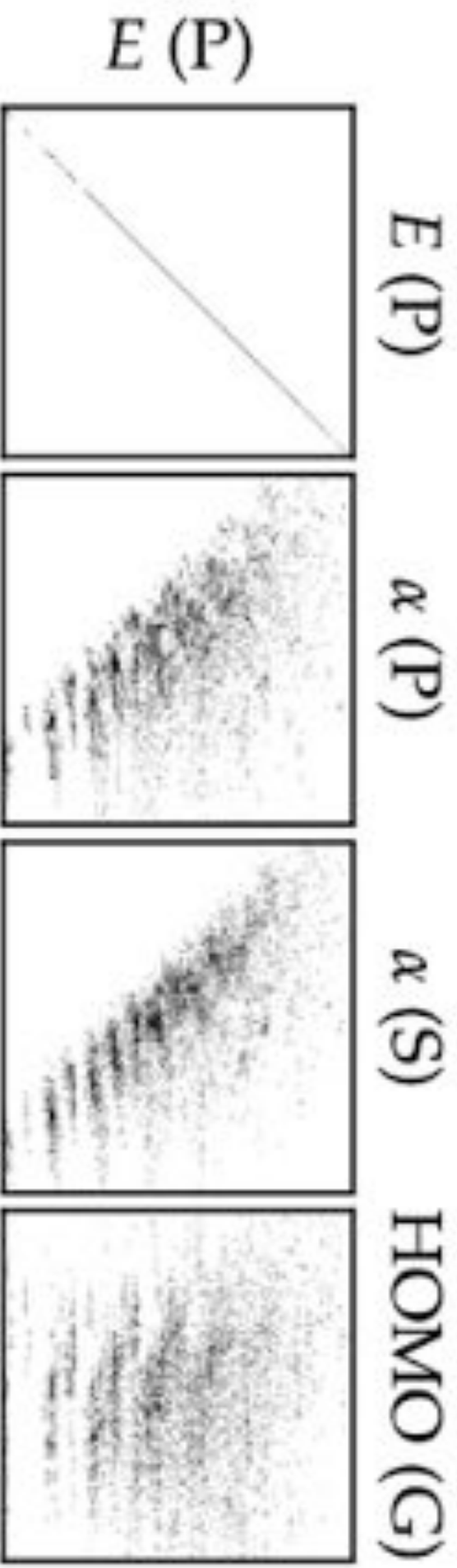
Machine learning for molecular big data



$\{R_i, Z_i\}$ maps to $\{P_1, P_2, P_3, P_4, \dots\}$

- **Descriptor**: what's a good representation of a molecule?
- **Metric**: how to define distance between two molecules?
- **Data selection**: Which molecules to use for training?
- **Properties**: which set of properties uniquely defines a molecule?
- **Degrees of freedom**: composition vs. conformation

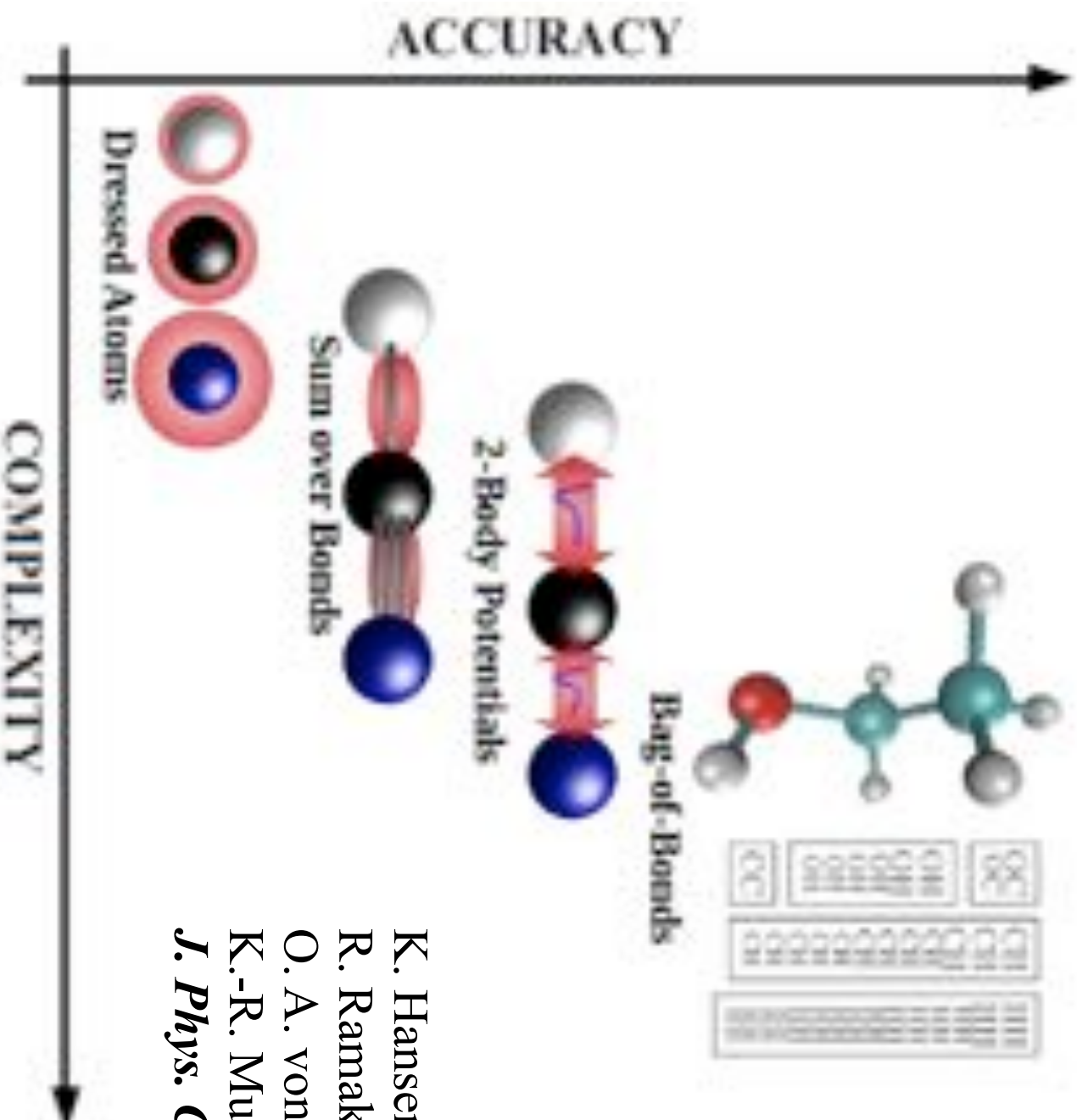
Chemical Compound Space: Freedom of design



G. Montavon, M. Rupp, V. Gobre, A. Vazquez-Mayagoitia, K. Hansen,
A. Tkatchenko, K.-R. Mueller, A. von Lilienfeld, *New J. Phys.* 15, 095003 (2013).

Predicting Molecular Properties:

Descriptors From “Dressed Atoms” to Bag-of-Bonds



K. Hansen, F. Biegler,
R. Ramakrishnan, W. Pronobis,
O. A. von Lilienfeld,
K.-R. Mueller, and A. Tkatchenko,
J. Phys. Chem. Lett. 6, 2326 (2015).

Predicting Molecular Properties: QM7 dataset

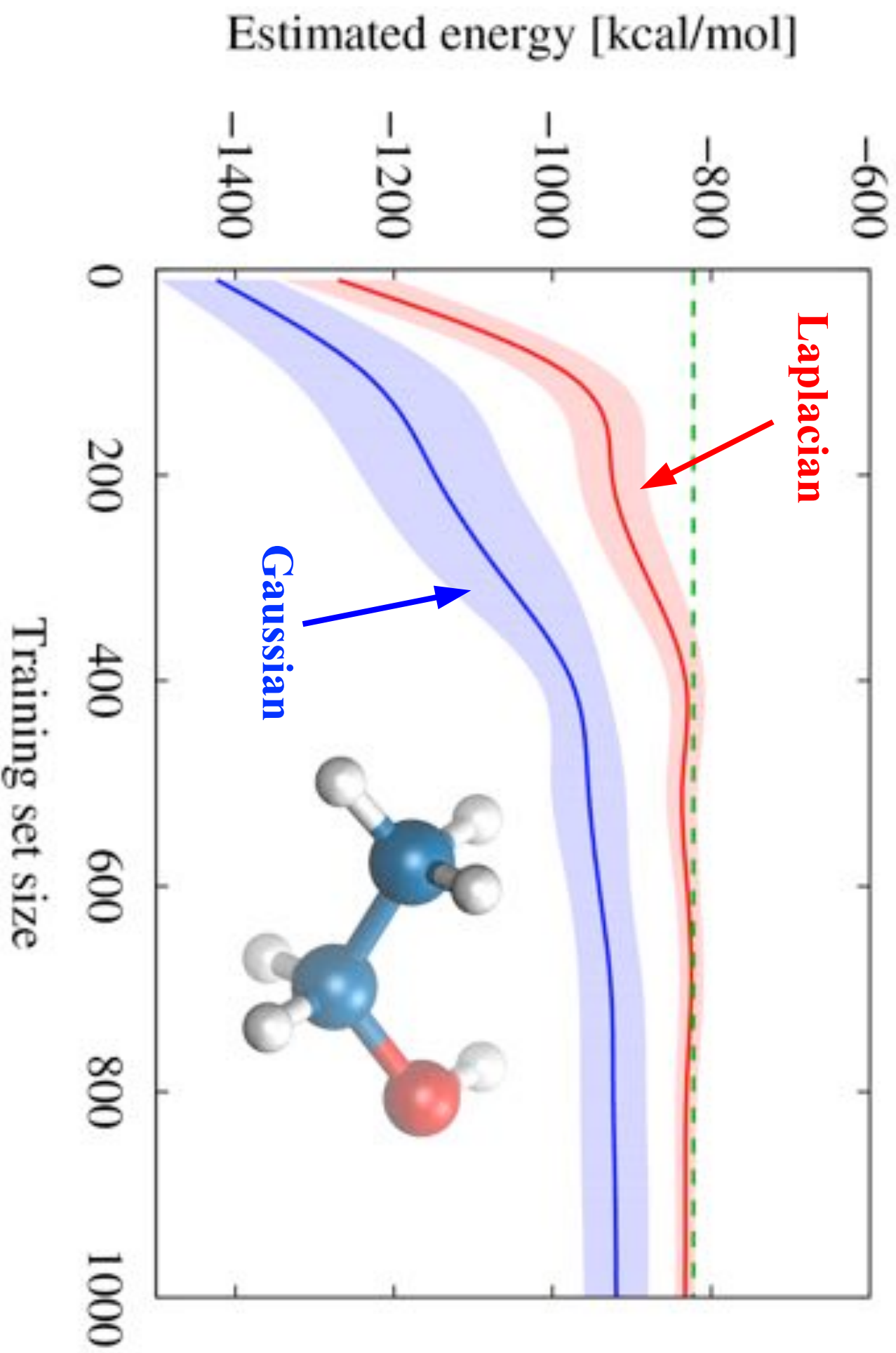
model	MAE [kcal/mol]
dressed atoms	15.1
sum-overbonds	9.9
Lennard-Jones potential	8.7
polynomial pot. ($n = 6$)	5.6
polynomial pot. ($n = 10$)	3.9
polynomial pot. ($n = 18$)	3.0
Bag of Bonds ($p = 2$, Gaussian)	4.5
Bag of Bonds ($p = 1$, Laplacian)	1.5
Coulomb matrix ($p = 2$, Gaussian) ¹⁷	10.0
Coulomb matrix ($p = 1$, Laplacian) ¹⁶	4.3

K. Hansen, F. Biegler, R. Ramakrishnan, W. Pronobis, O. A. von Lilienfeld, K.-R. Mueller, and A. Tkatchenko, *J. Phys. Chem. Lett.* 6, 2326 (2015).

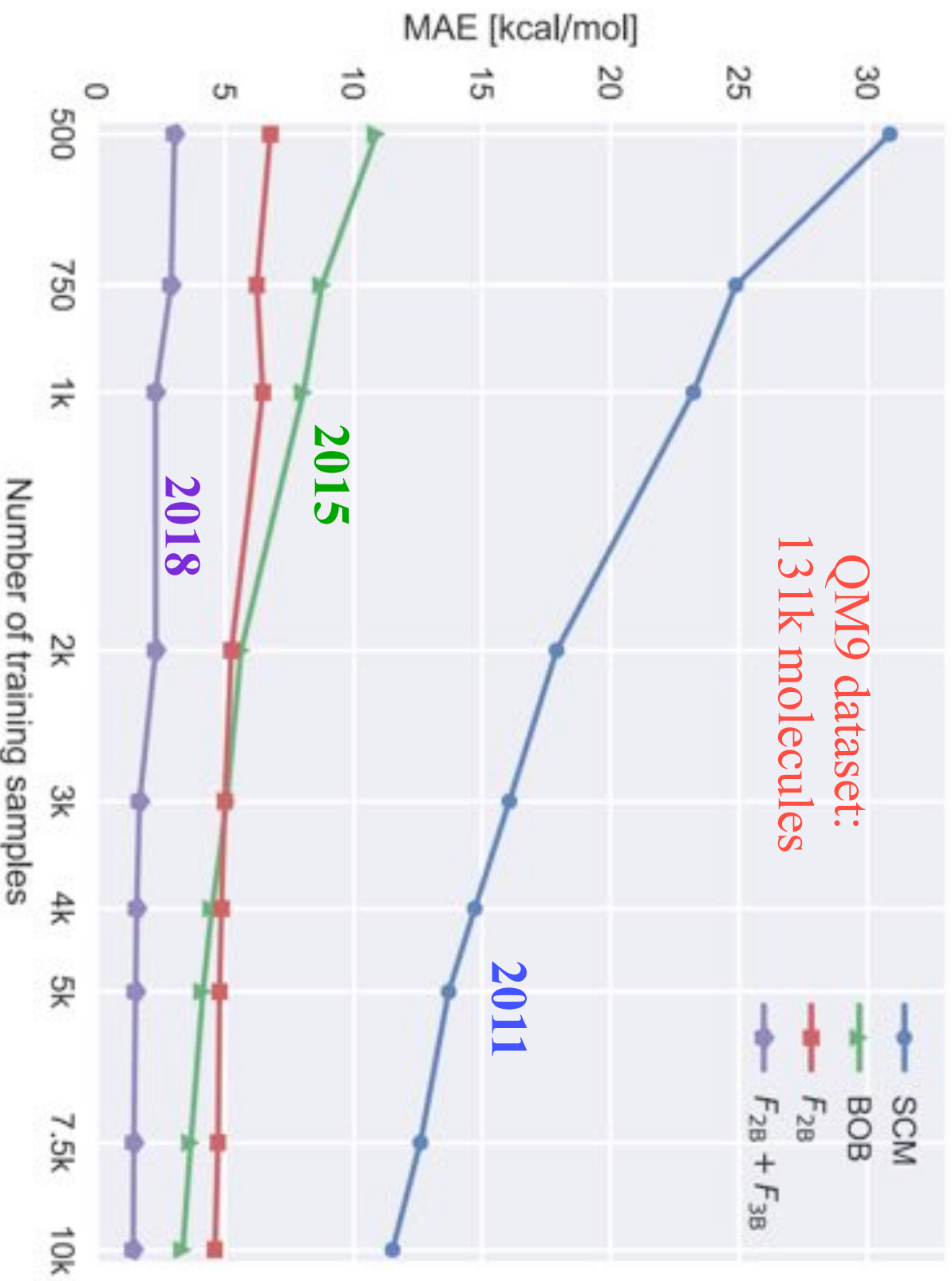
Predicting Molecular Properties: QM7 dataset

model	MAE [kcal/mol]
dressed atoms	15.1
sum-overbonds	9.9
Lennard-Jones potential	8.7
polynomial pot. ($n = 6$)	5.6
polynomial pot. ($n = 10$)	3.9
polynomial pot. ($n = 18$)	3.0
Bag of Bonds ($p = 2$, Gaussian)	4.5
Bag of Bonds ($p = 1$, Laplacian)	1.5
Coulomb matrix ($p = 2$, Gaussian) ¹⁷	10.0
Coulomb matrix ($p = 1$, Laplacian) ¹⁶	4.3
2+3body many-body expansion	0.8

Bag-of-Bonds (BoB): Non-Locality in Chemical Space

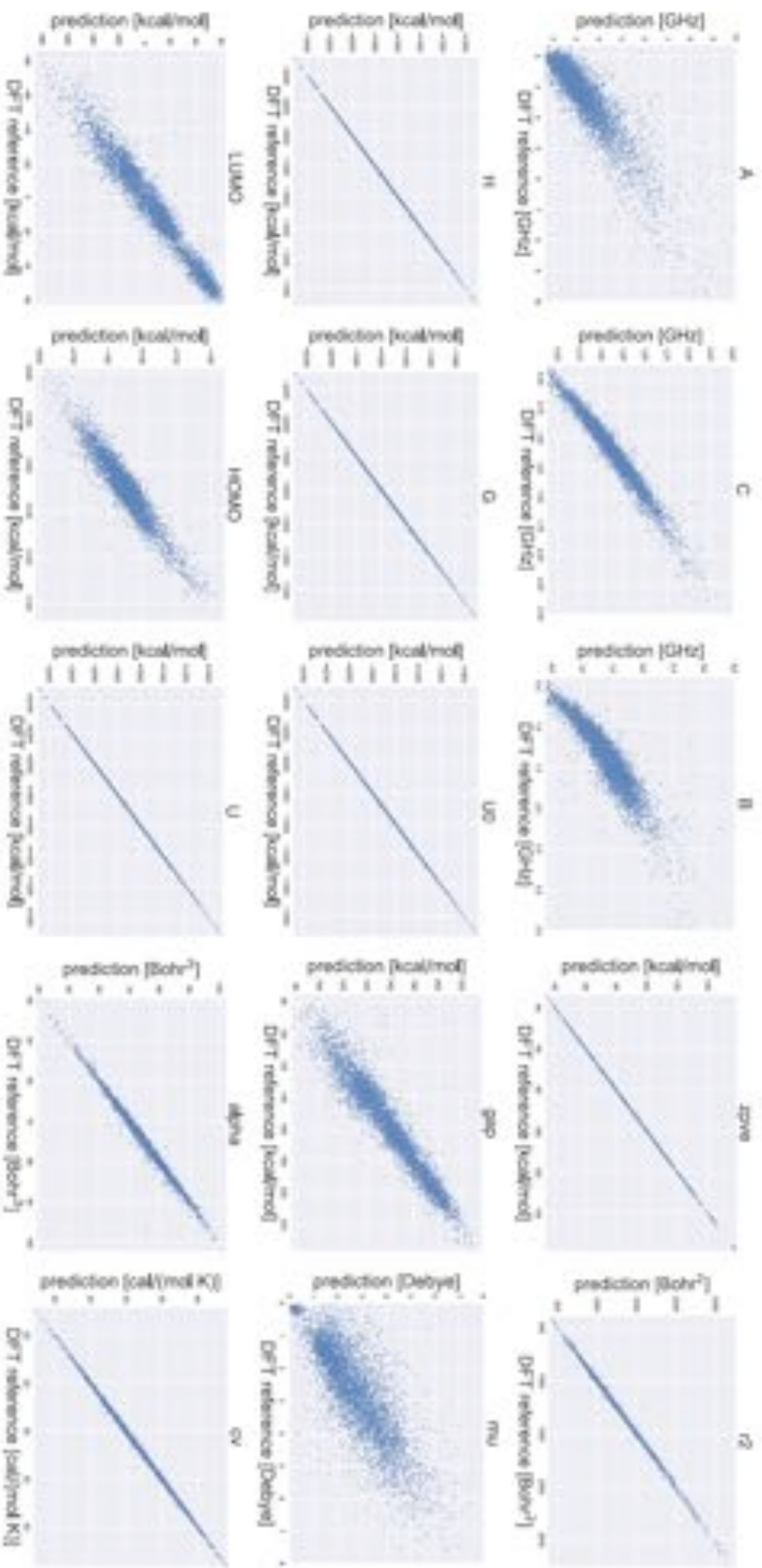


QM9 dataset: Evolution from Coulomb Matrix to Many-Body Representation



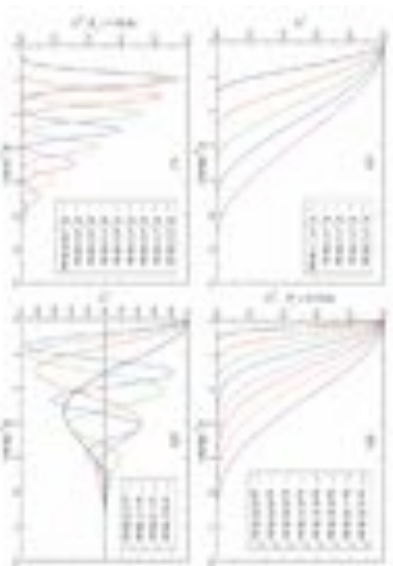
W. Pronobis, A. Tkatchenko, and K.-R. Mueller, *J. Chem. Theory Comput.* (2018).

QM9 dataset: Extensive and Intensive Properties



W. Pronobis, A. Tkatchenko, and K.-R. Mueller, *J. Chem. Theory Comput.* (2018).

Zoo of Descriptors for Molecules and Solids



Atom-centered symmetry functions
(Behler et al. 2007)

$$M_{ij} = \begin{cases} 0.5Z_i^{2.4} & \text{for } i = j \\ \frac{Z_i Z_j}{d_{ij}} & \text{for } i \neq j \end{cases}$$

Coulomb matrix
(Rupp et al. 2012)

$$\{Z_i, \mathbf{R}_i\}$$

$$\{Z_i, d_{ij}\}$$

$$x_{ij} = \begin{cases} 0.5Z_i^{2.4} & \text{if } i = j \\ \frac{Z_i Z_j}{\phi(r_{ij})} & \text{if } i \neq j \end{cases}$$

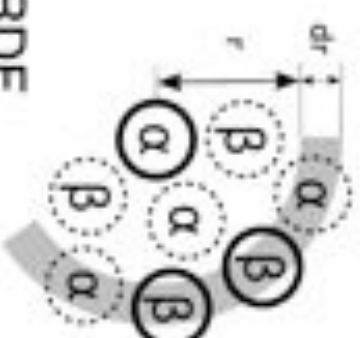
Sine matrix
(Faber et al. 2015)

$$k(\rho, \rho') = \int d\hat{R} |\rho(\mathbf{r})\rho'(\hat{R}\mathbf{r})|^n$$

SOAP
(Bartók et al. 2013)



Bag of bonds
(Hansen et al. 2015)

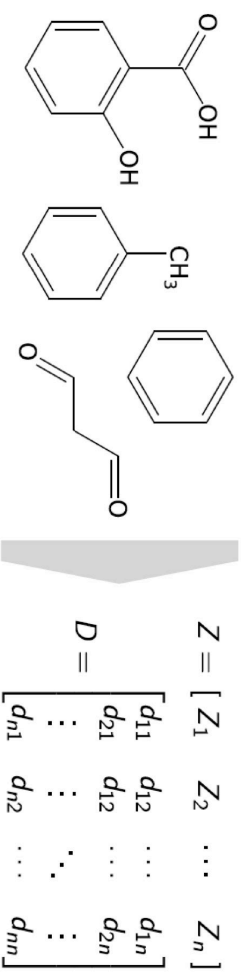


PRDF
(Schütt et al, 2014)

Learning the Representation: Deep Tensor Neural Networks (DTNN)

Deep Tensor Neural Networks (DTNN)

Input: Atomic numbers and interatomic distances



Embedding of based on atom types

$$\mathbf{x}_i^{(0)} = \mathbf{x}_{Z_i} \in \mathbb{R}^d$$

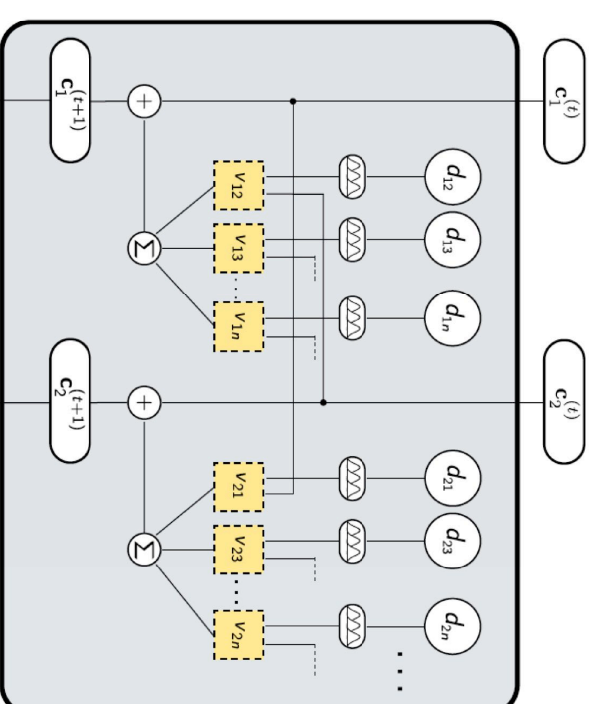
Add interaction with environment using $t = 1 \dots T$ sequential refinements $\mathbf{v}_i^{(t)}$

$$\mathbf{x}_i^{(t+1)} = \mathbf{x}_i^{(t)} + \mathbf{v}_i^{(t)} \left(\mathbf{x}_1^{(t)}, \dots, \mathbf{x}_{n_{\text{atoms}}}^{(t)}, d_{11}, \dots, d_{n_{\text{atoms}}} \right)$$

Prediction via atom-wise contributions:

$$\hat{E} = \sum_{i=1}^{n_{\text{atoms}}} f_{\text{out}}(\mathbf{x}_i^{(T)})$$

$$\hat{\mathcal{H}}\Psi = E\Psi$$

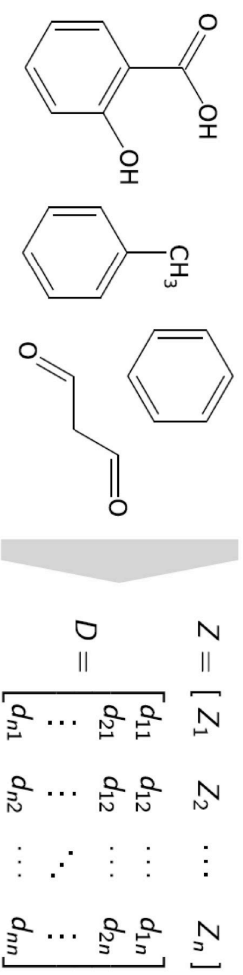


$$\mathbf{v}_{ij} = \tanh \left(W^{fc} (W^{cf} \mathbf{c}_j + \mathbf{b}^f) \circ (W^{df} \mathbf{d}_{ij} + \mathbf{b}^d) \right)$$

K. T. Schuett, F. Arbabzadah, S. Chmiela, K.-R. Mueller, and A. Tkatchenko, *Nature Commun.* 8, 13890 (2017).

Deep Tensor Neural Networks (DTNN)

Input: Atomic numbers and interatomic distances



Embedding of based on atom types

$$\mathbf{x}_i^{(0)} = \mathbf{x}_Z, \in \mathbb{R}^d$$

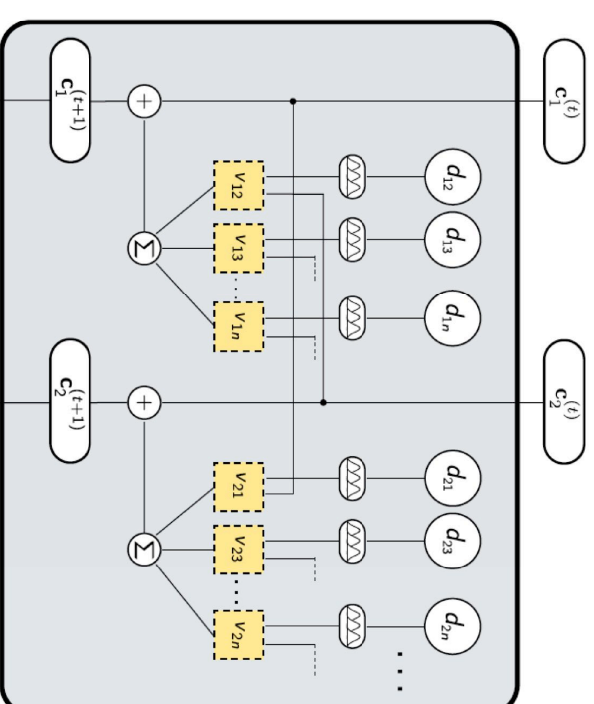
Add interaction with environment using $t = 1 \dots T$ sequential refinements $\mathbf{v}_i^{(t)}$

$$\mathbf{x}_i^{(t+1)} = \mathbf{x}_i^{(t)} + \mathbf{v}_i^{(t)} \left(\mathbf{x}_1^{(t)}, \dots, \mathbf{x}_{n_{\text{atoms}}}^{(t)}, d_{11}, \dots, d_{in_{\text{atoms}}} \right)$$

Prediction via atom-wise contributions:

$$\hat{E} = \sum_{i=1}^{n_{\text{atoms}}} f_{\text{out}}(\mathbf{x}_i^{(T)})$$

$$\hat{\mathcal{H}}\Psi = E\Psi$$

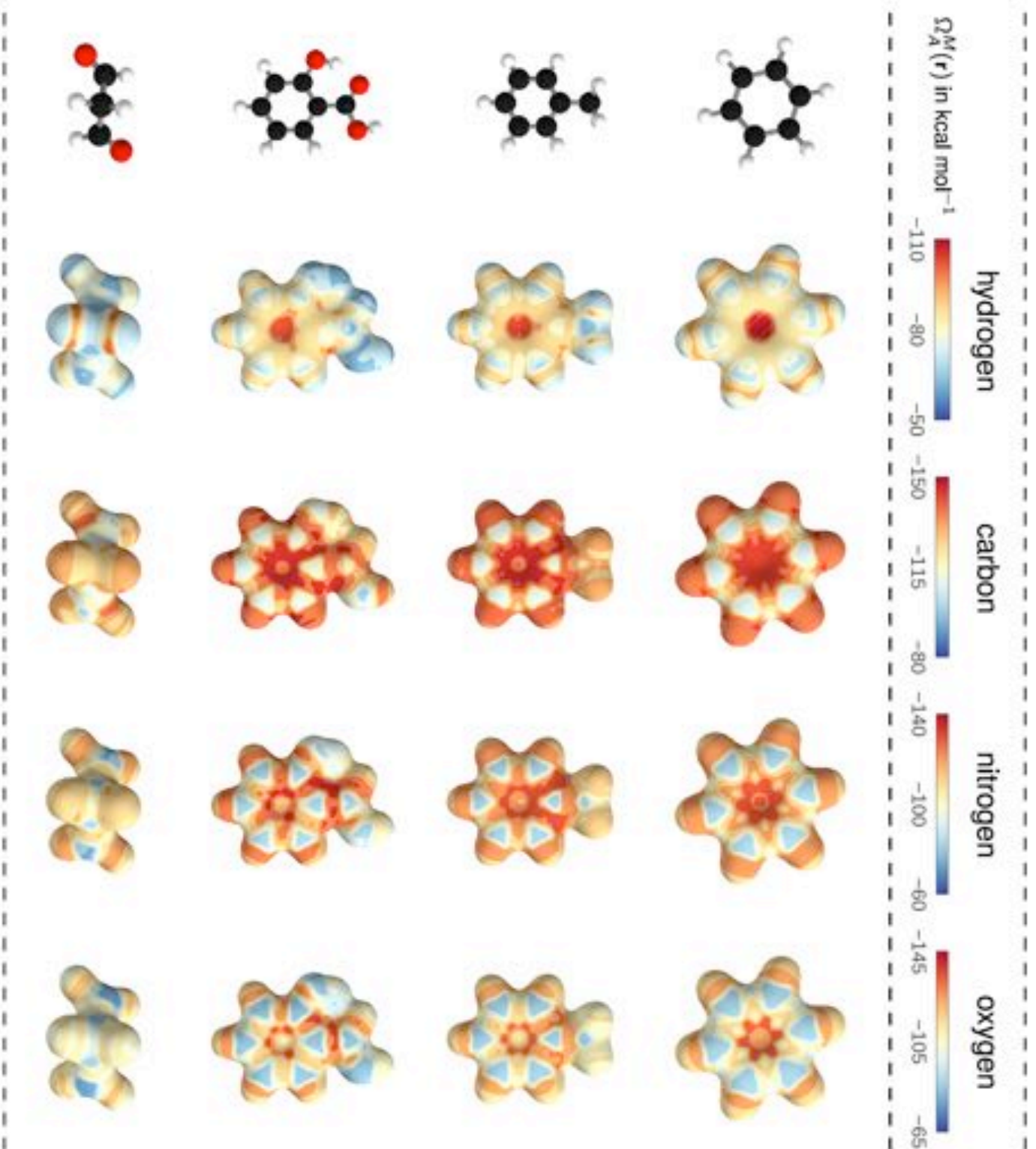


$$\mathbf{v}_{ij} = \tanh \left(W^{fc} (W^{cf} \mathbf{c}_j + \mathbf{b}^f) \circ (W^{df} \mathbf{d}_{ij} + \mathbf{b}^d) \right)$$

Mean absolute error on QM9: **0.2 kcal/mol**

K. T. Schuett, F. Arbabzadah, S. Chmiela, K.-R. Mueller, and A. Tkatchenko, *Nature Commun.* 8, 13890 (2017).

Molecular DTNN: What Did it Learn ?



Quantum Chemical Insights: Aromaticity

1 - 10



-859.9



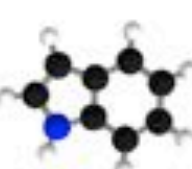
-858.3



-857.8



-857.4



-857.4



-857.3



-856.9



-856.8



-856.8



-856.6

E_{ring} in kcal mol⁻¹

281 - 290



-845.1



-843.8



-842.1



-841.9



-841.9



-841.7



-841.7



-841.4



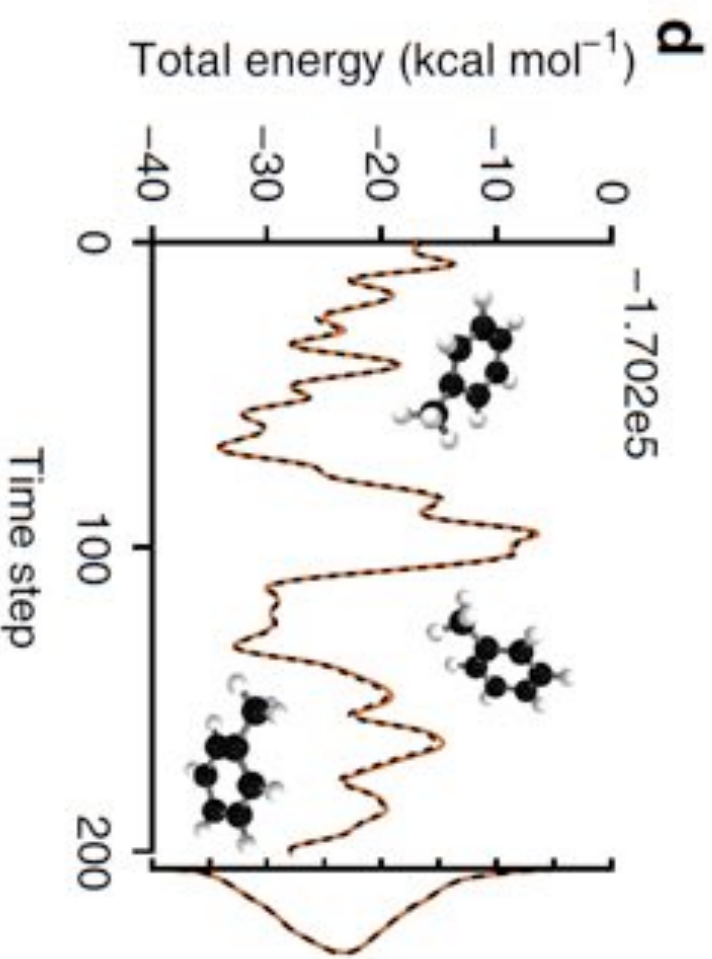
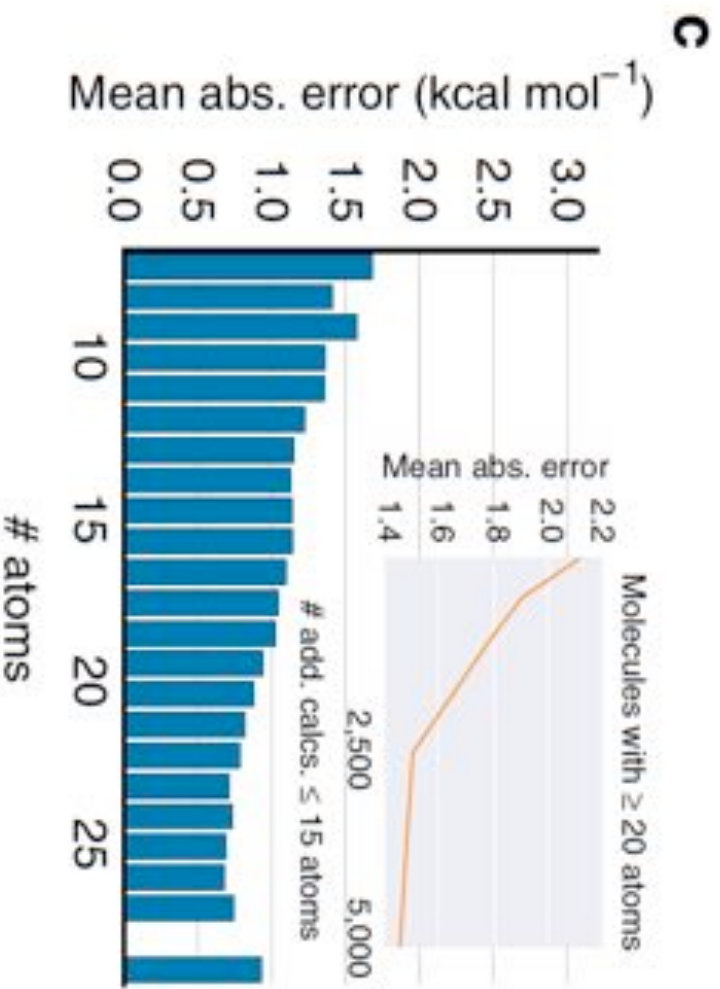
-841.2



-841.1

E_{ring} in kcal mol⁻¹

Learning Full Chemical Space with DTNN?



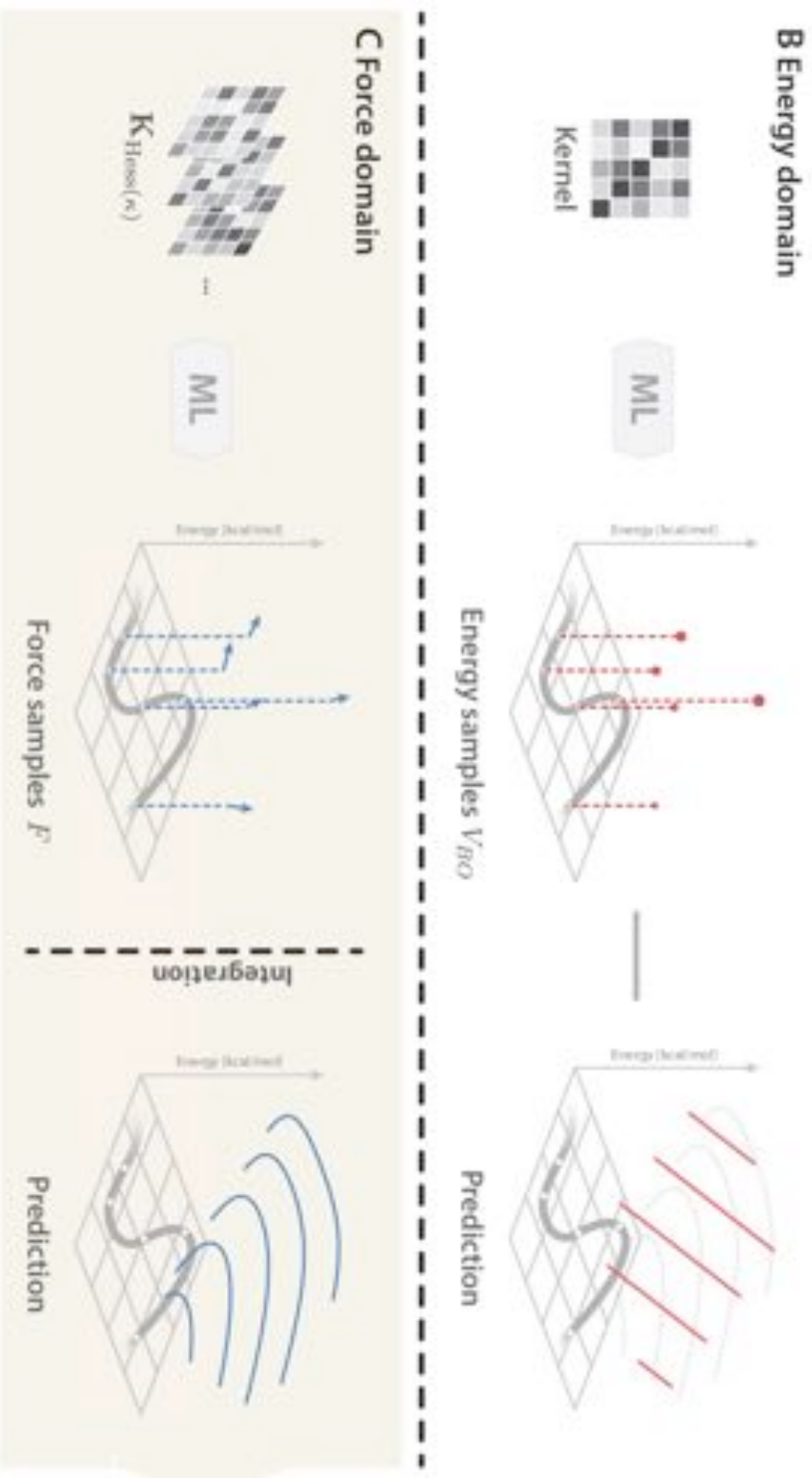
Accurately representing **BOTH** compositional and conformational degrees of freedom is difficult.

For C7O2H10 isomer and MD data, the error grows to > 1.0 kcal/mol

K. T. Schuett, F. Arbabzadah, S. Chmiela, K.-R. Mueller, and A. Tkatchenko, *Nature Commun.* 8, 13890 (2017).

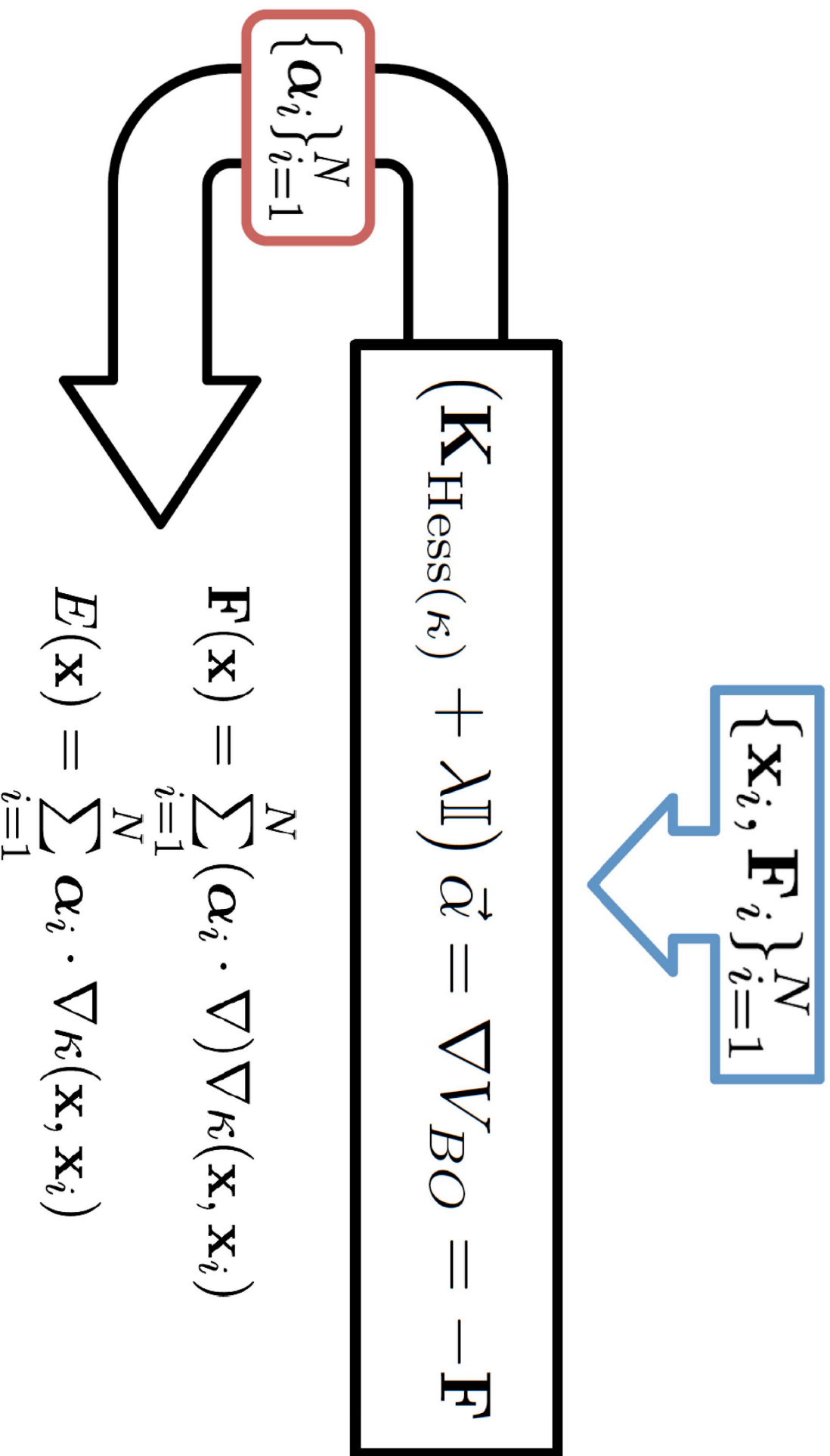
Beating the Hell out of Data: Gradient-Domain Machine Learning (GDML)

Beating the Hell out of Data: Gradient-Domain Machine Learning (GDML)



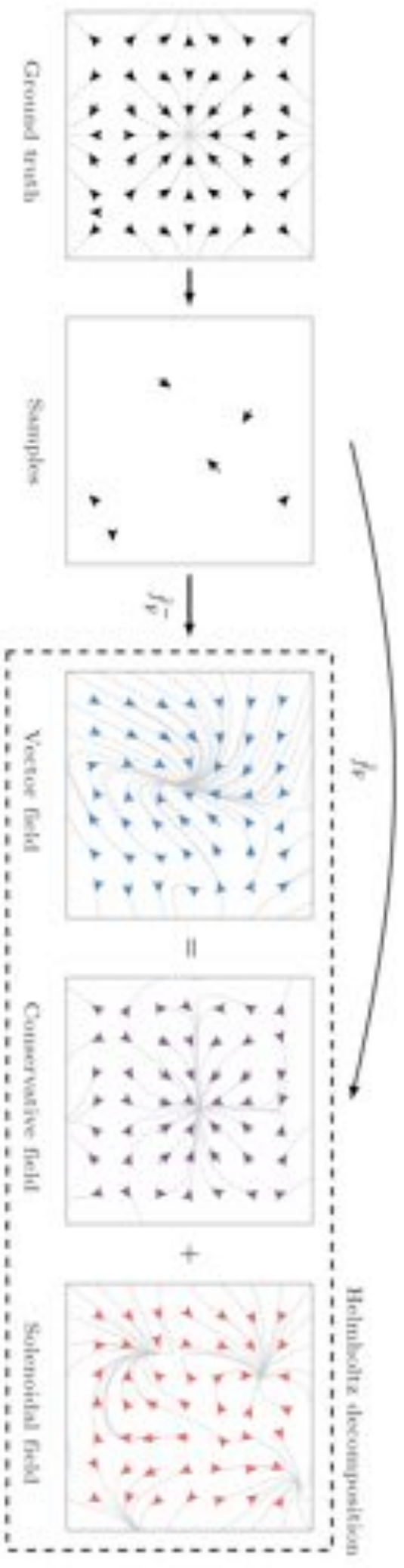
S. Chmiela, A. Tkatchenko, H. Sauceda, I. Poltavsky, K. T. Schuett, K.-R. Mueller, *Science Adv.* 3, e1603015 (2017).

Beating the Hell out of Data: Gradient-Domain Machine Learning (GDML)



S. Chmiela, A. Tkatchenko, H. Sauceda, I. Poltavsky, K. T. Schuett, K.-R. Mueller, *Science Adv.* 3, e1603015 (2017).

Beating the Hell out of Data: Gradient-Domain Machine Learning (GDML)



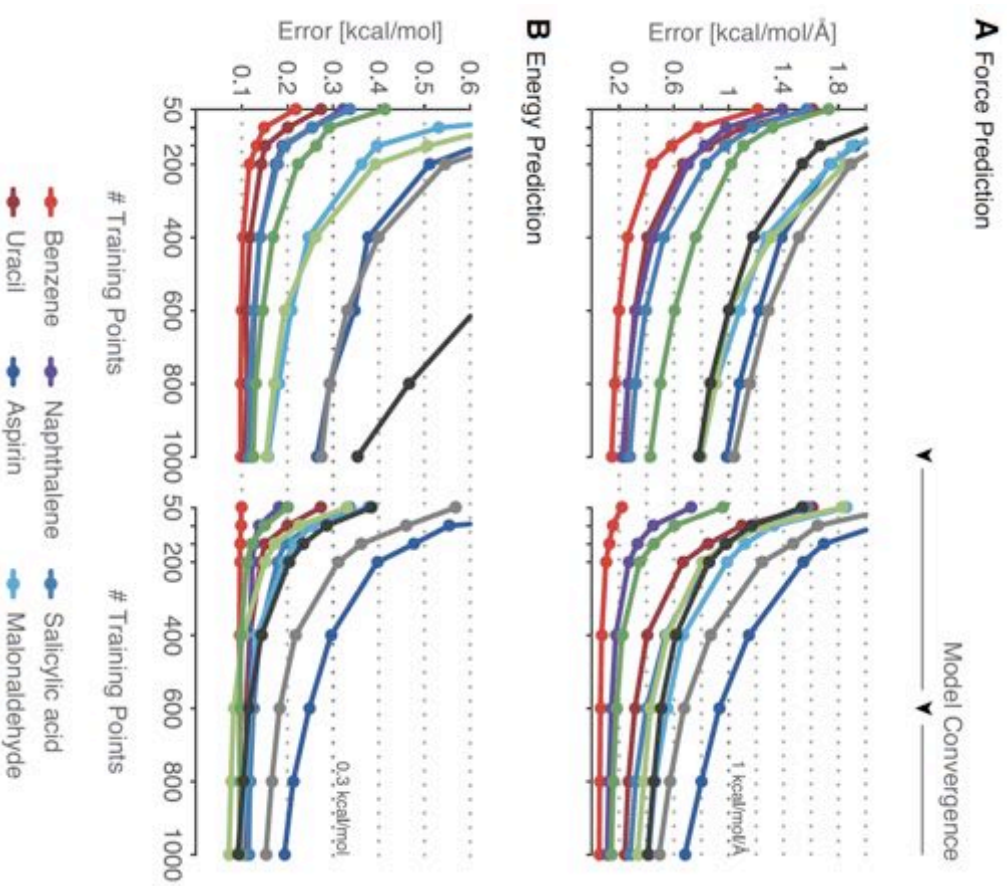
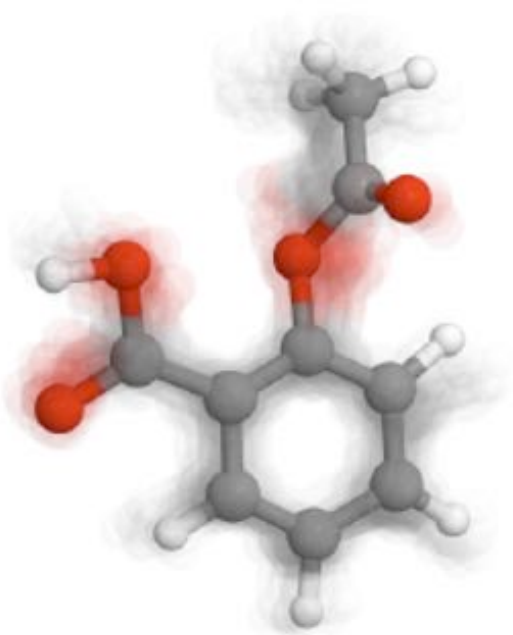
Wrong

Better

Error

S. Chmiela, A. Tkatchenko, H. Sauceda, I. Poltavsky, K. T. Schuett, K.-R. Mueller, *Science Adv.* 3, e1603015 (2017).

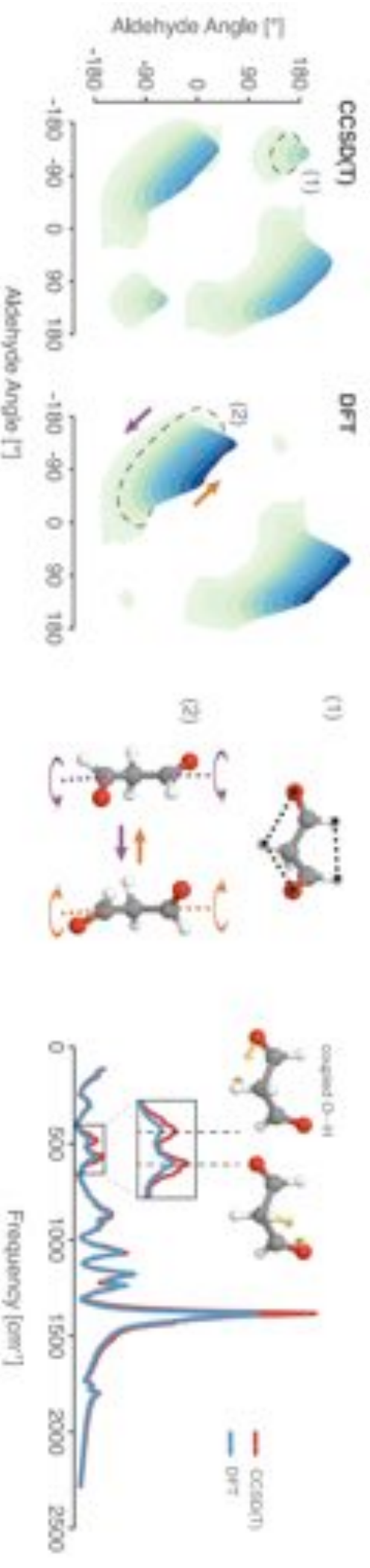
Symmetrized Gradient-Domain Machine Learning: Towards Exact Molecular Force Fields



Globally accurate force field from only 100s of conformations

Embarrassingly Quantum MD for Molecules: Quantized Electrons [CCSD(T)] and Nuclei [PIMD]

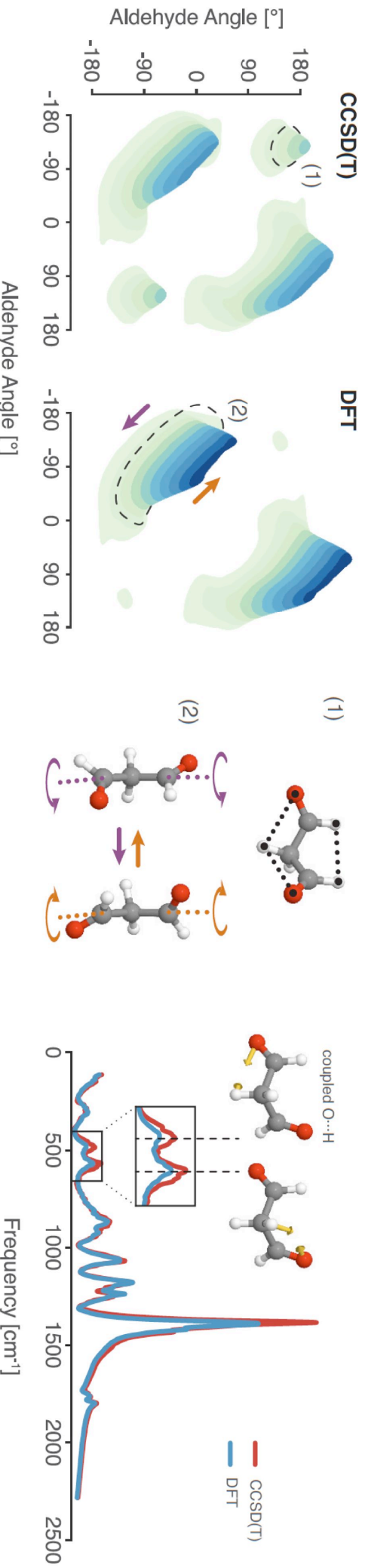
A Molecule's Probability Distribution & Vibrational Spectrum



S. Chmiela, H. Sauceda, K.-R. Mueller, and A. Tkatchenko
Nature Commun. 9, 3887 (2018).

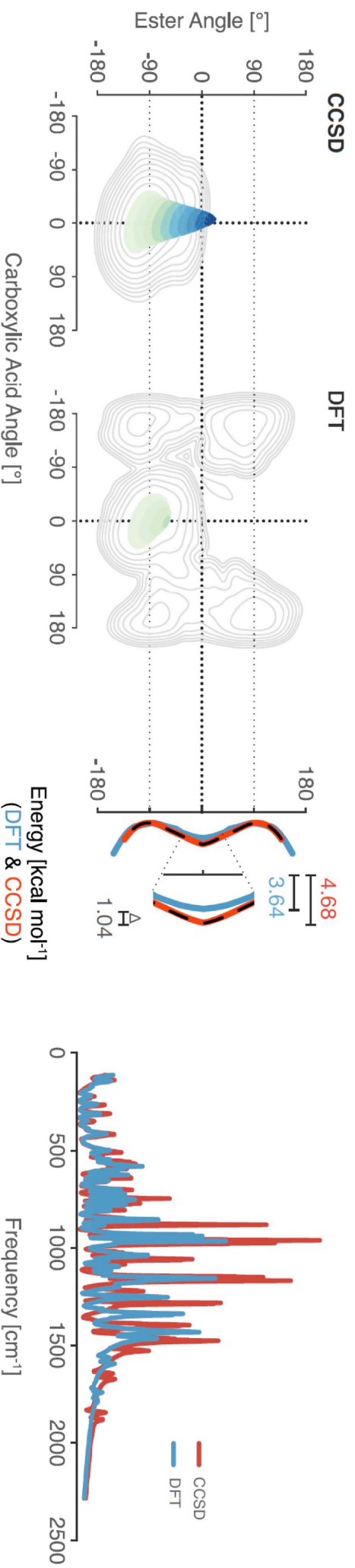
Embarrassingly Quantum MD for Molecules: Quantized Electrons [CCSD(T)] and Nuclei [PIMD]

A Malonaldehyde Probability Distribution & Vibrational Spectrum



B Aspirin Probability Distribution & Vibrational Spectrum

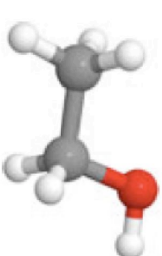
* The sGDML model for aspirin was trained on CCSD reference data.



S. Chmiela, H. Sauceda, K.-R. Mueller, and A. Tkatchenko
Nature Commun. 9, 3887 (2018).

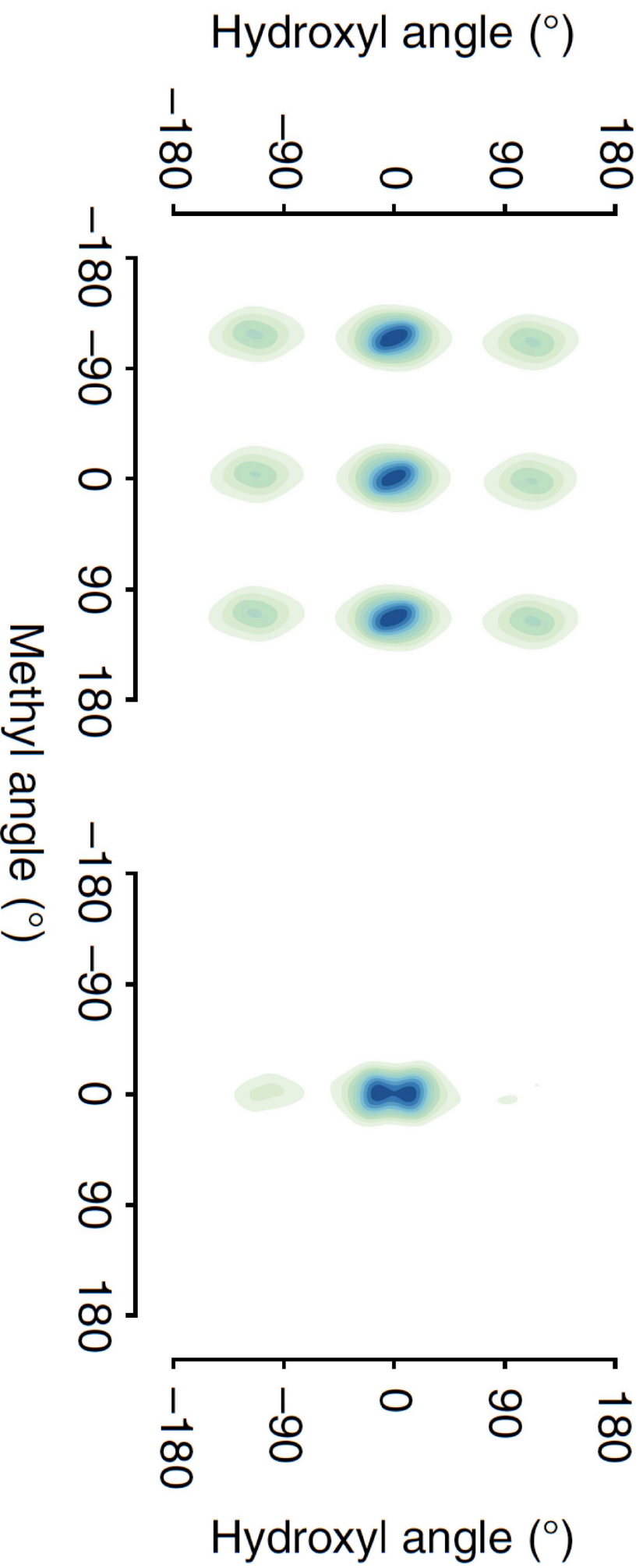
Exact Free Energy Surfaces vs. Empirical Force Fields

Ethanol probability distribution of dihedral angles



sgDML@CCSD(T)

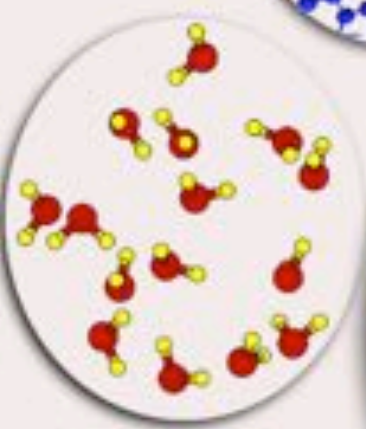
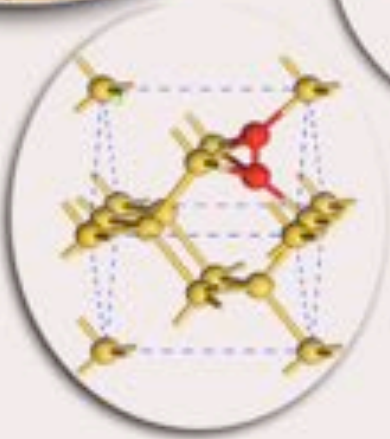
Amber



S. Chmiela, H. Sauceda, K.-R. Mueller, and A. Tkatchenko

Nature Commun. 9, 3887 (2018).

$$\hat{H}\Psi = E\Psi$$



Ambrosetti, Ferri, DiStasio Jr., and Tkatchenko, *Science* (2016).

Grand Challenges for Machine Learning in Physics/Chemistry

- *What is chemical space*: descriptors of molecules and materials, metric?
- *How to learn intensive properties*: energy levels, excited states, spectra?
- How to combine ML with physical laws (symmetries) and interaction models?
- Can we learn (approximate) Hamiltonians?
- Can ML suggest better approximations for $\hat{H}\Psi = E\Psi$?
- More and better (big) data

Towards rational design of molecules and materials in chemical space