

Constructing collective variables using Machine Learning and free energy biased simulations

Tony LELIEVRE, Thomas PIGEON and <u>Gabriel STOLTZ</u> (CERMICS, Ecole des Ponts & MATHERIALS team, Inria Paris)

Winter school GdR MatBio, 8-12 November 2021, CIRM

Outline

- A (short/biased) review of machine learning approaches for CV
- Free-energy biasing and iterative learning with autoencoders¹
 - Autoencoders: definition, training, interpretation
 - Extended adaptive biasing force method
 - General presentation of the iterative algorithm
 - Illustration/sanity checks on toy examples

• Applications to systems of interest (alanine dipeptide, chignolin, HSP90)

¹Z. Belkacemi, P. Gkeka, T. Lelièvre, G. Stoltz, arXiv preprint **2104.11061**

(A biased perspective on some) References

• ML reviews in MD

- A. Gliemlo, B. Husic, A. Rodriguez, C. Clementi, F. Noé, A. Laio, Chem. Rev. 121(16), 9722-9758 (2021)
- M. Chen, Eur. Phys. J. B 94, 211 (2021)
- P. Gkeka et al., J. Chem. Theory Comput. 16(8), 4757-4775 (2020)
- F. Noé, A. Tkatchenko, K.-R. Müller, C. Clementi, *Annu. Rev. Phys. Chem.* **71**, 361-390 (2020)
- A.L. Ferguson, J. Phys.: Condens. Matter 30, 04300 (2018)

• More general ML references

- P. Mehta, M. Bukov, C.-H. Wang, A.G.R.Day, C. Richardson, C.K.Fisher, D.J. Schwab, A high-bias, low-variance introduction to Machine Learning for physicists, *Physics Reports* 810, 1-124 (2019)
- I. Goodfellow, Y. Bengio, A. Courville *Deep Learning* (MIT Press, 2016) http://www.deeplearningbook.org

ML approaches for finding CV

Some representative approaches for finding CV (1)

- Chemical/physical intuition (distances, angles, RMSDs, coordination numbers, etc)
- Short list of data-oriented approaches (depending on the data at hand...)
 - [supervised learning] separate metastable states
 - [unsupervised/static] distinguish linear models (PCA) and nonlinear ones (e.g. based on autoencoders such as MESA²)
 - [unsupervised/dynamics] operator based approaches (VAC, EDMD, diffusion maps, MSM; incl. tICA and VAMPNets)

(Huge literature! I am not quoting precise references here because the list would be too long)

 \bullet Other classifications 3,4 possible, e.g. slow vs. high variance CV

²W. Chen and A.L. Ferguson, *J. Comput. Chem.* 2018; W. Chen, A.R. Tan, and A.L. Ferguson, *J. Chem. Phys.* 2018

- ³P. Gkeka et al., J. Chem. Theory Comput. (2020)
- ⁴A. Gliemlo et al., Annu. Rev. Phys. Chem. (2021)

Some representative approaches for finding CV (2)



CV construction with autoencoders

Autoencoders (1)



Autoencoders (2)

• Data space $\mathcal{X} \subseteq \mathbb{R}^D$, bottleneck space $\mathcal{A} \subseteq \mathbb{R}^d$ with d < D

$$f(x) = f_{\mathsf{dec}}\Big(f_{\mathsf{enc}}(x)\Big)$$

where $f_{enc}: \mathcal{X} \to \mathcal{A}$ and $f_{dec}: \mathcal{A} \to \mathcal{X}$

Collective variable = encoder part

$$\xi = f_{\rm enc}$$

 \bullet Fully connected neural network, symmetrical structure, 2L layers

• Parameters $\mathbf{p} = \{p_k\}_{k=1,\dots,K}$ (bias vectors b_ℓ and weights matrices W_ℓ)

$$f_{\mathbf{p}}(x) = g_{2L} \left[b_{2L} + W_{2L} \dots g_1 (b_1 + W_1 x) \right]$$
,

with activation functions g_ℓ

(examples: tanh(x), ReLU max(0, x), sigmoid $\sigma(x) = 1/(1 + e^{-x})$, etc)

Training autoencoders

• Theoretically: minimization problem in $\mathcal{P} \subset \mathbb{R}^{K}$

$$\mathbf{p}_{\mu} \in \operatorname*{argmin}_{\mathbf{p} \in \mathcal{P}} \mathcal{L}(\mu, \mathbf{p}),$$

with cost function

$$\mathcal{L}(\mu, \mathbf{p}) = \mathbb{E}_{\mu}(\|X - f_{\mathbf{p}}(X)\|^2) = \int_{\mathcal{X}} \|x - f_{\mathbf{p}}(x)\|^2 \ \mu(dx)$$

• In practice, access only to a sample: minimization of empirical cost

$$\mathcal{L}(\hat{\mu}, \mathbf{p}) = \frac{1}{N} \sum_{i=1}^{N} \|x^{i} - f_{\mathbf{p}}(x^{i})\|^{2}, \qquad \hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} \delta_{x^{i}}$$

• Typical choices: canonical measure μ , data points x^i postprocessed from positions q (alignement to reference structure, centering, reduction to backbone carbon atoms, etc)

Some elements on training neural networks (1/2)

• Many local minima...

• Actual procedure:

- Separate data set into training/validation: optimize on training set
- "Early stopping": stop when validation loss no longer improves⁵



- Computation of gradient performed with backpropagation
- Choice of optimization method⁶, here Adam

⁵See Section 7.8 in [Goodfellow/Bengio/Courville] ⁶See Chapter 8 in [Goodfellow/Bengio/Courville]

Some elements on training neural networks (2/2)

• Stochastic gradient descent (SGD): mini-batching (randomness)

$$\mathbf{p}_r = \mathbf{p}_{r-1} - \eta \nabla_{\mathbf{p}} \mathcal{L}_r(\mathbf{p}_{r-1}), \qquad \mathcal{L}_r(\mathbf{p}) = \frac{1}{m} \sum_{i=rm+1}^{(r+1)m} \|x^i - f_{\mathbf{p}}(x^i)\|^2$$

One epoch = $\lceil N/m \rceil$ gradient steps (in order to visit all the data) Stochastic approximation of the true gradient⁷

$$\nabla_{\mathbf{p}} \mathcal{L}_r(\mathbf{p}) = \nabla_{\mathbf{p}} \mathcal{L}(\mathbf{p}) + \sqrt{\varepsilon(r)} \Sigma(\mathbf{p})^{1/2} Z, \quad Z \approx \mathcal{N}(0, \mathrm{Id}), \quad \varepsilon(r) \approx \frac{N^2}{m}$$

• Adam (Adaptive Moment)⁸: uses a moving average of gradients, normalized by a moving average of gradient mean squares (element-wise) \rightarrow more robust to anisotropy

... but current debates for DeepNN about why SGD better for generalization although Adam better for optimization

⁷See e.g. Section 2.2 in I. Sekkat, G. Stoltz, *arXiv preprint* 2105.10347
⁸Kingma/Ba, 2015 (arXiv preprint 1412.6980); Balles/Hennig, *ICML* (2018)

A variance interpretation of autoencoders (Wei Zhang)

• Total variance $\operatorname{Var}(X) = \operatorname{Var}\left[\mathbb{E}(X|f_{\operatorname{enc}}(X))\right] + \mathbb{E}\left[\operatorname{Var}(X|f_{\operatorname{enc}}(X))\right]$

 $\underline{\operatorname{Proof:}} \text{ Use } \mathbb{E}\left[\mathbb{E}(h(X)|f_{enc}(X))\right] = \mathbb{E}(h(X)) \text{ to write } \operatorname{Var}\left[\mathbb{E}(X|f_{enc}(X))\right] = \mathbb{E}\left[\mathbb{E}(X|f_{enc}(X))^2\right] - \mathbb{E}(X)^2 \text{ and } \mathbb{E}\left[\operatorname{Var}(X|f_{enc}(X))\right] = \mathbb{E}(X^2) - \mathbb{E}\left[\mathbb{E}(X|f_{enc}(X))^2\right], \text{ the sum of these terms being } \mathbb{E}(X^2) - \mathbb{E}(X)^2.$

• Training w.r.t. decoder part performed analytically in principle as

$$\begin{split} \min_{f_{\text{enc}}} \left[\min_{f_{\text{dec}}} \int_{\mathcal{X}} |x - f_{\text{dec}}(f_{\text{enc}}(x))|^2 \, \mu(dx) \right] &= \min_{f_{\text{enc}}} \ell(f_{\text{enc}}) \\ \text{with} \quad \ell(f_{\text{enc}}) = \mathbb{E} \left[\left(X - \mathbb{E}(X|f_{\text{enc}}(X)) \right)^2 \right] = \mathbb{E} \left[\text{Var}(X|f_{\text{enc}}(X)) \right] \\ &= \text{Var}(X) - \text{Var} \left[\mathbb{E}(X|f_{\text{enc}}(X)) \right] \end{split}$$

• Minimizing $\ell(f_{enc})$ equivalent to maximizing Var $[\mathbb{E}(X|f_{enc}(X))]$ = minimizing intraclass dispersion vs. maximizing interclass dispersion = small spread of data points for f_{enc} given around the mean vs. the mean values associated with f_{enc} given should be as spread out as possible

Free energy biasing for complex CV

Extended systems

- Computing $\nabla \xi$ already difficult, higher order derivatives is worse
- Extended system strategy : $V_{\text{ext}}(q,\lambda) = V(q) + \frac{\kappa}{2} (\xi(q) \lambda)^2$
- Free energy for the (simple) collective variable $\xi_{\mathrm{ext}}(q,\lambda) = \lambda$

$$F_{\kappa}(z) = -\frac{1}{\beta} \ln \int_{\mathcal{D}} e^{-\beta V_{\text{ext}}(q,z)} dq + C$$

$$= -\frac{1}{\beta} \ln \int \left(\int_{\Sigma(\zeta)} e^{-\beta V(q)} \delta_{\xi(q)-\zeta}(dq) \right) e^{-\beta \kappa (\zeta-z)^2/2} d\zeta + C$$

$$= -\frac{1}{\beta} \ln \int e^{-\beta F(\zeta)} \chi_{\kappa}(z-\zeta) d\zeta + \widetilde{C}, \qquad \chi_{\kappa}(s) = \left(\frac{\beta \kappa}{2\pi}\right)^{d/2} e^{-\beta \kappa s^2/2}$$

$$\xrightarrow[\kappa \to +\infty]{} F(z)$$

Calls for taking κ large

Extended ABF

Extended overdamped Langevin dynamics ($\kappa \text{ limits } \Delta t...$)

$$\begin{cases} dq_t = \left[-\nabla V(q_t) + \kappa(\xi(q_t) - \lambda_t)\nabla\xi(q_t) \right] dt + \sqrt{2\beta^{-1}} dW_t^q \\ d\lambda_t = -[\lambda_t - \xi(q_t)] dt + \sqrt{2\beta^{-1}} dW_t^\lambda \end{cases}$$

Extended ABF overdamped Langevin dynamics

$$\begin{cases} dq_t = \left[-\nabla V(q_t) + \kappa(\xi(q_t) - \lambda_t)\nabla\xi(q_t) \right] dt + \sqrt{2\beta^{-1}} dW_t^q \\ d\lambda_t = \left[\xi(q_t) - \mathbb{E}(\xi(q_t) \mid \lambda_t) \right] dt + \sqrt{2\beta^{-1}} \, dW_t^\lambda \end{cases}$$

In practice, $\mathbb{E}(\xi(q_t)\,|\,\lambda_t)$ is estimated by $\Xi_t(\lambda_t)$ with

$$\Xi_t(\Lambda) = \frac{\int_0^t \delta_{\varepsilon}(\lambda_s - \Lambda)\xi(q_s) \, ds}{\max\left(\eta, \int_0^t \delta_{\varepsilon}(\lambda_s - \Lambda) \, ds\right)}$$

Unbiased estimate of the free energy in eABF

• Stationarity: configurations distributed according to $e^{-\beta(V_{ext}(q,\lambda)-F_{\kappa}(\lambda))}$

$$\rho(z,\lambda) = Z_{\kappa}^{-1} \exp\left(-\beta \left[F(z) + \frac{\kappa}{2}(z-\lambda)^2 - F_{\kappa}(\lambda)\right]\right)$$

• Unbiased estimator of the mean force (CZAR)⁹

$$F'(z) = -\frac{1}{\beta} \frac{d[\ln \overline{\rho}(z)]}{dz} + \kappa(\langle \lambda \rangle_z - z)$$

with
$$\overline{\rho}(z) = \int \rho(z,\lambda) d\lambda$$
 and $\langle \lambda \rangle_z = \frac{1}{\overline{\rho}(z)} \int \lambda \rho(z,\lambda) d\lambda$ (conditional dist.)

<u>Proof:</u> start from $F'(z) = -\frac{1}{\beta} \frac{\partial_z \rho(z, \lambda)}{\rho(z, \lambda)} - \kappa(z - \lambda)$, multiply both sides of the equality by $\rho(z, \lambda)/\overline{\rho}(z)$ and integrate with respect to λ

⁹A. Lesage, T. Lelièvre, G. Stoltz and J. Hénin, *J. Phys. Chem. B* (2017) Gabriel Stoltz (ENPC/Inria) November 2021

17 / 36

Joint distribution of (λ, z) (deca-alanine)



logarithmic scale

$$\sigma^2 = \frac{1}{\beta\kappa}$$

Marginal distribution in λ nearly uniform (as expected)

Iterative free energy biasing/ autoencoder learning

Training on modified target measures

• Interesting systems are metastable (no spontaneous exploration of phase space) Sample according to a biased distribution $\tilde{\mu}$ (importance sampling)

• Need for reweighting to learn the correct encoding!

$$w(x) = rac{\mu(x)}{\widetilde{\mu}(x)}$$

• Minimization problem: theoretical cost function

$$\mathcal{L}(\mu, \mathbf{p}) = \int_{\mathcal{X}} \|x - f_{\mathbf{p}}(x)\|^2 w(x) \widetilde{\mu}(dx),$$

actual cost function

$$\mathcal{L}(\widehat{\mu}_{\mathsf{wght}}, \mathbf{p}) = \sum_{i=1}^{N} \widehat{w}_i \| x^i - f_{\mathbf{p}}(x^i) \|^2, \qquad \widehat{w}_i = \frac{\mu(x^i) / \widetilde{\mu}(x^i)}{\sum_{j=1}^{N} \mu(x^j) / \widetilde{\mu}(x^j)}$$

 \bullet Only requires the knowledge of μ and $\widetilde{\mu}$ up to a multiplicative constant.

• Stochastic gradients in training: sampling with replacement according to multinomial distribution

Proof of concept (1)

• Gaussian distributions $\mu_i = \mathcal{N}(0, \Sigma_i)$ with

$$\Sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0.01 \end{pmatrix}, \qquad \Sigma_2 = \begin{pmatrix} 0.01 & 0 \\ 0 & 1 \end{pmatrix}$$

Datasets \mathcal{D}_i of $N = 10^6$ i.i.d. points

- \bullet Autoencoders with 2 layers of resp. 1 and 2 nodes, linear activation functions (\simeq PCA)
- Training on:
 - D_1
 - \mathcal{D}_2
 - \mathcal{D}_2 with reweighting $\widehat{w}_i \propto \mu_1/\mu_2$

Proof of concept (2)

Heat maps of $f_{\rm enc}$



Third encoder very similar to the first one: projection on x_1 . Second encoder projects on a direction close to x_2 .

Proof of concept with free energy biasing (1)

Two dimensional potential ("entropic switch")¹⁰

$$V(x_1, x_2) = 3e^{-x_1^2} \left(e^{-(x_2 - 1/3)^2} - e^{-(x_2 - 5/3)^2} \right)$$
$$- 5e^{-x_2^2} \left(e^{-(x_1 - 1)^2} + e^{-(x_1 + 1)^2} \right) + 0.2x_1^4 + 0.2(x_2 - 1/3)^4$$



Trajectory from $q^{j+1} = q^j - \nabla V(q^j)\Delta t + \sqrt{2\beta^{-1}\Delta t}G^j$ for $\beta = 4$ and $\Delta t = 10^{-3} \longrightarrow$ metastability in the x_1 direction

¹⁰S. Park, M.K. Sener, D. Lu, and K. Schulten (2003)

Proof of concept with free energy biasing (2)

• Free energy biasing: distributions $Z_i^{-1} \exp\left(-\beta \left[V(q) - F_i(\xi_i(q))\right]\right)$

$$F_1(x_1) = -\frac{1}{\beta} \ln\left(\int_{\mathbb{R}} e^{-\beta V(x_1, x_2)} dx_2\right), \qquad F_2(x_2) = -\beta^{-1} \ln\left(\int_{\mathbb{R}} \dots dx_1\right)$$

Three datasets: unbiased trajectory, trajectories biased using F_1 and F_2 (free energy biased trajectories are shorter but same number of data points $N = 10^6$)

 \bullet Autoencoders: 2-1-2 topology, activation functions \tanh (so that CV is in [-1,1]) then identity

- Five training scenarios:
 - training on long unbiased trajectory (reference CV)
 - ξ_1 -biased trajectory, with or without reweighting
 - ξ_2 -biased trajectory, with or without reweighting

Proof of concept with free energy biasing (3)



Full iterative algorithm (Free Energy Biasing and Iterative Learning with AutoEncoders)

Input: Initial condition q_0 , autoencoder topology and initialization parameters A_{init} , number of samples N, simulation procedure S and adaptive biasing procedure S_{AB} , maximum number of iterations I_{max} , minimum convergence score s_{\min}

Initialization

Sample traj₀ $\leftarrow S(q_0, N)$ Initialize autoencoder AE₀ $\leftarrow A_{\text{init}}$ Train AE₀ on traj₀ with weights $(\widehat{w}_0, \dots, \widehat{w}_N) = (1, \dots 1)$ Extract the encoder function $\xi_0 : x \mapsto \xi_0(x)$

Iterative update of the collective variable

Threshold s_{\min} to be determined

in our case: extended ABF

Convergence metric to be made precise

Production of output:

Sample traj_{final}, $F_{\text{final}} \leftarrow S_{\text{AB}}(q_0, N_{\text{final}}\xi_{\text{final}})$ with N_{final} large enough to ensure PMF convergence

Discussion on the convergence criterion (1/2)

• Check convergence of CV?

Quantify $\xi_i \approx \Phi(\xi_{i-1})$ for some monotonic function Φ

- \bullet Approach: approximate Φ by a linear model \rightarrow linear regression
- Regression score between ξ and ξ'
 - Two sets of values of CV $(\xi(q^1),\ldots,\xi(q^N))$ and $(\xi'(q^1),\ldots,\xi'(q^N))$
 - $\bullet\,$ Match them with a linear model M(z)=Wz+b

• Coefficient of determination $R^2 = 1 - \frac{\sum_{i=1}^{N} \left\| \xi'(q^i) - M(\xi(q^i)) \right\|^2}{\sum_{i=1}^{N} \left\| \xi'(q^i) - \bar{\xi'} \right\|^2}$

- Maximization of R^2 w.r.t. $W\!,b$ provides $\operatorname{regscore}(\xi',\xi)$
- \bullet Value of $\mathit{s_{\min}}$ computed using some bootstrap procedure

Discussion on the convergence criterion (2/2)



Histogram of the R^2 scores obtained using subsets of $N = 10^5$ points out of 10^6 points (vertical black line = 5% percentile). (Left: Alanine dipeptide. Right: Chignolin)

The iterative algorithm on the toy 2D example



Left: with reweighting Convergence to $CV \simeq x_1$

Right: without reweighting No convergence (cycles between two CVs)



29 / 36

Gabriel Stoltz (ENPC/Inria)

Applications to systems of interest

Alanine dipeptide

• Molecular dynamics:

openmm with openmm-plumed to link it with plumed colvar module for eABF and computation of free energies timestep 1 fs, friction $\gamma=1~{\rm ps}^{-1}$ in Langevin dynamics

• Machine learning:

keras for autoencoder training

input = carbon backbone (realignement to reference structure and centering) neural network: topology 24-40-2-40-24, tanh activation functions



Ground truth computation

Long trajectory (1.5 $\mu {\rm s}),~N=10^6$ (frames saved every 1.5 ps) CV close to dihedral angles Φ,Ψ



Quantify $s_{\rm min}=0.99$ for $N=10^5$ using a bootstraping procedure

For the iterative algorithm: 10 ns per iteration

(compromise between times not too short to allow for convergence of the free energy, and not too large in order to alleviate the computation cost)

Results for the iterative algorithm



iter.	regscore	(Φ, Ψ)
0	_	0.922
1	0.872	0.892
2	0.868	0.853
3	0.922	0.973
4	0.999	0.972
5	0.999	0.970
6	0.999	0.971
7	0.999	0.967
8	0.998	0.966
9	0.999	0.968





Chignolin (Folded/misfolded/unfolded states)



HSP90 (work in progress...)



Chaperone protein assisting other proteins to fold properly and stabilizing them against stress, including proteins required for tumor growth

 \longrightarrow look for inhibitors (e.g. targeting binding region of ATP; focus only on the N-terminus domain)

(picture from https://en.wikipedia.org/wiki/File:Hsp90_schematic_2cg9.png)

HSP90 (work in progress...)



6 conformational states, data from 10 \times 20 ns trajectories, input features = 621 C carbons, AE topology 621-100-5-100-621

Gabriel Stoltz (ENPC/Inria)

November 2021 36 / 36