ON THE STABILITY OF MULTIPLE STEADY PLANAR FLAMES WHEN THE LEWIS NUMBER IS LESS THAN 1

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Abstract

We consider the propagation of a planar premixed flame in an infinite tube, with one-step chemistry. We investigate the uniqueness and stability of the travelling-wave solutions for this problem, and show that there may exist two distinct stable solutions when the Lewis number is less than 1.

SUR LA STABILITE DES FLAMMES PLANES STATIONNAIRES DANS UN CAS DE SOLUTIONS MULTIPLES LORSQUE LE NOMBRE DE LEWIS EST INFERIEUR A 1

Résumé

Nous considérons la propagation d'une flamme plane prémélangée dans un tube infini, avec l'hypothèse de chimie simple. Etudiant l'unicité et la stabilité des solutions d'onde progressive, nous montrons que ce problème peut admettre deux solutions distinctes et stables lorsque le nombre de Lewis est inférieur à 1.

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1. INTRODUCTION

The propagation of a steady planar premixed flame in an infinite tube is one of the most fundamentals problem of combustion theory. This problem has attracted the attention of numerous physicists since the pioneer work of Zeldovich and Frank-Kamenetskii in 1938 [15]; in the past ten years, one can also notice an increase of the interest in the mathematical aspects of this problem, starting from the article of Berestycki, Nicolaenko and Scheurer [2].

Under classical hypotheses (and in particular with the assumption of a single one-step chemical reaction $A \to B$), a steady planar flame can be described with the following system of two differential equations (see e.g. [14]):

$$\begin{cases} -T'' + cT' = Yf(T), \\ \\ -\dfrac{Y''}{Le} + cY' = -Yf(T) \quad \text{ on } \mathbb{R} , \end{cases} \tag{1.1}$$

with the boundary conditions:

$$\begin{cases} T(-\infty) = 0 \ , \quad T(+\infty) = 1 \ , \\ Y(-\infty) = 1 \ , \quad Y(+\infty) = 0 \ ; \end{cases} \tag{1.2}$$

here, the unknowns are the normalised temperature and mass fraction profiles $T(x)$ and $Y(x)$ and the scalar c, which corresponds to the normalised mass flux across the flame, and can also be seen as the speed of the flame with respect to the fresh mixture. The boundary conditions at $-\infty$ (respectively: $+\infty$) correspond to the fresh mixture (resp. : to the burnt gases). The positive parameter Le is the Lewis number. We classically assume that the reaction term $Yf(T)$ has an ignition temperature $\theta \in (0,1)$, i.e.:

$$f(T) \equiv 0 \quad \text{on} \quad [0,\theta] \ , \quad f(T) > 0 \quad \text{on} \quad (\theta, 1] \ . \tag{1.3}$$

Our study deals with the questions of uniqueness and stability of the solutions of (1.1)-(1.2). From the works of Marion [9], we know that problem (1.1)-(1.2) has a unique solution (c, T, Y) when the Lewis number is greater than or equal to 1. The question of uniqueness for $Le < 1$ has remained open till the recent investigations of the first author [3], [4], who built an example for the reaction rate f such that problem (1.1)-(1.2) has three different solutions. A natural question is then to know if these three solutions, which correspond to three steady flames travelling at three different speeds, are physically relevant: are they stable or unstable steady solutions ? More precisely, we wish to know whether only one of these solutions is stable, or on the opposite if several of them are stable steady solutions.

This is the object of the present work, which supports the interest of the example of multiple solutions shown in [3], [4]: we show indeed below that, among the three solutions exhibited in [3], [4], two solutions are stable with respect to planar perturbations and one is unstable (the one with an intermediate speed of propagation). Therefore, we show that, in some situations, *with a Lewis number less than 1, system (1.1)-(1.2) can have several stable solutions.*

In order to investigate the stability of the solutions of (1.1)-(1.2), we will consider below the unsteady problem, which writes (see e.g. [7]):

$$(1.4) \qquad \begin{cases} T_t = T_{xx} + Yf(T), \\ \\ Y_t = \dfrac{Y_{xx}}{Le} - Yf(T), \end{cases}$$

with the initial data $T(x,0) = T_0(x)$, $Y(x,0) = Y_0(x)$.

The instabilities of the travelling-wave solutions of this system, i.e. of the solutions of (1.1)-(1.2), are well-known in the framework of the asymptotic analysis for high activation energies (see for instance Clavin [5], Sivashinsky [13]). In particular, two different types of instabilities may appear, depending on the sign of the difference $Le - 1$: for a Lewis number greater than 1, one may observe a planar pulsating instability; on the opposite, when the Lewis number is less than 1, the travelling-wave solution is stable with respect to planar perturbations, but cellular instabilities appear in the presence of non planar perturbations. These results do not directly apply to the situation we consider here, where we are going to analyse the stability of multiple solutions of (1.1)-(1.2) with respect to planar perturbations without any hypothesis of high activation energies.

In Section 2 below, we recall from [3], [4] the construction of multiple travelling-wave solutions. Then, we investigate the stability of these solutions, by numerically solving the unsteady system (1.4) in Section 3 and by carrying out a formal linear stability analysis in Section 4.

Remark 1.1: Since the Lewis number is less than 1, it might be the case that all three travelling-wave solutions are indistinctly unstable with respect to non planar perturbations, exhibiting a cellular instability. Nevertheless, investigating the stability of these solutions with respect to planar perturbations still has its own interest, in order to compare the present situation with the well-known case of high activation energies, and also because the planar stability is the one which determines the observability of a planar flame in a thin tube with a diameter smaller than the wavelength of the possible cellular instabilities. •

2. CONSTRUCTION OF THE MULTIPLE SOLUTIONS

We now recall how the multiple solutions of (1.1)-(1.2) are obtained; the reader is referred to [3], [4] for more details.

One chooses a piecewise constant function f given by:

$$\begin{cases} f(T) = 0 & \text{for } \; T < 0.0006 \; , \\ f(T) = 0.001 & \text{for } \; 0.0006 < T < 0.3 \; , \\ f(T) = 10 & \text{for } \; 0.3 < T < 1 \; . \end{cases} \tag{2.1}$$

In the sequel, we denote $s_2 = \theta = 0.0006$, $m_2 = 0.001$, $s_1 = 0.3$ and $m_1 = 10$; thus, the function f takes the value m_2 on the interval (s_2, s_1) and m_1 on $(s_1, 1)$. With this choice of f and with:

$$Le = 0.02 \; , \tag{2.2}$$

one can show that that there exist three solutions of (1.1)-(1.2), with the propagation speeds $c_1 \simeq 1.02$, $c_2 \simeq 1.12$ and $c_3 \simeq 1.22$. Their temperature and mass fraction profiles are shown on figures 1, 2 and 3.

Remark 2.1: Examples of multiple solutions can be constructed for higher values of the Lewis number (for instance, $Le \simeq 0.33$). Nevertheless, we will keep this low value of the Lewis number, $Le = 0.02$, because, in this case, the values of the propagation speeds c_i of the three travelling-wave solutions are 10% apart, which allows us to better distinguish these solutions from each other in the numerical simulations. Let us also notice that it is also possible to construct an example where system (1.1)-(1.2) has three different solutions with a smooth non linear function f. •

Let us be more specific about the construction of the multiple solutions. We first notice that it is easy to get an explicit analytic form for the solutions (T, Y) of (1.1) since f is piecewise constant: in each interval where $f(T)$ is constant and takes the value m, T and Y are obtained as a linear combinations of exponentials:

$$\begin{cases} T(x) = \dfrac{ma\, e^{\alpha x}}{-\alpha^2 + c\alpha} + \dfrac{mb\, e^{\beta x}}{-\beta^2 + c\beta} + g\, e^{cx} + j \; , \\ \\ Y(x) = a\, e^{\alpha x} + b\, e^{\beta x} \; , \end{cases} \tag{2.3}$$

where a, b, g and j are some real constants which are determined from the boundary conditions at $-\infty$ and $+\infty$ and from the continuity of T, Y and of their first derivatives. The constants α and β are the roots of $-\dfrac{1}{Le}z^2 + cz + m = 0$ (with $\alpha < \beta$).

For any $c > 0$, we can try to construct a solution of (1.1)-(1.2) in the following way: we begin by writing T and Y on an interval $(x_1, +\infty)$ where T remains greater than s_1; at the boundaries, we naturally impose the relations $T(x_1) = s_1$, $T(+\infty) = 1$ and $Y(+\infty) = 0$, which determine the values of the constants a, b, g, j in (2.3) (with $m = m_1$). Notice however that x_1 is undetermined, because of the translational invariance of the problem; with no loss of generality, we may set $x_1 = 0$. Then, we construct the solution on an interval (x_2, x_1), using again (2.3) with now $m = m_2$; here we impose that $T(x_2) = m_2$ and that T, Y and their first derivatives are continuous at x_1. These conditions allow us to determine the constants a, b, g, j and x_2. We have therefore defined T and Y on the interval $(x_2, +\infty)$. Then, we set:

$$h(c) = T'(x_2) \ . \tag{2.4}$$

Lastly, on $(-\infty, x_2)$, T and Y are necessarily exponentials:

$$T(x) = s_2 e^{c(x-x_2)} \ , \quad Y(x) = 1 - [1 - Y(x_2)]\, e^{\frac{c(x-x_2)}{Le}} \ . \tag{2.5}$$

Then, the continuity of T' and Y' at x_2 gives a necessary and sufficient condition on c:

$$h(c) = cs_2 \ . \tag{2.6}$$

The function $c \mapsto \dfrac{h(c)}{c}$ is shown on figure 4, where we see that the condition (2.6) is fulfilled for three values of c. For each of these three values, the above construction of T and Y gives a solution of (1.1)-(1.2), that is a travelling-wave solution of (1.4), or in other words a flame propagating with the speed c.

In fact, the S-shaped curve of figure 4 illustrates a classical situation of bifurcation. To see this clearly, we can modify the fresh mixture temperature and mass fraction and keep the same conditions for the burnt gases at $+\infty$. Taking $T(-\infty) = r$, $Y(-\infty) = 1 - r$ with $r < s_2$, we obtain, together with the equations (1.1) and the boundary conditions (1.2) at $+\infty$, a different problem which we call (P_r). For this problem, travelling-wave solutions are obtained if c satisfies:

$$\frac{h(c)}{c} = s_2 - r \ . \tag{2.7}$$

As a consequence, three different situations may occur:

(i) For $s_2 - r$ bigger than some value ρ_1 ($\rho_1 \simeq 0.00065$ on figure 4), then (2.7) has a unique solution c_1. There is a unique steady flame solution of (P_r), with the propagation speed c_1.

(ii) If $\rho_2 < s_2 - r < \rho_1$ (with $\rho_2 \simeq 0.0002$ on figure 4), then (2.7) has three solutions, and there exist three steady flames, with the speeds $c_1 < c_2 < c_3$.

(iii) Lastly, for $0 < s_2 - r < \rho_2$, (2.7) again has a unique solution c_3, and there is a unique solution to problem (P_r).

In this situation, one expects that the flames of speeds c_1 and c_3 are stable and that the flame of speed c_2 is unstable. This is in fact the result we will prove in the next sections.

Remark 2.2: We observe on Figure 1 that the temperature and mass fraction profiles associated with the solution of speed c_1 have the usual aspect (but with an order of magnitude of difference in the maximal temperature and mass fraction gradients, as it can be expected since the Lewis number is very far from unity). In contrast with this, the temperature profiles of the two other solutions, on figures 2 and 3, have a quite surprising aspect, which is due to the disparacy between the exponents α, β and c in the analytical expressions (2.3) (more precisely, in the interval (x_2, x_1), we have $\alpha \simeq -\dfrac{m_2}{c}$ and $\beta \simeq c\ Le$ since m_2 is very small). •

3. NUMERICAL INVESTIGATION OF THE STABILITY

In this section, we perform a numerical investigation of the stability of the three travelling-wave solutions described in the preceding section, by numerically solving the unsteady system (1.4).

The numerical method used in our experiments is very classical and simple, and we will omit the details. Let us just mention that, instead of simply solving (1.4), we add in the right-hand side of (1.4) a convective term which amounts to observing the solution in a reference frame moving with the flame; this allows us to observe a discrete steady solution instead of a travelling-wave solution propagating with constant speed; we refer to [8] for the details. Let us also add that the convergence of this numerical was proved in [1] (in a two-dimensional framework).

To investigate the stability of the three steady solutions, we have solved system (1.4) while taking as initial data each steady solution with some small perturbation. These experiments have shown that:

(i) the steady solutions of speeds c_1 and c_3 are stable;

(ii) the steady solution of speed c_2 is unstable.

More precisely, we observed that the numerical solution obtained when the (perturbed) solution of speed c_2 was used as initial data converged to the fastest steady solution (of speed c_3).

4. LINEAR STABILITY ANALYSIS

We now study the linear stability of the steady solutions of problem (1.1)-(1.2) with the particular choice (2.1) for f.

For a particular steady solution (c_0, T_0, Y_0) of (1.1)-(1.2), we rewrite the evolution problem (1.4) in the reference frame of the travelling flame:

$$\begin{cases} T_t = T_{xx} - c_0 T_x + Y f(T) \ , \\ \\ Y_t = \dfrac{Y_{xx}}{Le} - c_0 Y_x - Y f(T) \ . \end{cases} \tag{4.1}$$

Linearizing (4.1) amounts to perturbate the steady solution (T_0, Y_0) with exponentially time-dependent terms. We consider the perturbed solution:

$$T_p(x,t) = T_0(x) + \epsilon e^{\sigma t} T(x) \ , \quad Y_p(x,t) = Y_0(x) + \epsilon e^{\sigma t} Y(x) \ ; \tag{4.2}$$

as usual, we will assume below that the perturbations T and Y are bounded. In the limit $\epsilon \longrightarrow 0$, T and Y must be solutions of the linearized system:

$$\begin{cases} \sigma T = T'' - c_0 T' + Y f(T_0) + Y_0 f'(T_0) T \ , \\ \\ \sigma Y = \dfrac{Y''}{Le} - c_0 Y' - Y f(T_0) - Y_0 f'(T_0) T \ , \end{cases} \tag{4.3}$$

on each interval $(-\infty, x_2)$, (x_2, x_1) and $(x_1, +\infty)$. At x_1 and x_2, T and Y are continuous and their derivatives must satisfy the jump conditions:

$$\begin{cases} [T'](x_i) = -\dfrac{Y_0(x_i)}{T_0'(x_i)} T(x_i)[f](s_i) \ , \\ \\ [Y'](x_i) = Le \ \dfrac{Y_0(x_i)}{T_0'(x_i)} T(x_i)[f](s_i) \ , \end{cases} \tag{4.4}$$

where we denote $[g](y)$ the jump of a function g at point y. A justification of the jump conditions (4.4) is given in the Appendix below.

Classically we will say that the solution (T_0, Y_0) of (4.1) is linearly stable if there is no nontrivial solution of system (4.2)-(4.3) for any $\sigma \neq 0$ of non-negative real part.

To solve system (4.2)-(4.3), we use the fact that $f(T_0)$ is constant in each of the three intervals $(-\infty, x_2)$, (x_2, x_1) and $(x_1, +\infty)$: T and Y are thus solution of a system of constant-coefficient linear differential equations, which we can easily solve explicitly. In order to simplify the algebra, we define the constants (for $i = 1,\ 2$ or 3, with $m_3 = 0$):

$$\alpha_i = \frac{c_0 - \sqrt{c_0^2 + \frac{4}{Le}(m_i + \sigma)}}{\frac{2}{Le}} \ , \quad \beta_i = \frac{c_0 + \sqrt{c_0^2 + \frac{4}{Le}(m_i + \sigma)}}{\frac{2}{Le}} \ , \tag{4.5}$$

$$A_i = \frac{m_i}{\sigma - \alpha^2 + c_0 \alpha} \ , \quad B_i = \frac{m_i}{\sigma - \beta^2 + c_0 \beta} \ , \tag{4.6}$$

$$\gamma = \frac{c_0 - \sqrt{c_0^2 + 4\sigma}}{2} \ , \quad \delta = \frac{c_0 + \sqrt{c_0^2 + 4\sigma}}{2} \ . \tag{4.7}$$

We have then the following explicit solutions in each interval:

(i) on $(x_1, +\infty)$, T and Y are given by:

$$T(x) = A_1 e^{\alpha_1 x} + g_1 e^{\gamma x} \ , \quad Y(x) = e^{\alpha_1 x} \ , \tag{4.8}$$

where g_1 is an unknown constant;

(ii) on (x_2, x_1) we have:

$$T(x) = A_2 a_2 e^{\alpha_2 x} + B_2 b_2 e^{\beta_2 x} + g_2 e^{\gamma x} + d_2 e^{\delta x} \ , \quad Y(x) = a_2 e^{\alpha_2 x} + b_2 e^{\beta_2 x} \ , \tag{4.9}$$

where a_2, b_2, g_2 and d_2 are unknown constants;

(iii) on $(-\infty, x_2)$ we get:

$$T(x) = d_3 e^{\delta(x - x_2)} \ , \quad Y(x) = b_3 e^{\beta_3 (x - x_2)} \ , \tag{4.10}$$

where b_3 and d_3 are unknown constants.

Now, we have seen that T and Y are continuous at x_1 and x_2, and that their first derivatives T' and Y' should satisfy the jump conditions (4.4) at x_1 and x_2. These conditions give a system of eight equations for the seven unknowns g_1, a_2, b_2, d_2, g_2, b_3 and d_3:

$$
(4.11) \quad \begin{cases}
1 = a_2 + b_2 \ , \\[2ex]
\alpha_1 - (\alpha_2 a_2 + \beta_2 b_2) = Le \ \dfrac{Y_0(x_1)}{T_0'(x_1)}(m_1 - m_2)(A_1 + g_1) \ , \\[2ex]
A_1 + g_1 = A_2 a_2 + B_2 b_2 + g_2 + d_2 \ , \\[2ex]
A_1\alpha_1 + g_1\gamma - (A_2 a_2 \alpha_2 + B_2 b_2 \beta_2 + g_2\gamma + d_2\delta) = -\dfrac{Y_0(x_1)}{T_0'(x_1)}(m_1 - m_2)(A_1 + g_1) \ , \\[2ex]
a_2 e^{\alpha_2 x_2} + b_2 e^{\beta_2 x_2} = b_3 \ , \\[2ex]
a_2 \alpha_2 e^{\alpha_2 x_2} + b_2 \beta_2 e^{\beta_2 x_2} - b_3\beta_3 = Le \ \dfrac{Y_0(x_2)}{T_0'(x_2)} m_2 d_3 \ , \\[2ex]
A_2 a_2 e^{\alpha_2 x_2} + B_2 b_2 e^{\beta_2 x_2} + g_2 e^{\gamma x_2} + d_2 e^{\delta x_2} = d_3 \ , \\[2ex]
A_2 \alpha_2 a_2 e^{\alpha_2 x_2} + B_2 \beta_2 b_2 e^{\beta_2 x_2} + g_2 \gamma e^{\gamma x_2} + d_2 \delta e^{\delta x_2} - d_3\delta = -\dfrac{Y_0(x_2)}{T_0'(x_2)} m_2 d_3 \ .
\end{cases}
$$

Remark 4.1: Classically we should have ended with an homogeneous system of eight equations and eight unknowns. In fact, we already took advantage of this homogeneity property when we arbitrarily took the coefficient of $e^{\alpha_1 x}$ to be 1 in the last relation (4.8). •

The system (4.11) has a solution $(g_1, a_2, b_2, d_2, g_2, b_3, d_3)$ if and only if its determinant $D(\sigma)$ vanishes. Consequently, we have a necessary and sufficient condition for stability: the steady flame (T_0, Y_0) is linearly stable if and only if $D(\sigma)$ does not vanish for any $\sigma \neq 0$ with nonnegative real part.

In practice, finding the zeros of $D(\sigma)$ is not an easy task. We limit ourselves to a numerical study of $D(\sigma)$ on $\mathbb{R}_+$ and $i\mathbb{R}$. These numerical calculations give:

(i) for $c = c_2$ there is a zero of $D(\sigma)$ in $\mathbb{R}_+^*$;

(ii) for $c = c_1$ and $c = c_3$ $D(\sigma) \neq 0$ on $\mathbb{R}_+^*$ and $i\mathbb{R}$.

These numerical observations do not allow us to rigorously conclude about the stability of the flames of speed c_1 and c_3. However, together with the numerical simulations of Section

3, we may say that there is strong evidence towards the stability of the flames of speed c_1 and c_3.

For the flame of speed c_2, we can be more precise and state a rigorous result. Following the works of Evans [6] and Sattinger [10], who show that the stability for the evolution problem (1.4) boils down to that of the linearized problem, we can prove the:

Theorem 4.1:

The steady flame of speed c_2 is linearly and nonlinearly unstable. •

Remark 4.2: The determinant $D(\sigma)$ always vanishes for $\sigma = 0$ because of the translational invariance of the problem. Indeed, (T_0', Y_0') is a solution of (4.3)-(4.4) when $\sigma = 0$. •

APPENDIX

We give here the details about the derivation of the jump conditions (4.4) for the perturbations T and Y. These conditions can be obtained using two different methods.

The first method relies on regularizing f. If f is discontinuous at some point s_0 we introduce a smooth fonction f_η defined by:

$$(A.1) \qquad \begin{cases} f_\eta(s) = f(s) \text{ for } s \notin (s_0 - \eta, s_0 + \eta) \ , \\ f_\eta \text{ smooth and monotone in } (s_0 - \eta, s_0 + \eta) \ . \end{cases}$$

Substituting f_η for f in (4.1), we write the linearized equations as:

$$(A.2) \qquad \begin{cases} \sigma T = T'' - c_0 T' + Y f_\eta(T_0) + Y_0 f_\eta'(T_0) T \ , \\ \\ \sigma Y = \dfrac{Y''}{Le} - c_0 Y' - Y f_\eta(T_0) - Y_0 f_\eta'(T_0) T \ . \end{cases}$$

When $\eta \to 0$, $f_\eta'(T) \to [f](s_0)\delta_{T=s_0}$ (δ is the Dirac delta function). The integration of (A.2) in a neighborhood of the discontinuity followed by a straightforward change of variables readily leads to conditions (4.4).

The jump conditions (4.4) can also be derived without regularizing f. This second method relies on writing the evolution equations in a reference frame attached to the interface of the perturbed solution. This method is classical and has been used in several works related to flame instabilities (see e.g. [11], [12]); its application to the present case deserves however some particular comments.

We assume as above that f is discontinuous at a point s_0, and (for the sake of simplicity only) that the steady solution satisfies $T_0(0) = s_0$. We also assume that $T_0'(0) \neq 0$ (which is necessary for the jump conditions (4.4) to hold !). Then, considering a small perturbation of the steady solution and calling (T_p, Y_p) the perturbed solution, we define $\hat{x}(t)$ by:

$$T_p(\hat{x}(t)) = s_0 \ . \tag{A.3}$$

The method now consists in writing the evolution equation in the reference frame of the "interface" $T = s_0$, i.e. using the variable $\xi = x - \hat{x}(t)$. The equations (4.1) become:

$$\begin{cases} T_t = T_{\xi\xi} - c_0 T_\xi + \hat{x}'(t) T_\xi + Y f(T) \ , \\ \\ Y_t = \dfrac{Y_{\xi\xi}}{Le} - c_0 Y_\xi + \hat{x}'(t) Y_\xi - Y f(T) \ . \end{cases} \tag{A.4}$$

Writing the pertubed solution as:

$$T_p(\xi, t) = T_0(\xi) + \epsilon\ e^{\sigma t} \hat{T}(\xi),\ Y_p(\xi, t) = Y_0(\xi) + \epsilon\ e^{\sigma t} \hat{Y}(\xi) \ , \tag{A.5}$$

and also searching for $\hat{x}(t)$ under the form:

$$\hat{x}(t) = \gamma \epsilon\ e^{\sigma t} \ , \tag{A.6}$$

we obtain the equations for the perturbations as:

$$\begin{cases} \sigma \hat{T} = \hat{T}_{\xi\xi} - c_0 \hat{T}_\xi + \gamma \sigma T_0' + \hat{Y} f(T_0) + Y_0 \hat{T} f'(T_0) \ , \\ \\ \sigma \hat{Y} = \dfrac{\hat{Y}_{\xi\xi}}{Le} - c_0 \hat{Y}_\xi + \gamma \sigma Y_0' - \hat{Y} f(T_0) - Y_0 \hat{T} f'(T_0) \ . \end{cases} \tag{A.7}$$

Since the definition of $\hat{x}(t)$ and ξ implies that $\hat{T}(0) = 0$, it is clear that $\hat{T}$ and $\hat{Y}$ and their first derivatives are continuous (notice that the situation is different here from the above mentioned works [11], [12], where the same method is used but with a function f which is not just discontinuous, but converges in the high activation energy limit to a Dirac delta function). This nice property shows the advantage of the present method over the one used in Section 4 above, in the case where their exists a single discontinuity. In our problem however, with the choice (2.1) for f, we have two interfaces, and we cannot find a reference frame attached to both of them. This is why we need to stick to the reference frame of the travelling-wave solution (T_0, Y_0), as we did in Section 4. But the above approach (A.5) will nevertheless provide the jump relations (4.4).

Indeed, considering the two forms (4.2) and (A.5) of the perturbations, we see that there is a relation between T and $\hat{T}$, namely:

$$(A.8) \qquad T(x) = \hat{T}(x - \epsilon\gamma e^{\sigma t}) + \frac{T_0(x - \epsilon\gamma e^{\sigma t}) - T_0(x)}{\epsilon e^{\sigma t}} \ .$$

In the limit $\epsilon \to 0$, we get:

$$(A.9) \qquad T(x) = \hat{T}(x) + \gamma T_0'(x) \ ,$$

which implies that T' is discontinuous across the interface with the jump:

$$(A.10) \qquad [T'] = \gamma[T_0''] \ .$$

Moreover, (A.9) shows that $T(0) = \gamma T_0'(0)$ since $\hat{T}(0) = 0$. It is then straightforward to deduce the jump conditions (4.4) from (A.10).

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Figure 1: Temperature and mass fraction profiles for the steady solution of speed c_1.

Figure 2: Temperature and mass fraction profiles for the steady solution of speed c_2.

Figure 3: Temperature and mass fraction profiles for the steady solution of speed c_3.

Figure 4: Plotting $\dfrac{h(c)}{c}$ as a function of c (see (2.4)).