

# TRAVEL TIMES COMPUTATION FOR DYNAMIC ASSIGNMENT MODELLING<sup>1</sup>

*C. Buisson*<sup>b</sup>, *J.P. Lebacque*<sup>a</sup>, *J.B. Lesort*<sup>b</sup>

## **Affiliations:**

- a.** CERMICS, Centre d'Enseignement et de Recherche en Mathématique, Informatique et Calcul Scientifique. ENPC (Ecole Nationale des Ponts-et-Chaussées), La Courtine, 93167 NOISY-LE-GRAND Cedex, FRANCE.  
Tel: +33 1 49 14 35 81. email: lebacque@cermics.enpc.fr.
- b.** LICIT, Laboratoire d'Ingénierie Circulation Transport, ENTPE (Ecole Nationale des Travaux Publics de l'Etat)-INRETS (Institut National de la Recherche sur les Transports et leur Sécurité), 109 rue Salvador Allende, Case 24, 69675 BRON cedex, FRANCE.  
Tel: +33 4 72 36 23 00. emails: cbuisson@inrets.fr & lesort@inrets.fr.

## INTRODUCTION.

Travel time is the basic criterion used in most assignment modelling processes. However, few examples of systematic in-depth investigation exist in the literature concerning the definition and properties of travel time models that are in use. The aim of the present paper is twofold: to make a cursory review of various existing travel time formulas and concepts, and to introduce some new formulas for reactive dynamic assignment.

The contents of the paper are the following: first, a brief description of travel time functions and models used in dynamic assignment modelling (considering both the more theoretical formulations and the more application-oriented computational algorithms used in traffic flow simulation models); second, the analysis in the context of macroscopic traffic modelling of three travel time concepts: experienced travel time, predictive travel time and instantaneous travel time. Emphasis will bear on experienced and instantaneous travel times, which are most useful in the context of reactive dynamic assignment, and for which various computational formulas, many of them recursive, will be developed, suitable for both interrupted and uninterrupted traffic and most macroscopic models.

---

<sup>1</sup>Presented at the 1996 EURO Working Group on Transportation in Newcastle. Submitted for publication in the Proceedings of that conference.

## Definitions.

The trajectory of the individual vehicle constitutes the basis of the computation of travel times. For a link or a trip, the *experienced travel time (ETT)* of the vehicle can be deduced, estimated at the exit of the link or at the end of the trip. Conversely, at the beginning of the trip, or on the entry of the vehicle on the link, one might try to estimate a *predictive travel time (PTT)*, deduced from prevailing traffic conditions.

When stationary traffic conditions are assumed, both definitions concur and can be extended in an obvious way to links, since all vehicles crossing a link have the same travel time. Link travel times can then be defined and route travel times computed as the sum of travel times on the links forming the route. When dynamic traffic conditions are considered, the problem becomes more complex and the definition of link or route travel times is no longer unique or straightforward. If the above definitions remain valid for an individual vehicle, the definition and manipulation of link and route travel times depend very much on the type of assignment considered.

Definitions of various types of dynamic assignment have been widely discussed in the literature, (see for instance (Papageorgiou 1990) for a very clear presentation). The typology of travel times is closely related to that of assignment problems. These can roughly be classified according to the nature of the optimum, whether user or system optimum, and whether predictive or reactive. Furthermore, every assignment problem is related to a specific traffic flow model and a specific time-scale, which have great impact on the corresponding travel-time model.

For a system optimal assignment, the criterion used is the total time spent in the network (i.e. the sum of all individual travel times). This can be considered as an extension of the notion of mean travel time, and is strictly equivalent if only one origin/destination is considered. For a user optimal dynamic assignment, it is necessary to distinguish two cases. If a reactive optimum is considered (or the similar *Boston equilibrium* as defined by (Friesz et al. 1989)), a notion of *instantaneous travel time (ITT)* has to be defined, which has no physical meaning but characterizes traffic flow conditions at a given instant. This ITT might be constituted of link PTTs, link ETTs, or be a completely synthetic index of the link or network flow state. If a predictive optimum is considered, predictive travel times must be used, only known at the end of trip time, or predicted, or estimated through an iterative assignment procedure.

## The simple cases.

In traffic conditions that are stationary, or simplified as such, the most usual way to compute travel times is to use some function deriving link travel times from link flows. One can refer to (Branston, 1976) which presents an extensive review of travel time function in use at that time. Some more recent papers on that subject are indicated in the work of Weymann et al. (1994), in which travel time functions having finite values for over-saturated traffic conditions are briefly presented. Koutsopoulos and Habbal (1994) present a more detailed review, insisting on travel time functions used in practice for traffic equilibrium models.

On the contrary, when a system optimal assignment is computed, individual link travel times must be replaced by a global cost function: the total time spent on the network by all users. This quantity can be easily calculated by integrating the number of vehicles present in the whole network over the optimization horizon, as was initially proposed by Merchant and Nemhauser (1978a and b). This work has been re-used in quite a different context: in the case of route guidance, the discretized macroscopic METACOR model enables the operator to maintain a system optimum (Eloumi 1996).

A large part of the more theoretical literature concerned with predictive optima is represented by models in which the basic component of the traffic flow model itself is constituted of travel time models. These are supposed to be functions of the state of the network, and vehicles are propagated along links according to those travel times which must therefore be considered as PTTs (Friesz et al. 1993, Fernandez and De Cea 1994, Ran and Boyce 1994, Astarita 1996, Ran et al., 1996). The only real difficulty with such models is that of internal coherence. We shall say again a few words later about these models, but for the time being, let us note that they are characterized by: a large time-scale, uninterrupted traffic, the lack of explicit supply constraints (reputedly such constraints are implicit in the travel time function itself), travel times which are a function of the network state.

### **Interrupted traffic flow: temporal aggregation.**

When a traffic flow model is used in relation to reactive assignment (simulation of information or guidance on the basis of instantaneous travel times) the time scale of this model is typically shorter than a minute. Therefore, some kind of temporal aggregation or averaging must be applied, in order to smoothen the short-term travel time fluctuations resulting from traffic light cycles etc. . . .

Further, if the outflow of the link is nil, most natural travel time estimates yield infinite or unrealistic values. Predictive travel time, estimated on the basis of prevailing traffic conditions, will be underestimated. Experienced travel times will not take any effective value during the period in which the exiting flow is nil (but may be given conventional values, as will be seen later). Instantaneous travel times, when calculated on the basis of the link length divided by the mean speed on that link, will tend to be infinite.

These difficulties led most authors to compute directly average travel times, with the choice of the duration of the averaging period depending closely on the specific nature of the problem addressed to by the traffic model. We shall make in the sequel a brief review of the averaging methods mentioned in the literature, in various cases of optimum calculation. But first let us stress a point we deem important: that the computation of travel times and their averaging relate to two distinct processes and should be strictly separated. Indeed they belong to two distinct levels of the traffic model: travel time computation is related to *the network state estimation*, i.e. the basic level of the model, whereas travel time averaging concerns user information and/or traffic management and control, and is therefore related to *the control level* of the model, in some broad sense. Hence there are two very distinct problems that must be addressed: the choice of travel time estimates with reasonable properties, both physical and computational, and the temporal aggregation of these travel times. It is the former problem that will be examined in some more detail in the second part of the paper.

Let us turn towards the literature on temporal aggregation and examine a few cases. Mesoscopic traffic models divide travel time into two parts: the journey time and the queue waiting time. In the reactive assignment model DYNASMART (Jayakrishnan et al., 1994), the total travel time is calculated at each time step and, to the knowledge of the authors of the present paper, never averaged. In case of fixed time signals, the queue length is calculated on the basis of the effective status of the traffic signal. In the case of traffic actuated signals, at the end of each cycle, an averaged exit capacity of the link is used to compute the exiting flow.

CONTRAM is one of the earlier models developed to simulate the user optimum in the case of day-to-day variation (Leonard et al. 1978, Leonard et al. 1989). The peak hour is divided into a dozen periods. For each period and for each link controlled by a traffic signal an average queuing waiting time is calculated. The calculation is made according to time dependent queuing theory, thus reflecting the stochastic nature of vehicle arrivals. A queuing time is calculated for the whole period, based on the total demand during that

period and on the ratio of green over the cycle duration. In the more recent mesoscopic model of Weynmann et al. (1994), the delay is calculated for the fluid, desaturating and saturated cases of the traffic signal. The averaging period is taken to be the minimum common multiple of all cycle durations of the network. During this period, the inflows are supposed to be constant.

To cope with the traffic light signal problem, the recent macroscopic model STRADA (Buisson et al., 1995) uses also an averaging period that is taken to be the minimum common multiple of all cycle durations. During this period, demand can vary.

INTEGRATION uses a experimented travel time (ETT) to simulate the on-route guidance of drivers. This ETT is aggregated spatially in order to yield link travel times or route travel times. The link travel time is computed typically every five minutes. No mention is made in the most recent paper (Van Aerde, 1995) of the impact of traffic signal cycles on the average values of travel times or of really severe incidents (remaining flow nil) affecting more than one such averaging period.

Travel time estimation in the case of incidents is a relatively new and expanding subject, motivated notably by route guidance problems. If the incident totally blocks the flow on some network link, the travel time on that link tends to be infinite. But this is not a useful piece of information because the assignment model can compute the shortest path without considering that link (which obviously does not belong to the shortest path). On the other hand, for simulation models whose travel time estimate relate to the ETT category, such as INTEGRATION, using the travel time of the last vehicle to exit the link leads to a systematic underestimation of the mean travel time values when the exiting flow is nil. If there is a residual flow, the mean travel time resulting of ITT-like estimates might take on absurd values like one day or one week. Various heuristic methods have been proposed, as for instance in the work of Cremer et al. (1993). These authors propose to estimate the time necessary for drivers to exit the incident-impeded link by summing the total number of vehicles between the entrance and the incident bottleneck and dividing this sum by the remaining bottleneck outflow.

### **Interrupted traffic flow: spatial aggregation.**

All models must also address the problem of spatial aggregation, i.e. the computation of travel times along paths. Spatial aggregation is trivial for both static models and dynamic system optima, as mentioned above. For dynamic models using PTTs as their basic propagation model, the situation is straightforward enough. CONTRAM for instance (Leonard et al. (1978)), or the model described by Drissi-Kaïtouni et al. (1992), intend to reproduce the assignment of commuters on the basis of the travel time experienced the day before. Therefore, these models use a spatial aggregation based on the following rule. To compute the total travel time in a network from one origin to one destination, the mean travel time of the first link at the date of entrance in the network is used. The travel time of the second link is the one computed for the moment following the exit of the first link, and so on from one link to the next. This path travel time is effectively a predictive travel time and reproduces the conditions encountered by a vehicle.

Other models cannot easily compute such a predictive travel time and must rely on some link ITTs to be combined in order to yield path ITTs. No car actually experiences such a travel time, but it can be a good estimation of the conditions encountered on the network at that moment. This is the solution retained by the macroscopic assignment models (METACOR and STRADA) which aim to reproduce the effect of a guidance and/or information system. It is also, for the time being, used by DYNASMART. In the next sections we shall try to define such travel times in a rigorous manner.

**Introduction.**

Our aim in the following sections is to give rigorous definitions and computational procedures for different travel times within the *macroscopic approximation of interrupted traffic flow*. The starting point of our analysis of possible travel time expressions is the single link, say  $[a, b]$ , with its associated traffic flow modelled with the help of the usual macroscopic variables  $Q$  (flow),  $K$  (density) and  $V$  (speed).



Nearly everything we shall say in the sequel does *not* depend on the specific nature of the underlying traffic flow model, whether first- or second-order etc . . . . A crucial role will be played by the speed field  $\mathcal{V}(x, t) \stackrel{def}{=} (V(x, t), 1)$ , assumed to be integrable, with existence and unicity of the corresponding field-lines. Let us denote by  $\mathcal{X}$  the vehicle trajectory associated to such a field-line:  $\mathcal{X}(x_0, t_0; t)$  is the position at time  $t$  of the user whose position at time  $t_0$  is  $x_0$ .  $\mathcal{X}(x_0, t_0; t)$  is the solution at time  $t$  of:

$$\begin{aligned} \dot{x}(t) &= V(x(t), t) \\ x(t_0) &= x_0 \quad . \end{aligned}$$

Since the field-lines associated to trajectories  $\mathcal{X}$  do not intersect in the  $(x, t)$ -plane, this kind of description is intrinsically in agreement with the FIFO hypothesis: in accordance with this representation, vehicles exit the link in the precise order they entered it. Considering a vehicle entering the link at time  $t$ , exiting it at time  $E(t)$ , it follows  $\mathcal{X}(a, t; E(t)) = b$ . The function  $E$  is increasing and admits an inverse  $I$ :  $I(t)$  is the time at which a vehicle about to leave the link at time  $t$  has entered it. If  $V(b, \cdot) = 0$  during some time interval, the definition of  $E$  and the relationships between  $I$  and  $E$  are not completely straightforward since no vehicle may leave the link during such an interval. Indeed it is necessary to define  $E(t)$  as:

$$E(t) \stackrel{def}{=} \inf_s \{s / \mathcal{X}(a, t; s) > b\} \quad .$$

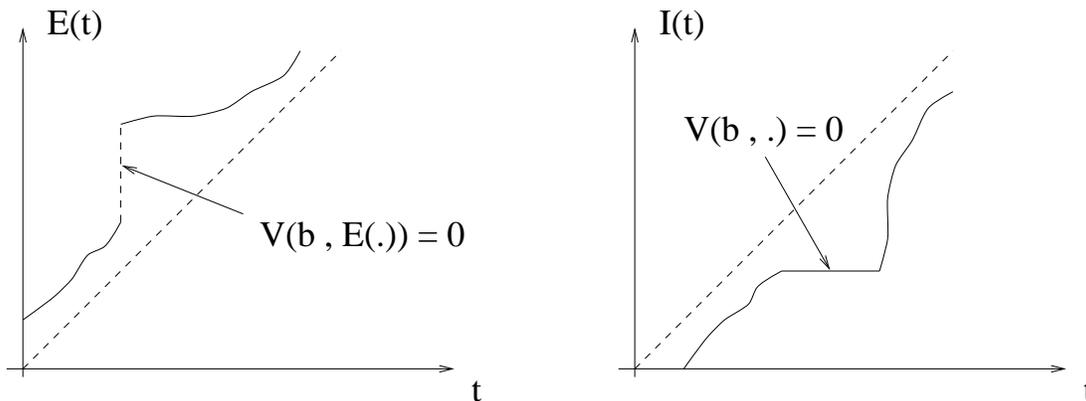
It follows that

$$I(E(t)) = t \quad \forall t \quad ,$$

but the converse is not true, indeed:

$$\begin{cases} E(I(t)) = t & \text{if } V(b, t) > 0 \\ E(I(t)) \geq t & \text{if } V(b, t) = 0 \end{cases} \quad .$$

This last inequality reflects the fact that the vehicle about to leave the link (after having entered it at time  $I(t)$ ) must wait till the speed at exit point  $b$  becomes  $> 0$  again. The graphs of  $E$  and  $I$  are illustrated hereafter; they are of course symmetric.



Another way yet to express the complications resulting from  $V(b, \cdot) = 0$  is to note that  $V(b, t) = 0$  implies  $\frac{dI}{dt}(t) = 0$ .

### Experienced and predictive travel times.

Then we define:

$$ETT(a, b; t) \stackrel{def}{=} t - I(t)$$

the *experienced travel time* (of the user about to exit the link at time  $t$ ). Let us note an important consequence of the preceding considerations:

$$(1) \quad V(b, t) = 0 \implies \frac{d}{dt} ETT(t) = 1 \quad .$$

Conversely,

$$PTT(a, b; t) \stackrel{def}{=} E(t) - t$$

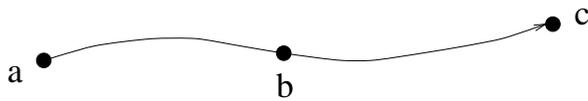
defines the *predictive travel time* of users entering the link at time  $t$ . In the case of a first-order model of the LWR (Lighthill-Whitham-Richards) type given by

$$\frac{\partial K}{\partial t} + \frac{\partial}{\partial x} Q_e(K, x) = 0$$

with  $Q_e$  the equilibrium flow-density relationship, the computation of  $PTT(a, b; t)$  at time  $t$  would require the initial condition  $K(\cdot, t)$  on the link and the downstream traffic supply  $\Sigma(b, r)$  for time  $r$  ranging from  $t$  to  $E(t)$ , since it would require the computation in the  $(x, t)$ -plane of the field-lines with origin  $(x, t)$  and  $x \in [a, b]$ . We refer to (Lebacque 1995) for a definition of the twin notions of local traffic supply and demand, and the related definition of boundary conditions for the LWR model. The upstream demand at point  $a$  is without influence on the field-line originating at  $(a, t)$ , because in the LWR model, the propagation speed of information,  $\frac{\partial}{\partial Q_e}(\cdot, x)$  is always less than the vehicle speed  $V_e(\cdot, x)$  (with  $V_e$  the equilibrium speed-density relationship). On the other hand, the computation of  $ETT(a, b; t)$  can be carried out by solving the following partial differential equation:

$$(2) \quad \left| \begin{array}{l} V \frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} = 1 \\ T(a, t) = 0 \quad (\forall t) \quad . \end{array} \right.$$

Indeed,  $dT$  is equal to  $dt$  along trajectories, hence  $T(x, t) = ETT(a, x; t)$ . This definition of the ETT is important in practice, because it implies that it is not necessary to store past data in order to compute the *ETT function*, i.e.  $T$ , as the formula (14) will illustrate later on.



Considering three points  $a, b, c$  in that order on a line, and considering the trajectory of a vehicle passing through these points, the following functional equations are satisfied:

$$PTT(a, c; t) = PTT(a, b; t) + PTT(b, c; t + PTT(a, b; t))$$

$$ETT(a, c; t) = ETT(b, c; t) + ETT(a, b; t - ETT(b, c; t)) \quad ,$$

which show how these travel times are to be combined. These are definitely not additive quantities! Hence the difficulties related to spatial aggregation.

## Instantaneous travel times.

Although experienced travel times may be used for reactive assignment (this is the case in the INTEGRATION model), they do not necessarily constitute the prime choice, since an experienced travel time reflects more what has just happened than what is about to happen. Another possibility is to define directly an *instantaneous travel time*  $ITT(a, b; t)$  for the link. A standard and natural definition (such as the one given by Ran and Boyce (1994)) specifies the instantaneous travel time as *the travel time that would result if prevailing traffic conditions remained unchanged*. In the present context of macroscopic models, this means that to compute  $ITT(a, b; t)$  we have to define a time-constant speed field say  $\mathcal{V}^t(x, r) \stackrel{def}{=} (V(x, t), 1)$  for all instants  $r \geq t$ , and compute its field-lines, which are of course invariant through translations parallel to the time-axis. Therefore the corresponding formula is given by:

$$(3) \quad ITT(a, b; t) \stackrel{def}{=} \int_a^b d\chi / V(\chi, t) \quad .$$

This formula is of course additive (a desirable feature if the estimation of trip travel times is required) but regrettably it is only applicable if there exists some strictly positive lower bound for the speed, as in some models. In the general case, the speed may become nil if the traffic is interrupted, and the above integral might diverge, or take on unrealistic values. Therefore, we construct the instantaneous travel time in order to satisfy some set of properties. The properties we retained are that, on a small scale, the  $ITT$  should be of the order  $-dx/V(x, t)$  at low density and high speed (yielding  $ITT(x, .; t) \approx \int_x d\chi / V(\chi, t)$ ) and of the order  $dt$  for strongly congested traffic. The former property reflects simply the idea that the instantaneous travel time should be close to its “natural definition” (3) at low density. The latter property is similar to the analogous property of  $ETT$ s (1). It reflects the fact that, for interrupted traffic,  $dITT(x, .; t) = dt$  yields the simplest estimate of the interruption duration, especially in the case of an incident in which this duration may not by definition be known beforehand. The simplest model satisfying to these properties is described by the following partial differential equation (Lebacque 1996):

$$(4) \quad \left\{ \begin{array}{l} -V \frac{\partial R}{\partial x} + (1 - \frac{V}{V_{max}}) \frac{\partial R}{\partial t} = 1 \\ R(b, t) = 0 \quad (\forall t) \quad , \end{array} \right.$$

with  $V_{max}$  the maximum speed and  $R(x, t) \stackrel{def}{=} ITT(x, b; t)$  (instantaneous travel time from  $x$  to  $b$  estimated at time  $t$ , labeled *backward ITT* in the above reference). To explain formula (4), let us note first that at high speed and low density, the model should yield

$$(5) \quad -V \frac{\partial R}{\partial x} \approx 1 \quad ,$$

and at low speed and high density, it should yield

$$(6) \quad \frac{\partial R}{\partial t} \approx 1 \quad .$$

Of course, in all cases, the boundary condition  $R(b, t) = 0$  should be satisfied. Now, (4) is nothing more than a linear interpolation between (5) and (6). Of course, (4) constitutes by no way the only possible model satisfying our requirements. Even the boundary condition might be changed to say

$$S(a, t) = 0 \quad ,$$

yielding an estimate  $S(x, t) \stackrel{def}{=} ITT(a, x; t)$  (instantaneous travel time from  $a$  to  $x$  estimated at time  $t$ , labeled *forward ITT* in the above reference). A partial differential equation for  $S$  can be constructed according to the same ideas as those used for  $R$ , yielding:

$$(7) \quad \begin{cases} V \frac{\partial S}{\partial x} + (1 - \frac{V}{V_{max}}) \frac{\partial S}{\partial t} = 1 \\ S(a, t) = 0 \quad (\forall t) \end{cases} .$$

The choice between  $R$  and  $S$  is essentially a matter of application. In the sequel, for brevity's sake, we shall concentrate our attention on  $R$ .

The  $ITT$  estimate  $R$  can be computed analytically. The method is the following. Denoting  $u$  the variable

$$(8) \quad u \stackrel{def}{=} t - \frac{x}{V_{max}} ,$$

and  $\mathcal{W}$  the field

$$(9) \quad \mathcal{W}(x, t) \stackrel{def}{=} (-V(x, t), 1 - \frac{V(x, t)}{V_{max}})$$

whose field-lines  $(\xi(u), \tau(u))$  are given by:

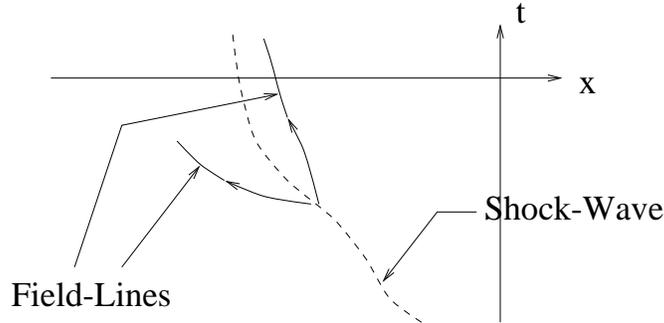
$$(10) \quad \begin{cases} \frac{d\xi}{du} = -V(\xi, \tau) \\ \frac{d\tau}{du} = 1 - \frac{V}{V_{max}}(\xi, \tau) \end{cases}$$

it follows that

$$(11) \quad dR = du$$

along such a field-line. Hence, to compute  $R$ , it suffices to compute the field-lines (10). The trivial graphical interpretation of  $u$  in the  $(x, t)$  plane extends to  $R$  along field-lines of  $\mathcal{W}$ .

A difficulty arises at shock-waves; it is indeed possible that two distinct field-lines of  $\mathcal{W}$  originate from the same point of a shock-wave, as is illustrated hereafter.



In that case, the shock-wave itself must be considered a field-line of  $\mathcal{W}$ , along which (11) applies. By considering three consecutive points of a field-line, the following three-point functional equation results:

$$ITT(a, c; t) = ITT(a, b; t) + ITT(b, c; t - \frac{a-b}{V_{max}} - ITT(a, b; t)) \quad .$$

It may be noted that, if the speed is near  $V_{max}$  on  $[a, b]$ , i.e.  $ITT(a, b; t)$  is nearly equal to  $\frac{b-a}{V_{max}}$ , then the above functional equation becomes nearly additive, i.e.:

$$ITT(a, c; t) \approx ITT(a, b; t) + ITT(b, c; t) \quad ,$$

which is precisely what is to be expected. Similar properties are satisfied by the forward ITTs.

## SEMIDISCRETIZED MODELS.

By semidiscretized models we mean models continuous in time and discretized in space, with the link as the space discretization unit. As indicated above, the link  $PTT(t)$  is essentially a function of the link state  $K(., t)$  at time  $t$  and the downstream traffic flow supply  $\Sigma(b, s)$  for  $s \in [t, E(t)]$ . In fluid traffic conditions (i.e. downstream traffic flow supply sufficient to accomodate the traffic demand of the link at all times), and at the zero-th order approximation, one might consider  $PTT(t)$  as a function of  $N(t)$ . This is the basis of some flow models for assignment problems (Fernandez and de Cea 1994, Friesz et al. 1993, Astarita 1996, Ran et al. 1996). In such models the link traffic flow dynamics are described by a model of the following kind:

$$\frac{dN}{dt}(t) = u(t) - v(t)$$

( $u(t)$  the link inflow and  $v(t)$  the link outflow), supplemented by a model for the  $PTT(t)$ , called here  $\tau(t)$ :

$$\tau(t) = f(N(t))$$

and the FIFO condition, which in the present case does *not* result naturally from the model. This last condition implies (Astarita 1996) that:

$$(12) \quad v(t + \tau(t)) = \frac{u(t)}{1 + \frac{d\tau}{dt}(t)} = \frac{u(t)}{\frac{dE}{dt}(t)}$$

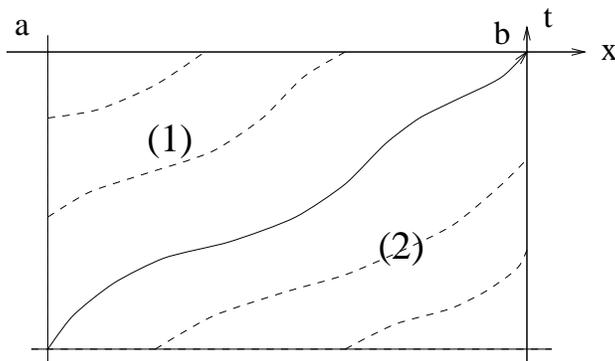
by expressing that users entering the link at time  $t$  exit it at time  $t + \tau(t)$ . With the FIFO hypothesis, the following integral relationships result:

$$N(t) = \int_{I(t)}^t u(s)ds = \int_t^{E(t)} v(s)ds \quad .$$

These are the same as the relationships that would be obtained within the framework of the preceding section. Indeed, by integrating the conservation equation

$$\frac{\partial K}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

over areas (1) and (2) depicted hereafter (and bounded diagonally by a field-line in the  $(x, t)$  plane),



the following relationships result:

$$N(t) \stackrel{def}{=} \int_a^b K(\chi, t)d\chi = \int_{I(t)}^t Q(a, s)ds = \int_t^{E(t)} Q(b, s)ds \quad ,$$

relating the link inflow and outflow to the number  $N(t)$  of vehicles contained in the link at a given time.

Let us note at this point some differences in the terminology, since in (Ran and Boyce 1994) for instance, the PTT is called *the actual travel time*.

It can be shown that the only consistent FIFO model of the above kind is the one associated to a linear travel time function:

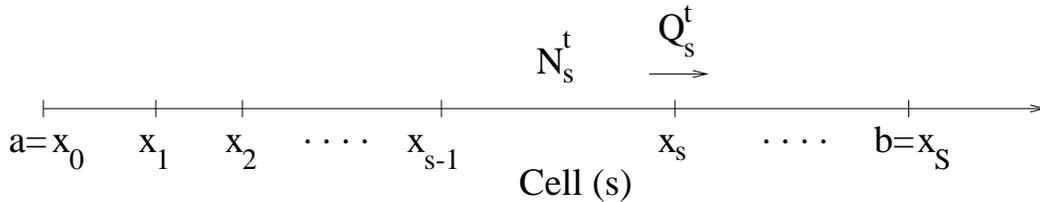
$$\tau(t) = \alpha + \beta N(t) \quad .$$

This result was suggested in (Daganzo 1995), the sufficiency of this linear form was demonstrated in (Friesz et al. 1993), and its necessity in (Lebacque and Lesort 1996). The linear part represents the average time lost in the queue at the exit of the link, which is somewhat at odds with the hypothesis that the downstream traffic supply can be neglected. It is not known to the authors of the present paper whether non-FIFO models of the above kind can be built. The analysis of such models might prove difficult since they would not admit any closed expressions such as (12) for the link outflow.

## FULLY DISCRETIZED MACROSCOPIC MODELS.

We shall now consider discretized macroscopic models and develop recursive formulas for ETTs and ITTs. The emphasis on recursive formulas is motivated by the need for formulas requiring as little computational effort as possible in order to be suitable for real-time applications. Unless stated otherwise, the report (Lebacque 1996) is the reference for all results described in the present section.

For discretized macroscopic models, either there exist no intrinsic estimates of speed (1st order models), or, if such estimates exist, they may lead to unrealistic values of the travel times. Hence we propose the following methodology. In most discretized macroscopic models, links are divided into cells say  $(s) = [x_{s-1}, x_s]$ , of length  $l_s$ , containing  $N_s^t$  vehicles at time  $t\Delta t$ , with the average cell exit flow  $Q_s^t$  during time-step  $[t\Delta t, (t+1)\Delta t]$ , estimated at the cell exit point  $x_s$ . A notable exception to this kind of approach to discretized macroscopic modelling is the particle discretization approach, as illustrated in INTEGRATION.



No hypothesis is made on the macroscopic model itself, nor on the manner in which the above quantities are computed. We denote by  $V_s^t$  the *cell exit speed* defined as

$$V_s^t = \frac{Q_s^t}{K_s^t} = \frac{Q_s^t l_s}{N_s^t}$$

with  $K_s^t \stackrel{def}{=} N_s^t / l_s$  the mean cell density at time  $t\Delta t$ . The significance of this choice is that it permits emulation of FIFO behaviour within each cell. We shall consider only what we call *proper discretizations* i.e. discretizations such that the cumulated cell outflow  $Q_s^t \Delta t$  during a time-step be less than the number of vehicles  $N_s^t$  contained in the cell at time  $t\Delta t$ :

$$Q_s^t \Delta t \leq N_s^t \quad .$$

Let us define now the coefficients

$$(13) \quad \begin{cases} \alpha_s &= V_{s,max} \Delta t / l_s, \\ \nu_s^t &= V_s^t / V_{s,max} = Q_s^t \Delta t / (\alpha_s N_s^t), \\ \beta_s^t &= Q_s^t \Delta t / N_s^t = \alpha_s \nu_s^t. \end{cases}$$

The fact that the discretization is proper can be translated as

$$\beta_s^t \leq 1$$

and in order for the discretization to work at all, it is necessary that

$$\alpha_s \leq 1 \quad .$$

Those inequalities are assumed to hold throughout the present section.

For a link containing cells  $s = 1$  to  $S$ , equation (2) for the experienced travel-time  $T$  may be discretized according to:

$$(14) \quad T_s^{t+1} = \beta_s^t T_{s-1}^t + (1 - \beta_s^t) T_s^t + \Delta t \quad ,$$

with  $T_s^t$  denoting the approximate experienced travel time from  $x_0$  to  $x_s$  at time  $t\Delta t$ . With  $S = 1$ , this formula is equivalent to the ITT formula introduced in (Buisson et al. 1995) on a completely heuristic basis. The boundary condition is  $T_0^t = 0$ . Introducing the cell travel times  $ETT_s^t = T_s^t - T_{s-1}^t$ , the above formula can be rewritten as:

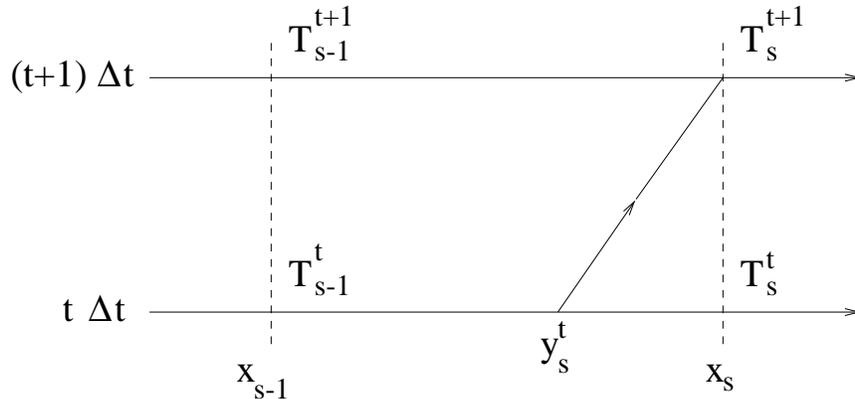
$$ETT_s^{t+1} = -(T_{s-1}^{t+1} - T_{s-1}^t) + (1 - \beta_s^t) ETT_s^t + \Delta t \quad .$$

This last formula is important in two ways. First it gives an indication of how to aggregate spatially travel-times. Second, it implies

$$ETT_s^{t+1} - \frac{l_s}{V_s} = -(T_{s-1}^{t+1} - T_{s-1}^t) + (1 - \beta_s^t) (ETT_s^t - \frac{l_s}{V_s}) \quad ,$$

showing that the cell-travel time relaxes towards  $l_s/V_s^t$  if the traffic flow is not too unstationary.

The recursive formula (14) is obtained in the following way.



$T(x, t) \stackrel{def}{=} ETT(a, x; t)$  is approximated by a continuous piecewise linear function whose values at points  $x_s$  are  $T_s^t$  at times  $t\Delta t$ . The field-lines of  $\mathcal{V}$  are approximated by lines, and we use the fact that  $dT = dt$  along such a line. Let  $(y_s^t, t\Delta t)$  denote the origin of the field-line ending at  $(x_s, (t+1)\Delta t)$ , then:

$$y_s^t = x_s - V_s^t \Delta t$$

at the first order approximation. It follows:

$$T_s^{t+1} = T(y_s^t, t\Delta t) + \Delta t$$

and the linear interpolation of  $T(y_s^t, t\Delta t)$  between  $T_s^t$  and  $T_{s-1}^t$  yields (14).

The discretization of (4) yields:

$$(15) \quad \begin{cases} R_{s-1}^{t+1} = \frac{\Delta t}{1-\alpha_s} + (1 - \frac{\alpha_s \nu_s^t}{1-\nu_s^t})R_{s-1}^t + \frac{\alpha_s \nu_s^t}{1-\nu_s^t}R_s^t & \text{if } \nu_s^t \leq \frac{1}{1+\alpha_s} \\ R_{s-1}^{t+1} = \frac{1}{\alpha_s \nu_s^t}[\Delta t + (1 - \nu_s^t)R_s^t + (\alpha_s \nu_s^t + \nu_s^t - 1)R_{s-1}^t] & \text{if } \nu_s^t \geq \frac{1}{1+\alpha_s} \end{cases}$$

The boundary condition is  $T_s^t = 0$ . Introducing the cell travel time  $ITT_s^t = R_s^t - R_{s-1}^t$ , it follows:

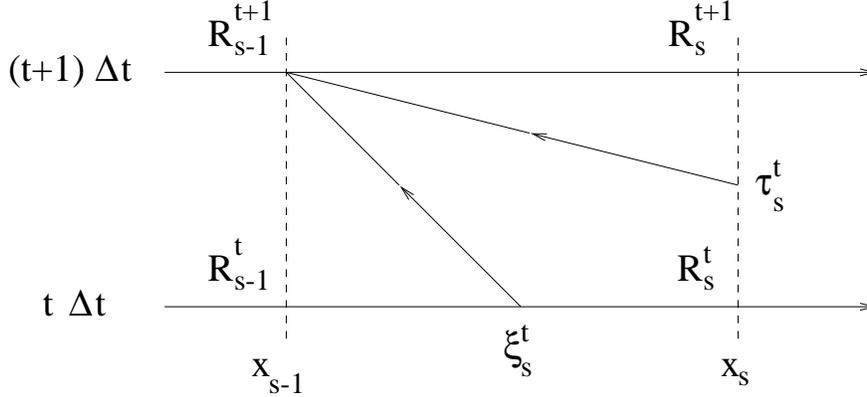
$$\begin{cases} ITT_s^{t+1} = \frac{\Delta t}{1-\alpha_s} + (1 - \frac{\alpha_s \nu_s^t}{1-\nu_s^t})ITT_s^t - (R_{s-1}^{t+1} - R_{s-1}^t) & \text{if } \nu_s^t \leq \frac{1}{1+\alpha_s} \\ ITT_s^{t+1} = \frac{\Delta t}{\alpha_s \nu_s^t} - \frac{1-\nu_s^t}{\alpha_s \nu_s^t}(R_{s-1}^{t+1} - R_{s-1}^t) & \text{if } \nu_s^t \geq \frac{1}{1+\alpha_s} \end{cases} .$$

It follows:

$$\begin{cases} ITT_s^{t+1} - \frac{l_s}{V_s^t} = (1 - \frac{\alpha_s \nu_s^t}{1-\nu_s^t})(ITT_s^t - \frac{l_s}{V_s^t}) - (R_{s-1}^{t+1} - R_{s-1}^t) & \text{if } \nu_s^t \leq \frac{1}{1+\alpha_s} \\ ITT_s^{t+1} - \frac{l_s}{V_s^t} = -\frac{1-\nu_s^t}{\alpha_s \nu_s^t}(R_{s-1}^{t+1} - R_{s-1}^t) & \text{if } \nu_s^t \geq \frac{1}{1+\alpha_s} \end{cases} .$$

showing that the cell-travel time relaxes towards  $l_s/V_s^t$  if traffic flow conditions approach stationary, and that this relaxation process is much faster for ITT estimates than for ETT estimates, and this is precisely the result that was hoped for: that the ITT should be as close to the ideal formula (3) as possible even in interrupted traffic conditions. It is in this restrictive sense that our definition of an ITT and the above-mentioned definition of Ran and Boyce (1994) may be said to concur.

The recursive formula (15) is obtained in the following way.



$R(x, t) \stackrel{def}{=} ITT(x, b; t)$  is approximated by a continuous piecewise linear function whose values at points  $x_s$  are  $R_s^t$  at times  $t\Delta t$ . The field-lines of  $\mathcal{W}$  are approximated by lines, and we use the fact that  $dR = du$  along such a line. Depending on whether  $\nu_s^t \geq 1/(1+\alpha_s)$ , or  $\nu_s^t \leq 1/(1+\alpha_s)$ , the field-line ending at  $(x_{s-1}, (t+1)\Delta t)$  originates at point  $(x_s, \tau_s^t)$  or at point  $(\xi_s^t, t\Delta t)$ , with:

$$\begin{cases} \tau_s^t = (t + 1 - \frac{1-\nu_s^t}{\beta_s^t})\Delta t \\ \xi_s^t = x_{s-1} + l_s \frac{\beta_s^t}{1-\nu_s^t} \end{cases} .$$

Applying  $dR = du$  and the linearity of  $R$ , it follows:

$$\begin{aligned} R_{s-1}^{t+1} &= R(x_s, \tau_s^t) + (t+1)\Delta t - \tau_s^t + (l_s/V_{s,max}) \\ &= R(x_s, \tau_s^t) + (\Delta t/\beta_s^t) \quad \text{if } \nu_s^t \geq 1/(1+\alpha_s) \end{aligned}$$

and

$$\begin{aligned} R_s^{t+1} &= R(\xi_s^t, t\Delta t) + \Delta t + (\xi_s^t - x_{s-1})/V_{s,max} \\ &= R(\xi_s^t, t\Delta t) + \Delta t/(1-\nu_s^t) \quad \text{if } \nu_s^t \leq 1/(1+\alpha_s) . \end{aligned}$$

Both formulas (14) and (15) are recursive, hence easy to implement and of reduced computational cost. Further, they can be extended to the case where the global flow is split into partial flows, according to destination, path, type of driver (informed or uninformed), depending on the assignment problem. They can also, within the framework developed for the STRADA model (Buisson et al. 1995), be extended to movements of intersections. Finally, when the traffic flow is nearly stationary, both formulas imply the convergence of the  $ETT_s^t$ s,  $ITT_s^t$ s, towards  $\frac{l_s}{V_s}$  on every cell  $s$ , as already noticed, with a better convergence for the ITT.

## CONCLUSION.

Much work remains to be done. Other ITT estimates are conceivable, depending on the properties one deems important for such quantities. The problem of the time-aggregation of travel times must be addressed, especially in the case of adaptative regulation schemes lacking periodicity. The impact of the proposed estimators on traffic assignment and management schemes must be studied as well. Especially since one of the motivations behind such an ‘‘axiomatic’’ definition of travel times as we have given here is to provide some solid ground for assignment computations. Finally, some experimental assesment of the proposed travel time estimators should be attempted, although it would seem difficult to separate the properties of the estimators from those of the associated macroscopic traffic flow model.

## REFERENCES.

- Astarita, V. (1996). A continuous-time link model for dynamic network loading based on travel time function. In: *Proceedings of the 13th ISTTT (International Symposium on Transportation and Traffic Theory), Lyon 1996* (J.B. Lesort, ed.), pp 79-102. Pergamon.
- Branston, D. (1976). Link travel time functions: a review. *Transportation Research* 10: 223-236.
- Buisson, C., J.P. Lebacque and J.B. Lesort (1995)
1. Macroscopic modelling of traffic flow and assignment in mixed networks. In: *Proceedings of the 6th International Conference on Computing in Civil and Building Engineering, Berlin 1995* (P.J. Pahl and H. Werner, ed.), pp 1367-1374.
  2. STRADA, a discretized macroscopic model of vehicular traffic flow in complex networks based on the Godunov scheme. In: *Proceedings of the IEEE-SMC IMACS'96 Multiconference, Lille 1996, Symposium on Modelling, Analysis and Simulation Vol 2*, pp 976-981.

- Cremer, M., F. Meissner and S. Schrieber (1993) On predictive control schemes in dynamic rerouting strategies. In: *Proceedings of the 12th ISTTT (International Symposium on Transportation and Traffic Theory)*, Berkeley, 1993 (C.F. Daganzo, ed.), pp 407-426.
- Daganzo, C.F. (1995). Properties of link travel time functions under dynamic loads. *Transportation Research B*, B29, 95-98.
- Drissi-Kaïtouni, O. and M. Gendreau (1992). A new dynamic traffic assignment model. Montréal, Centre de Recherche sur les Transports.
- Elloumi, N. (1996). Modélisation et commande du trafic sur un corridor - Application de la méthode LP. Orsay, Université Paris XI: 179.
- Fernandez, J.E. and J. de Cea (1994) Flow propagation description in dynamic network assignment models. In: *Proceedings of the Triennial Symposium in Transportation Analysis (TRISTAN)*, Capri 1994, pp 119-133.
- Friesz, T.L., J. Luque, R.L. Tobin and B.W. Wie (1989) Dynamic network traffic assignment considered as a continuous time optimal control problem. *Operations Research* 37, (6) pp 893-901.
- Friesz, T.L., D.H. Bernstein, T.E. Smith, R.L. Tobin and B.W. Wie (1993). A variational inequality formulation of the dynamic network user equilibrium problem. *Operations Research* 41, 179-191.
- Jayakrishnan, R., H.S. Mahmassani, T.Y. and T.Y. Hu (1994). An evaluation tool for advanced traffic information and management systems in urban networks. *Transportation Research* 2 C(3): 129-147.
- Koutsopoulos, H.N. and M. Habbal (1994). Effect of intersection delay modeling on the performance of traffic equilibrium models. *Transportation Research A*, 28A, 2, 133-149.
- Lebacque, J.P. (1995). The Godunov scheme and what it means for first order traffic flow models. Complete version: CERMICS Report 48-95. Short version: published in: *Proceedings of the 13th ISTTT (International Symposium on Transportation and Traffic Theory)*, Lyon 1996 (J.B. Lesort, ed.), pp 647-677. Pergamon.
- Lebacque, J.P. (1996). Instantaneous travel times for macroscopic traffic flow models. CERMICS Report 59-96.
- Lebacque, J.P. and J.B. Lesort (1996). Simplified traffic flow models for dynamic assignment. Groupe de Travail Modélisation du trafic CERMICS-INRETS.
- Leonard, D. R., P. Gower and N.B. Taylor (1989). CONTRAM: structure of the model. TRRL Report RR 178, Crowthorne.
- Leonard, D. R., J. B. Tough and P. C. Baguley (1978). CONTRAM: A traffic assignment model for predicting flows and queue during peak periods. Crowthorne, Transport and Road Research Laboratory.
- Merchant, D. K. and G. L. Nemhauser (1978a). A model and an algorithm for the dynamic traffic assignment problems. *Transportation Science* 12(3): 183-199.
- Merchant, D. K. and G. L. Nemhauser (1978b). Optimality conditions for a dynamic traffic assignment model. *Transportation Science* 12(3): 200-207.

- Papageorgiou, M. (1990). Dynamic modeling, assignment, and route guidance in traffic networks. *Transportation Research* 24 B(6): 471-495.
- Ran, B. and D.E. Boyce (1994) Dynamic urban transportation network models. Springer Verlag.
- Ran, B., R. W. Hall and D.E. Boyce (1996). A link-based variational inequality model for dynamic departure time/route choice. *Transportation Research* 30B (1): 31-46.
- Van Aerde, M. (1995). INTEGRATION release 2: User's guide. Kingston, Ontario, Canada, Transportation Systems Research Group.
- Weymann, J., J.L. Farges, Henry, J.J. (1994). Dynamic route guidance with queue and flow dependent travel time. *Transportation Research* 2 C(3): 165-183.