Optimal sequential decisions and variations of the value of information

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Abstract. For a sequential, two-period decision problem with uncertainty and under broad conditions (non-finite sample set, endogenous risk, active learning and stochastic dynamics), a general sufficient condition is provided to compare the optimal initial decisions with or without information arrival in the second period. More generally the condition enables the comparison of optimal decisions related to different information structures. It also ties together and clarifies many conditions for the 'irreversibility effect' that are scattered in the environmental economics literature. A numerical illustration with an integrated assessment model of climate-change economics is provided.

Key words: Value of Information, Uncertainty, Irreversibility effect, Climate change

1 Introduction

The study of sequential decisions problems with learning typically involves the comparison of the optimal initial decisions with different information structures. For instance, should we aim at more reductions of current greenhouse gases emissions if we assume some future improvement of our knowledge about the climate? Economic analysis has identified effects that go in opposite directions and make the conclusion elusive. This article proposes a simple and general relation between the value of future information on risks and the initial degree of precaution taken in a sequential decisions setting.

Seminal literature in environmental economics (Arrow and Fisher [2] and Henry [16, 17]) focused on the irreversible environmental consequences carried by the initial decision and showed that the possiblity of learning should lead to less irreversible current decisions ('irreversibility effect'). Yet, the conditions under which this result holds are rather restrictive and further contributions have insisted on the existence of an opposite economic irreversibility since environmental precaution imply sunk costs that may constrain future consumption (Kolstad [22], Pindyck [28], Fisher and Narain [10]). Finally, considering a specific payoff function, Gollier et al. [13] identified conditions on the utility function for the possibility of learning to have a 'precautionary effect' with and alternatively without the irreversibility constraint.

Initially, the literature on the irreversibility effect related this effect to the value of information through the concept of 'quasi-option' value, which was identified to the value of information 'conditional on the preservation decision' (Hanemann [15]). However, this identification does not hold in the more general case where the decision set is not binary and economic irreversibilities add on to the environmental ones so that the quasi-option value can be negative (Hanemann [15], Ha-Duong [14]).

The driving idea for linking the effect of learning and the value of information is the observation that, once an initial decision is made, the value of information can be defined as a *function* of that decision. Hints towards this can be found in Conrad [7], Hanemann [15] or Ha-Duong [14], but the functional dependance is either non explicit or limited to a binary decision set. More recently, Rouillon [29] defined for a specific model of climate-change the value of information as a function of the greenhouse gases (GHG) concentration. In a special case, he found that when this value of information (after the initial decision) is a monotone function of the pollution stock, then the optimal emission levels with and without information can be ordered.

We show that this result is very general and ties together different pieces of the literature on uncertainty and irreversibility. It can also be applied properly to integrated assessment models and thus connects two themes of the climate change literature, namely, the value of information and the irreversibility effect. Currently, in the economics of climate change, several studies have inquired the effect of learning (e.g. Kolstad [21], Ulph and Ulph [32], Fisher and Narain [10], Keller et al.[19]) or have centered around the value of information (e.g. Manne and Richels [24], Peck and Teisberg [27], Nordhaus [25], Ambrosi et al. [1]). But no exact and general link had been drawn between both researchs.

This paper considers two two-period decision problems with uncertainty, which are identical except for the information available at the second period (typically, one with and the other without information). For any arbitrarily fixed initial decision, there are thus two subproblems of expected utility maximization problems (still one with and one without information). They lead to define the value of information after an initial commitment and this value is now a *function of the initial decision*. We shall prove that, whenever this function is monotone, we are able to identify the problem with the lowest optimal initial decision. The result does not require any concavity assumptions. It is generalized to the value of exchanging one information structure for an other. In the literature, the ranking of optimal decisions is often obtained through conditions that depend upon at least one of those optimal solutions as in Epstein [8] or Ulph and Ulph [32]. We show how the monotonicity of the value of information allows to bypass those conditions.

Many of the specific models studied in the irreversibility literature from Arrow and Fisher [2] to Gollier et al. [13] can be seen as particular instances of our model. Since it avoids standard, restrictive assumptions on the utility and on the environment (like linear evolution or scalar dimension), it is general enough to be applied to the study of numerical integrated assessment models, like DICE [26]. Formally, it is not restricted to environmental problems.

Section 2 proposes a general analytical framework of sequential decisions under uncertainty. Section 3 gives our main result. It connects the comparison of the optimal initial decisions (with learning and without learning) to the variations of the value of information, considered as a function of the initial decision. Section 4 extends the result to decision problems with endogenous risk, active learning and stochastic dynamics. In section 5, the main result is shown to tie together several former results of the literature. Finally, section 6 uses Nordhaus' DICE model to provide a practical application and section 7 concludes.

2 The standard model of decision with learning

2.1 The decision problem

We consider in this section a rather general model of optimal control under uncertainty, where decisions u_0 and u_1 are taken at two periods of time, namely $t = 0$ and $t = 1$. In the first period, the decision-maker chooses a policy $u_0 \in \mathcal{U}_0 \subset \mathbb{R}$ that influences a state variable $x_t \in \mathbb{R}^n$, for example a stock of pollution. The state variable evolves according to $x_1 = f(x_0, u_0)$. The decision-maker then chooses a second policy $u_1 \in \mathcal{U}_1(x_1) \subset \mathbb{R}^m$. The total payoff is:

$$l_0(u_0) + l_1(u_1, x_1, \gamma), \tag{1}$$

where l_t is the discounted utility or benefit enjoyed in period t and γ is some unknown random variable over a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Before choosing u_1, the observation of the signal Φ (another random variable over the same sample space as γ), allows the decision-maker to revise her prior probability distribution about γ.

The decision maker aims at maximizing the expected present benefit[1]

$$\begin{gathered} \max_{u_0} \; l_0(u_0) + \mathbb{E}\left[\max_{u_1} \mathbb{E}\left[l_1(u_1, x_1, \gamma) \mid \Phi\right]\right] \\ \text{with} \quad x_1 = f(u_0) \quad \text{and} \quad u_t \in \mathcal{U}_t(x_t),\; t = 0, 1. \end{gathered} \tag{2}$$

'Irreversibility' of the initial decision u_0 may materialize through the dependance on x_1 of both utility, l_1, and available second period decisions, $\mathcal{U}_1(x_1)$.

The information structure is defined[2] by the signal Φ. At time $t = 1$, the decision u_1 can be seen as a function from Ω to $\mathcal{U}_1(x_1)$ and should be measurable with respect to the σ-algebra induced by the signal function Φ.

For the problem with information structure Φ, define the 'expected optimal benefit in state $x_1 = x$' as the value function at $t = 1$:

$$V_\Phi(x) \overset{\text{def}}{=} \mathbb{E}\left[\max_{u_1 \in \mathcal{U}_1(x)} \mathbb{E}\left[l_1(u_1, x, \gamma) \mid \Phi\right]\right] \tag{3}$$

which allows to rewrite the decision problem (2) at $t = 0$ as:

$$\max_{u_0 \in \mathcal{U}_0} \left[l_0(u_0) + V_\Phi(f(u_0))\right]. \tag{4}$$

'No information' at time $t = 1$ can be represented by a constant signal over Ω or, equivalently, by the trivial σ-algebra $\{\Omega, \emptyset\}$. In the following, we shall denote by $\perp$ a non-informative structure[3].

2.2 Second-period value of the information structure

After any initial decision u_0, the decision maker knows from the deterministic dynamics f what subsequent state of the system, x_1, will enter her new decision problem at time $t = 1$.

[1] In what follows, we shall always assume that, for the problems we consider, the *sup* is attained and we shall use the notation *max*.

[2] The irreversibility literature (for instance in [11, 22]) relies on a description of information through partitions. However partitions are less general in the non-finite case. More generally, information is a σ-algebra (the one generated by the signal, $\sigma(\Phi)$ in the case hereabove).

[3] Bottom $\perp$ of the lattice of subfields of $\mathcal{F}$.

If she thinks she will not learn about γ (corresponding to information structure $\perp$), she may be ready to pay to obtain information from a signal Φ. When buying Φ, she does not know which information she will receive, but she will be able to move from the expected benefit $V_\perp(x_1) = \max_{u_1 \in \mathcal{U}_1(x)} \mathbb{E}\left[l_1(u_1, x, \gamma)\right]$ to the expected benefit $V_\Phi(x_1)$. Let us define therefore[4]

$$I_\Phi(x) \stackrel{\text{def}}{=} V_\Phi(x) - V_\perp(x) \tag{5}$$

as the *second-period value of the information* structure Φ when the system is in state x in $t = 1$. This value is clearly always non-negative. Note also that the second-period value of the information is a function of the state of the system. In the following, we shall indifferently use the expressions ‘value of information’ or ‘value of the information structure’. More generally, when the state of the system in $t = 1$ is $x_1 = x$, the value of having an information structure Ψ rather than the information structure Φ is:

$$\Delta_{\Psi\Phi}(x) \stackrel{\text{def}}{=} I_\Psi(x) - I_\Phi(x) \tag{6}$$

If Ψ is finer than Φ, meaning that the σ-algebra induced by Φ is included in the one induced by Ψ or, equivalently, that Φ is a (measurable) function of Ψ, than Φ, this value is also positive.

3 Effect of learning and value of information

3.1 The value of information in the decision problem

From Eq. (4) applied to the non-informative structure $\perp$, the decision problem (2) with a non-informative information structure becomes:

$$\max_{u_0 \in \mathcal{U}_0} \left[l_0(u_0) + V_\perp(f(u_0))\right] \tag{7}$$

From Eq. (4) and (7) and the definition of the second-period value of information in Eq. (5), the decision problem (2) with information structure Φ writes:

$$\max_{u_0 \in \mathcal{U}_0} \left[l_0(u_0) + V_\perp(f(u_0)) + I_\Phi(f(u_0))\right] \tag{8}$$

Comparing programs (7) and (8), it appears that the decision maker who expects information optimizes the same objective as the uninformed decision maker *plus* the value of the information, which depends on her initial decision. Her optimal decision can achieve a trade-off: it can be suboptimal from the point of view of the non-informed decision maker but compensate for this by an increase of the value of information.

Note also that the second-period value of information, I_Φ, depends on the initial decision even though there is no active learning, i.e. what one expects to learn does not depend on u_0.

More generally, replacing the information structure Φ by the the information structure Ψ leads to a reformulation of the problem (4) as

$$\max_{u_0 \in \mathcal{U}_0} \left[l_0(u_0) + V_\Phi(f(u_0)) + \Delta_{\Psi\Phi}(f(u_0))\right].$$

[4] With general utility functions (instead of benefit functions), the value of information is measured in utility units. Equivalent or compensating variations in monetary values can also be defined [23].

3.2 Comparison of initial and second-period values of information

Before comparing first period optimal decisions with and without future information, it is easier to compare the second-period values of information resulting from these decisions.

Proposition 1

Denote by I^0 the initial *value of acquiring the information structure Φ before any decision u_0 is made:*

$$I^0 \stackrel{\text{def}}{=} \max_{u_0 \in \mathcal{U}_0} [l_0(u_0) + V_\perp(f(u_0)) + I_\Phi(f(u_0))] - \max_{u_0 \in \mathcal{U}_0} [l_0(u_0) + V_\perp(f(u_0))] \ . \tag{9}$$

Let $u_0^\star$ be an optimal solution of (7), the problem without learning, and u_0^Φ be an optimal solution of (8), the problem with learning. Then,

$$I_\Phi(f(u_0^\star)) \leq I^0 \leq I_\Phi(f(u_0^\Phi)) \, . \tag{10}$$

The proof is in Appendix A.1. This comparison generalizes the relation between the initial value of information and the option value given by Hanemann [15], who defines option value as $I_\Phi(f(u_0^\Phi)) - I_\Phi(f(u_0^\star))$ for a family of problems where $I_\Phi(f(u_0^\star)) = 0$.

The hereabove inequalities show that a decision maker who knows she will receive information in the future *chooses her first decision so as to increase the value of information*, whereas a decision maker who neglects the fact that she will receive information makes a decision that reduces the value she would be ready to pay for information.

We next derive sufficient conditions for the comparison of initial optimal decisions, a problem at the centre of the literature on irreversibility and uncertainty.

3.3 Comparison of optimal solutions

The goal of this section is to identify how the presence of learning will affect the first period behavior.

From Proposition 1, we obtain immediately:

$$(\forall u > u_0^\star, \ I_\Phi(f(u)) < I_\Phi(f(u_0^\star))\,) \Rightarrow \ u_0^\Phi \leq u_0^\star.$$

Hence, *a practical sufficient condition for comparison of optimal solutions is the strict monotonicity of $u_0 \mapsto I_\Phi(f(u_0))$.*

More generally, our main result is the following proposition.

Proposition 2

Let Φ and Ψ be two information structures (not necessarily comparable in the sense that one is finer than the other).

Let u_0^Φ be any optimal initial decision with information structure Φ,

$$u_0^\Phi \in \arg\max_{u_0 \in \mathcal{U}_0} [l_0(u_0) + V_\Phi(f(u_0))] \, ,$$

and let u_0^Ψ be any optimal initial decision with information structure Ψ,

$$u_0^\Psi \in \arg\max_{u_0 \in \mathcal{U}_0} [l_0(u_0) + V_\Psi(f(u_0))] \, .$$

If the value of substituting information structure Ψ for Φ, namely the mapping $u_0 \mapsto \Delta_{\Psi\Phi}(f(u_0))$, is a strictly decreasing function, then the effect of learning is precautionary in the sense that

$$u_0^\Psi \leq u_0^\Phi .$$

The result[5] comes immediately from the generalization of Proposition 1 (Eq. (16), in Appendix A.1). It also holds under the weaker assumption that $u_0 \mapsto \Delta_{\Psi\Phi}(f(u_0))$ is strictly decreasing when $u_0 < u_0^\Psi$ (respectively strictly increasing when $u_0 > u_0^\Psi$.) A more general proposition can be made for non-strictly decreasing (or increasing) functions.

Proposition 3
If the value of substituting Ψ for Φ, $u_0 \mapsto \Delta_{\Psi\Phi}(f(u_0))$, is a decreasing function, then comparisons are still possible as follows

$$\sup \arg \max_{u_0 \in \mathcal{U}_0} [l_0(u_0) + V_\Psi(f(u_0))] \leq \sup \arg \max_{u_0 \in \mathcal{U}_0} [l_0(u_0) + V_\Phi(f(u_0))] .$$

The proof derives from Proposition 6 (see Appendix A.2).

As a consequence, if u_0^Φ is unique, it is sufficient that $u_0 \mapsto \Delta_{\Psi\Phi}(f(u_0))$ be decreasing to conclude that $u_0^\Psi \leq u_0^\Phi$.

4 Extension to active learning and stochastic evolution

Possible extensions of the standard case appear in the literature. This section shows that the main result still apply in the general, extended case.

Stochastic dynamics. From period $t = 0$ on, the state of the system $\tilde{x}_t$ is a random variable. Its evolution may depend on an other random variable w_t: $\tilde{x}_{t+1} = f(\tilde{x}_t, u_t, w_t)$. The model in Conrad [7] is an occurrence of stochastic dynamics in the irreversibility literature.

Endogenous risk. An example of endogenous risk can be found in Gjerde, Grepperud and Kverndokk [12] where the date of a climate catastrophe is a random variable and its probability distribution depends on the emission reductions. Endogenous risk arises when the random variable γ depends on the previous decisions, u_0 and u_1. In stochastic control theory, γ is treated as a state variable. Endogenous risk is thus viewed as a particular case of stochastic dynamics.

Active learning. Active learning (or dependent learning) takes place when the initial decision can modify the signal the decision maker will receive. It means that in addition to ω, Φ depends on u_0, or more generally on $\tilde{x}_1$ (then the modification is also random). Rouillon [29] studies a model of active learning in climate change economics and uses the variations of the value of information to conclude about the irreversibility effect.

[5] Freixas and Laffont [11] give sufficient conditions for the monotonicity of $\Delta_{\Psi\Phi}$ in a setting where the dynamics is reduced to $x_{t+1} = u_t$ and where the state of the system does not enter the benefits l_t but only the admissibility set. However, they do not provide the interpretation of Δ in terms of value of substituting information structures. Kolstad [22] obtains necessary and sufficient conditions for a problem which is actually a sub-case of Freixas and Laffont though this does not appear at first glance from his notations but has to be derived from his hypotheses.

Comparison in the general model

Consider the problem :

$$\max_{u_0,u_1} \mathbb{E}\left[l_0(u_0,\tilde{x}_0)+l_1(u_1,\tilde{x}_1)\right]$$
$$\text{with} \quad \tilde{x}_1 = f(\tilde{x}_0,u_0,w_0) \quad \text{and} \quad u_t \in \mathcal{U}_t(y_t),\ t=0,1$$

where w_t is a random variable (r.v.) and y_t a non-stochastic subcomponent of $\tilde{x}_t$, so that the decision maker knows the admissible set $\mathcal{U}(y_1)$ when she makes her choice[6] u_1.

At time $t=1$, when the state of the system is the r.v. $\tilde{x}$, the information structure Φ delivers a signal that depends on $\tilde{x}$. We denote by $\Phi_{\tilde{x}}$ the corresponding signal function $\Phi_{\tilde{x}} : \omega \mapsto s(\omega, \tilde{x}(\omega))$. The decision-problem can be written as:

$$\max_{u_0\in\mathcal{U}_0(y_0)} \ \mathbb{E}\left[l_0(u_0,\tilde{x}_0)+V_\Phi(f(\tilde{x}_0,u_0,w_0))\right].$$
$$\text{with} \quad V_\Phi(\tilde{x}) \stackrel{\text{def}}{=} \mathbb{E}\left[\max_{u_1\in\mathcal{U}_1(y)} l_1(u_1,\tilde{x}) \mid \Phi_{\tilde{x}}\right].$$

As in previous section, the decision problem with information can be put under the form:

$$\max_{u_0\in\mathcal{U}_0} \mathbb{E}\left[l_0(u_0,\tilde{x}_0)+V_\perp(f(\tilde{x}_0,u_0,w_0))+I_\Phi(f(\tilde{x}_0,u_0,w_0))\right]$$

and the comparison of initial decisions now relies on the expectation of I_Φ or $\Delta_{\Psi\Phi}$ as follows.

Proposition 4

If $u_0 \mapsto \mathbb{E}\left[\Delta_{\Psi\Phi}(f(\tilde{x}_0,u_0,w_0))\right]$ is monotone, comparison of the optimal decisions for the general problems with information structures Φ and Ψ will be possible. Precise conditions are the same as in Proposition 2.

It is self-explanatory that $\mathbb{E}\, I_\Phi(f(\tilde{x}_0,u_0,w_0))$ is the expected value of information after decision u_0, and $\mathbb{E}\,\Delta_{\Psi\Phi}(f(\tilde{x}_0,u_0,w_0))$ the expected value of exchanging the information structure Φ for Ψ. It is also possible to define the value of information conditional on a realization of w_0 or of $\tilde{x}_1$.

To end this section, let us point out that the introduction of the state x_t is by no means necessary. In all generality, a decision maker takes two successive decisions and solves

$$\max_{u_0} \max_{u_1\preceq\mathcal{G}(u_0),u_1\in\mathcal{U}(u_0)} \mathbb{E}(L(u_0,u_1,\omega)) \tag{11}$$

where L is a utility function depending upon decisions $u_0 \in \mathbb{U}_0 \subset \mathbb{R}$, $u_1 \in \mathbb{U}_1$ and as well upon a random element ω in $(\Omega,\mathcal{F},\mathbb{P})$; and $u_1 \preceq \mathcal{G}(u_0)$ means that $u_1 : \Omega \to \mathbb{U}_1$ is $\mathcal{G}(u_0)$-measurable. At the first period, the decision maker takes a deterministic decision u_0 while, at the second period, the information available is described by a subfield $\mathcal{G}(u_0)$ of $\mathcal{F}$. Given two decision problems characterized by utility functions L_+ and L_-, by domains $\mathcal{U}_+(u_0)$, $\mathcal{U}_-(u_0)$ and by information mappings $u_0 \hookrightarrow \mathcal{G}_+(u_0)$ and $u_0 \hookrightarrow \mathcal{G}_-(u_0)$, ranking of optimal initial decisions u_0^+

[6]It is sufficient to assume that the decision maker gets full information at time $t=1$ on a stochastic subcomponent $\tilde{y}_1$; then this information, $\tilde{y}_1$ should be explicitly included for conditioning the problem, even in the case where no additional information arrives.

and u_0^- is possible whenever the function

$$u_0 \hookrightarrow \mathbb{E}[\max_{u_1 \in \mathcal{U}_+(u_0)} \mathbb{E}(L_+(u_0, u_1, \omega) \mid \mathcal{G}_+(u_0)) - \max_{u_1 \in \mathcal{U}_-(u_0)} \mathbb{E}(L_-(u_0, u_1, \omega) \mid \mathcal{G}_-(u_0))] \tag{12}$$

is monotone. When $L_+ = L_-$ and $\mathcal{G}_+(u_0)$, $\mathcal{G}_-(u_0)$ are fixed subfields, this quantity may be interpreted as the variation of the value of information

5 Value of information as a key to the irreversibility literature

A goal of the literature on irreversibility and uncertainty consists in identifying hypotheses or conditions under which it is possible to compare efficient decisions made with different information structures. Two kinds of conditions can be examined. A first thread follows Epstein [8] and concentrates on determining the direction of the effect of learning for all possible random vectors γ over a finite sample set and for all comparable information structures. As Ulph and Ulph [32] noted, this restricts the conclusion to limited classes of problems, for example those later identified by Gollier et al. [13]. An other thread looks for specific problems where some kind of comparison is possible though Epstein's conditions do not apply, as in Ulph and Ulph [32].

In the above literature, the ranking of optimal initial decisions is obtained through conditions that depend upon at least one of the two solutions. We show here how such conditions may be bypassed by making use of the monotonicity of the value of information.

5.1 Epstein's Theorem and the value of information

Epstein's Theorem [8] may be stated as follows.

Let $(\Omega, \mathbb{P})$ be a finite probability space. Let Ψ and Φ be two information structures, with Ψ finer than Φ (in [8], these are random variables). Let u_0^Ψ and u_0^Φ denote the corresponding solutions (assuming unicity for simplicity) of $\max_{u_0} \mathbb{E}[\max_{u_1 \in \mathcal{U}(u_0)} \mathbb{E}(L(u_0, u_1, \cdot) \mid \Psi)]$ and $\max_{u_0} \mathbb{E}[\max_{u_1 \in \mathcal{U}(u_0)} \mathbb{E}(L(u_0, u_1, \cdot) \mid \Phi)]$.

For any distribution law ρ on Ω, let us define

$$J(u_0, \rho) \stackrel{\text{def}}{=} \max_{u_1 \in \mathcal{U}(u_0)} \mathbb{E}_\rho(L(u_0, u_1, \cdot)) = \max_{u_1 \in \mathcal{U}_1(x)} \int_\Omega L(u_0, u_1, \omega) \rho(d\omega) \tag{13}$$

In [8], any distribution law ρ on Ω is identified with an element of the simplex with dimension the number of elements of Ω.

Epstein's theorem states that, if $\frac{\partial J}{\partial u_0}(u_0^\Psi, \rho)$ exists and is concave (resp. convex) in ρ, then $u_0^\Psi \leq u_0^\Phi$ (resp. $\geq$). If $\frac{\partial J}{\partial u_0}(u_0^\Psi, \rho)$ is neither concave nor convex, the ranking is ambiguous.

We now replace the condition on $\frac{\partial J}{\partial u_0}(u_0^\Psi, \rho)$, which requires to know u_0^Ψ, with one which does not. We also relax the discrete probability and differentiability assumptions.

Proposition 5
Assume that

1. *for any* $u_0^+ \geq u_0^-$, $J(u_0^+, \rho) - J(u_0^-, \rho)$ *is convex (concave) in* ρ,
2. Ψ *is finer than* Φ.

Then the value of substituting Ψ *for* Φ, $u_0 \mapsto \Delta_{\Psi\Phi}(u_0)$, *is an increasing (a decreasing) function.*

Thus, initial decisions may be compared (see the remarks following Proposition 2). The proof is in Appendix A.4.

5.2 ‘ All or nothing ’ decision set (linear dynamics and costs)

The seminal literature as well as more recent contributions often considers linear dynamics and costs — which imply all or nothing decisions — or hinges directly on a binary decision set (see for instance [2, 16, 14, 9] and [17, part 2]). With a binary decision set, the monotonicity of the value of information becomes trivial. Moreover, the direction of variation is easily determined under the hypothesis of total irreversibility, *i.e.* when one of the two possible initial decisions affects the second period cost so that it does not depend any longer on the second period decision. This is for example the case with the model of Arrow and Fisher [2].

5.3 Value of information in Ulph and Ulph, 1997

Ulph and Ulph [32] developed a simple model of global warming where Epstein's conditions cannot apply. They proposed a specific condition that implies the irreversiblity effect. We show that their assumptions imply the monotonicity of the second-period value of information and can be generalized to any information structure.

The model examined in [32] can be rewritten with our formalism as follows

$$\max_{u_0}\left[l_0(u_0)+\mathbb{E}\max_{u_1}\left\{l_1(u_1)-\mathbb{E}[\gamma\mid\Phi]D(\delta x_1+u_1)\right\}\right] \tag{14}$$
$$\text{with}\quad x_{t+1}=\delta x_t+u_t\quad\text{and}\quad u_t\in[0,A_t],$$

where u are greenhouse gases (GHG) emissions, x GHG concentrations, l_t utilities, and D a damage function. A_t is the unrestricted level of emissions[7]. Functions l_t are assumed to be strictly increasing and strictly concave, and D strictly increasing and strictly convex. The r.v. γ is assumed to be non-negative.

The authors compare $u_0^\star$, the initial decision without information, and $u_0^{\star\star}$, the initial decision with perfect information structure (for instance $\Phi=\gamma$). With our notations, their theorem 3 states that:

if $(u_0^\star,u_1^\star)$ is such that $u_1^\star=0$, then $u_0^{\star\star}\leq u_0^\star$.

Two features are essential to this result. On the one hand, the assumption that the optimal policy, $u_1^\star=0$, is a corner solution in the second period. On the other hand, the shape of the payoff, which is linear in the random variable.

In fact, in Ulph and Ulph's model, the condition $u_1^\star=0$ implies that the second-period value of any information structure Φ is a decreasing function for $u_0\geq u_0^\star$ (the proof[8] is given in Appendix B). As a consequence, $u_0^\Phi\leq u_0^\star$ for any information structure Φ.

[7]Ulph and Ulph do not make this assuption which is implicit for the problem considered (greenhouse gases emissions cannot be infinite) and makes the demonstration easier.

[8]The intuition for monotonicity is as follows. The condition $u_1^\star=0$ implies that when no information is available, it is optimal to cut emissions to zero in $t=1$ if the GHG concentration x_1 is above a level $x^\star=\delta x_0+u_0^\star$. Conversely, when information is obtained when $x_1\geq x^\star$, it might open the opportunity to emit. The value of the information is then equal to the benefit of additional emissions in $t=1$ minus the expected additional damages. From the envelope theorem, these expected additional damages are strictly increasing at the margin for a small increase of concentration x_1, whereas benefits do not directly depend on concentration x_1. As a consequence, the value of information diminishes and Proposition 2 applies.

Ulph and Ulph however noted that the condition $u_1^\star = 0$ seems unlikely to be verified. They also checked with a numerical model that for different parameter values (discount rate, probabilities, damage cost parameter), the opposite of the irreversibility effect was generally obtained ($u_0^{\star\star} > u_0^\star$). In terms of value of information, the question arises whether emitting more today decreases or increases the value of future learning about the climate.

6 Near-term emission reductions and value of future information on the climate

As Ulph and Ulph [32] noted, it is not possible to conclude in advance and "as a matter of principle" about the direction of the effect of learning for the climate change issue. This would require the condition identified by Epstein, which is not met even in the "simplest model of global warming" that they set out.

Moreover, most of the theoretical models, including theirs', can hardly be used to help and interpret the results of integrated-assessment models (IAM) of climate and economics such as DICE [25, 26]. For instance, theoretical models tend to represent environment by a scalar and its dynamics by a linear function, whereas in DICE 98, the environment is a five-component vector and the dynamics for atmospheric temperature is non-linear.

By contrast, DICE can fit into the framework proposed in section 4, provided the simplification that the policy adopted before information occurs is summarized by a scalar.

6.1 Description and extension of the stochastic DICE model

The model is a stochastic optimal-growth model of the world economy. It is designed to maximize the discounted expected value of utility from consumption. The decisions $\mathbf{v}$ are the rate of investment and the rate of emissions reduction in greenhouse gases, so that $\mathbf{v}_\tau \in [0,1] \times [0,1]$. The state variable $\mathbf{z}_\tau$ comprises the stock of capital; concentrations of carbon in three reservoirs (atmosphere; biosphere and surface ocean; deep ocean); and oceanic and atmospheric global mean temperature rises with respect to pre-industrial times. The temperature components of $\mathbf{z}$ are stochastic. Uncertainty enters their dynamics through the *climate sensitivity* γ, which remains unobserved until year 2040. This random variable is constant through time with values 2.5 °C, 3.5 °C and 4.5 °C corresponding to the atmospheric temperature rise for a permanent doubling of the carbon concentration in the atmosphere. These values are within the range reported by the IPPC [18, chapter IX]. In time step $\tau = 0$, the true atmospheric temperature rise is also uncertain. The model operates in time steps of 10 years. Perfect information on climate sensitivity γ is obtained in 2040.

The stochastic version of the DICE model [25, chap. 8] with learning in 2040 (time step $\tau = 4$) has the following structure:

$$\max_{\mathbf{v}_0,\ldots,\mathbf{v}_3} \mathbb{E}\left\{ \sum_{\tau<4} L_\tau(\mathbf{v}_\tau, \mathbf{z}_\tau) + \mathbb{E}\left[\max_{(\mathbf{v}_\tau)_{\tau\geq 4}} \sum_{\tau=4}^{T} L_\tau(\mathbf{v}_\tau, \mathbf{z}_\tau) \,\middle|\, \gamma \right] \right\} \tag{15}$$
$$\text{with} \quad \mathbf{z}_{\tau+1} = G(\mathbf{z}_\tau, \mathbf{v}_\tau, \gamma) \ \in \mathbb{R}^6$$

Some of the detailed climate-economy equations in G depart from the original DICE model. The temperature increase equation is an updated calibration that provides a better description of warming over forthcoming decades. A threshold damage function replaces the original quadratic one. Both climate module and damage function are taken from Ambrosi et al. [1]. The full description for the original DICE model can be found in Nordhaus [25] or Nordhaus and Boyer [26].

In order to apply the formalism of section 4, we summarize the initial policy $(\mathbf{v}_0, \ldots, \mathbf{v}_3)$ with a function $R : [0,1] \mapsto \mathbb{R}_+^{4\times 2}$, constructed so that optimal decisions $(\mathbf{v}_0^{\#}, \ldots, \mathbf{v}_3^{\#})$ in the original model can be approximated with $R(\mathrm{v}_3^{\#\,\text{abat. rate}})$ under different scenarios. Details for this parameterization are in Appendix C. The model in Eq. (15) is then solved with the constraint $(\mathbf{v}_0, \ldots, \mathbf{v}_3) = R(\mathrm{v}_3^{\text{abat. rate}})$. The model is formulated in GAMS and solved with MINOS. The code is available from the corresponding author upon request.

Finally, 'first period $t = 0$' in section 4 shall refer to the time steps $\tau = 0, \ldots, 3$ and 'second period $t = 1$' shall refer to the time steps $\tau \geq 4$. The initial policy is summarized through $u_{\text{initial}} \stackrel{\text{def}}{=} 1 - \mathrm{v}_3^{\text{abat. rate}}$, the *rate of emissions allowance* at time step $\tau = 3$ and we will plot the second-period value of information as a function of the initial policy choice u_{initial}.

6.2 Results

Figure 1 plots the expected value of information as a function of the initial emission policy. Available initial decisions range from no effort until 2039 (100% emissions allowance) to targeting the maximum effort in 2039 (0% allowance). Three cases are presented corresponding to three different probability distributions for γ: optimistic case, centered case and pessimistic case (see Appendix C).

In all cases, the prospect of learning the true value of γ in 2040 is an opportunity to allow initially more emissions (less reduction efforts) than in the never-learn situation ($u_{\text{initial}}^{\star\star} > u_{\text{initial}}^{\star}$). Here, the effect of learning is not precautionary. This is usually found in the empirical literature [25, 32, 1], but as far as we know, empirical models like this one remain out of bounds for the existing analytical literature about irreversibility, learning and climate change. Therefore it could offer no explanation for such results. However, in all three cases, the direction of the effect of learning can be related to the monotonicity of the expected value of information. In the DICE model — with a modified threshold damage function — it turns out that allowing more emissions now tends to increase the desirability of getting information in the future.

The 'effect of learning' (the difference between $u_{\text{initial}}^{\star}$ and $u_{\text{initial}}^{\star\star}$) that we find increases emissions allowance in 2030 from 71 to 76% with centered probabilities, from 66 to 69% with pessimistic probabilities, 75 to 80% with optimistic ones. The corresponding decrease in the abatement rate (from 28 to 24%; 34 to 31%; 25 to 20%), ranges from a factor 0.91 to 0.79. In terms of abatement costs[9] it yields a reduction which is roughly between 18% and 40%. Clearly, learning has an effect on decision which is not negligible. This is in contrast with earlier results by Nordhaus [25] or Ulph and Ulph.

In an analytical framework with a linear dynamics, Gollier et al. [13] showed that logarithmic utility implies that the structure of information has no effect on the initial decision. They wondered whether this was the explanation for the little or nonexistent effect of learning found in these earlier results. Our model departs from Nordhaus' DICE with some specifications of the dynamics (temperature model and damage function). But the utility function is logarithmic as it is in DICE. Thus, our findings answer the question raised in [13] and show that the weak effect of learning found by Nordhaus is also determined by his temperature dynamics and damage function and not solely by the logarithmic utility function. This also emphasizes the difficulty to apply results from the analytical literature for interpreting integrated assessment models.

[9] In DICE 98, the abatement costs are proportional to the rate of abatement raised at the power 2.15.

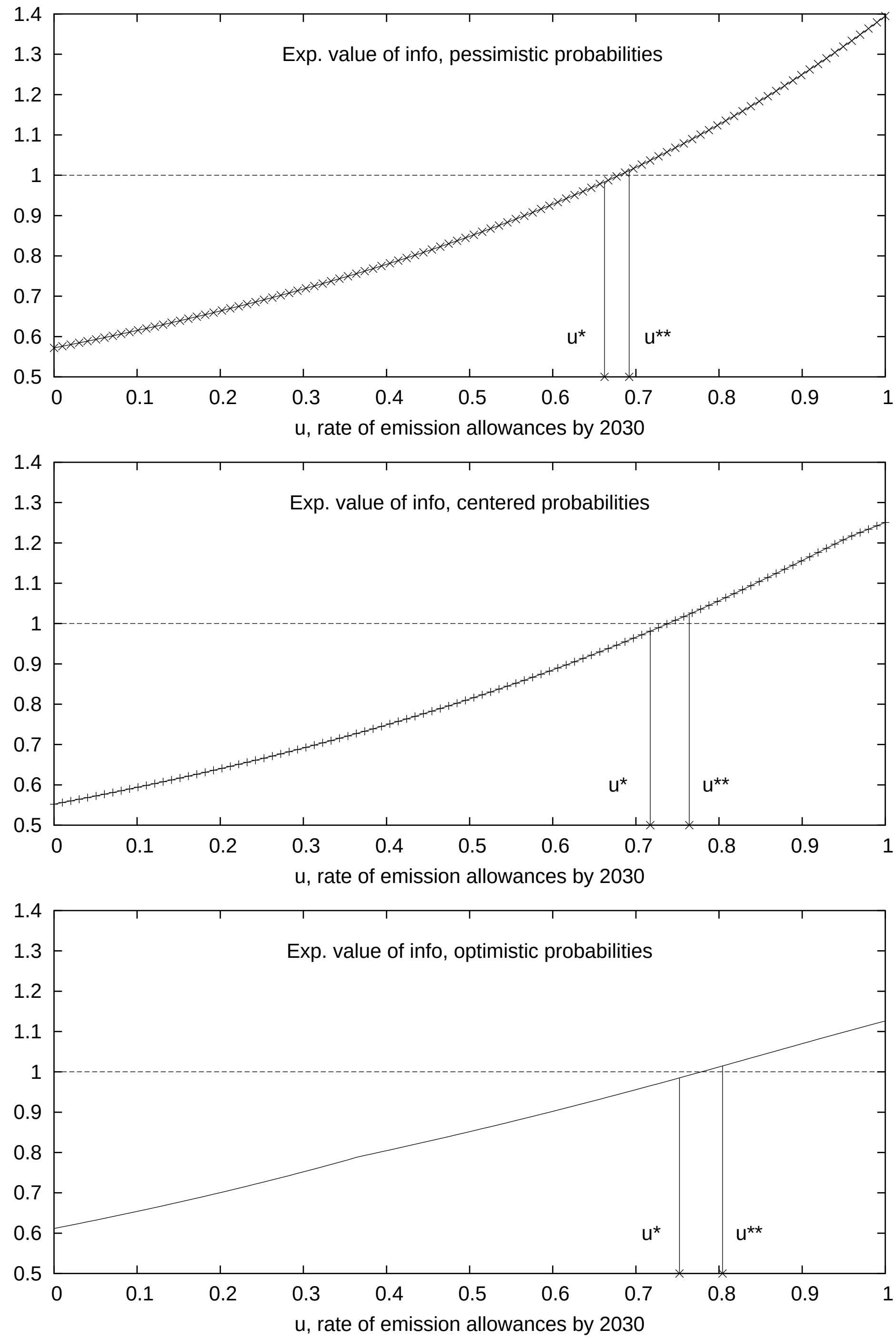

Figure 1: Expected value of information in 2040 as a function of initial policy.
Initial policy is summarized by u_{initial}, the rate of emissions allowance in 2030–2039. In each case, expected value of information increases with the emission allowance: the more we emit before 2040, the more we are willing to pay for information on the climate in 2040. As a consequence, the optimal initial policy with future learning, $u^{\star\star}_{\text{initial}}$, allows more emissions than the optimal initial policy without future learning, $u^{\star}_{\text{initial}}$ (Proposition 2). The expected value of information has been normalized with $\mathbb{E}I^0$, the expected value of information before any decision is made. Note that this normalization is different in each case. It also shows that $\mathbb{E}I^0$ is upper bounded by the expected value of information after decision $u^{\star\star}_{\text{initial}}$ and lower bounded by the expected value of information after decision $u^{\star}_{\text{initial}}$ (Proposition 1).

7 Conclusion

This article put forward the usefulness of the concept of value of information in the analysis of sequential decision problems. The difference between value of future information before and after an initial decision is taken was made explicit. The *second-period value of information* should be viewed as a function of the initial decision and its monotonicity is sufficient for making a conclusion about the direction of the effect of learning. Interestingly, many of the conditions given in the literature concerning the irreversibility effect can be related to this monotonicity. However, the present analysis shares a common limitation with the irreversibility literature: the initial decision is assumed to be scalar. But extension is available in theory. As long as the set of admissible initial decisions can be ordered even incompletely, supermodularity results as in [30] lead to a similar conclusion. However, the difficulty is to find a meaningful order over the decision set.

Consequently, in the case of the DICE integrated assessment model[10] we have parameterized the decisions over 2000–2039 with the level of emission abatement by 2030. The value of the information on the climate is defined as a function of this scalar. With different probability distributions over the climate sensitivity, we find that the more we allow emissions over 2000-2039, the more the value of information forthcoming in 2040 increases. In those cases, it sheds light on why the effect of learning is not precautionary, a result often found in the climate change literature[11].

Appendix

A Proofs

A.1 Proof of Proposition 1

By definition, the initial value of information is

$$I^0 \overset{\text{def}}{=} \overbrace{\max_{u_0 \in \mathcal{U}_0} \left[l_0(u_0) + V_\perp(f(u_0)) + I_\Phi(f(u_0)) \right]}^{\mathcal{J}_\Phi \overset{\text{def}}{=}} \\ - \underbrace{\max_{u_0 \in \mathcal{U}_0} \left[l_0(u_0) + V_\perp(f(u_0)) \right]}_{\mathcal{J}_\perp \overset{\text{def}}{=}} .$$

Since $u_0^\star$ is an optimal solution of the problem without information and since u_0^Φ is an optimal solution of the problem with information, we have, on the one hand,

$$\mathcal{J}_\perp = l_0(u_0^\star) + V_\perp(f(u_0^\star)) \geq \underbrace{l_0(u_0^\Phi) + V_\perp(f(u_0^\Phi))}_{\mathcal{I}_\Phi - I_\Phi(f(u_0^\Phi))}$$

so that $\mathcal{I}_\Phi - \mathcal{I}_\perp \leq I_\Phi(f(u_0^\Phi))$.

[10] With a modified threshold damage function and a modified climate module.

[11] The question remains whether certainty about the future evolution of the climate could be obtained as soon as 2040. Kelly and Kolstad [20] suggest that certainty on the true value of the climate sensitivity with less than 5% rejection might be available only after 2090. For this reason, Keller et al. [19] explore a case where perfect knowledge is gained in 2085 only. It has however the opposite limitation to assume that our knowledge does not improve from 2000 to 2085.

On the other hand,

$$\mathcal{I}_\Phi = l_0(u_0^\Phi) + V_\perp(f(u_0^\Phi) + I_\Phi(f(u_0^\Phi)) \geq \underbrace{l_0(u_0^\star) + V_\perp(f(u_0^\star))}_{\mathcal{J}_\perp} + I_\Phi(f(u_0^\star))$$

so that $\mathcal{I}_\Phi - \mathcal{I}_\perp \geq I_\Phi(f(u_0^\star))$. Combining both inequalities, we obtain

$$I_\Phi(f(u_0^\star)) \leq I^0 = \mathcal{J}_\Phi - \mathcal{J}_\perp \leq I_\Phi(f(u_0^\Phi))$$

which is Proposition 1.

Similarly we obtain easily:

$$\Delta_{\Psi\Phi}(f(u_0^\Phi)) \leq \mathcal{J}_\Psi - \mathcal{J}_\Phi \leq \Delta_{\Psi\Phi}(f(u_0^\Psi)) \tag{16}$$

where u_0^Ψ (respectively u_0^Φ) is any optimal initial decision for the problem with the information structure Ψ (respectively Φ). Note that, without specific hypothesis on the relative informativeness of Φ and Ψ, Δ can assume negative values and $\mathcal{J}_\Psi - \mathcal{J}_\Phi$ can be negative.

A.2 General results on comparison of arg max

We recall here some results on comparisons between the arg max of two optimization problems. They may be seen as particular instances of results from a general theory with supermodular functions or functions with increasing differences, as developed in [31].

PROPOSITION 6

Let $\mathcal{D} \subset \mathbb{R}$, let $g : \mathcal{D} \to \mathbb{R}$ and $h : \mathcal{D} \to \mathbb{R}$. We denote

$$\mathcal{D}_g \stackrel{\text{def}}{=} \arg\max_{u\in\mathcal{D}} g(u) \subset \mathcal{D} \quad \textit{and} \quad \mathcal{D}_{g+h} \stackrel{\text{def}}{=} \arg\max_{u\in\mathcal{D}} (g+h)(u) \subset \mathcal{D}\,,$$

and we assume that $\mathcal{D}_g \neq \emptyset$ and $\mathcal{D}_{g+h} \neq \emptyset$.

1. *If h is strictly increasing on $]-\infty, \sup\mathcal{D}_g]$, then*
$$\sup\mathcal{D}_g \leq \inf\mathcal{D}_{g+h}\,.$$
2. *If h is increasing on $]-\infty, \sup\mathcal{D}_g]$, then*
$$\sup\mathcal{D}_g \leq \sup\mathcal{D}_{g+h}\,.$$
3. *If h is strictly decreasing on $[\inf\mathcal{D}_g, +\infty[$, then*
$$\sup\mathcal{D}_{g+h} \leq \inf\mathcal{D}_g\,.$$
4. *If h is decreasing on $[\inf\mathcal{D}_g, +\infty[$, then*
$$\inf\mathcal{D}_{g+h} \leq \inf\mathcal{D}_g\,.$$

Proof. We prove the first statement, the others being minor variations.

Let $u_g^\sharp \in \mathcal{D}_g$. For any $u \in \mathcal{D}$, we have $g(u) \leq g(u_g^\sharp)$. For any $u \in]-\infty, u_g^\sharp[$, we have $h(u) < h(u_g^\sharp)$ if h is strictly increasing. Thus

$$u \in]-\infty, u_g^\sharp[\Rightarrow g(u) + h(u) < g(u_g^\sharp) + h(u_g^\sharp)\,.$$

We conclude that $\mathcal{D}_{g+h} \subset [u_g^\sharp, +\infty[$, so that

$$\mathcal{D}_{g+h} \subset \bigcap_{u_g^\sharp \in \mathcal{D}_g} [u_g^\sharp, +\infty[= [\sup \mathcal{D}_g, +\infty[\,,$$

which proves that $\sup \mathcal{D}_g \leq \inf \mathcal{D}_{g+h}$. □

A.3 Proof of Proposition 2

The proof of Proposition 2 is a straightforward consequence of Proposition 6 with $u_0 \mapsto l_0(u_0) + V_\Phi(f(u_0)) + \Delta_{\Psi\Phi}(f(u_0))$ as function g and $u_0 \mapsto -\Delta_{\Psi\Phi}(f(u_0))$ as function h.

Freixas et Laffont [11] propose a similar proof for a case with simplified dynamics and criteria (see section 3.3).

A.4 Proof of Proposition 5

Let $\mathcal{P}(\Omega)$ be the set of all distributions on Ω, the set of the states of the world. By classical arguments [5, p. 77] (as soon as Ω is a complete separable metric space for instance), there exists a regular conditional probability of $\mathbb{P}$ given Φ, denoted by $\mathbb{P}^\Phi : \Omega \times \mathcal{F} \to [0,1]$ and characterized by:

1. $\forall \omega \in \Omega$, $\mathbb{P}^\Phi(\omega, \cdot) \in \mathcal{P}(\Omega)$;
2. $\forall A \in \mathcal{F}$, $\omega \hookrightarrow \mathbb{P}^\Phi(\omega, \cdot)$ is measurable with respect to Φ;
3. for all bounded random variable Z, $\mathbb{E}(Z \mid \Phi)(\omega) = \int_\Omega Z(\omega')\mathbb{P}^\Phi(\omega, d\omega')$, for $\mathbb{P}$-almost all ω.

The sensor[12] associated to $\mathbb{P}$ and Φ is the random measure $S^\Phi \in \mathcal{P}(\mathcal{P}(\Omega))$ image of the measure $\mathbb{P}$ by the mapping

$$\omega \in \Omega \hookrightarrow \mathbb{P}^\Phi(\omega, \cdot) \in \mathcal{P}(\Omega)\,. \tag{17}$$

It is shown in Artstein and Wets [4] that, as soon as measurability and integrability assumptions hold true,

$$\begin{aligned}
\mathbb{E}\left(\max_{u_1 \in \mathcal{U}_1(x)} \mathbb{E}\left[L(u_0, u_1, \omega) \mid \Phi\right]\right) &= \int_\Omega \mathbb{P}(d\omega)\left(\max_{u_1 \in \mathcal{U}_1(x)} \int_\Omega L(u_0, u_1, \omega')\mathbb{P}^\Phi(\omega, d\omega')\right) \\
&= \int_{\mathcal{P}(\Omega)} dS^\Phi(\rho)\left(\max_{u_1 \in \mathcal{U}_1(x)} \int_\Omega L(u_0, u_1, \omega')\rho(d\omega')\right) \\
&= \int_{\mathcal{P}(\Omega)} dS^\Phi(\rho) J(u_0, \rho)\,.
\end{aligned}$$

Thus, by (6) and (5), we have

$$\begin{aligned}
\Delta_{\Psi\Phi}(u_0) &= \mathbb{E}\left(\max_{u_1 \in \mathcal{U}_1(x)} \mathbb{E}\left[L(u_0, u_1, \omega) \mid \Psi\right]\right) - \mathbb{E}\left(\max_{u_1 \in \mathcal{U}_1(x)} \mathbb{E}\left[L(u_0, u_1, \omega) \mid \Phi\right]\right) \\
&= \int_{\mathcal{P}(\Omega)} dS^\Psi(\rho) J(u_0, \rho) - \int_{\mathcal{P}(\Omega)} dS^\Phi(\rho) J(u_0, \rho)\,.
\end{aligned}$$

[12] A sensor is a probability law on the set $\mathcal{P}(\Omega)$ of all distributions on the states of the world, *i.e.* an element of $\mathcal{P}(\mathcal{P}(\Omega))$, the Borel space of probability measures on $\mathcal{P}(\Omega)$. Following [3], an information structure can be defined by a sensor since it governs which posterior beliefs will be materialized at the time of decision.

Still following [4] and [3], if Ψ is finer than Φ, then S^{Ψ} is more refined than S^{Φ} in the sense that for all $\phi : \mathcal{P}(\Omega) \to \mathbb{R}$ convex,

$$\int_{\mathcal{P}(\Omega)} \phi(\rho) dS^{\Psi}(\rho) \geq \int_{\mathcal{P}(\Omega)} \phi(\rho) dS^{\Phi}(\rho) \,. \tag{18}$$

Thus, under the assumptions, the value of substituting Ψ for Φ, $u_0 \mapsto \Delta_{\Psi\Phi}(u_0)$, is an increasing (a decreasing) function.

B Extension of Ulph and Ulph's result, 1997

We express $\dfrac{dI_{\Phi}}{dx_1} = \dfrac{dV_{\Phi}}{dx_1} - \dfrac{dV_{\perp}}{dx_1}$ for the problem (14).

The optimal feedback without information is given by

$$\hat{u}_1(x_1) \stackrel{\mathrm{def}}{=} \arg \overbrace{\max_{u_1 \geq 0} \left[l_1(u_1) - \mathbb{E}\gamma \, D(u_1 + \delta x_1) \right]}^{V_{\perp}(x_1)} \,.$$

Unicity of the arg max results from the strict concavity of the mapping $u_1 \mapsto l_1(u_1) - \mathbb{E}\gamma \, D(u_1 + \delta x_1)$ since, by assumption, l_1 is strictly concave, D is strictly convex, and $\gamma \geq 0$.

Denoting $x_1^{\star} \stackrel{\mathrm{def}}{=} \delta x_0 + u_0^{\star}$, we have $u_1^{\star} = \hat{u}_1(x_1^{\star})$ by definition. From Euler's characterization of the maximum of a concave function, the assumption $u_1^{\star} = 0$ implies that $l'(0) - \delta \mathbb{E}\gamma D'(\delta x_1^{\star}) \leq 0$. Now, since $-D'$ is decreasing (D is convex), we have for any $x_1 \geq x_1^{\star}$,

$$l'(0) - \delta \mathbb{E}\gamma D'(\delta x_1) \leq l'(0) - \delta \mathbb{E}\gamma D'(\delta x_1^{\star}) \leq 0$$

Thus, by Euler's condition, we get $\hat{u}_1(x_1) = 0$. Replacing in $V_{\perp}(x_1)$ and differentiating with respect to x_1, we obtain

$$\frac{dV_{\perp}}{dx_1}(x_1) = -\mathbb{E}\left[\gamma\right] \delta D'(\delta x_1).$$

We now turn to $\frac{dV_{\Phi}}{dx_1}(x_1)$. Let us define

$$u_1^{\Phi}(x_1) \stackrel{\mathrm{def}}{=} \arg \max_{u_1} l_1(u_1) - \mathbb{E}[\gamma \mid \Phi] D(u_1 + \delta x_1)$$

which is a random variable.

By the Danskin theorem (see [6]), we get

$$\frac{d}{dx_1} \max_{u_1} l_1(u_1) - \mathbb{E}[\gamma \mid \Phi] D(u_1 + \delta x_1) = -\mathbb{E}[\gamma \mid \Phi] \delta D'(\delta x_1 + u_1^{\Phi}(x_1)) \,.$$

By differentiating under the integral sign, it comes

$$\frac{dV_{\Phi}}{dx_1}(x_1) = \mathbb{E}\big[-\mathbb{E}[\gamma \mid \Phi] \delta D'(\delta x_1 + u_1^{\Phi}(x_1))\big]$$

Finally,

$$\begin{aligned}
\frac{dI_\Phi}{dx_1}(x_1) &= \mathbb{E}\big[-\mathbb{E}[\gamma \mid \Phi]\delta D'(\delta x_1 + u_1^\Phi(x_1))\big] + \mathbb{E}\left[\gamma\right]\delta D'(\delta x_1) \\
&= \mathbb{E}\big[-\mathbb{E}[\gamma \mid \Phi]\delta D'(\delta x_1 + u_1^\Phi(x_1))\big] + \mathbb{E}\big[\mathbb{E}\left[\gamma \mid \Phi\right]\big]\,\delta D'(\delta x_1) \\
&= \mathbb{E}\big[\mathbb{E}\left[\gamma \mid \Phi\right](D'(\delta x_1) - D'(\delta x_1 + u_1^\Phi(x_1)))\big]
\end{aligned}$$

which is non-positive since $u_1^\Phi(x_1, s) \geq 0$ and D is convex. Therefore $u_0 \mapsto I_\Phi(\delta x_0 + u_0)$ is decreasing for all u_0 greater than $u_0^\star$: if initial GHG emissions are above their optimal level without information, increasing these emissions diminishes the value of information.

C Details for the numerical model

A more comprehensive description of the model, parameterization and results is included in a supplementary document available from the corresponding author.

C.1 Parameterization of the initial policy

An initial policy $(\mathbf{v}_\tau)_{\tau=0,\ldots,3}$ comprises emission abatement rates, $(\mathrm{v}_\tau^{\mathrm{abat.}})_{\tau=0,\ldots,3}$ and investment rates $(\mathrm{v}_\tau^{\mathrm{inv.}})_{\tau=0,\ldots,3}$. We call u_{initial} the emission allowance rate in time step $\tau = 3$ and define the initial policy summarized by u_{initial} and by function $R : [0,1] \mapsto \mathbb{R}_+^{4\times 2}$: it is given by the constraint $(\mathbf{v}_\tau)_{\tau=0,\ldots,3} = R(1 - u_{\mathrm{initial}})$ where $\mathrm{v}_3^{\mathrm{abat.}} = 1 - u_{\mathrm{initial}}$.

The function R is constructed in order to approximate the optimal policies obtained by DICE under different scenarios on information and climate sensitivity. For instance, if $\mathrm{v}_3^{\#\,\mathrm{abat.}}$ is the optimal value for $\mathrm{v}_3^{\mathrm{abat.}}$ in the DICE model with early information in 2000 and climate sensitivity of 2.5 °C, we would like the values of the 8-tuple $R(\mathrm{v}_3^{\#\,\mathrm{abat.}})$ to be as close as possible to the optimal values for $(\mathrm{v}_\tau^{\mathrm{abat.}}, \mathrm{v}_\tau^{\mathrm{inv.}})_{\tau=0,\ldots,3}$ in the same scenario.

Four scenarios have been used for the construction of R:

- H1, H2, H3 (certainty scenarios): the sensitivity of the climate, γ, is known from time step $\tau = 0$ and assumes respectively the values 2.5 °C, 3.5 °C and 4.5 °C .
- H4 (uncertainty with learning): the true value of γ is unkown in the near-term. Uncertainty is resolved at time step $\tau = 4$. Pessimistic probabilities (see section C.2 below) are used. below).

Figure 2 displays as dots the optimal policy $(\mathbf{v}_\tau^{\#})_{\tau=0,\ldots,3}$ for each scenario H1–H4 (abatement rate in the left panel, investment rate in the right panel). The initial policies defined by $R(\mathrm{v}_3^{\#\,\mathrm{abat.}})$ are shown as lines for each optimal value of $\mathrm{v}_3^{\mathrm{abat.}}$ obtained under scenarios H1 to H4.

C.2 Probability distributions

.

	Climate sensitivity		
	2.5 °C	3.5 °C	4.5 °C
optimistic	2/3	1/3	1/3
centered	1/3	2/3	1/3
pessimistic	1/3	1/3	2/3

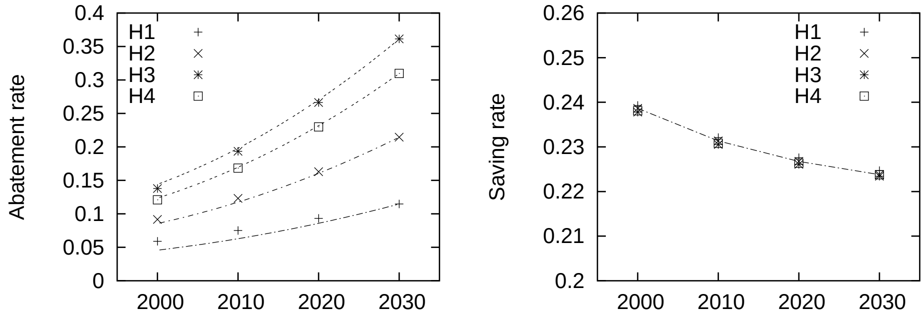

Figure 2: Parameterization of policy before 2040

The left panel shows the abatement rate component of the initial policy (time steps $\tau = 0, \ldots, 3$), and the right panel the investment rate component. Optimal values of the policy under hypotheses H1–4 are shown as dots. The lines trace the initial policy values defined by the parameterization $R(\mathrm{v}_3^{\#\,\text{abat. rate}})$.

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