Méthode de quasi-Newton en interaction fluide-structure

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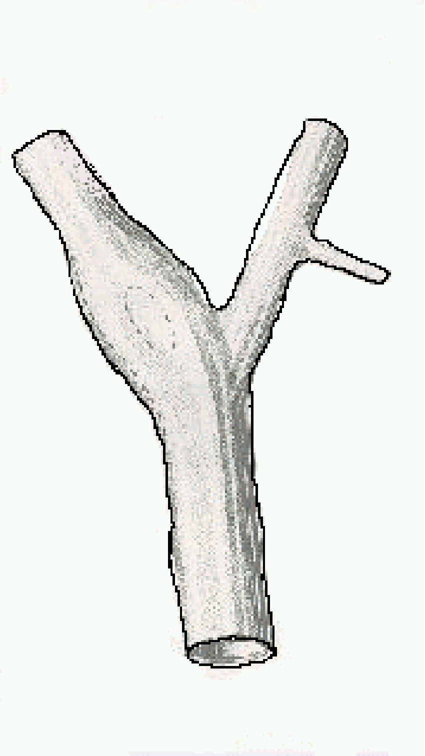
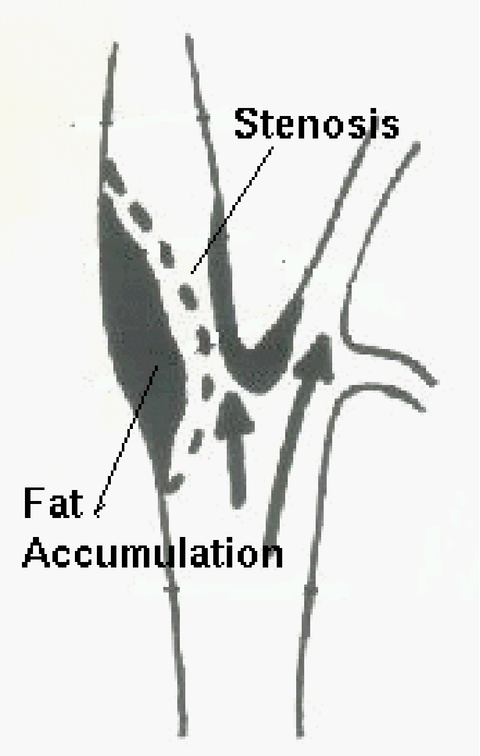
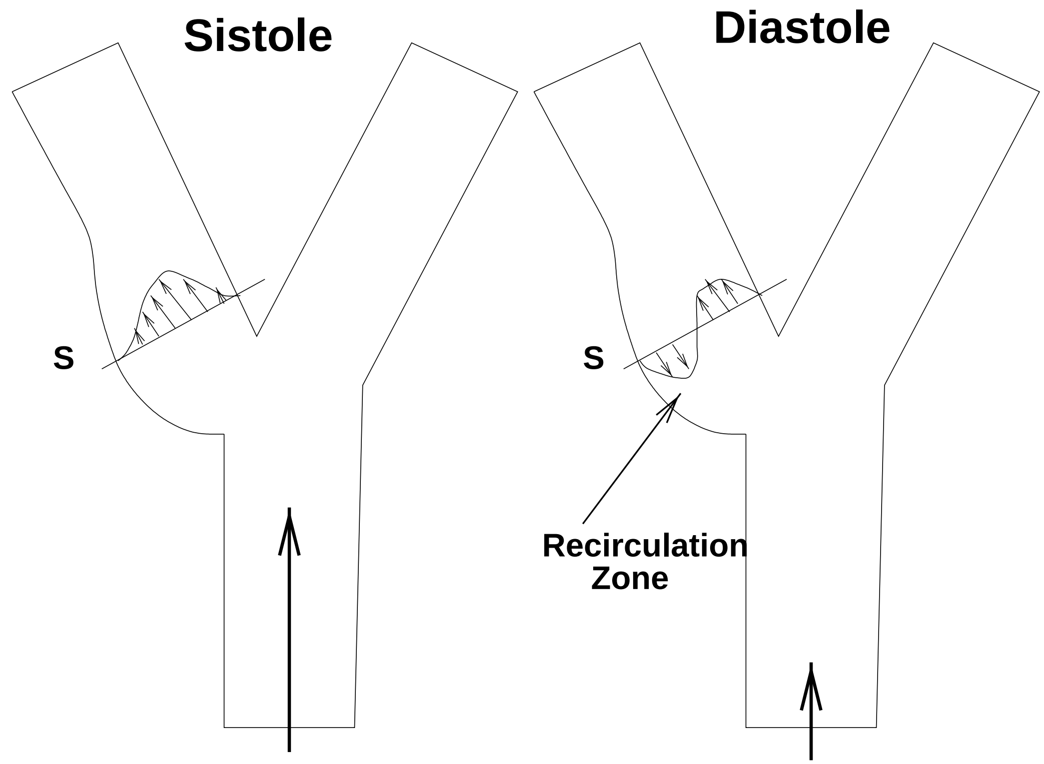
Motivations médicales

Athérosclérose :





La formation des plaques est liée, entre autre, la dynamique du sang et aux contraintes exercées sur la paroi.

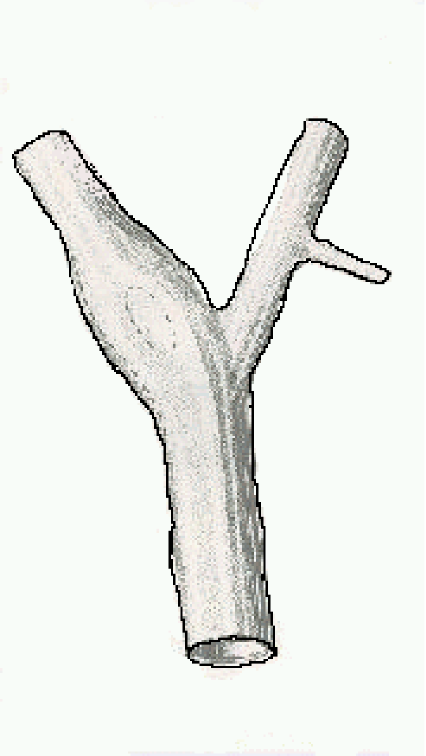


Ce que pourrait permettre la simulation :

- mieux comprendre le phénomène
- prévoir les conséquences d'opérations chirurgicales
- améliorer les prothèses

1. Problem statement

Purpose : simulate the mechanical interaction between the wall and the blood in a portion of large vessels



1.1. General equations Fluid models :

Navier-Stokes 2D or 3D ($\boldsymbol{u}, p$) in Arbitrary Lagrange Euler formulation ($\boldsymbol{w}$)

Structure models :

- Structure 1D ($\boldsymbol{d} = d_r \boldsymbol{e}_r$) : cylindrical geometry, linear elasticity

$$\rho_w h \frac{\partial^2 d_r}{\partial t^2} - kGh \frac{\partial^2 d_r}{\partial z^2} + \frac{Eh}{1-\nu^2} \frac{d_r}{R_0^2} - \gamma \frac{\partial^3 d_r}{\partial z^2 \partial t} = \boldsymbol{f}_\Sigma$$

- Structure 2D ($\boldsymbol{d}$). Shell model in large displacements (MITC)

Coupling Fluid-Structure interface Σ_t (reference configuration $\hat{\Sigma}$) :

$$\boldsymbol{u}|_{\Sigma_t} = \frac{\partial \boldsymbol{d}}{\partial t}|_{\hat{\Sigma}}$$

$$< \boldsymbol{f}_\Sigma, \boldsymbol{\varphi} >= \int_{\Sigma_t} (p\boldsymbol{n} - 2\mu\epsilon(\boldsymbol{u}) \cdot \boldsymbol{n}) \cdot \boldsymbol{\varphi} \, d\sigma$$

1.2. Variational formulation

Let $\hat{\boldsymbol{v}}$ and $\boldsymbol{\varphi}$ test fonctions independent of t on $\hat{\Omega}$, with $\boldsymbol{v} = 0$ on Σ_t.

$$\begin{aligned}
\frac{d}{dt}\int_{\Omega_F(t)} \rho_F \boldsymbol{u}\cdot\boldsymbol{v}\,dx + \int_{\Omega_F(t)} \rho_F(\boldsymbol{u}-\boldsymbol{w})\cdot\boldsymbol{\nabla}\boldsymbol{u}\cdot\boldsymbol{v}\,dx & \\
-\int_{\Omega_F(t)} \rho_F\boldsymbol{u}\cdot\boldsymbol{v}\ \mathrm{div}\ \boldsymbol{w}\,dx - \int_{\Omega_F(t)} p\ \mathrm{div}\ \boldsymbol{v}\,dx + \int_{\Omega_F(t)} 2\mu\epsilon(\boldsymbol{u})\cdot\epsilon(\boldsymbol{v})\,dx &= 0\\
\int_{\Omega_F(t)} q\,\mathrm{div}\ \boldsymbol{u} &= 0\\
\int_{\Omega_F(t)} \boldsymbol{\nabla}\boldsymbol{w}\cdot\boldsymbol{\nabla}\boldsymbol{v} &= 0\\
\int_{\hat{\Omega}_S} \rho_S\frac{\partial^2\boldsymbol{d}}{\partial t^2}\cdot\boldsymbol{\varphi}\,d\hat{x} + a(\boldsymbol{d},\boldsymbol{\varphi}) = <\boldsymbol{f}_{\Sigma_t},\boldsymbol{\varphi}>
\end{aligned}$$

1.3. Time discretization. Fluid. (implicit Euler)

$$
\begin{aligned}
\frac{1}{\delta t}\int_{\Omega_F^{n+1}} \rho_F \boldsymbol{u}^{n+1}\cdot\boldsymbol{v} + \int_{\Omega_F^{n+1}} \rho_F(\boldsymbol{u}^{n+1}-\boldsymbol{w}^{n+1})\cdot\boldsymbol{\nabla}\boldsymbol{u}^{n+1}\cdot\boldsymbol{v} - \\
\int_{\Omega_F^{n+1}} \rho_F \boldsymbol{u}^{n+1}\cdot\boldsymbol{v}\ \mathrm{div}\ \boldsymbol{w}^{n+1} - \int_{\Omega_F^{n+1}} p\ \mathrm{div}\ \boldsymbol{v} + \int_{\Omega_F^{n+1}} 2\mu\epsilon(\boldsymbol{u}^{n+1})\cdot\epsilon(\boldsymbol{v}) &= \frac{1}{\delta t}\int_{\Omega_F^{n}} \rho_F \boldsymbol{u}^{n}\cdot\boldsymbol{v} \\
\int_{\Omega_F^{n+1}} q\ \mathrm{div}\ \boldsymbol{u}^{n+1} &= 0 \\
\boldsymbol{u}^{n+1}|_{\Sigma^{n+1}} &= \frac{\boldsymbol{d}^{n+1}-\boldsymbol{d}^{n}}{\delta t} \\
\int_{\Omega^{n+1}} \boldsymbol{\nabla}\boldsymbol{w}^{n+1}\cdot\boldsymbol{\nabla}\boldsymbol{v} &= 0 \\
\boldsymbol{w}^{n+1}|_{\Sigma^{n+1}} &= \boldsymbol{u}^{n+1}|_{\Sigma^{n+1}}
\end{aligned}
$$

$$\boxed{\boldsymbol{f}_{\Sigma^{n+1}} = \mathcal{F}(\boldsymbol{d}^{n+1})}$$

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1.4. Time discretization. Structure. (mid-point)

$$\int_{\hat{\Omega}_S} \rho_S \frac{\boldsymbol{v}_S^{n+1} - \boldsymbol{v}_S^n}{\delta t} \cdot \boldsymbol{\varphi} \, d\hat{x} + \frac{1}{2} a(\boldsymbol{d}^n, \boldsymbol{\varphi}) + \frac{1}{2} a(\boldsymbol{d}^{n+1}, \boldsymbol{\varphi}) = < \boldsymbol{f}_{\Sigma^{n+1}}, \boldsymbol{\varphi} >$$

$$\frac{\boldsymbol{d}^{n+1} - \boldsymbol{d}^n}{\delta t} = \frac{\boldsymbol{v}_S^{n+1} + \boldsymbol{v}_S^n}{2}$$

where $< \boldsymbol{f}_{\Sigma^{n+1}}, \boldsymbol{\varphi} >$ is the “residual” of the fluid problem

$$\boxed{\boldsymbol{d}^{n+1} = \mathcal{S}(\boldsymbol{f}_{\Sigma^{n+1}})}$$

1.5. Fluid/structure coupling.

Two main approaches :

(i) Staggered schemes *(Piperno, Fahrat, Larrouturou, ...)*

At each time step : basically one resolution of $\mathcal{F}$ and one resolution of $\mathcal{S}$.

$\longrightarrow$ this seems to be unsuitable for blood flows

(ii) Implicit schemes *(Le Tallec & Mouro)*

At each time step a nonlinear problem to be solved :

$$\boxed{\boldsymbol{d}^{n+1} = \mathcal{S} \circ \mathcal{F}(\boldsymbol{d}^{n+1})}$$

Advantages : stability (energy balance)

Drawback : expensive !

2. Numerical algorithms

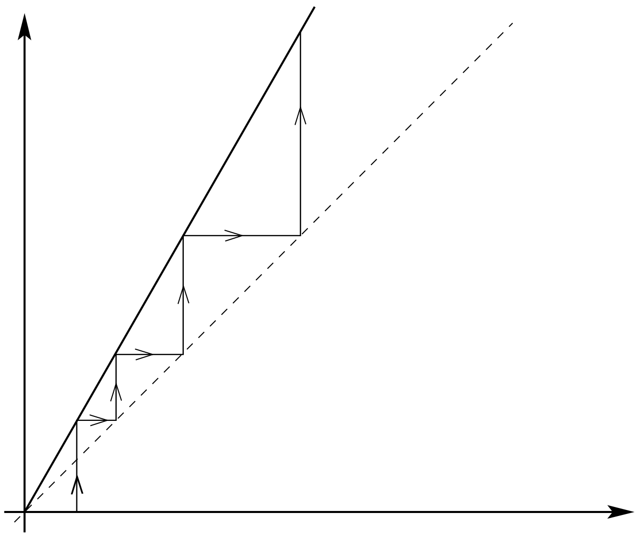
2.1. Fixed point algorithms

"Naive" fixed point algorithm

(i) $\boldsymbol{d}_0 = \boldsymbol{d}^n$

(ii) $\boldsymbol{d}_{k+1} = \mathcal{S} \circ \mathcal{F}(\boldsymbol{d}_k)$

$\longrightarrow$ does not converge !



Relaxed fixed point algorithm with prediction

(i) $\boldsymbol{d}_0 = \boldsymbol{d}^n + \dfrac{3\delta t}{2}\boldsymbol{v}_S^n - \dfrac{\delta t}{2}\boldsymbol{v}_S^{n-1}$

(ii) $\tilde{\boldsymbol{d}}_{k+1} = \mathcal{S} \circ \mathcal{F}(\boldsymbol{d}_k)$

(iii) $\boldsymbol{d}_{k+1} = \omega_k \tilde{\boldsymbol{d}}_{k+1} + (1 - \omega_k)\boldsymbol{d}_k$

How to choose ω_k ? :
(i) "by hand" : $\omega_k = \omega$

(ii) Domain decomposition *(Le Tallec - Mouro)* : reformulate the fixed point algorithm as a preconditioned gradient method applied to an interface problem

(iii) Aitken-like acceleration formula *(Mok - Wall - Ramm)* : heuristic generalization in n dimensions, of the formula that gives the exact solution for affine functions in 1D :

$$\omega_k = \frac{(\boldsymbol{d}_k - \boldsymbol{d}_{k-1}) \cdot (\boldsymbol{d}_k - \mathcal{S} \circ \mathcal{F}(\boldsymbol{d}_k) - \boldsymbol{d}_{k-1} + \mathcal{S} \circ \mathcal{F}(\boldsymbol{d}_{k-1}))}{|\boldsymbol{d}_k - \mathcal{S} \circ \mathcal{F}(\boldsymbol{d}_k) - \boldsymbol{d}_{k-1} + \mathcal{S} \circ \mathcal{F}(\boldsymbol{d}_{k-1})|^2}$$

Mok & co. concluded that (iii) gave slightly better results compared to (ii).

Domain decomposition method (formal)

$$\begin{bmatrix} A^F_{II} & A^F_{I\Sigma} & 0 \\ A^F_{\Sigma I} & A^F_{\Sigma\Sigma} + A^S_{\Sigma\Sigma} & A^S_{\Sigma I} \\ 0 & A^S_{I\Sigma} & A^S_{II} \end{bmatrix} \begin{bmatrix} d^F_I \\ d_\Sigma \\ d^S_I \end{bmatrix} = \begin{bmatrix} b^F_I \\ b_\Sigma \\ b^S_I \end{bmatrix} \quad (d^F_I = \delta t\, u^F_I)$$

(i) Fluid solver with Dirichlet boundary conditions : $A^F_{II} d^F_{I,k+1} = b^F_I - A^F_{I\Sigma} d_{\Sigma,k}$

(ii) Residual (force) computation : $F_{\Sigma,k+1} = A^F_{\Sigma I} d^F_{I,k+1} + A^F_{\Sigma\Sigma} d_{\Sigma,k}$

(iii) Structure solver with Neumann boundary conditions :

$$\begin{bmatrix} A^S_{\Sigma\Sigma} & A^S_{\Sigma I} \\ A^S_{I\Sigma} & A^S_{II} \end{bmatrix} \begin{bmatrix} \tilde{d}_{\Sigma,k+1} \\ d^S_{I,k+1} \end{bmatrix} = \begin{bmatrix} b_\Sigma - F_{\Sigma,k+1} \\ b^S_I \end{bmatrix}$$

(iv) $d_{\Sigma,k+1} = \omega_k \tilde{d}_{\Sigma,k+1} + (1 - \omega_k) d_{\Sigma,k}$

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- Eliminate $d^F_{I,k+1}$ in (i) and (ii) ($S^F = A^F_{\Sigma\Sigma} - A^F_{\Sigma I}(A^F_{II})^{-1}A^F_{I\Sigma}$)

$$F_{\Sigma,k+1} = S^F d^F_{\Sigma,k} + A^F_{\Sigma I}(A^F_{II})^{-1}b^F_I$$

- Eliminate $d^S_{I,k+1}$ in (iii) ($S^S = A^S_{\Sigma\Sigma} - A^S_{\Sigma I}(A_{II})^{-1}A^S_{I\Sigma}$)

$$S^S\tilde{d}_{\Sigma,k+1} = b_\Sigma - A^S_{\Sigma I}(A^S_{II})^{-1}b^S_I - F_{\Sigma,k+1}$$

$$S^S\tilde{d}_{\Sigma,k+1} = -S^F d_{\Sigma,k} + b_\Sigma - A^S_{\Sigma I}(A^S_{II})^{-1}b^S_I - A^F_{\Sigma I}(A^F_{II})^{-1}b^F_I$$

$$\boxed{\tilde{d}_{\Sigma,k+1} = (S^S)^{-1}(\tilde{b} - S^F d_{\Sigma,k})}$$

• Relaxation :

$$\begin{aligned} d_{\Sigma,k+1} &= \omega_k \tilde{d}_{\Sigma,k+1} + (1-\omega_k) d_{\Sigma,k} \\ &= d_{\Sigma,k} + \omega_k (\tilde{d}_{\Sigma,k+1} - d_{\Sigma,k}) \\ &= d_{\Sigma,k} + \omega_k \left[(S^S)^{-1} (\tilde{b} - S^F d_{\Sigma,k}) + d_{\Sigma,k} \right] \\ &= d_{\Sigma,k} + \omega_k (S^S)^{-1} \left[\tilde{b} - (S^F + S^S) d_{\Sigma,k} \right] \end{aligned}$$

• We recognize a gradient method for the solution of

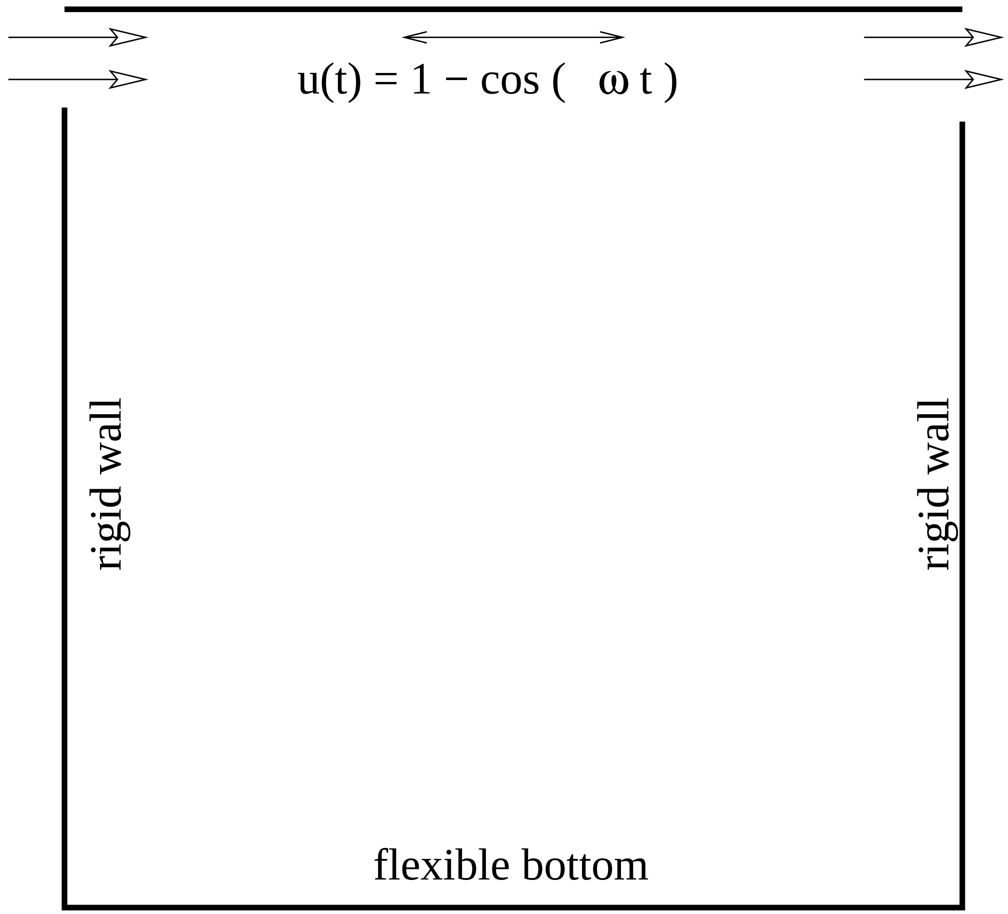
$$(S^F + S^S) d_\Sigma = \tilde{b}$$

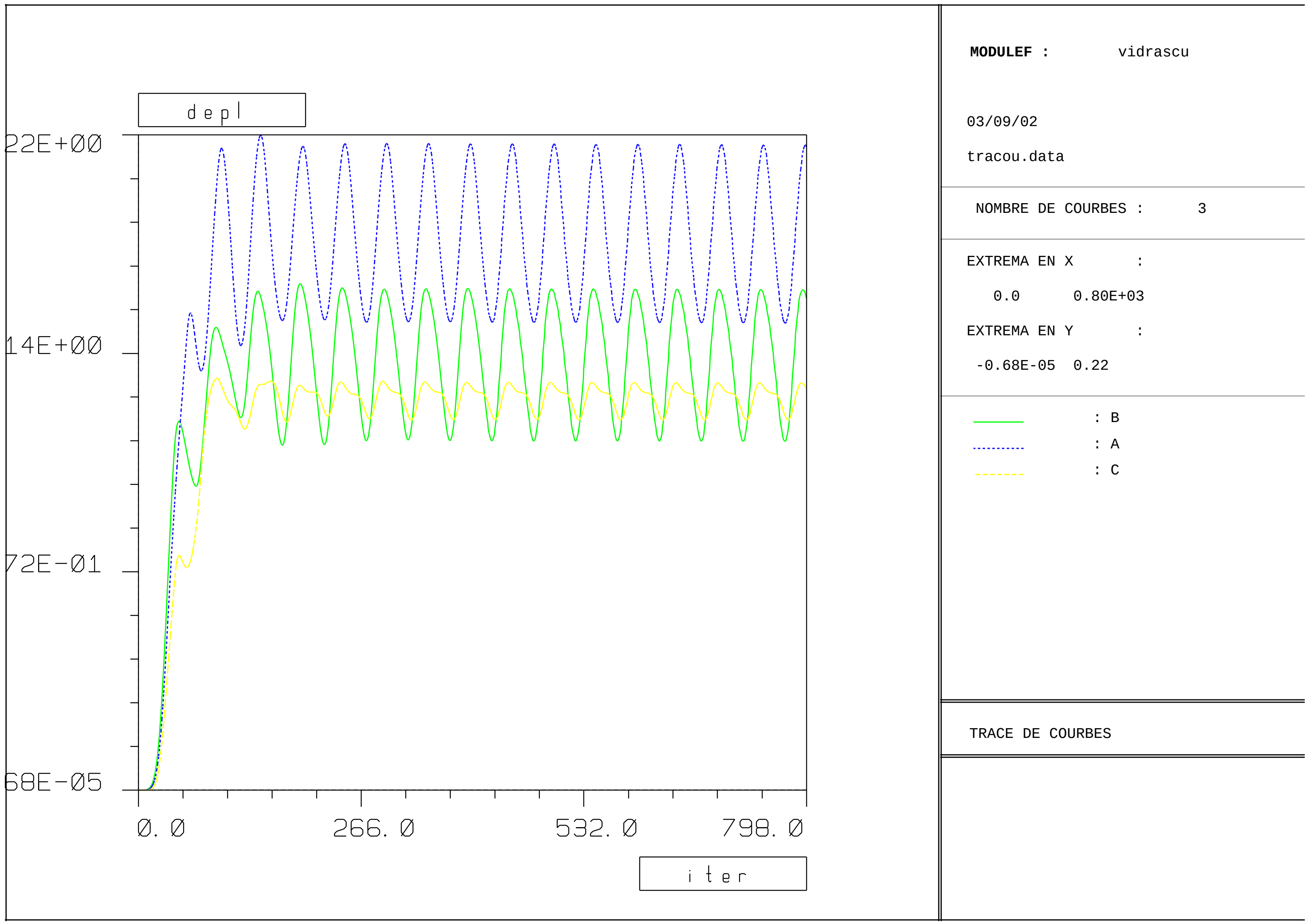
preconditionned by S^S.

$\longrightarrow$ this gives a way to design ω_k (e.g. steepest descent)

Benchmark test (*Mok, Wall, Ramm* in 2D)

Fluid : $\rho_F = 1$, $\mu = 0.01$ Structure : $\rho_S = 500$, $E = 250$, $h = 0.002$

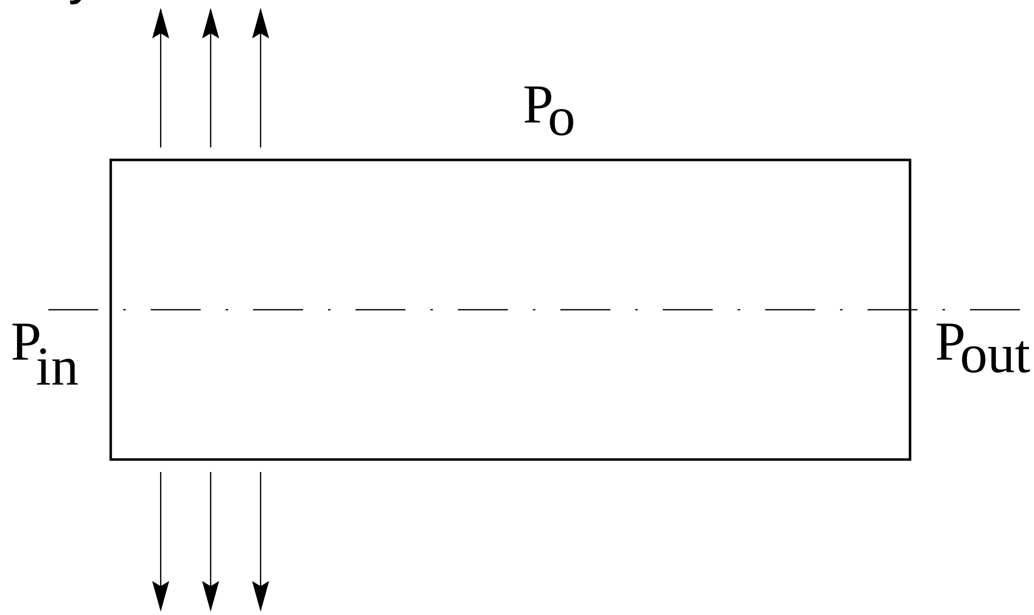



depl
22E+ØØ
14E+ØØ
72E-Ø1
68E-Ø5
Ø. Ø
266. Ø
532. Ø
798. Ø
iter
MODULEF : vidrascu
03/09/02
tracou.data
NOMBRE DE COURBES : 3
EXTREMA EN X :
0.0 0.80E+03
EXTREMA EN Y :
-0.68E-05 0.22
: B
: A
: C
TRACE DE COURBES

Pressure wave in artery

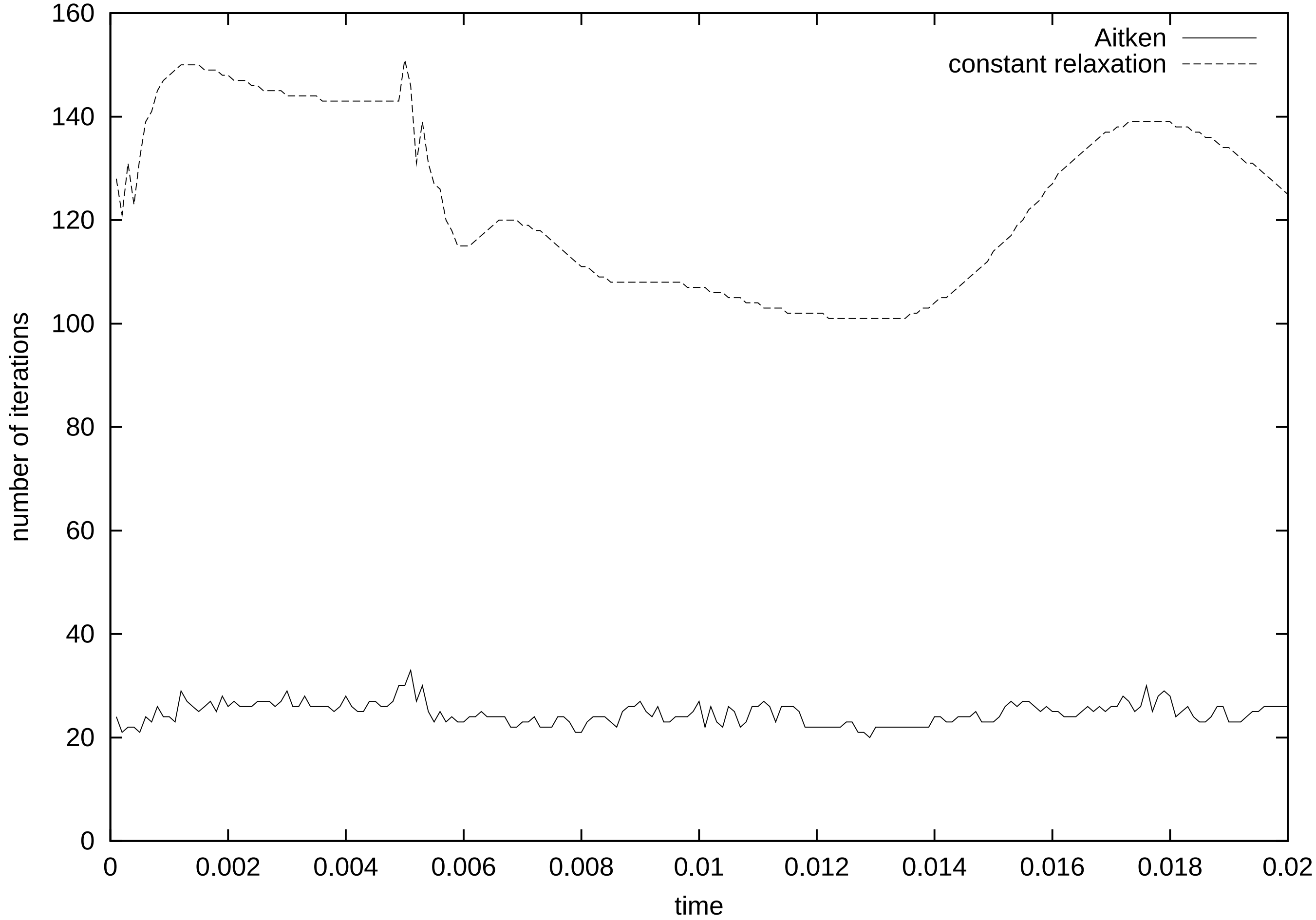
Fluid $\approx$ blood

Structure $\approx$ artery



$$t = 0 \quad : \quad \mathbf{u} = 0, \quad P = P_0 = 0.$$

$$\begin{cases} 0 < t < 5 \text{ ms} & : \quad P_{in} = 10 \text{ mmHg} \\ t > 5 \text{ ms} & : \quad P_{in} = 0 \end{cases}$$



Aitken
constant relaxation
number of iterations
time
0
20
40
60
80
100
120
140
160
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02

2.2. Quasi-Newton algorithms

$$\mathcal{R} = \mathcal{I} - \mathcal{S} \circ \mathcal{F}$$

$$\boxed{\mathcal{R}\boldsymbol{d} = 0}$$

(i) $\boldsymbol{d}_0 = \boldsymbol{d}^n + \frac{3\delta t}{2}\boldsymbol{v}_S^n - \frac{\delta t}{2}\boldsymbol{v}_S^{n-1}$

(ii) $\tilde{\mathcal{R}}'(\boldsymbol{d}_k)\delta\boldsymbol{d}_k = -\mathcal{R}(\boldsymbol{d}_k)$

(iii) $\boldsymbol{d}_{k+1} = \boldsymbol{d}_k + \lambda_k\delta\boldsymbol{d}_k$

$\tilde{\mathcal{R}}'$ is a suitable approximation of $\mathcal{R}'$

λ_k is determined *via* a linesearch strategy.

First possibility : automatic differentiation ...

Second possibility : *(Brown & Saad)*

The linear system at step (ii) is solved approximatively with a few iterations of GMRES. The matrix/vector products $\mathcal{R}'(\boldsymbol{d}_k)z$ are evaluated by computing :

$$\frac{\mathcal{R}(\boldsymbol{d}_k + hz) - \mathcal{R}(\boldsymbol{d}_k)}{h}$$

or (better)

$$\frac{\mathcal{R}(\boldsymbol{d}_k + hz) - \mathcal{R}(\boldsymbol{d}_k - hz)}{2h}$$

It works, but it's expensive and it's difficult to choose h.

Third possibility : evaluate the continuous Jacobian of the fluid structure problem *(Fernandez & Moubachir)*. To be tested !

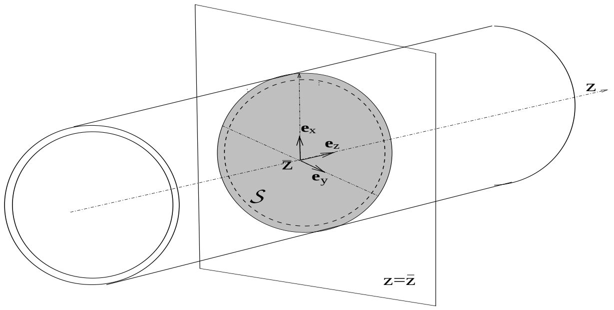
Fourth possibility :

use a reduced model.

3. Reduced FSI models

3.1. Classical 1D model :

– cylinder
– axial symmetry
– no radial velocity



- $\boxed{A = |\mathcal{S}|}$ (area of $\mathcal{S}$)

- $\boxed{Q = \int_{\mathcal{S}} u_z \, d\sigma}$ (flux through $\mathcal{S}$) *ou* $\boxed{\overline{u} = \dfrac{Q}{A}}$

- $\boxed{\overline{p} = \dfrac{1}{A(z)} \int_{\mathcal{S}} p \, d\sigma}$ (mean pressure moyenne)

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mass conservation :

$$\text{div } \mathbf{u} = 0 \qquad \Longrightarrow \qquad \boxed{\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial z} = 0}$$

momentum along z :

$$\frac{\partial u_z}{\partial t} + \text{div } (u_z \mathbf{u}) + \frac{1}{\rho_F}\frac{\partial p}{\partial z} - \nu \Delta u_z = 0 \Longrightarrow \boxed{\frac{\partial Q}{\partial t} + \frac{\partial}{\partial z}\left(\alpha \frac{Q^2}{A}\right) + \frac{A}{\rho_F}\frac{\partial \overline{p}}{\partial z} + K_R \frac{Q}{A} = 0}$$

wall law : relation between $\overline{p}$ and A

e.g. : $$\Phi(A) = p_0 + \frac{\sqrt{\pi} h_0 E}{(1-\nu^2)A_0}\left(\sqrt{A} - \sqrt{A_0}\right) \Longrightarrow \qquad \boxed{\overline{p} = \beta_0 + \beta\sqrt{A}}$$

Main difficulties :

– geometry : cylinder
– mean value variables : 3D $\rightarrow$ 1D $\rightarrow$ 3D

3.2. A new reduced model

An important feature in fluid structure interaction problems in blood flow is the so-called “mass added effect”.

This effect can be captured with a non-viscous linear fluid model :

$$\left\{ \begin{array}{rcll} \rho_F \partial_t \boldsymbol{u} + \boldsymbol{\nabla} p &=& 0 & \text{on } \bar{\Omega}_F \\ \mathrm{div}\ \boldsymbol{u} &=& 0 & \text{on } \bar{\Omega}_F \\ \boldsymbol{u} \cdot \boldsymbol{n} &=& \partial_t \boldsymbol{d} \cdot \boldsymbol{n} & \text{on } \bar{\Sigma} \end{array} \right.$$

or,

$$\boxed{\left\{ \begin{array}{rcll} -\triangle p &=& 0 & \text{on } \bar{\Omega}_F \\ \dfrac{\partial p}{\partial n} &=& -\rho_F \dfrac{\partial \boldsymbol{u}}{\partial t} \cdot \boldsymbol{n} & \text{on } \bar{\Sigma} \end{array} \right.}$$

3.3. The special case of the generalized wave equation

Notation : $\left\{ \begin{array}{rcll} -\triangle p & = & 0 & \text{on } \Omega_F \\ \dfrac{\partial p}{\partial n} & = & g & \text{on } \Sigma \\ p & = & p^d & \text{on } \Gamma \end{array} \right. \longrightarrow \boxed{p|_\Sigma = \mathcal{M}_\mathcal{A}\, g}$

Generalized wave equation for the structure :

$$\rho_S \frac{\partial^2 d_r}{\partial t^2} - a\frac{\partial^2 d_r}{\partial z^2} + b d_r - c\frac{\partial^3 d_r}{\partial z^2 \partial t} = p_\Sigma$$

Simplified fluid model : $p_\Sigma = -\rho_F \mathcal{M}_\mathcal{A} \dfrac{\partial \boldsymbol{u}}{\partial t} \cdot \boldsymbol{n} = -\rho_F \mathcal{M}_\mathcal{A} \dfrac{\partial^2 d_r}{\partial t^2}$

Simplified fluid-structure problem :

$$\boxed{(\rho_S + \rho_F \mathcal{M}_\mathcal{A})\frac{\partial^2 d_r}{\partial t^2} - a\frac{\partial^2 d_r}{\partial z^2} + b d_r - c\frac{\partial^3 d_r}{\partial z^2 \partial t} = 0}$$

3.4. Three possible applications of the reduced model

With 1D model structure (wave equation) :

(i) A good candidate for studying fluid-structure algorithms
(ii) A possible alternative for classical FSI 1D models

With general structure models :

(iii) A good candidate for approximating the jacobian in Quasi-Newton algorithms

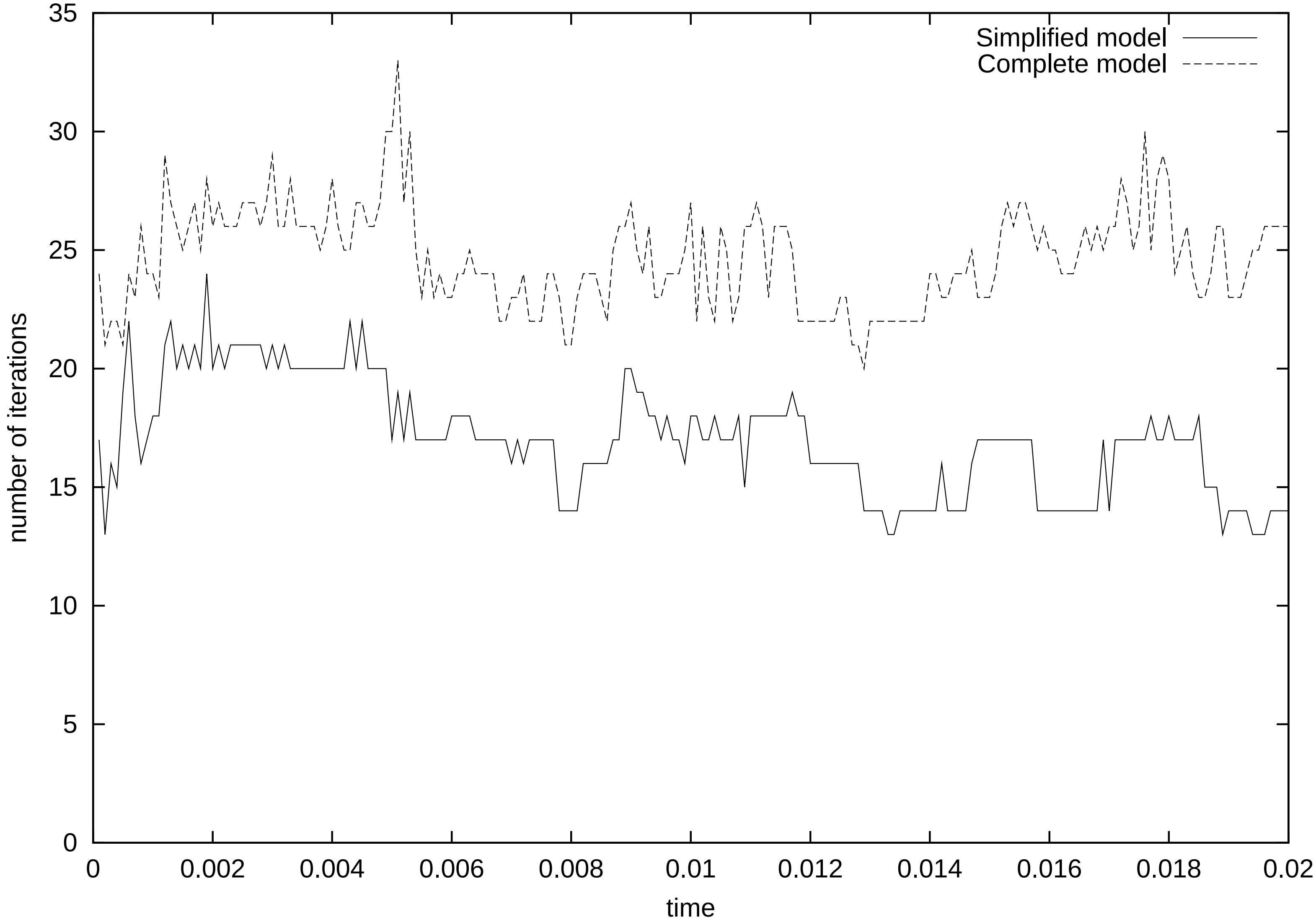
(i) A good candidate for studying fluid-structure algorithms

– it is rather simple (linear)

– it shares two important features with the real model :

$\longrightarrow$ lots of sub-structuring iterations are needed to converge

$\longrightarrow$ analogous property of propagation



Simplified model
Complete model
number of iterations
time
35
30
25
20
15
10
5
0
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02

Why staggered algorithm does not work in blood flows ?

Implicit algorithm :

$$\boldsymbol{u}^{n+1} \cdot \boldsymbol{n} = \boldsymbol{v}_S^{n+1} \cdot \boldsymbol{n}$$

$$(\rho_S + \rho_F \mathcal{M}_{\mathcal{A}}) \frac{\boldsymbol{v}_S^{n+1} - \boldsymbol{v}_S^n}{\delta t} - a \frac{\partial^2 d_r^{n+1/2}}{\partial z^2} + b d_r^{n+1/2} - c \frac{\partial^3 d_r^{n+1/2}}{\partial z^2 \partial t} = 0$$

Staggered algorithm :

$$\boldsymbol{u}^{n+1} \cdot \boldsymbol{n} = \boldsymbol{v}_S^{n} \cdot \boldsymbol{n}$$

$$\rho_S \frac{\boldsymbol{v}_S^{n+1} - \boldsymbol{v}_S^n}{\delta t} + \rho_F \mathcal{M}_{\mathcal{A}} \frac{\boldsymbol{v}_S^{n} - \boldsymbol{v}_S^{n-1}}{\delta t} - a \frac{\partial^2 d_r^{n+1/2}}{\partial z^2} + b d_r^{n+1/2} - c \frac{\partial^3 d_r^{n+1/2}}{\partial z^2 \partial t} = 0$$

$$\begin{bmatrix} D_{n+1} \\ V_{n+1} \\ W_{n+1} \end{bmatrix} = P \begin{bmatrix} D_n \\ V_n \\ W_n \end{bmatrix}$$

Spectral property of the iteration matrix P :

- For $\rho_F/\rho_S > 0.001$ (roughly) : $|\lambda_k| >> 1$
- $|\lambda_k|$ increases when δt decreases

(ii) A possible alternative for classical FSI 1D models

$$\boxed{\frac{\partial}{\partial t}\begin{bmatrix} A \\ Q \end{bmatrix} + \frac{\partial}{\partial z}\begin{bmatrix} Q \\ \dfrac{Q^2}{A} + \dfrac{\beta}{3}A^{3/2} \end{bmatrix} = \begin{bmatrix} 0 \\ -K\dfrac{Q}{A} \end{bmatrix}}$$

versus

$$\boxed{(\rho_S + \rho_F \mathcal{M}_{\mathcal{A}})\frac{\partial^2 \boldsymbol{d}}{\partial t^2} - a\frac{\partial^2 \boldsymbol{d}}{\partial z^2} + b\boldsymbol{d} - c\frac{\partial^3 \boldsymbol{d}}{\partial z^2 \partial t} = 0}$$

What has been lost ?

– nonlinearity
– viscous effect

What has been gained ?

– same variables as the complete model (3D-3D)
– realistic structure response

(iii) Approximate the tangent operator of the real problem

(i) Prediction : $\boldsymbol{d}_0 = \boldsymbol{d}^n + \frac{3\delta t}{2}\boldsymbol{v}_S^n - \frac{\delta t}{2}\boldsymbol{v}_S^{n-1}$

(ii) Resolution of $\tilde{\mathcal{R}}'(\boldsymbol{d}_k)\delta\boldsymbol{d}_k = -\mathcal{R}(\boldsymbol{d}_k)$

- evaluate $-\mathcal{R}(d_k)$ (with $\mathcal{R} = \mathcal{I} - \mathcal{S} \circ \mathcal{F}$)
- Solve the linear system with GMRES (tolerance 10^{-3}, about 8 iterations). The matrix/vector products being evaluated by :

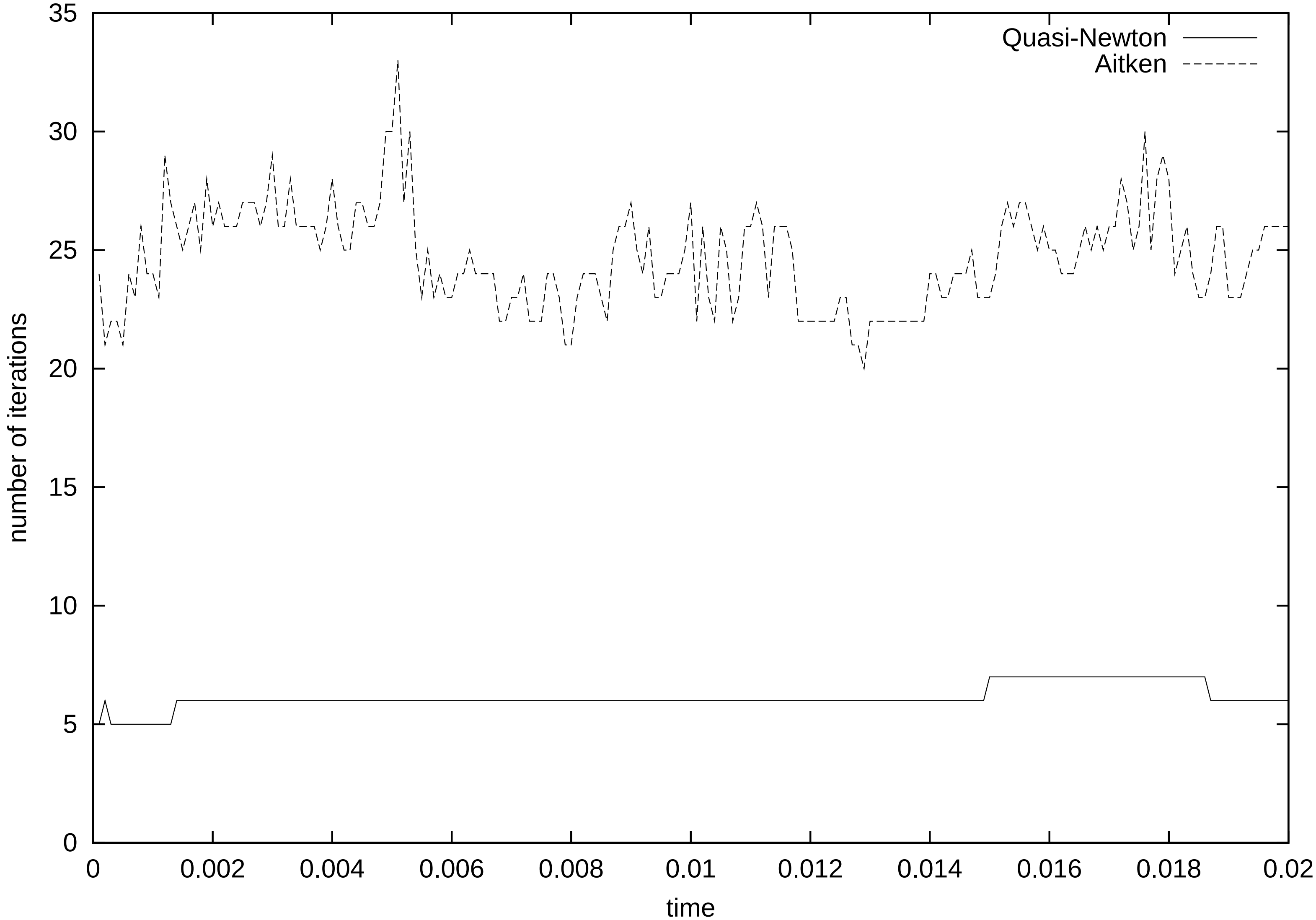
$$\tilde{\mathcal{R}}'(d_k)z = z - \mathcal{S}'(\mathcal{F}(d_k)) \cdot \tilde{\mathcal{F}}'(d_k)z$$

$\longrightarrow$ one resolution of a scalar Poisson problem (instead of Navier-Stokes)

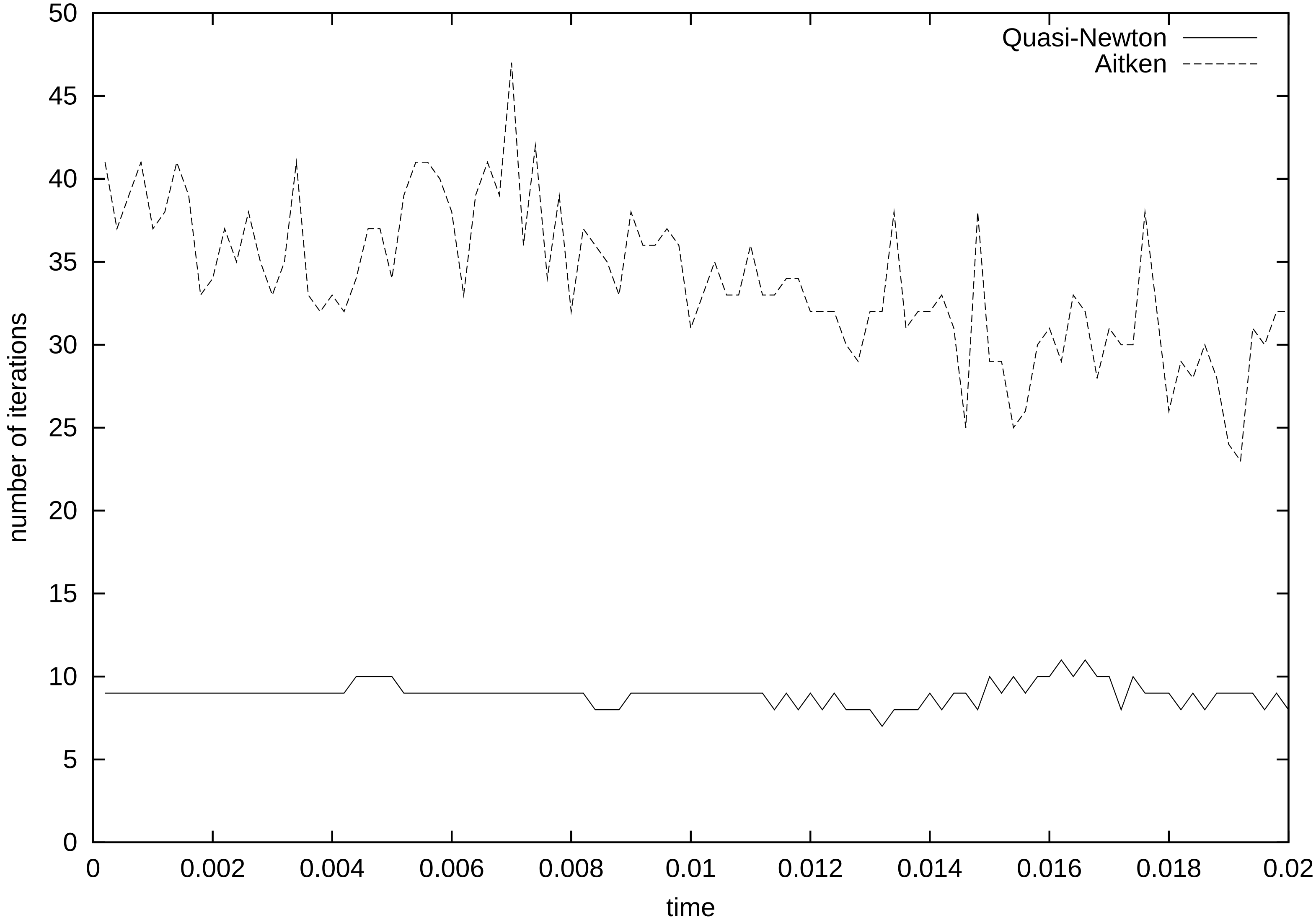
$\longrightarrow$ one resolution of the linearized structure (already computed and factorized during the structure solution)

(iii) $\boldsymbol{d}_{k+1} = \boldsymbol{d}_k + \lambda_k \delta\boldsymbol{d}_k$

In practice the linesearch procedure is never activated ($\lambda_k = 1$) : the approximated jacobian is not so bad !



Quasi-Newton
Aitken
number of iterations
time
35
30
25
20
15
10
5
0
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02



Quasi-Newton
Aitken
number of iterations
time
0
5
10
15
20
25
30
35
40
45
50
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02

Mean number of fluid-structure evaluations :

	Aitken	Quasi-Newton	Speed up
2D	24.7	6.1	4.
3D	33.9	8.9	3.8

Computational time :

	Aitken	Quasi-Newton	Speed up
2D	43 min	16 min	2.7
3D	12 h 16	5 h 13	2.4