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Mixed Volatility Dynamics: numerical calibration of cross currency models

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Overview

- Model requirements
  - Standard approaches
  - Lattice versus Monte Carlo
  - Smile models
- Market requirements
  - FX vol smile
  - Smile dynamics
  - Market data issues
  - Examples of exotic structures
- Mixed Volatility Dynamics (MVD)
  - Constructing a Hybrid model
  - Formulation
  - Calibration
  - MVD parameters
  - Risk measures
  - Limitations
  - Conclusions

Cross currency models

- Model formulation:
  - Use a single factor for forward FX rate with a diffusion process.
  - Perhaps acceptable for short maturity.
  - However even then no possible information on cross currency correlations.
  - Need to include second factor, the domestic (JPY) bond since in swap we are receiving floating Libor.
  - Full ‘standard’ model also models foreign (USD) bond giving three factors, but now model spot FX process
  - Displaced (CEV) models for rates skew
  - Local vol approaches for matching static FX smile
  - Require a fourth factor for the Stochastic volatility process
  - Multifactor rate models for complex hybrid products
  - Multi-currency pairs XCCY require more factors
  - Correlations

Cross currency models (2)

- Long term FX products are also interest rate products:
  - For no-arbitrage it is clear that the *forward* FX rate is composed of the *spot* rate and the interest rates for the 2 currencies.
  - To illustrate assume 'standard' approach of lognormal spot FX rate with lognormal zero coupon bonds $P_T(t)$ (i.e. normal short rates) for the respective domestic and foreign interest rates (IR), with correlation linking the 3 processes
  - Spot FX process:

$$dS = S\sigma(t)\grave{}dW_S + (r_{dom} - r_{for})dt$$

  - We wish the *forward* FX to be a Martingale in T-forward measure.

Cross currency models (3)

- Hence construct:

$$F_T(t) = S(t)\frac{P_T^{for}(t)}{P_T^{dom}(t)}$$

- From Ito's lemma and change of measure:

$$dF = \Omega(t)FdW^{Q_T}$$

- We have a lognormal forward FX rate and Black-Scholes formula for Option prices to match market and allow simple calibration by bootstrap over time periods.

- Note even in this case, there are times when no solution for Ω(t) exists, a 'Vol squeeze'.

Cross currency models (4)

- Model construction:
  - Use Gauss-Markov lattice construction as with the pure interest rate model.
  - Lognormal spot FX is easily incorporated as exp(x)
  - Integrate over Gaussian densities using correct Numeraire.
  - Fast integration methods with measure change only Libor settings dates are required (large time steps).
  - However larger FX vol (around 10-20%) requires larger number of lattice points and Std dev span for convergence.
- Model use:
  - Calibration of separate currency bonds are identical for each single currency cases respectively, typically match swaption vols.
  - Strip spot FX vol for ATM only (cannot take smile in described model).
  - Correlation (3x3 matrix) inputs from user based on trading criteria.

Cross currency models with Smile

- Model extensions
  - Markov Functional approach to incorporate interest rate vol skew, i.e. integrate *backwards* to construct distribution
  - This is similar to Local Vol (LV) methods for short dated FX models typically using PDEs (short time steps).
  - But these methods can only fit static smiles
  - For realistic smile dynamics, require process for stochastic volatility (SV) process for the FX vol.
  - Particularly important for long dated FX products

- Model considerations:
  - Non-markov vol of vol process requires Monte Carlo (even Markov approximations of SV would need a 4-factor PDE.
  - Use ‘Markov-Functional’ method to calibrate within *forward* MC paths.
  - Does not restrict choice of Interest Rate dynamics
  - Only ‘large’ time steps needed at Libor setting dates.
  - Incorporate LV and SV or a ‘Hybrid’ model within MC method.

Cross Currency models with Smile (2)

- FX Smile model construction
  - Hybrid LV and SV processes

$$dS = S\xi\sigma_{local}(S)dW_S + (....)dt$$
$$d\xi = \mu(1-\xi)dt + \nu\xi dW_V$$
$$\langle dW_V \,|\, dW_S \rangle = \rho$$

- Hybrid Model considerations:
  - Choice of process for SV (in this case SABR with exponent 1)
  - Parameterisation of LV
  - Standard approach to use ‘quadratic form’ for LV, thus 3 free parameters
  - For SV choose constant Vvol and Rho
  - This creates a simple hybrid approach, with LV used to best fit FX Smile given SV.

Cross Currency models with Smile (3)

- Fitting to market FX smiles (USD/JPY. AUD/JPY, EUR/USD) challenging
  - Very steep especially at low strikes
  - Long dated, e.g. typically 30 years, semi-annual resets
  - Price matching within vega bid-offer (e.g. 20bp) still creates much noise in risk
- Require model with more degrees of freedom
  - More accurate fit to static Smile over long dates
  - However, aim to maintain relatively more stable parameters
  - Realistic combination of LV and SV, with control of the mix
- MVD Model considerations:
  - Determine sequence of strikes (per maturity) to calibrate
  - LV parameters are piece-wise linear and mapped to strikes
  - Introduce explicit ‘mixture’ parameter
  - Introduce decay constant for Vvol

MVD model (2)

- As model parameters vary over the MC evolution, we need to match market option prices:

$$Call_{T,i} = \left\langle P_T^{dom} \left(F_T - K_i\right)^+ \right\rangle$$

- These prices can be inverted to obtain simulation implied vols
- For each time-step T, we can define an error,

$$Error\left(\nu^T, \rho^T, L^T{}_0, L^T{}_1, \ldots, L^T{}_N\right) = \sum \left(\sigma_{T,i}^{sim} - \sigma_{T,i}^{mkt}\right)^2$$

- The simulation vol is the product of Local and Stoch vol
- Constrain the LV to be flat and solve for SV parameters Vvol (v) and correlation (ρ) to minimize error.
- Once SV parameters are fixed, second solve finds the LV parameters (L_i) to minimize error.
- Furthermore the Vvol parameter is scaled such that

$$\nu_{MVD}^T = c_0 \exp\left(-c_1 T\right)\nu^T$$

- This allows explicit control of degree of mixing between LV and SV through c_0 and c_1 which represents the mixing and decay constants for the volatility process respectively. We normalise such that $0 < c_0 < 1$

MVD model

- Evolution equations after discretisation:

$$\xi_{T+1} = (\xi_T)^{1-\mu dt} \exp\left(-\frac{1}{2}\nu^2 + \nu dW_\nu\right)$$

$$\sigma'_T = \xi_{T+1}\sigma_T^{local}(S_T)$$

$$S_{T+1} = S_T \exp\left(-\frac{1}{2}\sigma'^2_T dt + \sigma'_T dW_S\right)$$

- Monte Carlo evolution of processes incorporates Smile calibration
- Drifts are corrected to maintain no-arbitrage conditions e.g. FX forwards are re-priced.
  - Note as IR processes may be generic there are no closed form ‘quanto’ corrections, so drift corrections are part of numerical calibration
- There is also a 2-step evolution to calibrate:
  - Find flat level LV parameters which minimize error for pure SV case by solving for Vvol and Rho
  - Then LV contribution is solved for by minimizing parameters L_i

MVD model (3)

- Define digital probabilities as

$$\text{Prob}\left(Fwd, K, T, \sigma^{mkt}(K,T)\right) = N\left(\frac{\ln\left(Fwd/K\right) - \sigma^2 T/2}{\sigma\sqrt{T}}\right)$$

- Strikes are chosen such that the implied digital probabilities are equally spaced.
- Using a set of strikes K_i, and local vol parameters L_i, we define the LV as:

$$\sigma_T^{local}(S) = \sigma_T^{mkt}\frac{\left(\ln(K_{i+1}) - S\right)L_i + \left(S - \ln(K_i)\right)L_{i+1}}{\ln(K_{i+1}) - \ln(K_i)}$$

- Important to remember that the IR processes are still calibrated separately and that the MC evolution for the MVD incorporates this. These can remain as single factor Hull-White dynamics or multi-factor BGM.
- The latter requires more thought on correlation, but is not much slower in performance (the FX calibration component of MVD dominates), while giving much richer yield curve dynamics which might be required for other Hybrid exotics

Powered Reverse Dual products

- Payoff function at option maturity date *m* can be written as:

$$\max(0, \sum_{k=m}^{N} \theta_k D_k^d (L_k - \max(0, (X_k/X_0)K_f - K_d + \beta) - \alpha))$$

where:

L_k is the Libor (e.g. 12M Libor rate) set at date k

K_f is a fixed *foreign* or USD rate

K_d is a fixed *domestic* or JPY rate

X_k is the forward FX rate set at date k

D_k^d is the *domestic* or JPY discount factor at date k

θ_k is the accrual factor at date k

β and α are spreads

k is the index which counts swap payment dates

N is the number of swap payment dates.

- Callable/Putable ('PRDCs')
- Target/Trigger redemption note ('FX Tarns')

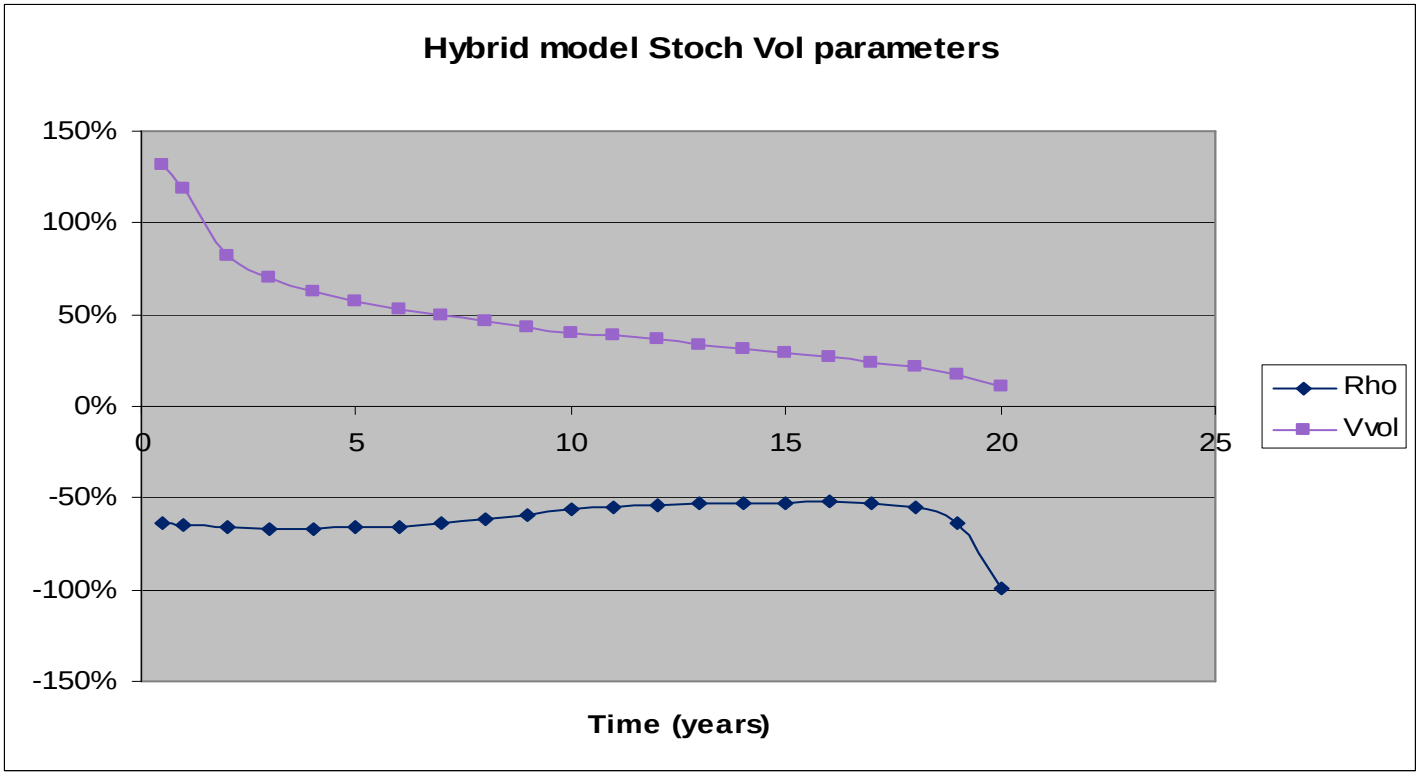
Powered Reverse Dual products (2)

- Underlying swap structure is:
  - Bank receives JPY Libor plus margin with Act/360
  - Bank pays spread on fixed USD coupon and fixed JPY coupon floored at zero in Bond basis.
  - This Swap is then callable at coupon dates.
  - Typical swap maturity is 10-30 years.
  - Possible funding is in foreign ccy (USD) or even third ccy (e.g. EUR)
  - Possible foreign redemption

- Target Redemption Notes
  - Swap terminates when target accrued coupon based on FX is hit
  - Also a trigger variant of this
  - Can extend to multi-ccy pairs, ‘Chooser Tarns’

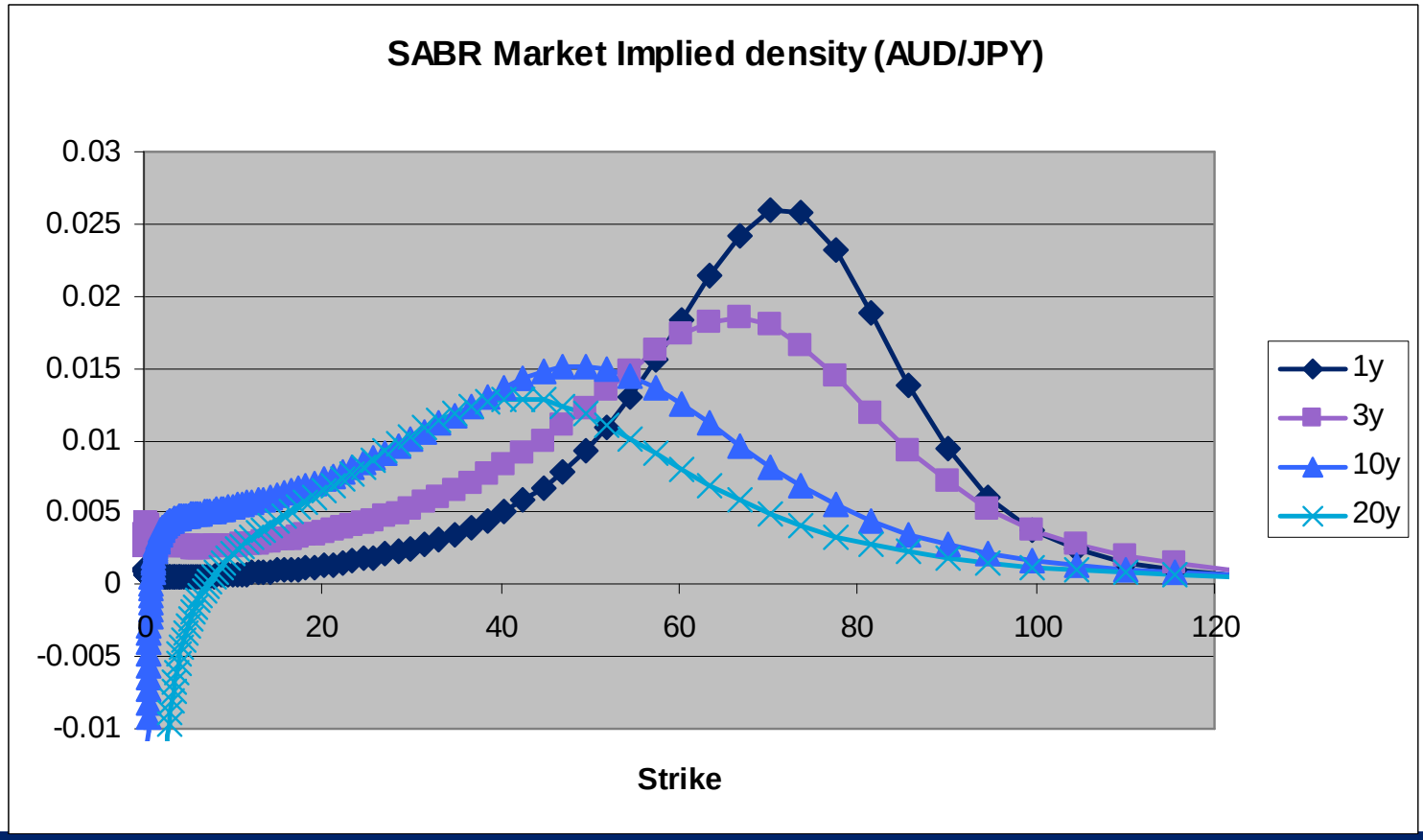
Market data issues

- SABR parameters defining market FX vol smile
- Higher Vvol at short dates, Rho more constant till long dates, but these are terminal parameter values



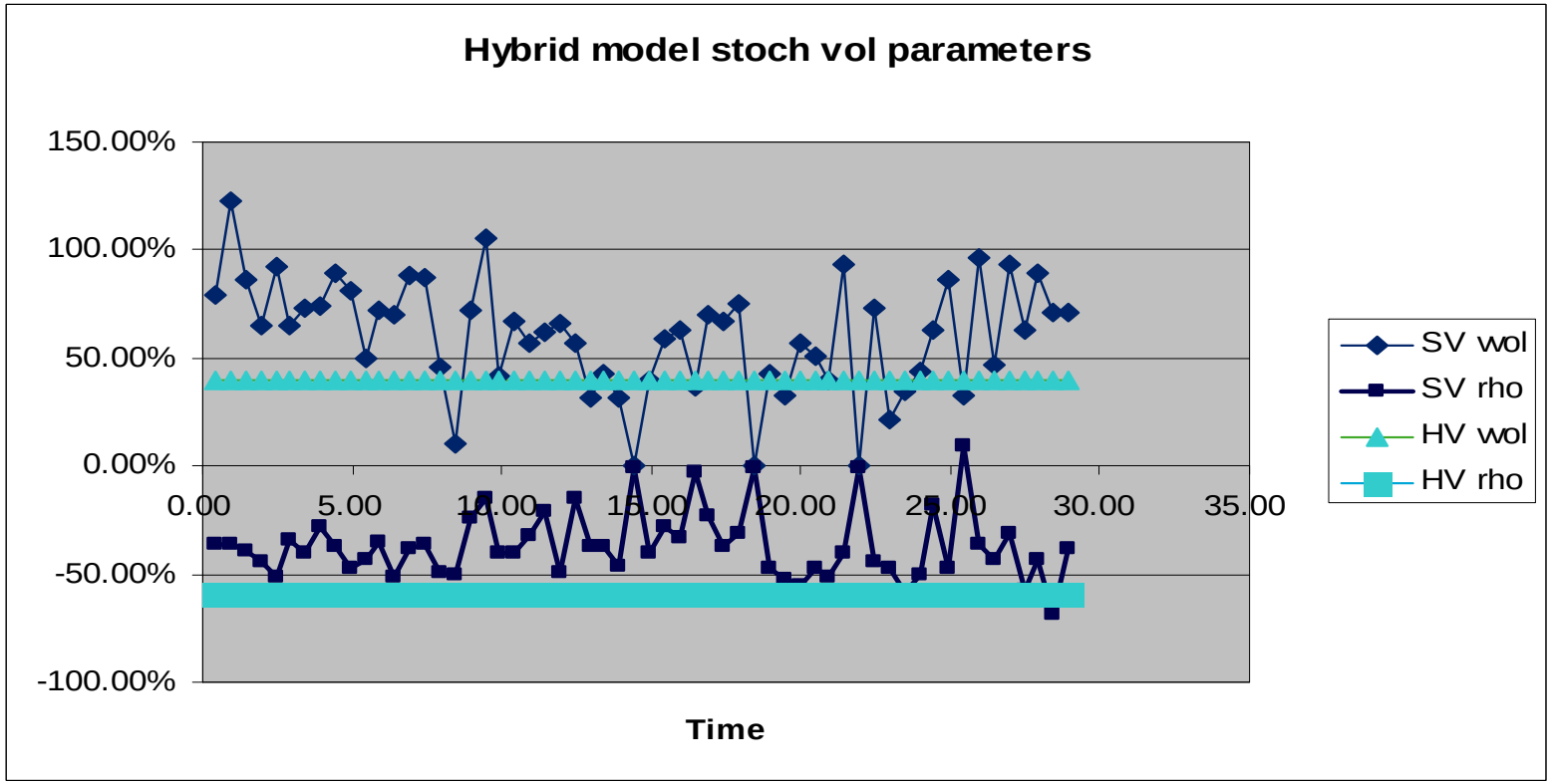
Market data issues (2)

- SABR term parameters for USD/JPY surface
- Can at long maturities lead to arbitrageable surfaces



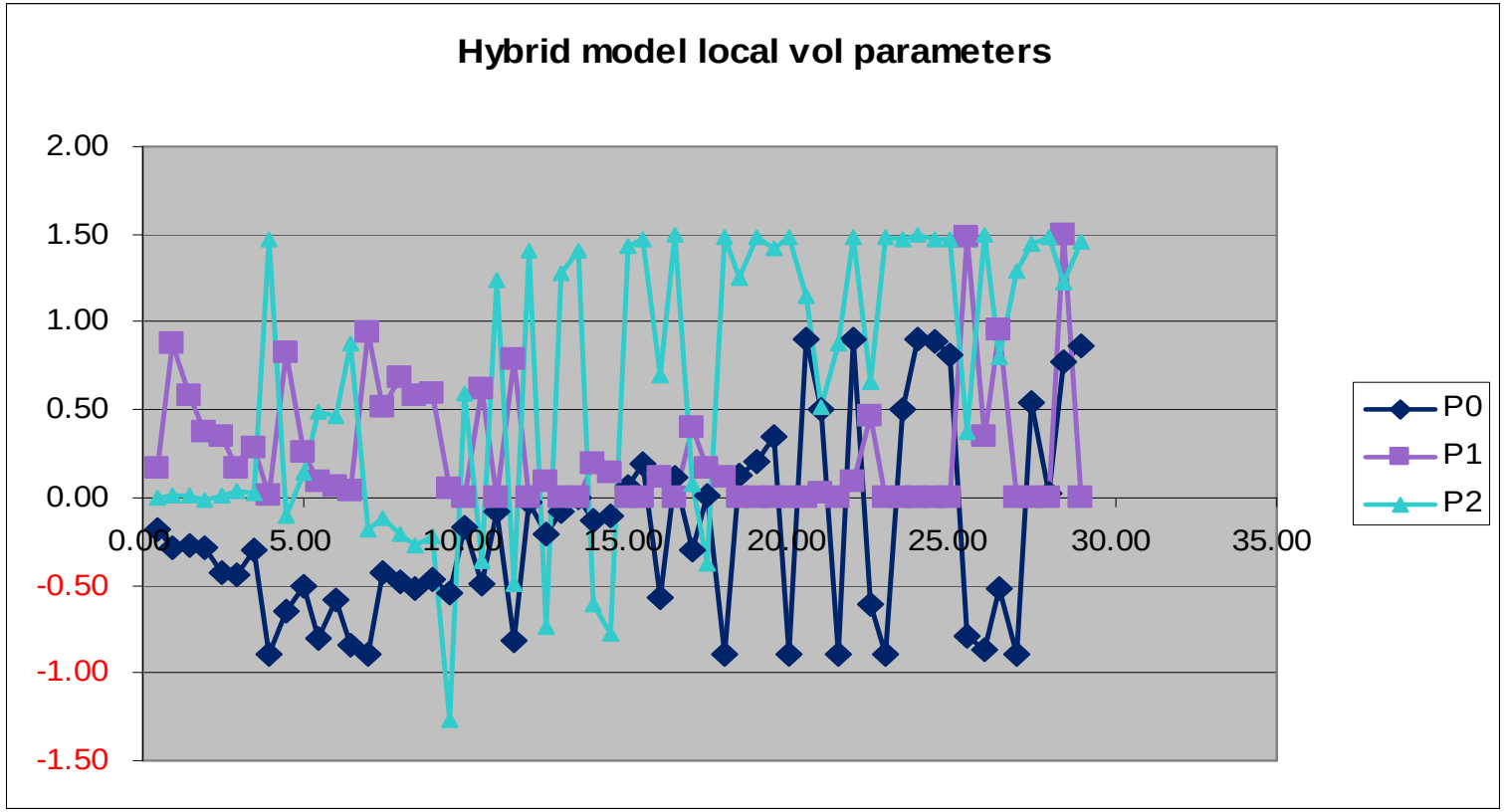
Model parameters

- ‘Hybrid’ model implied parameters show high degree of non-stationarity
- Stoch Vol component parameters term structure, compare to flat mark at 10 years



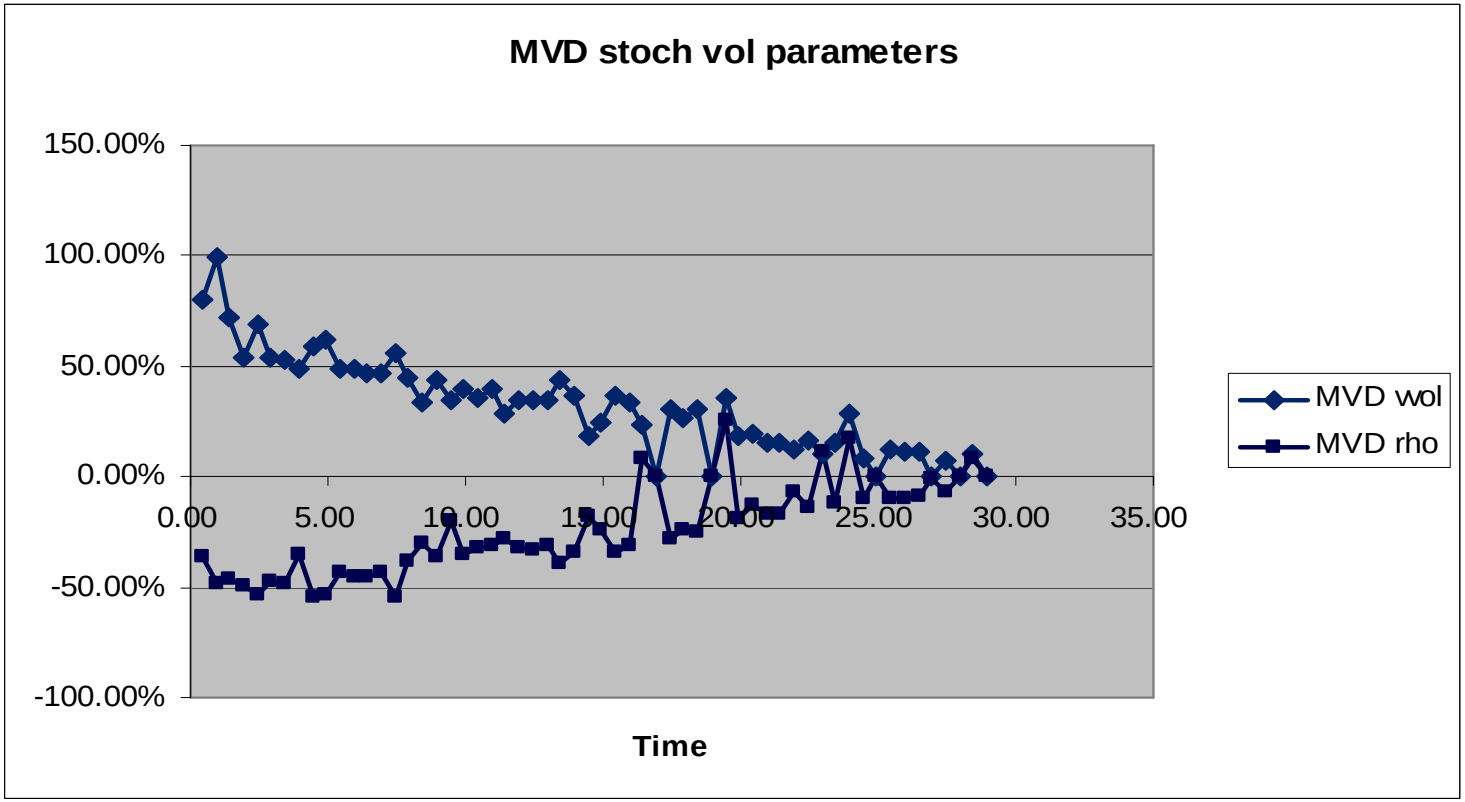
Model parameters (2)

- Local Vol 3-parameter implied fit is also non-stationary



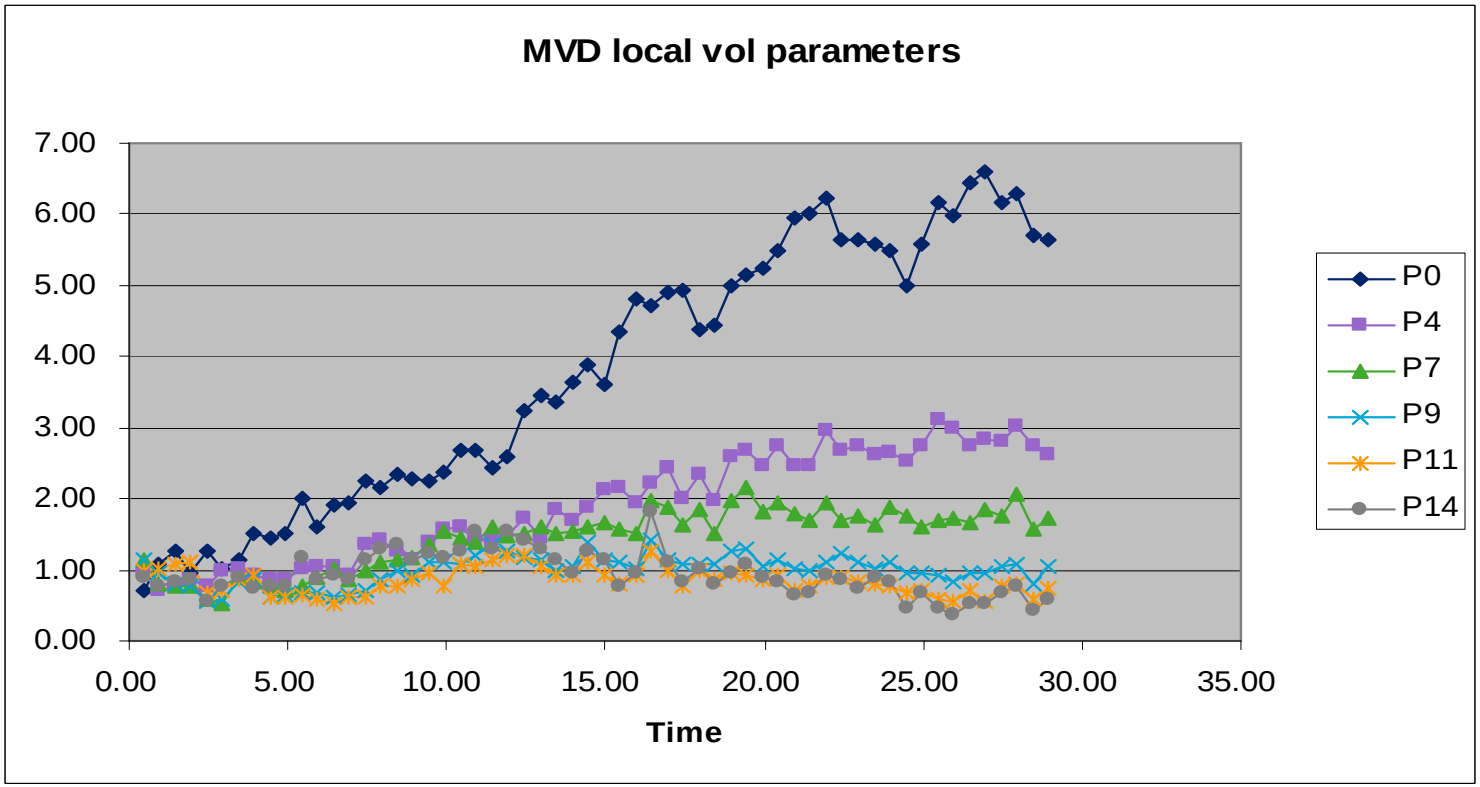
Model parameters (3)

- Contrast to MVD Stoch Vol parameters



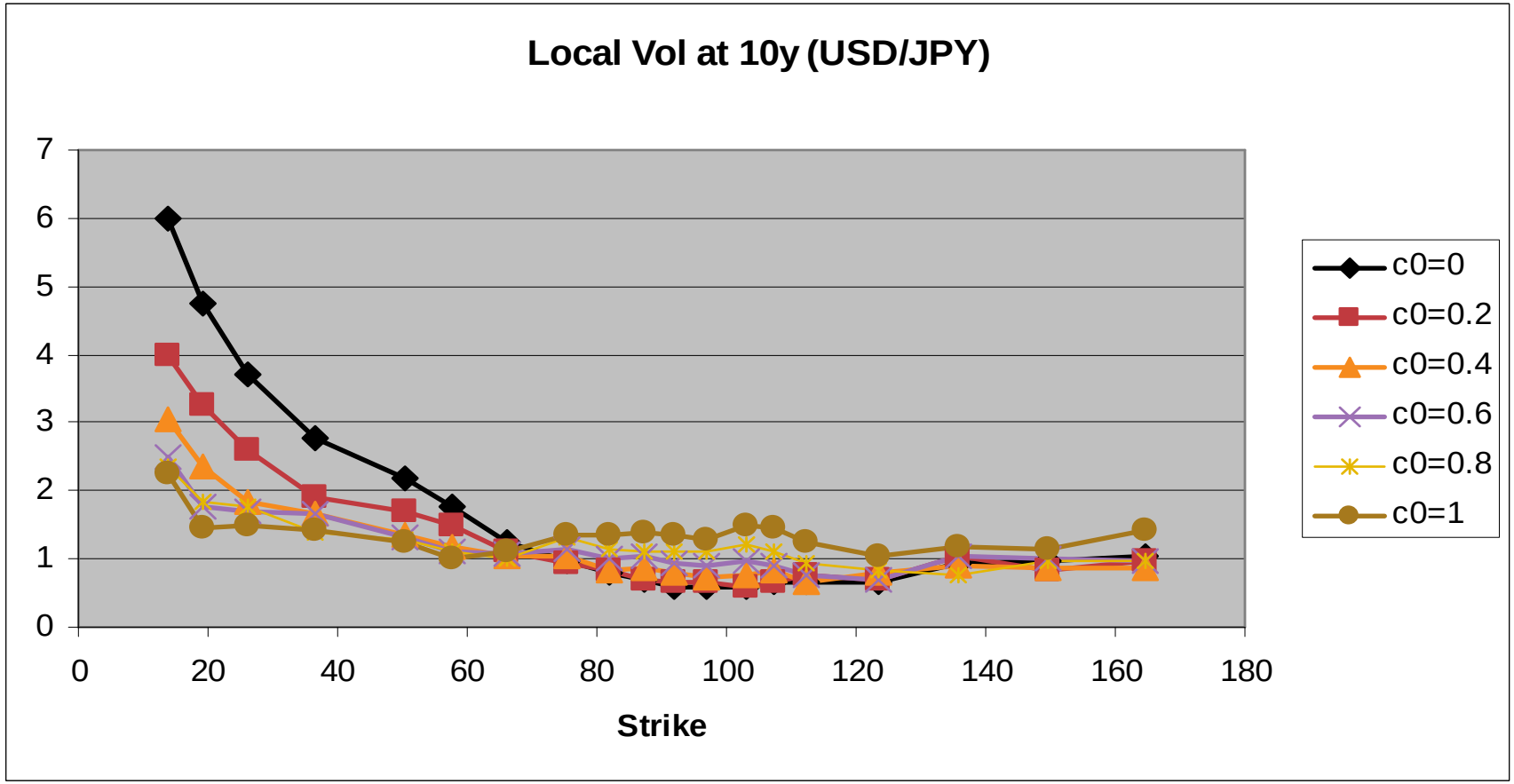
Model parameters (4)

- Similarly for MVD Local Vol parameters



Model parameters (5)

- Effect of changing mixing parameter c_0 on LV



Calibration across strikes and maturity

- pure ‘Stoch Vol’ Model fit (implied Vvol and Rho)

	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165
16-Sep-09	5.08%	4.60%	4.11%	3.27%	2.45%	1.28%	0.40%	0.09%	0.03%	0.06%	0.06%	0.07%	0.51%	0.98%	1.50%	1.80%	1.80%
16-Mar-10	2.91%	0.83%	0.36%	0.60%	0.61%	0.51%	0.30%	0.14%	0.04%	0.08%	0.08%	0.02%	0.28%	0.43%	0.45%	0.53%	0.62%
16-Sep-10	4.21%	3.43%	2.70%	2.03%	1.45%	0.89%	0.44%	0.10%	0.14%	0.10%	0.20%	0.09%	0.32%	0.62%	0.81%	0.88%	0.98%
16-Mar-11	6.11%	4.20%	3.04%	2.18%	1.47%	0.85%	0.37%	0.02%	0.14%	0.08%	0.17%	0.12%	0.22%	0.47%	0.66%	0.75%	0.90%
16-Sep-11	4.94%	3.47%	2.48%	1.68%	1.04%	0.60%	0.25%	0.03%	0.17%	0.05%	0.19%	0.07%	0.19%	0.40%	0.51%	0.53%	0.53%
16-Mar-12	4.82%	3.15%	2.17%	1.51%	0.95%	0.50%	0.16%	0.07%	0.16%	0.02%	0.20%	0.06%	0.26%	0.46%	0.58%	0.65%	0.71%
18-Sep-12	5.00%	3.37%	2.21%	1.42%	0.87%	0.44%	0.08%	0.14%	0.15%	0.02%	0.17%	0.01%	0.21%	0.32%	0.35%	0.35%	0.30%
18-Mar-13	5.57%	3.82%	2.62%	1.72%	1.03%	0.49%	0.07%	0.20%	0.21%	0.06%	0.24%	0.09%	0.18%	0.35%	0.45%	0.48%	0.51%
17-Sep-13	4.34%	2.89%	1.90%	1.20%	0.63%	0.20%	0.07%	0.18%	0.13%	0.10%	0.20%	0.01%	0.22%	0.40%	0.55%	0.68%	0.80%
17-Mar-14	3.87%	2.69%	1.77%	1.09%	0.60%	0.23%	0.05%	0.20%	0.13%	0.10%	0.21%	0.05%	0.12%	0.23%	0.29%	0.33%	0.38%
16-Sep-14	3.06%	2.06%	1.34%	0.82%	0.43%	0.12%	0.08%	0.16%	0.08%	0.10%	0.16%	0.00%	0.13%	0.16%	0.15%	0.12%	0.11%
16-Mar-15	2.64%	1.83%	1.23%	0.77%	0.38%	0.08%	0.14%	0.16%	0.05%	0.13%	0.12%	0.04%	0.18%	0.23%	0.24%	0.22%	0.22%
16-Sep-15	1.88%	1.40%	0.90%	0.49%	0.16%	0.04%	0.15%	0.11%	0.01%	0.14%	0.07%	0.10%	0.23%	0.26%	0.26%	0.24%	0.23%
16-Mar-16	2.13%	1.56%	1.04%	0.57%	0.22%	0.03%	0.17%	0.13%	0.01%	0.12%	0.04%	0.11%	0.16%	0.15%	0.12%	0.07%	0.02%
16-Sep-16	1.25%	0.95%	0.61%	0.29%	0.06%	0.07%	0.09%	0.04%	0.03%	0.05%	0.06%	0.14%	0.09%	0.03%	0.07%	0.19%	0.31%
16-Mar-17	0.29%	0.32%	0.16%	0.03%	0.04%	0.05%	0.01%	0.05%	0.08%	0.05%	0.27%	0.42%	0.45%	0.42%	0.37%	0.34%	0.30%
19-Sep-17	0.36%	0.14%	0.11%	0.04%	0.01%	0.04%	0.10%	0.13%	0.09%	0.13%	0.44%	0.63%	0.67%	0.68%	0.67%	0.66%	0.64%
16-Mar-18	0.65%	0.66%	0.47%	0.21%	0.00%	0.13%	0.08%	0.05%	0.11%	0.03%	0.24%	0.34%	0.33%	0.31%	0.27%	0.23%	0.18%
18-Sep-18	1.70%	1.40%	0.94%	0.48%	0.08%	0.20%	0.19%	0.00%	0.11%	0.02%	0.14%	0.23%	0.22%	0.18%	0.13%	0.09%	0.07%
18-Mar-19	1.25%	0.96%	0.51%	0.15%	0.04%	0.11%	0.06%	0.05%	0.08%	0.07%	0.27%	0.36%	0.35%	0.30%	0.25%	0.21%	0.19%
17-Sep-19	1.23%	0.96%	0.54%	0.16%	0.10%	0.18%	0.10%	0.03%	0.08%	0.04%	0.19%	0.23%	0.18%	0.11%	0.07%	0.03%	0.02%
16-Mar-20	0.64%	0.63%	0.41%	0.18%	0.02%	0.02%	0.03%	0.05%	0.02%	0.21%	0.40%	0.46%	0.44%	0.37%	0.32%	0.28%	0.26%
16-Sep-20	1.09%	0.93%	0.49%	0.07%	0.17%	0.13%	0.00%	0.10%	0.06%	0.12%	0.31%	0.37%	0.34%	0.28%	0.23%	0.19%	0.15%
16-Mar-21	0.90%	0.67%	0.28%	0.02%	0.12%	0.09%	0.00%	0.07%	0.00%	0.17%	0.31%	0.32%	0.21%	0.07%	0.05%	0.17%	0.26%
16-Sep-21	0.52%	0.38%	0.12%	0.05%	0.07%	0.01%	0.08%	0.06%	0.04%	0.24%	0.37%	0.36%	0.22%	0.06%	0.10%	0.24%	0.35%
16-Mar-22	0.30%	0.35%	0.23%	0.13%	0.11%	0.13%	0.12%	0.02%	0.18%	0.42%	0.59%	0.60%	0.51%	0.37%	0.24%	0.12%	0.02%
16-Sep-22	0.18%	0.44%	0.38%	0.32%	0.27%	0.26%	0.17%	0.03%	0.30%	0.58%	0.78%	0.80%	0.73%	0.60%	0.49%	0.38%	0.30%
16-Mar-23	0.24%	0.55%	0.55%	0.49%	0.41%	0.32%	0.18%	0.06%	0.38%	0.71%	0.92%	0.96%	0.90%	0.79%	0.70%	0.62%	0.56%
19-Sep-23	0.75%	0.61%	0.38%	0.33%	0.36%	0.34%	0.25%	0.05%	0.25%	0.53%	0.72%	0.75%	0.66%	0.51%	0.37%	0.24%	0.13%
18-Mar-24	0.61%	0.57%	0.44%	0.39%	0.35%	0.26%	0.09%	0.19%	0.54%	0.85%	1.04%	1.08%	1.00%	0.88%	0.76%	0.64%	0.54%
17-Sep-24	0.58%	0.66%	0.60%	0.57%	0.50%	0.38%	0.17%	0.14%	0.53%	0.87%	1.08%	1.14%	1.08%	0.97%	0.86%	0.75%	0.66%
17-Mar-25	0.45%	0.38%	0.39%	0.48%	0.50%	0.42%	0.24%	0.05%	0.41%	0.75%	0.99%	1.06%	1.03%	0.94%	0.83%	0.74%	0.65%
16-Sep-25	0.18%	0.25%	0.30%	0.38%	0.38%	0.27%	0.07%	0.24%	0.62%	0.95%	1.18%	1.26%	1.24%	1.16%	1.06%	0.98%	0.90%
16-Mar-26	0.17%	0.44%	0.54%	0.57%	0.53%	0.40%	0.16%	0.19%	0.60%	0.97%	1.22%	1.34%	1.34%	1.28%	1.19%	1.11%	1.04%
16-Sep-26	0.54%	0.47%	0.40%	0.50%	0.55%	0.47%	0.28%	0.02%	0.33%	0.60%	0.74%	0.76%	0.69%	0.55%	0.41%	0.27%	0.14%
16-Mar-27	0.49%	0.28%	0.15%	0.29%	0.38%	0.35%	0.19%	0.05%	0.32%	0.53%	0.63%	0.62%	0.52%	0.35%	0.17%	0.00%	0.17%
16-Sep-27	0.48%	0.20%	0.03%	0.13%	0.28%	0.29%	0.17%	0.06%	0.31%	0.49%	0.59%	0.60%	0.51%	0.34%	0.16%	0.04%	0.22%
16-Mar-28	0.31%	0.19%	0.11%	0.17%	0.24%	0.21%	0.06%	0.17%	0.41%	0.57%	0.63%	0.62%	0.51%	0.34%	0.14%	0.07%	0.27%
19-Sep-28	0.23%	0.21%	0.17%	0.21%	0.23%	0.17%	0.01%	0.24%	0.47%	0.61%	0.64%	0.58%	0.44%	0.24%	0.01%	0.22%	0.44%
16-Mar-29	0.30%	0.39%	0.36%	0.34%	0.28%	0.14%	0.07%	0.34%	0.60%	0.78%	0.85%	0.84%	0.74%	0.58%	0.39%	0.19%	0.01%
18-Sep-29	0.26%	0.40%	0.41%	0.38%	0.31%	0.16%	0.05%	0.30%	0.52%	0.66%	0.70%	0.64%	0.50%	0.31%	0.08%	0.15%	0.37%
18-Mar-30	0.22%	0.40%	0.40%	0.36%	0.26%	0.10%	0.12%	0.35%	0.53%	0.64%	0.64%	0.57%	0.41%	0.20%	0.05%	0.30%	0.55%
17-Sep-30	0.27%	0.55%	0.55%	0.46%	0.31%	0.11%	0.15%	0.41%	0.64%	0.80%	0.85%	0.82%	0.71%	0.56%	0.36%	0.16%	0.04%
17-Mar-31	0.37%	0.62%	0.60%	0.52%	0.37%	0.15%	0.12%	0.39%	0.62%	0.80%	0.88%	0.88%	0.79%	0.65%	0.46%	0.27%	0.08%
16-Sep-31	0.39%	0.59%	0.58%	0.50%	0.35%	0.14%	0.12%	0.38%	0.60%	0.77%	0.83%	0.82%	0.73%	0.59%	0.41%	0.22%	0.04%
16-Mar-32	0.17%	0.34%	0.33%	0.23%	0.07%	0.14%	0.41%	0.65%	0.85%	0.99%	1.04%	1.00%	0.90%	0.76%	0.58%	0.39%	0.21%
16-Sep-32	0.22%	0.44%	0.42%	0.30%	0.11%	0.13%	0.41%	0.68%	0.90%	1.07%	1.13%	1.13%	1.06%	0.94%	0.79%	0.62%	0.44%
16-Mar-33	0.23%	0.43%	0.38%	0.26%	0.07%	0.19%	0.49%	0.77%	1.01%	1.21%	1.33%	1.37%	1.34%	1.27%	1.16%	1.02%	0.88%
16-Sep-33	0.31%	0.50%	0.46%	0.33%	0.15%	0.09%	0.35%	0.56%	0.70%	0.77%	0.77%	0.69%	0.55%	0.37%	0.16%	0.08%	0.32%
16-Mar-34	0.12%	0.29%	0.23%	0.09%	0.11%	0.35%	0.61%	0.80%	0.93%	0.98%	0.96%	0.87%	0.72%	0.55%	0.33%	0.10%	0.14%
19-Sep-34	0.14%	0.31%	0.27%	0.13%	0.09%	0.36%	0.64%	0.85%	0.99%	1.05%	1.04%	0.96%	0.83%	0.67%	0.48%	0.26%	0.05%
16-Mar-35	0.14%	0.35%	0.30%	0.15%	0.07%	0.32%	0.57%	0.75%	0.85%	0.85%	0.79%	0.65%	0.47%	0.26%	0.02%	0.24%	0.49%
18-Sep-35	0.03%	0.15%	0.07%	0.10%	0.32%	0.59%	0.83%	1.01%	1.09%	1.09%	1.02%	0.89%	0.72%	0.52%	0.29%	0.04%	0.20%
17-Mar-36	0.08%	0.08%	0.01%	0.16%	0.42%	0.70%	0.96%	1.14%	1.23%	1.24%	1.19%	1.07%	0.91%	0.73%	0.52%	0.29%	0.07%
16-Sep-36	0.01%	0.12%	0.01%	0.22%	0.50%	0.82%	1.10%	1.31%	1.45%	1.52%	1.53%	1.47%	1.37%	1.24%	1.10%	0.93%	0.74%
16-Mar-37	0.16%	0.08%	0.21%	0.44%	0.74%	1.06%	1.33%	1.54%	1.67%	1.73%	1.73%	1.66%	1.57%	1.44%	1.30%	1.13%	0.96%
16-Sep-37	0.15%	0.09%	0.23%	0.46%	0.77%	1.11%	1.39%	1.61%	1.74%	1.81%	1.83%	1.78%	1.70%	1.60%	1.48%	1.34%	1.19%
16-Mar-38	0.29%	0.27%	0.43%	0.68%	0.99%	1.34%	1.61%	1.81%	1.93%	1.99%	2.00%	1.94%	1.86%	1.76%	1.64%	1.51%	1.35%

Calibration across strikes and maturity (2)

- Hybrid Model fit (constant Vvol and Rho and LV with 3-parameters)

	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165
16-Sep-09	5.08%	1.70%	2.35%	1.92%	1.11%	0.80%	0.58%	0.27%	0.01%	0.12%	0.06%	0.03%	0.15%	0.07%	0.20%	0.49%	0.63%
16-Mar-10	2.22%	2.15%	1.98%	1.42%	1.02%	0.73%	0.38%	0.08%	0.02%	0.06%	0.09%	0.05%	0.10%	0.36%	0.56%	0.60%	0.47%
16-Sep-10	3.59%	2.46%	1.67%	1.25%	0.89%	0.57%	0.30%	0.11%	0.06%	0.09%	0.11%	0.02%	0.04%	0.12%	0.25%	0.29%	0.30%
16-Mar-11	2.57%	1.49%	1.16%	0.84%	0.54%	0.31%	0.14%	0.01%	0.12%	0.05%	0.22%	0.00%	0.11%	0.01%	0.06%	0.08%	0.02%
16-Sep-11	2.59%	1.78%	1.35%	0.98%	0.64%	0.37%	0.16%	0.00%	0.11%	0.05%	0.24%	0.02%	0.13%	0.15%	0.06%	0.05%	0.10%
16-Mar-12	3.51%	2.08%	1.47%	1.09%	0.78%	0.49%	0.22%	0.03%	0.09%	0.06%	0.20%	0.01%	0.11%	0.05%	0.08%	0.18%	0.26%
18-Sep-12	3.42%	2.04%	1.32%	0.84%	0.47%	0.26%	0.09%	0.04%	0.10%	0.01%	0.26%	0.00%	0.15%	0.15%	0.06%	0.02%	0.09%
18-Mar-13	3.39%	2.05%	1.39%	0.88%	0.46%	0.20%	0.06%	0.05%	0.11%	0.02%	0.27%	0.01%	0.15%	0.14%	0.07%	0.00%	0.06%
17-Sep-13	3.65%	2.27%	1.54%	1.00%	0.60%	0.28%	0.06%	0.07%	0.11%	0.04%	0.28%	0.03%	0.14%	0.19%	0.16%	0.13%	0.10%
17-Mar-14	3.87%	2.40%	1.62%	1.01%	0.56%	0.26%	0.05%	0.10%	0.12%	0.04%	0.25%	0.02%	0.12%	0.21%	0.24%	0.27%	0.29%
16-Sep-14	3.90%	2.50%	1.64%	1.05%	0.63%	0.29%	0.07%	0.09%	0.12%	0.05%	0.20%	0.00%	0.10%	0.13%	0.13%	0.13%	0.10%
16-Mar-15	3.92%	2.34%	1.54%	1.02%	0.59%	0.29%	0.06%	0.10%	0.13%	0.07%	0.18%	0.01%	0.09%	0.13%	0.14%	0.16%	0.16%
16-Sep-15	4.01%	2.40%	1.56%	0.96%	0.54%	0.23%	0.01%	0.12%	0.12%	0.08%	0.17%	0.02%	0.09%	0.13%	0.17%	0.21%	0.20%
16-Mar-16	4.05%	2.41%	1.52%	0.92%	0.48%	0.18%	0.02%	0.14%	0.10%	0.10%	0.17%	0.02%	0.11%	0.21%	0.30%	0.35%	0.38%
16-Sep-16	4.03%	2.29%	1.42%	0.85%	0.41%	0.12%	0.06%	0.15%	0.09%	0.15%	0.23%	0.08%	0.06%	0.16%	0.26%	0.34%	0.40%
16-Mar-17	4.02%	2.20%	1.37%	0.80%	0.40%	0.10%	0.10%	0.17%	0.08%	0.15%	0.22%	0.09%	0.05%	0.15%	0.23%	0.29%	0.33%
19-Sep-17	3.90%	2.12%	1.27%	0.73%	0.33%	0.02%	0.16%	0.19%	0.05%	0.17%	0.21%	0.10%	0.03%	0.12%	0.17%	0.19%	0.20%
16-Mar-18	3.75%	2.02%	1.17%	0.63%	0.25%	0.01%	0.17%	0.18%	0.03%	0.21%	0.22%	0.09%	0.04%	0.16%	0.24%	0.29%	0.32%
18-Sep-18	3.59%	1.92%	1.06%	0.57%	0.21%	0.04%	0.17%	0.17%	0.01%	0.22%	0.24%	0.11%	0.04%	0.17%	0.25%	0.32%	0.35%
18-Mar-19	3.42%	1.75%	0.94%	0.43%	0.11%	0.09%	0.19%	0.16%	0.02%	0.23%	0.23%	0.10%	0.02%	0.13%	0.20%	0.25%	0.27%
17-Sep-19	3.38%	1.69%	0.86%	0.35%	0.03%	0.16%	0.22%	0.13%	0.06%	0.24%	0.25%	0.12%	0.03%	0.13%	0.20%	0.24%	0.27%
16-Mar-20	3.29%	1.54%	0.77%	0.27%	0.03%	0.19%	0.23%	0.12%	0.09%	0.25%	0.24%	0.13%	0.00%	0.12%	0.18%	0.23%	0.26%
16-Sep-20	3.06%	1.38%	0.68%	0.23%	0.03%	0.18%	0.20%	0.10%	0.08%	0.20%	0.18%	0.07%	0.04%	0.11%	0.12%	0.11%	0.09%
16-Mar-21	3.05%	1.35%	0.62%	0.20%	0.07%	0.21%	0.21%	0.09%	0.09%	0.21%	0.20%	0.10%	0.02%	0.10%	0.15%	0.17%	0.18%
16-Sep-21	2.86%	1.27%	0.54%	0.13%	0.12%	0.23%	0.19%	0.06%	0.11%	0.22%	0.20%	0.09%	0.04%	0.13%	0.18%	0.21%	0.21%
16-Mar-22	2.72%	1.09%	0.41%	0.02%	0.19%	0.25%	0.20%	0.03%	0.15%	0.24%	0.20%	0.08%	0.06%	0.15%	0.19%	0.21%	0.20%
16-Sep-22	2.67%	1.01%	0.34%	0.07%	0.25%	0.28%	0.18%	0.00%	0.19%	0.29%	0.26%	0.13%	0.02%	0.13%	0.20%	0.24%	0.27%
16-Mar-23	2.45%	0.87%	0.22%	0.14%	0.28%	0.29%	0.18%	0.02%	0.22%	0.30%	0.27%	0.14%	0.02%	0.14%	0.22%	0.27%	0.30%
19-Sep-23	2.24%	0.72%	0.09%	0.23%	0.33%	0.29%	0.16%	0.05%	0.22%	0.30%	0.26%	0.13%	0.01%	0.12%	0.19%	0.24%	0.27%
18-Mar-24	2.06%	0.54%	0.04%	0.29%	0.35%	0.29%	0.13%	0.09%	0.26%	0.33%	0.29%	0.17%	0.02%	0.10%	0.21%	0.27%	0.33%
17-Sep-24	2.07%	0.56%	0.00%	0.27%	0.34%	0.28%	0.12%	0.09%	0.26%	0.33%	0.30%	0.18%	0.05%	0.08%	0.20%	0.29%	0.34%
17-Mar-25	2.04%	0.54%	0.00%	0.25%	0.32%	0.26%	0.12%	0.08%	0.24%	0.31%	0.30%	0.19%	0.06%	0.07%	0.18%	0.26%	0.31%
16-Sep-25	2.02%	0.57%	0.00%	0.24%	0.30%	0.23%	0.08%	0.08%	0.20%	0.23%	0.18%	0.08%	0.04%	0.13%	0.19%	0.22%	0.23%
16-Mar-26	2.06%	0.60%	0.04%	0.21%	0.30%	0.24%	0.09%	0.07%	0.17%	0.20%	0.16%	0.06%	0.04%	0.13%	0.20%	0.24%	0.27%
16-Sep-26	2.11%	0.61%	0.05%	0.19%	0.26%	0.22%	0.09%	0.06%	0.15%	0.18%	0.13%	0.04%	0.06%	0.14%	0.21%	0.24%	0.24%
16-Mar-27	2.10%	0.61%	0.08%	0.16%	0.24%	0.21%	0.10%	0.04%	0.13%	0.16%	0.14%	0.05%	0.05%	0.11%	0.17%	0.21%	0.22%
16-Sep-27	2.06%	0.61%	0.07%	0.18%	0.27%	0.23%	0.10%	0.04%	0.14%	0.19%	0.17%	0.11%	0.03%	0.05%	0.12%	0.16%	0.20%
16-Mar-28	1.97%	0.54%	0.03%	0.21%	0.28%	0.23%	0.09%	0.03%	0.12%	0.17%	0.16%	0.12%	0.06%	0.00%	0.06%	0.10%	0.13%
19-Sep-28	1.93%	0.52%	0.05%	0.19%	0.25%	0.20%	0.08%	0.03%	0.09%	0.13%	0.12%	0.07%	0.01%	0.04%	0.09%	0.13%	0.13%
16-Mar-29	1.87%	0.51%	0.04%	0.20%	0.25%	0.19%	0.07%	0.03%	0.08%	0.12%	0.11%	0.06%	0.01%	0.05%	0.09%	0.13%	0.14%
18-Sep-29	1.93%	0.54%	0.05%	0.18%	0.24%	0.18%	0.07%	0.01%	0.08%	0.11%	0.11%	0.07%	0.01%	0.04%	0.08%	0.12%	0.14%
18-Mar-30	1.91%	0.56%	0.08%	0.15%	0.20%	0.16%	0.08%	0.01%	0.04%	0.07%	0.09%	0.06%	0.02%	0.01%	0.04%	0.06%	0.08%
17-Sep-30	1.92%	0.56%	0.09%	0.14%	0.20%	0.16%	0.09%	0.02%	0.03%	0.07%	0.08%	0.06%	0.02%	0.02%	0.06%	0.10%	0.12%
17-Mar-31	1.94%	0.60%	0.12%	0.10%	0.18%	0.15%	0.08%	0.02%	0.03%	0.08%	0.09%	0.06%	0.01%	0.03%	0.08%	0.13%	0.17%
16-Sep-31	1.86%	0.55%	0.08%	0.13%	0.18%	0.15%	0.09%	0.05%	0.01%	0.02%	0.04%	0.04%	0.03%	0.03%	0.03%	0.01%	0.01%
16-Mar-32	1.88%	0.58%	0.12%	0.09%	0.16%	0.14%	0.09%	0.06%	0.04%	0.01%	0.00%	0.00%	0.01%	0.02%	0.01%	0.02%	0.02%
16-Sep-32	1.74%	0.49%	0.07%	0.11%	0.17%	0.15%	0.10%	0.06%	0.04%	0.03%	0.02%	0.01%	0.02%	0.02%	0.03%	0.05%	0.05%
16-Mar-33	1.72%	0.50%	0.08%	0.11%	0.16%	0.14%	0.09%	0.05%	0.05%	0.04%	0.03%	0.02%	0.02%	0.02%	0.03%	0.04%	0.05%
16-Sep-33	1.66%	0.49%	0.10%	0.08%	0.14%	0.13%	0.10%	0.06%	0.05%	0.05%	0.04%	0.02%	0.00%	0.01%	0.02%	0.02%	0.02%
16-Mar-34	1.63%	0.49%	0.12%	0.06%	0.12%	0.11%	0.09%	0.08%	0.09%	0.08%	0.07%	0.06%	0.04%	0.02%	0.01%	0.00%	0.00%
19-Sep-34	1.60%	0.51%	0.13%	0.06%	0.13%	0.12%	0.10%	0.09%	0.08%	0.07%	0.07%	0.06%	0.05%	0.03%	0.00%	0.01%	0.02%
16-Mar-35	1.61%	0.48%	0.10%	0.07%	0.14%	0.13%	0.11%	0.10%	0.09%	0.08%	0.07%	0.06%	0.04%	0.03%	0.01%	0.01%	0.01%
18-Sep-35	1.57%	0.49%	0.13%	0.05%	0.13%	0.13%	0.11%	0.09%	0.09%	0.08%	0.07%	0.07%	0.05%	0.02%	0.00%	0.02%	0.02%
17-Mar-36	1.49%	0.42%	0.06%	0.09%	0.13%	0.12%	0.11%	0.09%	0.09%	0.08%	0.07%	0.04%	0.02%	0.01%	0.04%	0.06%	0.08%
16-Sep-36	1.45%	0.40%	0.05%	0.10%	0.13%	0.11%	0.09%	0.09%	0.09%	0.09%	0.10%	0.09%	0.08%	0.07%	0.05%	0.04%	0.03%
16-Mar-37	1.40%	0.37%	0.01%	0.14%	0.15%	0.12%	0.09%	0.08%	0.08%	0.07%	0.07%	0.06%	0.05%	0.04%	0.02%	0.01%	0.01%
16-Sep-37	1.38%	0.34%	0.02%	0.10%	0.11%	0.09%	0.09%	0.09%	0.07%	0.06%	0.06%	0.04%	0.01%	0.01%	0.04%	0.06%	0.08%
16-Mar-38	1.33%	0.33%	0.02%	0.08%	0.10%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.07%	0.06%	0.04%	0.02%	0.01%	0.05%

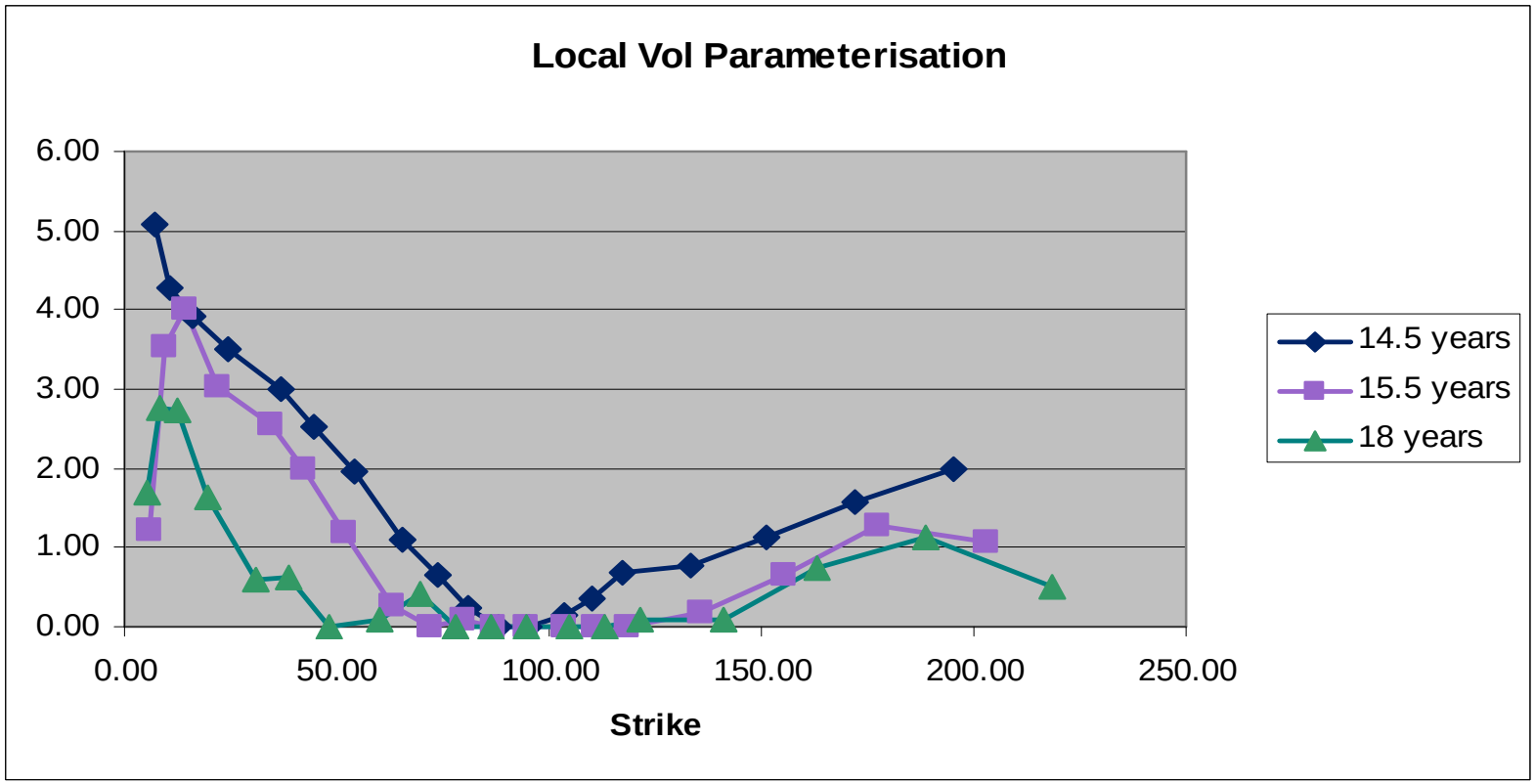
Calibration across strikes and maturity (3)

- MVD Model fit (c0 = 0.75, c1 = 0.1)

	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165
16-Sep-09	5.08%	1.52%	1.64%	1.50%	0.84%	0.22%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%	0.04%	0.15%	0.61%	1.36%
16-Mar-10	0.88%	0.09%	0.26%	0.23%	0.18%	0.02%	0.00%	0.01%	0.00%	0.00%	0.01%	0.01%	0.01%	0.08%	0.39%	0.64%	1.09%
16-Sep-10	0.52%	0.31%	0.12%	0.06%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	0.21%	0.31%
16-Mar-11	3.21%	1.59%	0.84%	0.27%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.07%	0.10%	0.14%
16-Sep-11	0.73%	0.36%	0.31%	0.08%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.10%	0.23%	0.33%
16-Mar-12	0.19%	0.03%	0.18%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%
18-Sep-12	0.73%	0.35%	0.19%	0.02%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.02%	0.05%	0.04%
18-Mar-13	0.86%	0.44%	0.11%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.02%	0.02%	0.02%
17-Sep-13	0.19%	0.14%	0.05%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.03%	0.08%	0.14%
17-Mar-14	0.07%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%
16-Sep-14	0.30%	0.03%	0.01%	0.00%	0.00%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.07%	0.13%
16-Mar-15	0.04%	0.05%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%
16-Sep-15	0.10%	0.10%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.06%	0.09%
16-Mar-16	0.06%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
16-Sep-16	0.41%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.03%
16-Mar-17	0.25%	0.03%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
19-Sep-17	0.44%	0.03%	0.01%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
16-Mar-18	0.31%	0.02%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
18-Sep-18	0.17%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
18-Mar-19	0.29%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
17-Sep-19	0.16%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%
16-Mar-20	0.12%	0.00%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
16-Sep-20	0.20%	0.01%	0.00%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.00%
16-Mar-21	0.17%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.01%	0.02%	0.01%
16-Sep-21	0.14%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.01%	0.01%
16-Mar-22	0.13%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
16-Sep-22	0.08%	0.01%	0.01%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%
16-Mar-23	0.10%	0.01%	0.00%	0.01%	0.01%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%
19-Sep-23	0.08%	0.01%	0.00%	0.00%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
18-Mar-24	0.03%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%
17-Sep-24	0.04%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
17-Mar-25	0.04%	0.01%	0.00%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%
16-Sep-25	0.05%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.02%	0.01%	0.02%
16-Mar-26	0.04%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%
16-Sep-26	0.04%	0.01%	0.01%	0.00%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
16-Mar-27	0.04%	0.01%	0.01%	0.00%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%
16-Sep-27	0.03%	0.01%	0.00%	0.01%	0.01%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
16-Mar-28	0.02%	0.02%	0.00%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%
19-Sep-28	0.02%	0.02%	0.00%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%
16-Mar-29	0.01%	0.01%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%
18-Sep-29	0.02%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%
18-Mar-30	0.02%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
17-Sep-30	0.02%	0.01%	0.00%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%
17-Mar-31	0.01%	0.02%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%
16-Sep-31	0.02%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.01%
16-Mar-32	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
16-Sep-32	0.02%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
16-Mar-33	0.01%	0.02%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
16-Sep-33	0.01%	0.01%	0.01%	0.02%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%
16-Mar-34	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
19-Sep-34	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
16-Mar-35	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%
18-Sep-35	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
17-Mar-36	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
16-Sep-36	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%
16-Mar-37	0.00%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
16-Sep-37	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
16-Mar-38	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%

Vol Squeeze

- Increase in USD swaption vol
- No model vol solution exists (for given correlation) to match market FX vol
- Local vol shows a clear pathology in order to adapt to the vol squeeze



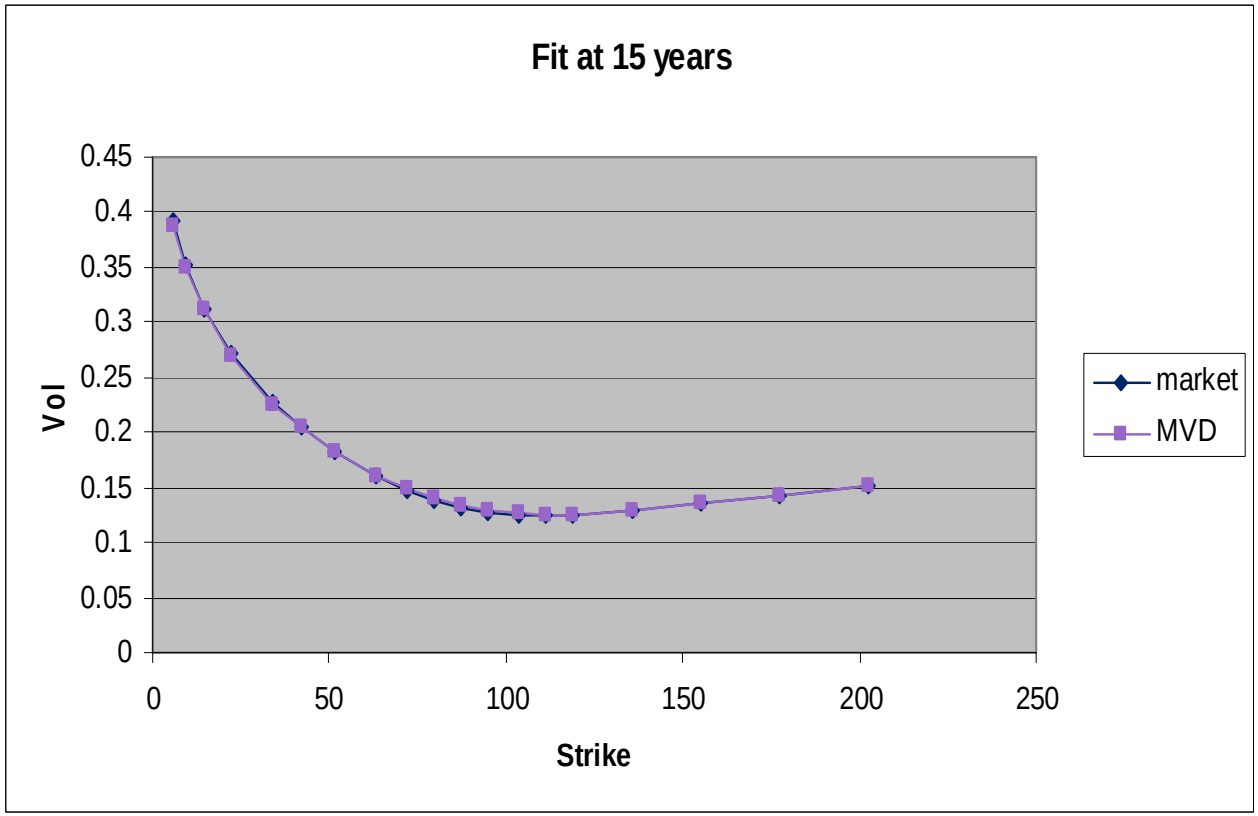
RBS
The Royal Bank of Scotland

Vol Squeeze: MVD model fit

	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165
16-Sep-09	5.08%	3.71%	0.47%	0.78%	0.57%	0.37%	0.04%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.11%	0.28%	0.53%	0.86%
16-Mar-10	5.03%	3.05%	0.99%	0.45%	0.19%	0.01%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.03%	0.08%	0.04%	0.54%
16-Sep-10	3.02%	1.28%	0.67%	0.31%	0.11%	0.02%	0.01%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.02%	0.09%	0.14%
16-Mar-11	0.25%	0.75%	0.33%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.01%	0.00%	0.02%	0.06%	0.07%
16-Sep-11	0.27%	0.33%	0.24%	0.06%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
16-Mar-12	2.24%	1.13%	0.47%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.01%	0.00%	0.09%	0.22%	0.28%
18-Sep-12	0.58%	0.39%	0.17%	0.02%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.07%	0.15%	0.18%
18-Mar-13	0.60%	0.36%	0.12%	0.00%	0.01%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.04%	0.09%	0.18%
17-Sep-13	0.38%	0.06%	0.04%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%
17-Mar-14	0.07%	0.06%	0.05%	0.01%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.02%	0.01%
16-Sep-14	0.40%	0.04%	0.00%	0.01%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.02%	0.03%
16-Mar-15	0.38%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.08%	0.17%
16-Sep-15	0.68%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.02%
16-Mar-16	0.18%	0.04%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.04%
16-Sep-16	0.38%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%
16-Mar-17	0.59%	0.07%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.06%
19-Sep-17	0.21%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.01%	0.04%
16-Mar-18	0.28%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.03%
18-Sep-18	0.16%	0.00%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%
18-Mar-19	0.25%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.03%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%
17-Sep-19	0.14%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.04%	0.02%	0.01%	0.01%	0.00%	0.01%	0.01%	0.01%
16-Mar-20	0.23%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.04%	0.01%	0.01%	0.00%	0.00%	0.01%	0.01%	0.00%
16-Sep-20	0.18%	0.00%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.00%	0.02%	0.00%	0.01%	0.00%	0.00%	0.01%	0.00%	0.01%
16-Mar-21	0.14%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.03%	0.05%	0.06%	0.07%	0.09%	0.12%	0.14%
16-Sep-21	0.16%	0.01%	0.01%	0.00%	0.00%	0.01%	0.01%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
16-Mar-22	0.16%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.01%	0.05%	0.01%	0.00%	0.01%	0.02%	0.01%	0.00%	0.01%
16-Sep-22	0.13%	0.02%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.05%	0.10%	0.03%	0.01%	0.00%	0.01%	0.03%	0.04%	0.03%
16-Mar-23	0.09%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.00%	0.05%	0.13%	0.04%	0.01%	0.03%	0.01%	0.02%	0.01%	0.01%
19-Sep-23	0.07%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.01%	0.02%	0.06%	0.01%	0.01%	0.02%	0.01%	0.01%	0.01%	0.01%
18-Mar-24	0.02%	0.00%	0.01%	0.00%	0.01%	0.00%	0.01%	0.01%	0.04%	0.10%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
17-Sep-24	0.39%	0.17%	0.11%	0.08%	0.06%	0.03%	0.01%	0.09%	0.21%	0.28%	0.18%	0.09%	0.04%	0.01%	0.01%	0.03%	0.03%
17-Mar-25	0.57%	0.28%	0.17%	0.09%	0.04%	0.02%	0.10%	0.23%	0.42%	0.51%	0.40%	0.27%	0.19%	0.12%	0.09%	0.08%	0.07%
16-Sep-25	0.34%	0.11%	0.09%	0.07%	0.03%	0.04%	0.17%	0.38%	0.62%	0.72%	0.61%	0.45%	0.33%	0.22%	0.15%	0.10%	0.08%
16-Mar-26	0.59%	0.36%	0.27%	0.19%	0.12%	0.03%	0.24%	0.52%	0.80%	0.91%	0.81%	0.63%	0.46%	0.32%	0.20%	0.12%	0.07%
16-Sep-26	0.85%	0.43%	0.26%	0.17%	0.12%	0.06%	0.32%	0.69%	0.99%	1.10%	1.01%	0.82%	0.62%	0.44%	0.28%	0.16%	0.08%
16-Mar-27	1.10%	0.61%	0.41%	0.28%	0.16%	0.11%	0.45%	0.87%	1.20%	1.32%	1.24%	1.05%	0.83%	0.63%	0.46%	0.33%	0.24%
16-Sep-27	1.30%	0.79%	0.54%	0.39%	0.20%	0.15%	0.58%	1.03%	1.38%	1.51%	1.45%	1.26%	1.03%	0.82%	0.63%	0.47%	0.34%
16-Mar-28	1.44%	0.93%	0.66%	0.45%	0.19%	0.23%	0.77%	1.23%	1.57%	1.71%	1.66%	1.48%	1.26%	1.03%	0.82%	0.64%	0.48%
19-Sep-28	1.63%	1.08%	0.76%	0.51%	0.16%	0.34%	0.94%	1.41%	1.77%	1.92%	1.89%	1.72%	1.52%	1.29%	1.06%	0.86%	0.68%
16-Mar-29	1.74%	1.15%	0.84%	0.54%	0.10%	0.45%	1.09%	1.58%	1.94%	2.11%	2.09%	1.95%	1.75%	1.53%	1.29%	1.08%	0.89%
18-Sep-29	1.95%	1.31%	0.94%	0.57%	0.05%	0.59%	1.25%	1.76%	2.13%	2.30%	2.30%	2.17%	1.98%	1.77%	1.52%	1.30%	1.09%
18-Mar-30	2.15%	1.46%	1.04%	0.58%	0.03%	0.73%	1.41%	1.93%	2.32%	2.49%	2.51%	2.40%	2.22%	2.01%	1.78%	1.54%	1.33%
17-Sep-30	2.34%	1.59%	1.11%	0.56%	0.12%	0.87%	1.57%	2.11%	2.50%	2.69%	2.73%	2.63%	2.47%	2.27%	2.04%	1.80%	1.59%
17-Mar-31	2.50%	1.71%	1.16%	0.53%	0.22%	1.02%	1.73%	2.29%	2.67%	2.87%	2.93%	2.86%	2.71%	2.52%	2.30%	2.07%	1.84%
16-Sep-31	2.65%	1.81%	1.19%	0.48%	0.34%	1.18%	1.89%	2.46%	2.85%	3.06%	3.13%	3.08%	2.95%	2.78%	2.57%	2.34%	2.12%
16-Mar-32	2.79%	1.89%	1.20%	0.41%	0.47%	1.34%	2.05%	2.62%	3.01%	3.23%	3.32%	3.30%	3.18%	3.02%	2.83%	2.61%	2.39%
16-Sep-32	2.91%	1.96%	1.20%	0.33%	0.60%	1.49%	2.21%	2.78%	3.17%	3.40%	3.50%	3.50%	3.41%	3.26%	3.08%	2.88%	2.67%
16-Mar-33	3.02%	2.01%	1.18%	0.23%	0.74%	1.64%	2.37%	2.94%	3.32%	3.56%	3.68%	3.70%	3.63%	3.50%	3.34%	3.15%	2.95%
16-Sep-33	3.11%	2.05%	1.14%	0.13%	0.89%	1.80%	2.53%	3.10%	3.48%	3.73%	3.86%	3.90%	3.84%	3.74%	3.59%	3.42%	3.23%
16-Mar-34	3.20%	2.08%	1.08%	0.01%	1.04%	1.95%	2.68%	3.24%	3.62%	3.88%	4.04%	4.09%	4.05%	3.96%	3.83%	3.68%	3.50%
19-Sep-34	3.28%	2.09%	1.00%	0.13%	1.21%	2.12%	2.85%	3.40%	3.77%	4.04%	4.20%	4.27%	4.25%	4.17%	4.06%	3.92%	3.75%
16-Mar-35	3.35%	2.09%	0.92%	0.27%	1.37%	2.27%	2.99%	3.53%	3.90%	4.17%	4.35%	4.42%	4.41%	4.34%	4.25%	4.12%	3.96%
18-Sep-35	3.41%	2.07%	0.82%	0.42%	1.53%	2.43%	3.15%	3.67%	4.04%	4.32%	4.50%	4.58%	4.58%	4.53%	4.44%	4.33%	4.18%
17-Mar-36	3.46%	2.04%	0.71%	0.58%	1.69%	2.59%	3.29%	3.80%	4.18%	4.45%	4.64%	4.73%	4.74%	4.71%	4.63%	4.53%	4.40%
16-Sep-36	3.50%	2.00%	0.59%	0.74%	1.86%	2.75%	3.44%	3.94%	4.31%	4.59%	4.78%	4.88%	4.91%	4.89%	4.83%	4.74%	4.62%
16-Mar-37	3.54%	1.94%	0.46%	0.90%	2.02%	2.90%	3.57%	4.06%	4.44%	4.72%	4.91%	5.02%	5.06%	5.05%	5.00%	4.93%	4.82%
16-Sep-37	3.56%	1.87%	0.32%	1.06%	2.17%	3.04%	3.70%	4.19%	4.56%	4.85%	5.04%	5.16%	5.21%	5.21%	5.18%	5.12%	5.02%
16-Mar-38	3.57%	1.78%	0.17%	1.23%	2.33%	3.19%	3.84%	4.31%	4.68%	4.98%	5.18%	5.30%	5.36%	5.37%	5.36%	5.31%	5.22%

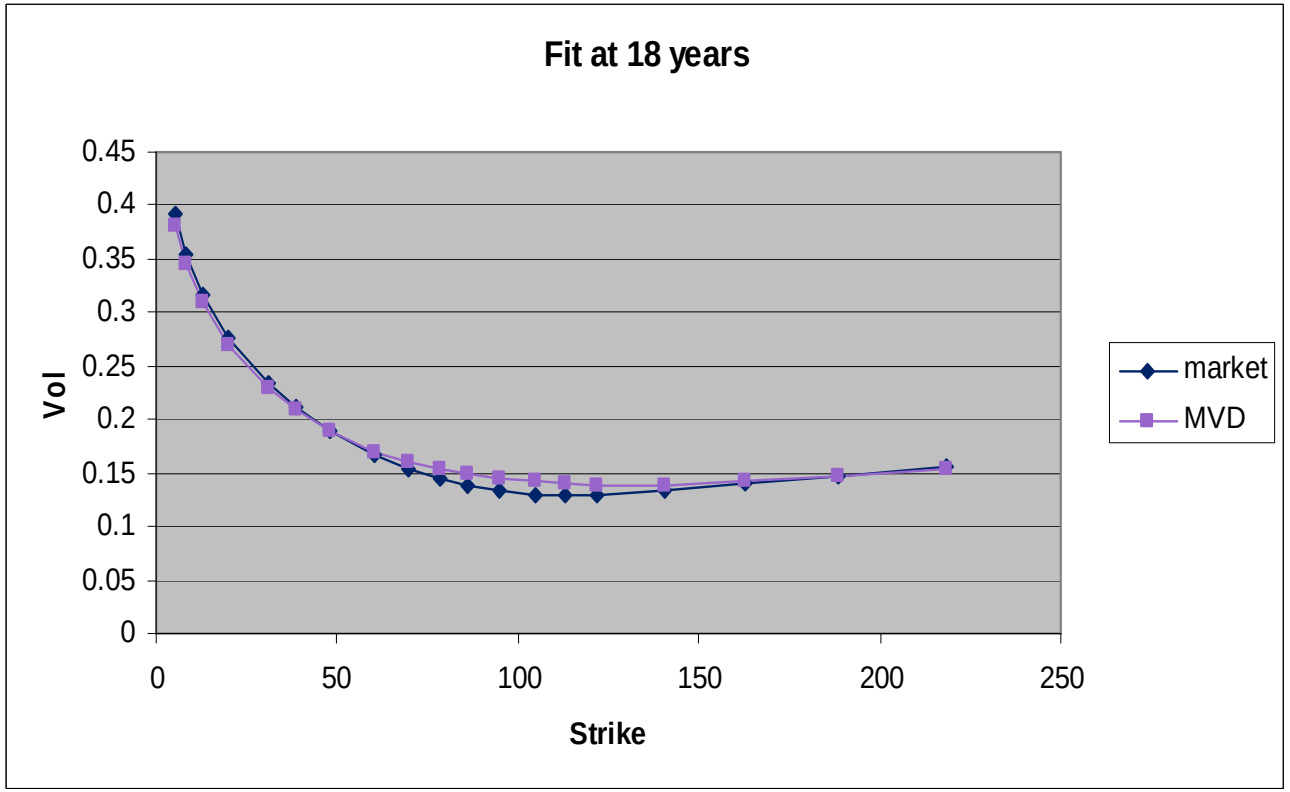
MVD smile fit at maturity

- USDJPY at 15 years



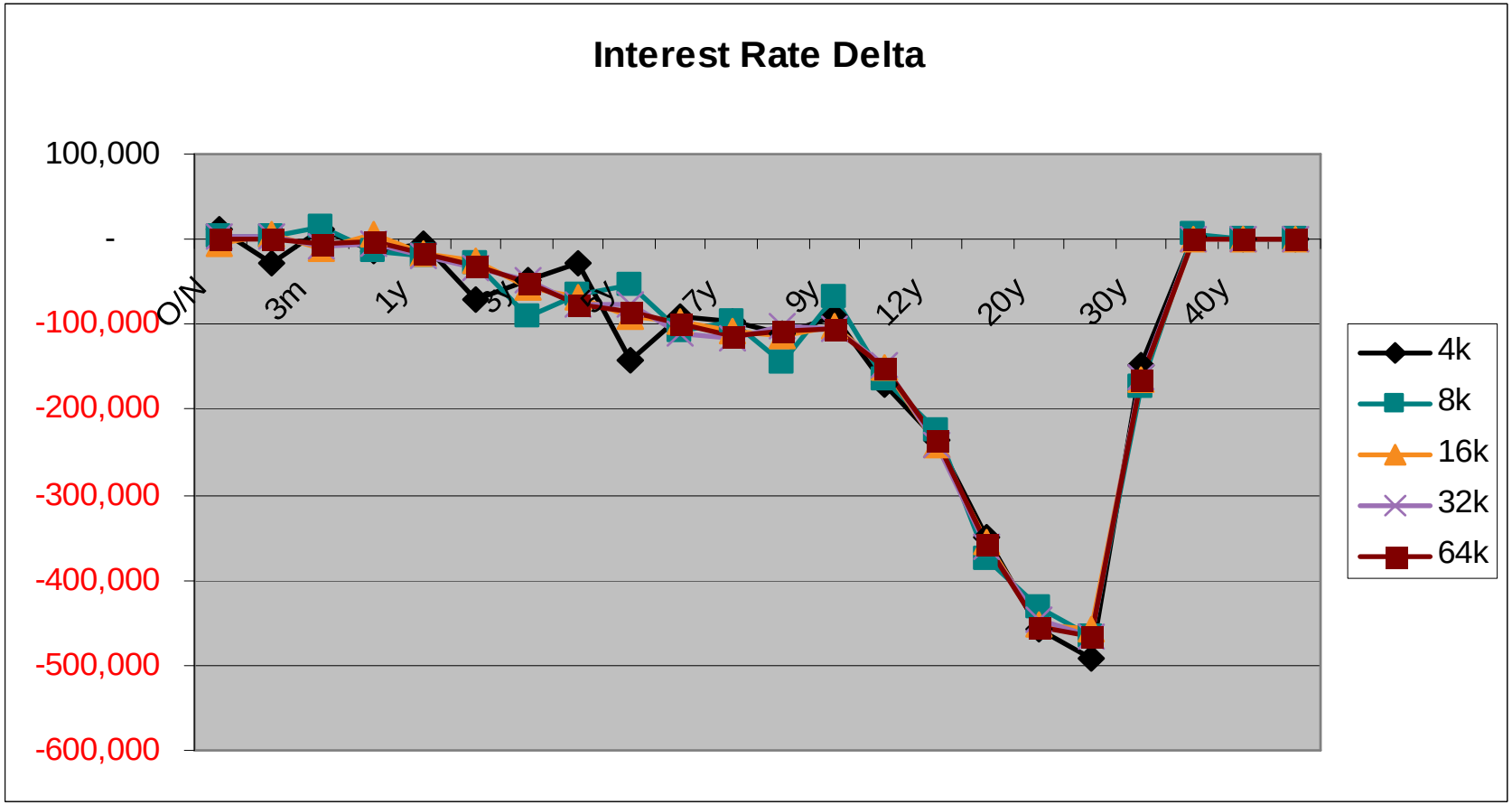
MVD smile fit at maturity (2)

- USDJPY at 18 years



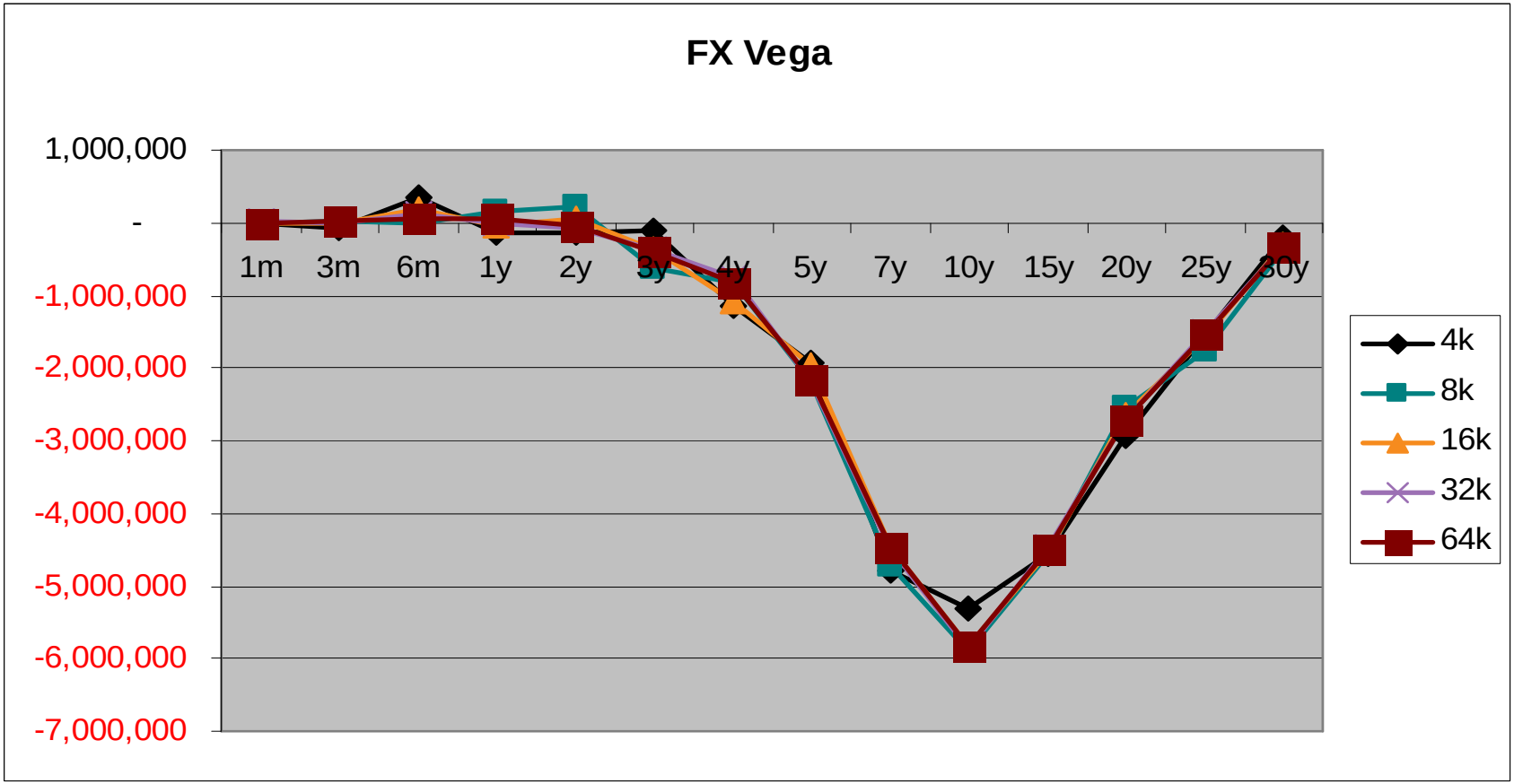
Risk sensitivities

- PRDC Delta as a function of MC paths



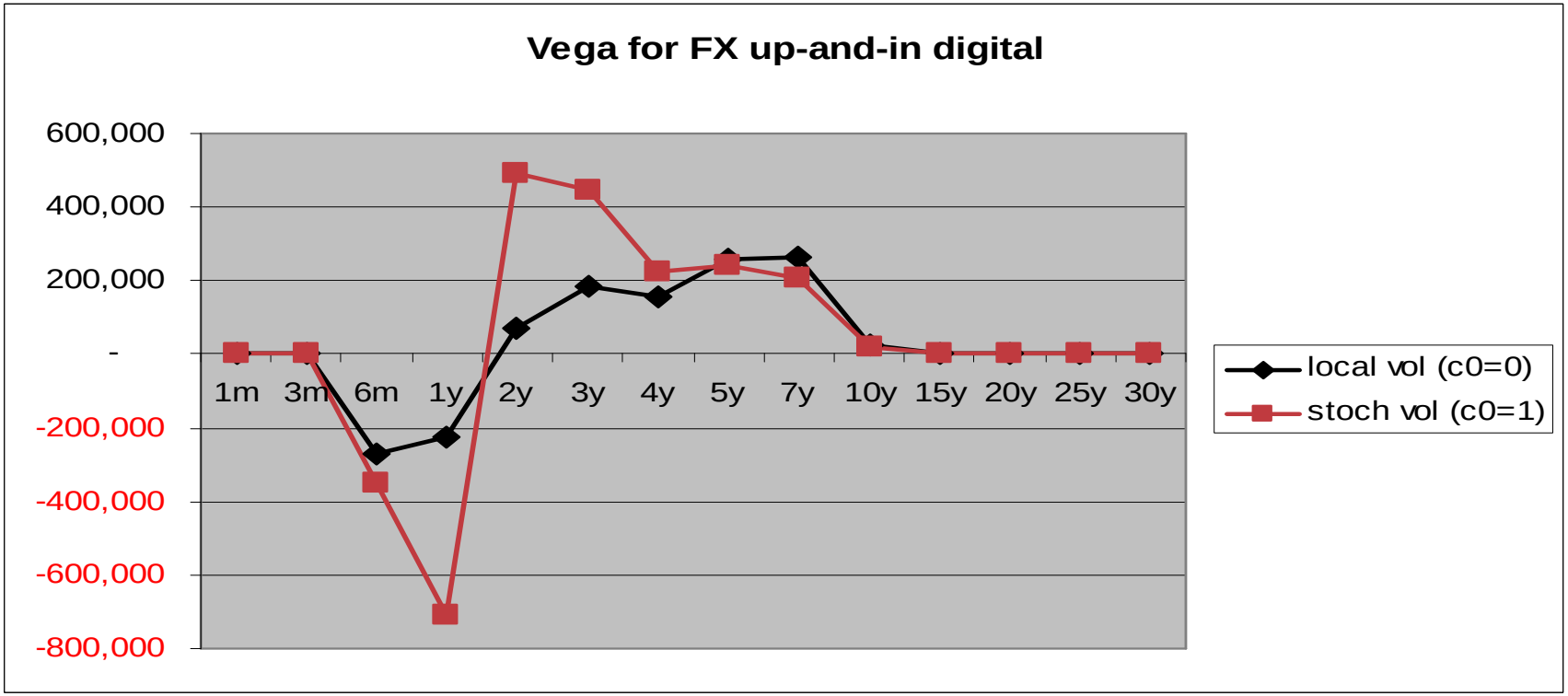
Risk sensitivities (2)

- PRDC FX Vega as a function of MC paths



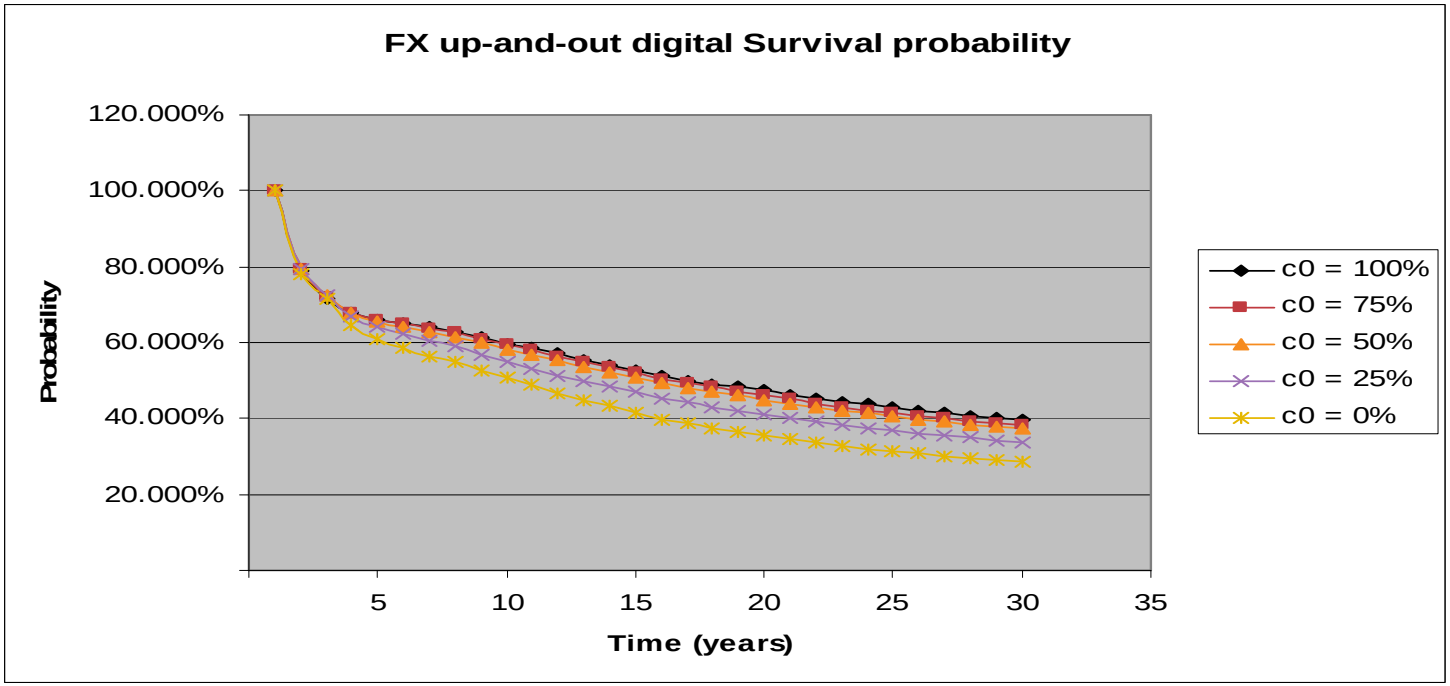
Pricing effects from mixture

- FX Vega risk for long dated up-and-in barrier
  - Difference between pure LV and pure SV limits
  - Vega crosses over in sign as it moves from in to out of the money



Pricing effects from mixture (2)

- FX up-and-out digital as a function of c_0
  - Higher the amount of SV => higher the terminal survival probability.
  - In SV, path-wise vol depends strongly on its previous value. Surviving paths thus have a lower vol and will continue to survive
  - In LV, vol depends only on strike, so surviving paths have a similar vol to the average



Limitations: MVD in practise

- Numerical efficiencies crucial:
  - Monte Carlo generation
  - Solver algorithms
  - Use of GRID computing for Risk sensitivities

- Other (non-trivial) considerations:
  - Correlations across multifactor ‘blocks’ e.g. when Rates processes are multifactor
  - Calibration of Mixing parameters, using prices of liquid (and short dated) FX exotics e.g. American digitals, or Barrier options
  - Product variations stretch the model if smooth Risks are required (for given computing power)
  - Given near exact fitting of Smile, how to deal with inconsistent market data, e.g. Vol Squeeze, SABR surfaces implying negative densities, negative forward interest rates

Conclusion

- Cross currency models covering important dynamics for long-dated exotic FX products, require multi-factors that incorporate Smile replication and dynamics.
- Formulation of the MVD model using a Monte Carlo implementation, with calibration constructed as part of evolution.
- We show how the MVD model can match market smiles even under extreme market conditions, while allowing extra parameters to mix the desired combination of LV and SV within the process.
- Model can be used in practise (i.e. in production running PV and Risk) but requires significant GRID computing.
- Model can be easily adapted for other ‘Hybrid’ assets and the pricing of these exotic structures.