Dynamics of disloctions lines

We are interested in the modeling of the dynamics of line defects in crystals, known as dislocations. We describe the dislocation line as the level set zero of an auxiliary function, generally the signed distance from the curve. The dynamics is obtained by the propagation of the auxiliary function by a speed normal to the front. The speed is a non-local function with sign changing in space and time. The model we obtain is a non-local Hamilton Jacobi equation.

We discretize the equation by a first order finite difference scheme. The scheme is based on a monotone numerical Hamiltonian, proposed by Osher and Sethian.

We present numerical simulations of the scheme. We consider a line propagating in normal direction with a speed given by the sum of a piecewise constant function and a convolution in space between the characteristic function of the set, which boundary is the dislocation line, and an anisotropic kernel, behaving as $\frac{-1}{|x|^3}$. The discrete convolution is computed by a simple rectangular formula applied on the numerical domain. The approximation is meaningful since the kernel tends to zero as x tends to infinity. The constant piecewise function is chosen positive outside of a circular region centered in the origin of the domain, where the speed is negative. Physically this means, that the dislocation line will propagate with positive speed meeting an obstacle in the center of the domain. In the simulation, we see in red the dislocation line and in blue the obstacle. The dislocation starts to propagate on the left and when it meets the obstacle the speed decreases, then the dislocation breaks, changing topology. One part of the dislocation continues the propagation on the left. Another small part is captured by the obstacle and collapses rapidly.