Investing in bonds or in stock market?

Michel De Lara

October 10, 2017

Contents

1 A look at annual data from stock French market

1.1 Annual historical returns

Here are data from `http://www.euronext.com`

```
 Indice CAC40  Performance

Fin année  Indice prix  % Indice prix  Rentabilité nette  % Rentabilité nette

31/12/1987  1000,00  -      1000,00  -
30/12/1988  1573,94  57,39%  1606,82  60,68%
29/12/1989  2001,08  27,14%  2082,58  29,61%
28/12/1990  1517,93  -24,14% 1609,91  -22,70%
31/12/1991  1765,66  16,32%  1916,19  19,02%
31/12/1992  1857,78  5,22%  2060,31  7,52%
```

```
31/12/1993  2268,22  22,09%  2570,48  24,76%
30/12/1994  1881,15  -17,06% 2176,64  -15,32%
29/12/1995  1871,97  -0,49%  2213,92  1,71%
31/12/1996  2315,73  23,71%  2795,90  26,29%
31/12/1997  2998,91  29,50%  3684,87  31,80%
30/12/1998  3942,66  31,47%  4908,03  33,19%
30/12/1999  5958,32  51,12%  7515,81  53,13%
29/12/2000  5926,42  -0,54%  7550,48  0,46%
28/12/2001  4624,58  -21,97% 5975,25  -20,86%
31/12/2002  3063,91  -33,75% 4032,77  -32,51%
31/12/2003  3557,90  16,12%  4785,01  18,65%
31/12/2004  3821,16  7,40%  5268,31  10,10%
30/12/2005  4715,23  23,40%  6669,63  26,60%
29/12/2006  5541,76  17,53%  8061,85  20,87%
```

Question 1 *Draw different graphical representations of the historical annual rentability of French stock market (in percentage).*

```
//

years=1988:2006 ;

cac40_perf_percent=[60.68 29.61 -22.70 19.02 7.52 ...
24.76 -15.32 1.71 26.29 31.80 33.19 53.13 0.46 -20.86 ...
-32.51 18.65 10.10 26.60 20.87] ;

xset('window',40) ; xbasc();
plot2d2(years,cac40_perf_percent);
xtitle('Annual rentability of French stock market (percentage)',...
'year','%')

xset('window',41) ; xbasc();
plot2d(sort(cac40_perf_percent));
xtitle('Sorted annual rentability of French stock market (percentage)')

xset('window',42) ; xbasc();
histplot((-40):20:70,cac40_perf_percent);

//
```

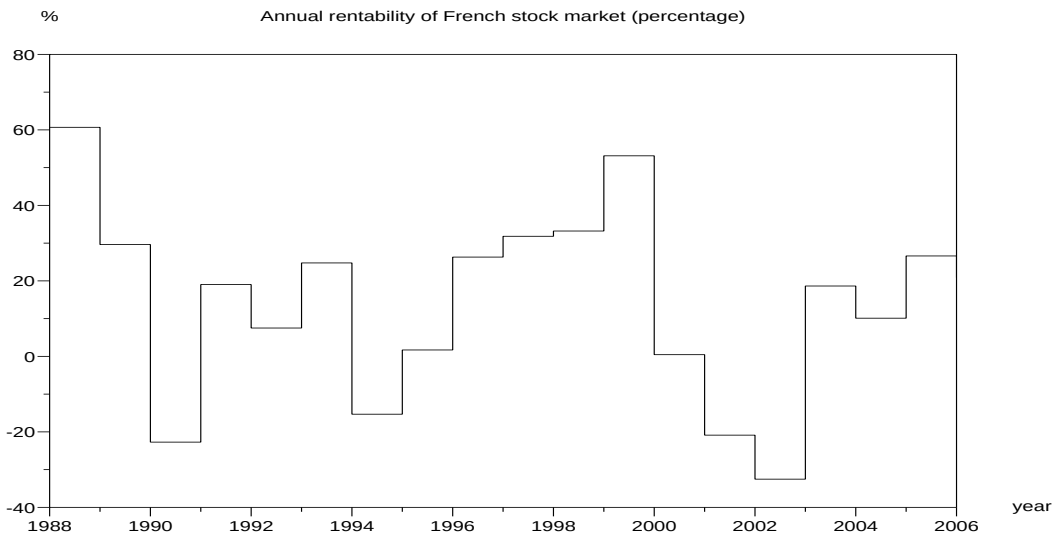


Figure 1: Annual rentability of French stock market (percentage)

1.2 Equivalent annual historical returns

Suppose that you choose an investment duration of T years. You invest at the beginning of any year between 1988 and $2006 - T + 1$, and you compare the final value with what an investment with safe return would have given on the same duration T. We shall call the return for which both are equivalent the *historical equivalent annual rentability*. Letting the year between 1988 and $2006 - T + 1$ vary, we obtain, for each investment duration T, an array of historical equivalent annual rentabilities.

Question 2 *Compute the historical equivalent annual rentabilities of French stock market over 1,2, . . . , 19 consecutive years. Draw a graphic with the number of consecutive years as horizontal axis and the historical rentabilities as vertical axis.*

//

```
t0=years(1);
tf=years($);
duration=1:(tf-t0+1);
cac40_equi_returns=list();
// cac40_equi_returns(k) is the vector of
// equivalent annual returns over k consecutive years

xset('window',43) ; xbasc() ;
for i=duration;
  loc_vec=[];
```

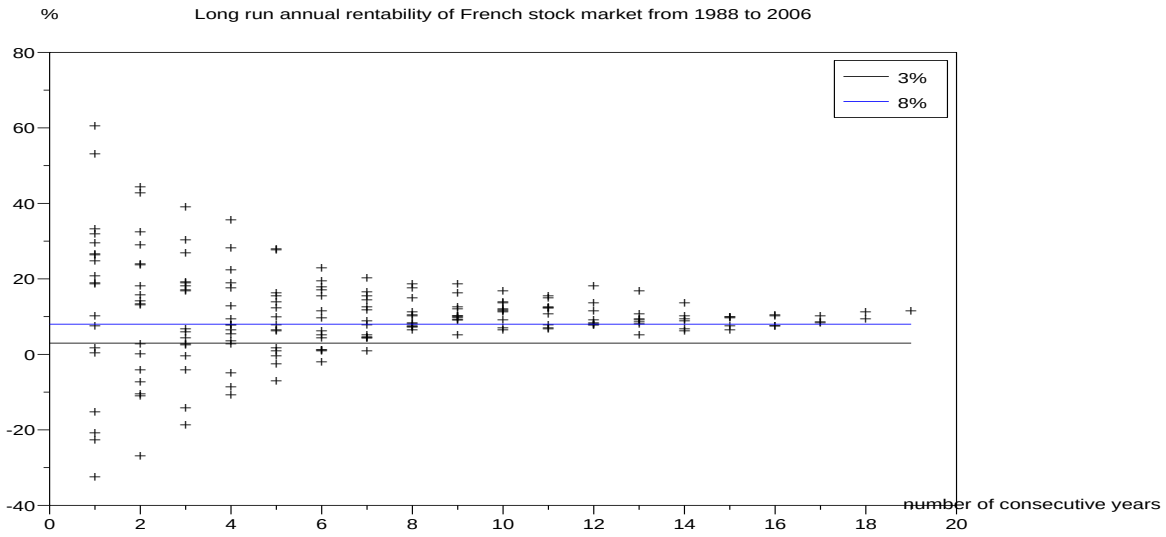


Figure 2: Historical equivalent annual rentabilities (percentage) of French stock market as function of the duration of investment

```
  for j=1:(tf-t0-i+2);
loc=cumprod(1 + 0.01 *cac40_perf_percent(j:(j+i-1)));
loc_vec=[loc_vec,(loc($))^{1/i}-1];
  end
  plot2d(i*ones(loc_vec),100*loc_vec,style=-1)
  cac40_equi_returns(i)=100*loc_vec;
end
xtitle('Long run annual rentability of French stock market from '...
+string(t0) +' to ' +string(tf) ,...
   'nb consecutive years','%')
plot2d([0 duration], ones([0 duration]') * [3 8] ); // ,style=[2 3]);
legends([ '3%'; '8%' ] , [1,2], 'ur' );

//
```

1.3 Regular investment

Consider $t_f - t_0 + 1$ periods t_0, $t_0 + 1$, ..., t_f (which will be years in this context). Investing a unitary amount of money at the beginning of period t is supposed to return $1 + \mathcal{R}(t)$ at the beginning of period $t+1$. Thus, if your planning strategy is to invest $\mathcal{I}(t)$ at the beginning of every period $t \in \{t_0, t_0 + 1, \ldots, t_f\}$, you can evaluate your fortune at the horizon $T = t_f + 1$.

//

```
function [capital,plus_values]=reg_invest(invest_tab,return_tab)
periods=cumsum(ones(invest_tab)) ;
cumul_invest=cumsum(invest_tab);
// invest_tab : array of regular investment amounts
// return_tab : array of returns
// capital : at last period 1+periods($)
capital=[];
plus_values=[];
for t=periods
  capital_loc= ( capital($) + invest_tab(t) ) * (1+return_tab(t))  ;
  capital=[capital capital_loc];
  plus_values=[plus_values capital_loc-cumul_invest(t) ] ;
end
endfunction

//
```

Question 3 *Suppose now that the same strategy is applied with a fixed return $\mathcal{R}$, giving fortune at the horizon $T = t_f + 1$*

$$(1+\mathcal{R})^{t_f-t_0+1} + \cdots + (1+\mathcal{R}) = \frac{1+\mathcal{R}}{\mathcal{R}}\left((1+\mathcal{R})^{t_f-t_0+1} - 1\right).$$

Which is the fixed rate $\mathcal{R}_s$ which gives the same final fortune than with a time-varying return $\mathcal{R}(\cdot)$?

```
  //
function retour=equi_return(nb_periods,fortune)
function y=f(rate)
  y=( 1 + (1 ./rate) ) .* ( (1+rate).^(nb_periods) -1 ) - fortune ;
endfunction
retour=fsolve(0.03,f)
endfunction

//
```

1.4 Equivalent annual historical returns for yearly investment strategy

Question 4 *Suppose you invest a unitary amount of money at the beginning of every year in the stock market. Evaluate the equivalent fixed rates, on different horizons, of the strategy consisting of investing the same amount every year on the stock market.*

```
  //
```

```
t0=years(1);
tf=years($);
duration=1:(tf-t0+1);
cac40_equi_yearly_returns=list();
// cac40_equi_returns(i) is the vector of
// equivalent annual returns for regular annual investment
// over i consecutive years

xset('window',63) ; xbasc() ;
for i=duration;
  loc_vec=[];
  for j=1:(tf-t0-i+2);
// loop when the investment horizon i is fixed
return_tab=0.01*cac40_perf_percent(j:(j+i-1));
invest_tab=ones(return_tab);
// unitary amount of regular annual investment
[capital,plus_values]=reg_invest(invest_tab,return_tab);
nb_periods=sum(ones(invest_tab));
fortune=capital($);
retour=equi_return(nb_periods,fortune);
loc_vec=[loc_vec,retour];
  end
  plot2d(i*ones(loc_vec),100*loc_vec,style=-2);
  cac40_equi_yearly_returns(i)=100*loc_vec;
end
xtitle('Yearly investment in French stock market from '...
+string(t0) +' to ' +string(tf) ...
   +': annual rentability',...
'nb consecutive years','%')
plot2d([0 duration], ones([0 duration]') * [3 8] );
legends([ '3%'; '8%' ] , [1,2], 'ur' );

//
```

2 Perception of the risk of stock market on different investment durations

2.1 Historical returns on fixed investment duration

Suppose that you choose an investment duration of T years and an initial capital of 1 000 euros. Consider that the investment may have started at the beginning of any year between

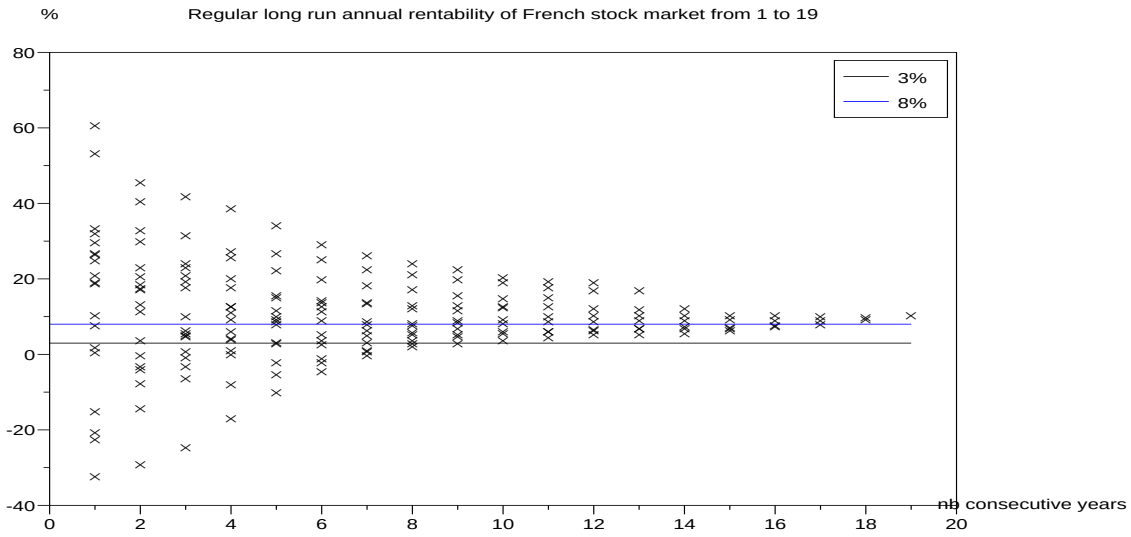


Figure 3: Historical equivalent annual rentabilities (percentage) of French stock market as function of the duration of investment in case of yearly investment

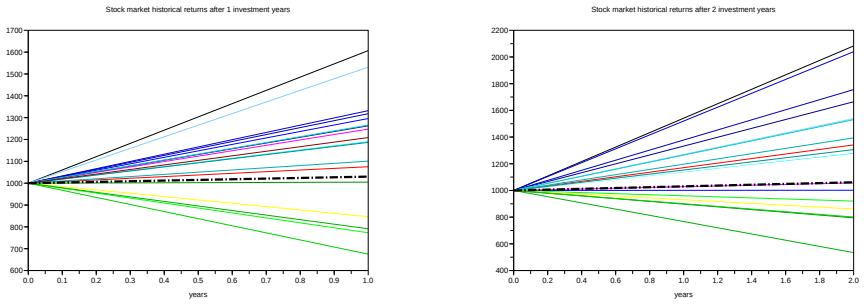
1988 and $2006 - T + 1$. Letting the year between 1988 and $2006 - T + 1$ vary, we obtain, for each investment duration T, an array of historical returns. For $T = 1$ year, we obtain $2006 - 1988 + 1 = 19$ historical returns; for $T = 2$ years, 18 years; ...; for $T = 19$, only 1 historical return.

Question 5 *Draw graphics for $T = 1$, $T = 2$, $T = 10$ with years between 0 and T in abcisse and a straight line joining 1 000 euros to the different historical returns. The straight lines ignore the variability during the investment period to focus only on the final return.*

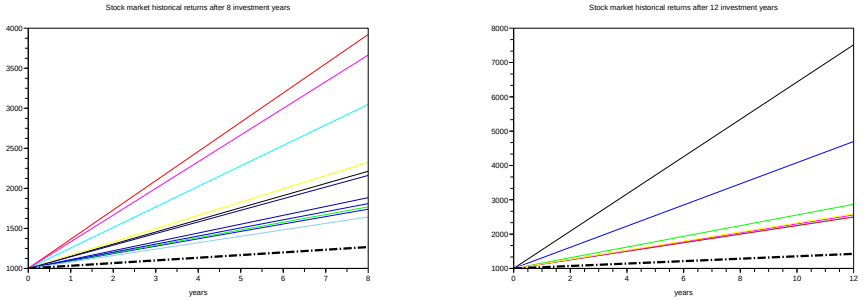
```
//

t0=years(1);
tf=years($);
duration=1:(tf-t0+1);
cac40_hist_returns=list();
// cac40_hist_returns(i) is the vector of
// returns over i consecutive years for 1000 euros investment

for i=duration;
  loc_vec=[];
  for j=1:(tf-t0-i+2);
loc=1000*cumprod(1 + 0.01 *cac40_perf_percent(j:(j+i-1)));
// for 1000 euros investment
loc_vec=[loc_vec,loc($)];
```



(a) 1 year (b) 2 years



(c) 8 years (d) 12 years

Figure 4: French stock market historical returns for same initial investment under different sequences of consecutive years

```
  end
  cac40_hist_returns(i)=loc_vec;
end

for i=[1 2 5 8 12]
  xset('window',70+i) ; xbasc() ;
  plot2d([0 i],[1000*ones(cac40_hist_returns(i)')...
cac40_hist_returns(i)' ]')
  xtitle('Stock market historical returns after ' +string(i) +...
   ' investment years','years')
xpoly([0 i],[1000 1000*(1+.03)^i] ,"lines",0)
  p=get("hdl"); //get handle on current entity (here the polyline entity)
  p.thickness=6;
  p.line_style=5;
end


//
```

2.2 CPT evaluation of risky historical returns on fixed investment durations

For each number T of consecutive years, one has an array of historical returns, supposed to be the values taken by a prospect $X(T)$ with equal probabilities. We shall evaluate $X(T)$ with the so called *Cumulative prospect theory* or CPT (see [KT79, TK92]), presented in Risk and Decision.

Question 6 *Draw the evaluation of the prospect $X(T)$ as a function of investment duration T. Where do you see that the stock market is not chosen if its returns are only perceived on an annual basis? What is the investment time needed to prefer stock market to a safe investment between 2% and 3% [BT95]?*

```
//
// exec Code/CPT.sci

anchors_percent=0:0.25:10 ;
Values=zeros(anchors_percent'*duration);

for i=duration;
values=cac40_hist_returns(i);
lotery=[ values ; (1/sum(ones(values)))*ones(values) ] ;
// deff('y=f(x)','y=CPT_eval(lotery,x)');
// indif_anch(i)=fsolve(3,f);
loc_Values=[];
for k=1:prod(size(anchors_percent));
anchor=1000* (1+0.01*anchors_percent(k))^i ;
  loc_Values=[loc_Values;CPT_eval(lotery,anchor)];
end
// Values(k,:)=loc_Values;
Values(:,i)=loc_Values;
end

xset('window',65) ; xbasc() ;
plot2d(duration,[zeros(duration)' Values([1 9 13],:)']);
xtitle('CPT evaluation of historical stock market returns', ...
'investment length (years)');
legends([ 'anchor '+string(anchors_percent(1))+'%'; ...
'anchor '+string(anchors_percent(9))+'%' ; ...
  'anchor '+string(anchors_percent(13))+'%'],...
[2 3 4], 'ul');
```

```
[lhs,ind]=mini(abs(Values),"r");
CPT_indif=anchors_percent(ind);

xset('window',66) ;  xbasc() ;
plot2d([0 duration], ...
   [ [0 CPT_indif]' ones([0 duration]') * [3 8] ],...
rect=[0,-4,duration($),14]);
legends([ 'CPT indifferent safe investment return' ; '3%'; '8%' ] , ...
[1,2,3], 'lr' );
xstring(1,11,["Safe prefered to stock"],0,1)
xstring(12,5,["Stock prefered to safe"],0,1)
xtitle('Safe/stock CPT indifference curve ...
as function of investment length','investment length (years)',...
'safe return (%)');

//
```

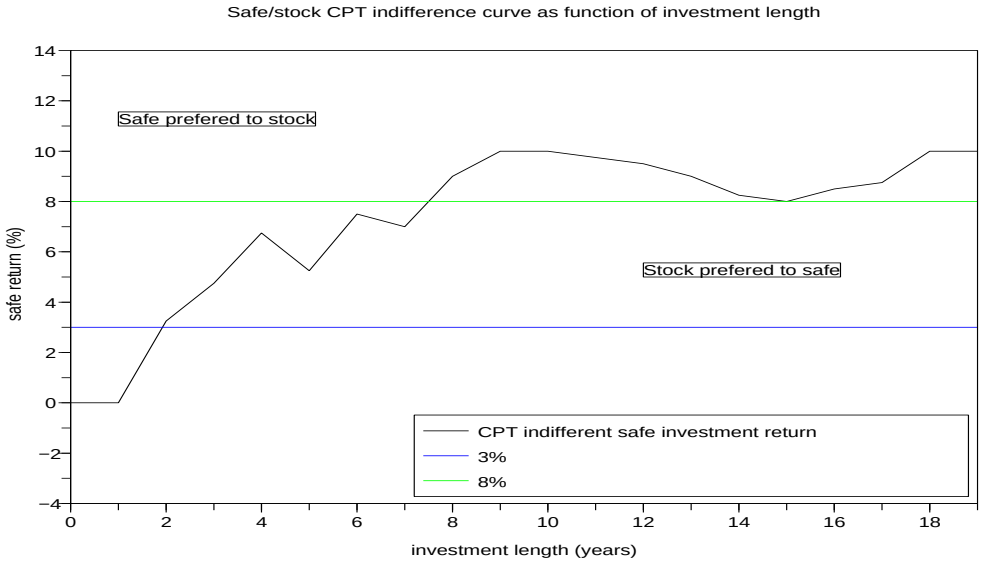


Figure 5: CPT indifference curve between safe or historical stock market rentability as function of investment length. For instance, a CPT investor is indifferent between investing 5 years in stock markets or 5 years at 6% safe return; a CPT investor prefers investing 10 years in stock markets than 10 years at 9% safe return.

3 Perception of the risk of regular investment in stock market

3.1 Historical returns for regular investment in stock market

Suppose that you choose an investment duration of T years and a regular investment of 1 000 euros at the beginning of each year. Consider that the investment may have started at the beginning of any year between 1988 and $2006 - T + 1$. Letting the year between 1988 and $2006 - T + 1$ vary, we obtain, for each investment duration T, an array of historical returns. For $T = 1$ year, we obtain $2006 - 1988 + 1 = 19$ historical returns; for $T = 2$ years, 18 years; ...; for $T = 19$, only 1 historical return.

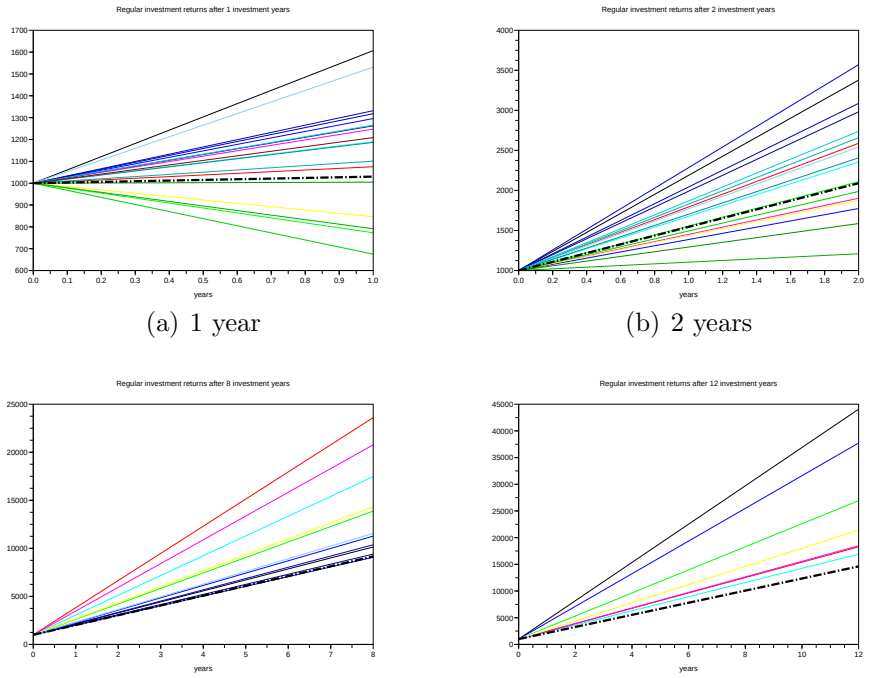
Question 7 *Draw graphics for $T = 1$, $T = 2$, $T = 10$ with years between 0 and T in abcisse and a straight line joining 1 000 euros to the different historical returns. The straight lines ignore the variability during the investment period to focus only on the final return.*

```
   //

t0=years(1);
tf=years($);
duration=1:(tf-t0+1);
cac40_yearly_invest_returns=list();
// cac40_yearly_invest_returns(i) is the vector of
// returns for regular annual investment
// over i consecutive years

for i=duration;
  loc_vec=[];
  for j=1:(tf-t0-i+2);
// loop when the investment horizon i is fixed
return_tab=0.01*cac40_perf_percent(j:(j+i-1));
invest_tab=ones(return_tab);
// unitary amount of regular annual investment
[capital,plus_values]=reg_invest(invest_tab,return_tab);
loc_vec=[loc_vec,capital($)];
  end
  cac40_yearly_invest_returns(i)=1000*loc_vec;
   // for 1000 euros annual investment
end

for i=[1 2 5 8 12]
  xset('window',90+i) ; xbasc() ;
  plot2d([0 i],[1000*ones(cac40_yearly_invest_returns(i)')  ...
```



(a) 1 year (b) 2 years

(c) 8 years (d) 12 years

Figure 6: French stock market historical returns for same initial investment under different sequences of consecutive years

```
cac40_yearly_invest_returns(i)' ]')
  xtitle('Regular investment returns after ' +string(i) +...
   ' investment years','years')
  return_tab=0.03*ones(1:i);
  // investment of length i
  invest_tab=ones(return_tab);
  // unitary amount of regular annual investment
  [capital,plus_values]=reg_invest(invest_tab,return_tab);
xpoly([0 i],[1000 1000*capital($)] ,"lines",0)
  p=get("hdl"); //get handle on current entity (here the polyline entity)
  p.thickness=6;
  p.line_style=5;
end

//
```

3.2 CPT evaluation of historical returns for regular investment in stock market

Suppose that you choose an investment duration of T years and a regular investment of 1 000 euros at the beginning of each year. For each number T of consecutive years, one has an array of historical returns, supposed to be the values taken by a prospect $Y(T)$ with equal probabilities. We shall evaluate $X(T)$ with the CPT.

Question 8 *Draw the evaluation of the prospect $Y(T)$ as a function of investment duration T. Where do you see that the stock market is not chosen if its returns are only perceived on an annual basis? What is the investment time needed to prefer stock market to a safe investment between 2% and 3% [BT95]?*

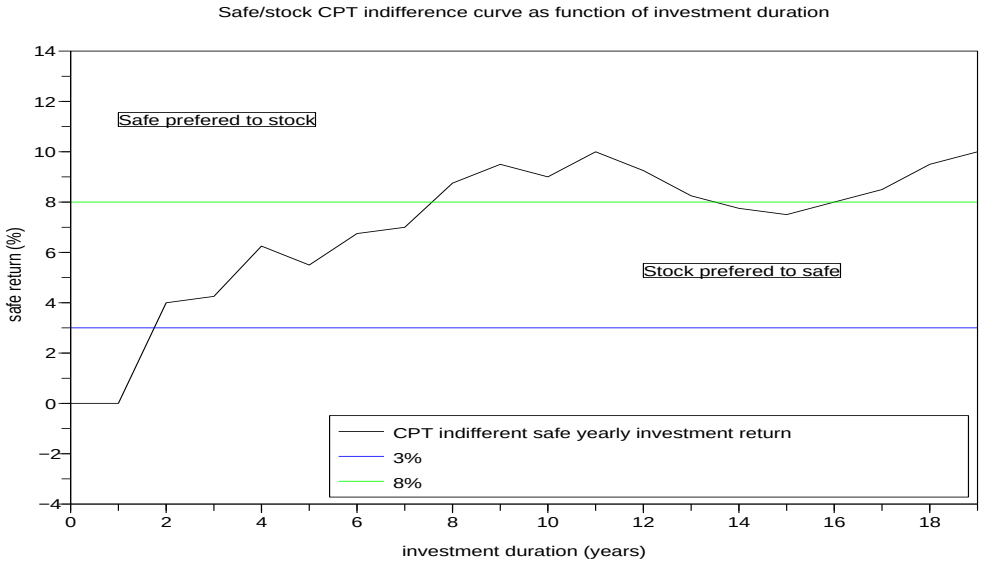


Figure 7: CPT indifference curve between safe or historical stock market rentability as function of investment length for yearly investment strategy. For instance, a CPT yearly investor is indifferent between investing 4 years in stock markets or 4 years at 5% safe return; a CPT investor prefers investing 10 years in stock markets than 10 years at 8% safe return.

```
//
// exec Code/CPT.sci

anchors_percent=0:0.25:10 ;
Values=zeros(anchors_percent'*duration);

for i=duration;
```

```
values=cac40_yearly_invest_returns(i);
lotery=[ values ; (1/sum(ones(values)))*ones(values) ] ;
// deff('y=f(x)','y=CPT_eval(lotery,x)');
// indif_anch(i)=fsolve(3,f);
loc_Values=[];
for k=1:prod(size(anchors_percent));
  return_tab=0.01*anchors_percent(k)*ones(1:i);
  // investment of duration i
  invest_tab=ones(return_tab);
  // unitary amount of regular yearly investment
  [capital,plus_values]=reg_invest(invest_tab,return_tab);
  anchor=1000* capital($);
  // for 1000 euros annual investment
  loc_Values=[loc_Values;CPT_eval(lotery,anchor)];
end
// Values(k,:)=loc_Values;
Values(:,i)=loc_Values;
end

xset('window',105) ; xbasc() ;
plot2d(duration,[zeros(duration)' Values([1 9 13],:)']);
xtitle('CPT evaluation of stock market returns ...
   with yearly investment', ...
'investment duration (years)');
legends([ 'anchor '+string(anchors_percent(1))+'%'; ...
'anchor '+string(anchors_percent(9))+'%' ; ...
  'anchor '+string(anchors_percent(13))+'%'],...
[2 3 4], 'ul');


[lhs,ind]=mini(abs(Values),"r");
CPT_indif_yearly=anchors_percent(ind);

xset('window',106) ;  xbasc() ;
plot2d([0 duration], ...
   [ [0 CPT_indif_yearly]' ones([0 duration]') * [3 8] ],...
rect=[0,-4,duration($),14]);
legends([ 'CPT indifferent safe yearly investment return' ; '3%'; '8%' ] , ...
[1,2,3], 'lr' );
xstring(1,11,["Safe prefered to stock"],0,1)
xstring(12,5,["Stock prefered to safe"],0,1)
xtitle('Safe/stock CPT indifference curve ...
```

```
as function of investment duration','investment duration (years)',...
'safe return (%)');

xset('window',166) ;  xbasc() ;
plot2d([0 duration], ...
   [ [0 CPT_indif]' [0 CPT_indif_yearly]' ...
 ones([0 duration]') * [3 8] ],...
rect=[0,-4,duration($),14]);
legends([ 'CPT indifferent safe investment return' ; ...
  'CPT indifferent safe yearly investment return' ; '3%'; '8%' ] , ...
[1,2,3,4], 'lr' );
xtitle('Safe/stock CPT indifference curves ...
as function of investment duration','investment duration (years)',...
'safe return (%)');

//
```

References

[BT95] Shlomo Benartzi and Richard H Thaler. Myopic loss aversion and the equity premium puzzle. *The Quarterly Journal of Economics*, 110(1):73–92, February 1995. available at http://ideas.repec.org/a/tpr/qjecon/v110y1995i1p73-92.html.

[KT79] D. Kahneman and A. Tversky. Prospect theory: an analysis of decision under risk. *Econometrica*, 47:263–291, 1979.

[TK92] Amos Tversky and Daniel Kahneman. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5(4):297–323, October 1992. available at http://ideas.repec.org/a/kap/jrisku/v5y1992i4p297-323.html.