Optimal Resources Allocation in Ecology: to Grow or to Reproduce? (I) The Constant Environment Case

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*with the help of Alain Rapaport, Anselme Le Van, Romain Casey

1 Life History Patterns Exhibit Large Variety

" Herbs often flower in their first year and then die, roots and all, after setting seed. Plants that flower once and then die are *monocarpic*.

Many monocarps are annual but a few species have long lives. Bamboos are grasses but they grow to unusually large size. One Japanese species, *Phyllostachys bambusoides*, waits 120 years to flower (Janzen, 1976).

Most trees flower repeatedly. However, Foster (1977) has characterized *Tachigalia versicolor* as a 'suicidal neotropical tree'. After reaching heights of 30-40 m, it flowers once and then dies. "

(Cited from Mark Kot, *Elements of Mathematical Ecology*, Cambridge University Press, 2001)

Mammals and other organisms present *determinate growth*: they stop growth when they become mature and start to reproduce. But many animals and plants, such as fishes, snakes, clams, etc. experience *indeterminate growth*: they life-history shows trade-offs between growth and reproductiwe havelong their lifetime.

2 The Basic Model of an Annual Plant Growth by Amir and Cohen (1990)

The model of single plant growth of [1] depends on three factors:

- a (deterministic) integer T, the *maximum length of the growing period* (number of days, weeks or months in one year, for instance);
- an integer random variable θ, interpreted either as an *individual lifetime length* or as a *growing season length*;
- a *growth function* $f : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$: if, at the beginning of time interval $[t, t+1[$, x is the vegetative biomass of the plant and e is the state of the environment, then $f(x, e) \geq 0$ is the maximum total biomass (vegetative and reproductive) which may be generated by the plant during $[t, t+1[$ (depends on roots, leaves, environmental factors, etc.).

The dynamics is a follows.

1. At the beginning of each time interval $[t, t+1[$, the plant
   - is characterized by its vegetative biomass $x_t \in [0, +\infty[$ (state);
   - is able to mobilize resource to generate the total biomass $f(x_t, e_t)$;
   - allocates biomass $f(x_t, e_t) - u_t$ as reproductive biomass in the interval $[t, t+1[$, where $0 \leq u_t \leq f(x_t, e_t)$.

2. At the end of each time interval $[t, t+1[$,

    - the annual reproductive yield is $S_{t+1} = \sum_{s=0}^{t}[f(x_s, e_s) - u_s] = S_t + [f(x_t, e_t) - u_t]$;
    - the vegetative biomass is $x_{t+1} = u_t$.

In the first part of the paper, Amir and Cohen consider one source of stochasticity, namely θ, before introducing also the environment e as random. The dynamics above is randomly aborted at θ giving the *annual reproductive yield per plant*:

$$S_\theta := \sum_{t=0}^{\theta}[f(x_t, e_t) - u_t] \, . \tag{1}$$

Note that the constraint $0 \leq u_t \leq f(x_t, e_t)$ implies two biological as:

- $u_t = 0$ is a possible decision, meaning that the plant may "decide" to die;
- $u_t \leq x_t$ is a possible decision, meaning that the plant may transfer a fraction of its vegetative biomass to reproductive biomass, by reducing its present size (for some organisms, as fishes, for which size may not decrease with time, we shall rather impose $x_t \leq u_t$).

3 Optimal Allocation in Constant Environnement (Theory)

3.1 Optimisation model in constant environnement

Dynamics

Consider an annual plant which grows and reproduces according to the following dynamics

$$y_t + x_{t+1} = f(x_t) \, , \quad t = 0, ..., T-1 \tag{2}$$

where

- T is the *maximum length of the growing period* (number of days, weeks or months in one year, for instance), with $T < +\infty$;
- x_t is the vegetative biomass at the beginning of period $[t, t+1[$;
- y_t is the reproductive biomass produced during period $[t, t+1[$.

Here are the general assumptions on the *growth function* :

- f is C^2 on $]0, +\infty[$ *[technical assumption]*;
- $f(0) = 0$ and $f(x) > 0$ for $x > 0$ *[the growth function measures a biomass]*;
- f is nondecreasing: $f' \geq 0$ *[the bigger you are, the more biomass you generate]*;
- f is concave: $f'' \leq 0$ *[the bigger you are, the lower your rate of biomass generation]*.

Random mortality

Let us assume that the plant lives for a random number θ of periods, with the random variable θ following a geometric law with parameter β, *survival probability* :

$$\forall t \in \mathbb{N}\,, \quad \mathbb{P}(\theta \geq t) = \beta^t\,. \tag{3}$$

We assume that death occurs at the end of a period. Thus, on the event $\{\theta = t\}$, the plant dies at the end of period $[t, t+1[$, leaving reproductive biomass $f(x_t) = f(x_\theta)$ and vegetative biomass $x_{t+1} = x_{\theta+1} = 0$.

Decisions and strategies

Consider a plant with vegetative biomass x_t at the beginning of period $[t, t+1[$. It will generate total biomass $f(x_t)$ at the end of the period. This biomass will be split between reproductive biomass y_t and future vegetative biomass x_{t+1}. What we call here a "decision" for the plant corresponds to the choice of an allocation between reproductive biomass and future vegetative biomass.

Let us introduce the *decision* variable

$$u_t = x_{t+1} \in [0, f(x_t)]\,. \tag{4}$$

We define a *strategy* for the plant as a sequence of decisions

$$(u_0, u_1, ..., u_{T-1}) \quad \text{with} \quad 0 \leq u_0 \leq f(x_0), ..., 0 \leq u_{T-1} \leq f(x_{T-1})\,. \tag{5}$$

Mathematically speaking, a strategy is a mapping

$$u() : \{0, ..., T-1\} \times \mathbb{R}_+ \to \mathbb{R}_+ \tag{6}$$

such that $u(t, x) \in [0, f(x)]$.

Criterion

We shall say that a strategy is *optimal* if it maximizes the mathematical expectation of the *fitness* (individual contribution to future generations):

$$J(u(.)) = \mathbb{E}\Big(\sum_{t=0}^{(T-1)\wedge\theta} [f(x_t) - u_t]\Big)\,, \quad u(.) = (u_0, u_1, ..., u_{T-1})\,. \tag{7}$$

Here, the fitness is defined as the cumulated reproductive biomass at the end of the growing season (the annual reproductive yield for an annual plant).

An *optimal trajectory* is any sequence $(x_0, ..., x_T)$ generated by

$$x_{t+1} = f(x_t) - u_t\,, \quad u_t = u^\sharp(t, x_t)\,, \quad t = 0, ..., T-1 \tag{8}$$

where $u^\sharp$ is an optimal strategy.

Why should natural selection favor such optimal strategies?

Consider various plants with various strategies. At the end of the growing season, those plants with an optimal strategy should, in the mean (because a mathematical expectation is maximized), be relatively more numerous than the others. Season after season, their relative number would grow.

When variability is between individuals within the same season (independent lifetimes) and when the populations are sufficiently large, the law of large numbers applies and this justifies the expected value of the annual reproductive yield as appropriate measure of fitness. Other approaches define an optimal strategy as one not susceptible to be invadable by a mutant.

A deterministic optimization problem

Here, the expectatiwe above is with respect to the only source of randomness, namely θ.

Question 1 *Show that*

$$J(u(.)) = \mathbb{E}(\sum_{t=0}^{(T-1)\wedge\theta} [f(x_t) - u_t]) = \sum_{t=0}^{T-1} \beta^t [f(x_t) - u_t] \,, \tag{9}$$

Thus, identifying optimal strategies amounts to solving

$$\begin{array}{c} \sup\limits_{u_0, u_1, \ldots, u_{T-1}} \qquad \sum\limits_{t=0}^{T-1} \beta^t (f(x_t) - u_t) \\ \begin{cases} x_{t+1} & = & u_t \\ 0 \leq & u_t & \leq f(x_t) \end{cases} \end{array} \tag{10}$$

This is a deterministic dynamic optimization problem with additive criterion characterized by

- instantaneous cost $l(t, x, u) = \beta^t (f(x) - u)$,
- zero final cost (there is no term with x_T in the criterion).

3.2 Resolution by dynamic programming

The dynamic programming equation (DPE) is

$$V(x, t) = \sup_{0 \leq u \leq f(x)} (\beta^t (f(x) - u) + V(u, t+1)) \quad \text{with} \quad V(x, T) = 0. \tag{11}$$

Denoting

$$\overline{V}(x, t) := \frac{1}{\beta^t} V(x, t) \tag{12}$$

the DPE may equivalently be written as

$$\overline{V}(x,t) = \sup_{0 \le u \le f(x)} \left(f(x) - u + \beta \overline{V}(u, t+1) \right) \quad \text{with} \quad \overline{V}(x,T) = 0. \tag{13}$$

Optimal strategies $u^\sharp$ are solution of

$$u^\sharp(t,x) \in \mathcal{U}^\sharp(t,x) := \arg\max_{0 \le u \le f(x)} \left(f(x) - u + \beta \overline{V}(u, t+1) \right). \tag{14}$$

Question 2 *Show that $\overline{V}(x, T-1) = f(x)$ and that the last optimal decision $u^\sharp(T-1, x)$ consists in allocating all available biomass at the end of period $[T-1, T[$ into reproductive biomass, and thus die at T.*

There is no gain in terms of offspring to keep vegetative biomass at the end of the last period.

Linear growth function

Let us suppose here that

$$f(x) = rx\,. \tag{15}$$

Question 3

- *Assume that $\beta r \le 1$. Show that, for $t = 0, ..., T-1$, $\overline{V}(t,x) = rx$ and $\mathcal{U}^\sharp_t(t,x) = \{0\}$. Describe the profile of an optimal trajectory.*
- *Assume that $\beta r > 1$, Show that, for $t = 0, ..., T-1$, $\overline{V}(t,x) = (\beta r)^{T-t-1} rx$ and $\mathcal{U}^\sharp_t(t,x) = \{rx\}$. Describe the profile of an optimal trajectory.*

Strictly concave growth function with low slope

Let us suppose here that

$$\forall x \ge 0\,, \quad f'(x) \le \frac{1}{\beta}\,. \tag{16}$$

Question 4 *Show that, for $t = 0, ..., T-1$, $\overline{V}(t,x) = f(x)$ and $\mathcal{U}^\sharp_t(t,x) = \{0\}$. Describe the profile of an optimal trajectory.*

Marginalist economic interpretation: whatever the plant size, the expected marginal gain in future biomass ($\beta f'(x)$) is always less than the direct gain in offspring ($+1$ by renouncing to growing by one unit).

Strictly concave growth function with high slope

Let us suppose here that

$$\forall x \geq 0\,, \quad f'(x) > \frac{1}{\beta}\,. \tag{17}$$

Question 5 *Show that, for $t = 0, ..., T-1$, $\mathcal{U}_t^\sharp(t,x) = \{f(x)\}$. Describe the profile of an optimal trajectory.*

Marginalist economic interpretation: whatever the plant size, the expected marginal gain in future biomass ($\beta f'(x)$) is always greater than the direct gain in offspring (+1). An example is given by the 'suicidal neotropical tree' *Tachigalia versicolor*.

Strictly concave growth function with moderate slope

Let us suppose here that

$$\exists x_+ > 0\,, \quad \beta f'(x_+) = 1\,. \tag{18}$$

Together with the *target* x_+, let us define the *threshold*

$$x_- := f^{-1}(x_+)\,. \tag{19}$$

Thus, the function $u \hookrightarrow -u + \beta V(u,t)$ is nondecreasing up to x^+, then nonincreasing.

Question 6 *Show that an optimal decision rule at period $T-2$ is*

- *if the vegetative biomass at the beginning of period $[T-2, T-1[$ is less than the threshold x_-, allocate total biomass into vegetative biomass;*
- *if the vegetative biomass at the beginning of period $[T-2, T-1[$ is greater than the threshold x_-, allocate part of total biomass to obtain vegetative biomass x_+; the rest $f(x) - x^+ \geq 0$ is allocated to reproductive biomass.*

Question 7 *Show that*

1. *$\overline{V}(x, T-2)$ is continuous and continuously differentiable at $x = x_-$, and that $\beta \frac{\partial V}{\partial x}(x_+, T-2) = 1$;*
2. *$\overline{V}(x, T-2)$ is continuously differentiable and that $\frac{\partial \overline{V}}{\partial x}(x, T-2)$ is decreasing.*

Deduce from this that optimal decision rules at periods $T-2$ and $T-3$ coincide.

Going backward, one may show that, for all $t = 0, ..., T-2$, growing without reproducing when vegetative biomass is under threshold x^- and growing up to the target x^+ if not is the optimal strategy (except at the last season).

Marginalist economic interpretation: at the target x^+, the expected marginal gain in future biomass ($\beta f'(x^+)$) is equal to the direct gain in offspring (+1). Examples are given by mammals (including whales and humans).

4 Comparison of Strategies in Constant Environment (PW Scilab)

Let us take

$$f(x,r) = rx^{\gamma} \quad \text{with} \quad 0 < \gamma \leq 1\,. \tag{20}$$

To facilitate computer programming, the control is no longer $u_t \in [0, f(x_t)]$, but

$$v_t := \frac{u_t}{f(x_t)} \in [0,1] \quad (v_t = 0 \quad \text{if} \quad f(x_t) = 0)\,. \tag{21}$$

Thus, this control v belongs to a fixed interval $[0,1]$, contrarily to $u \in [0, f(x)]$.

We incorporate the random time θ directly in a stochastic dynamics as follows

$$x_{t+1} = F(x_t, v_t, s_t)\,, \quad t = 0, ..., T-1\,, \quad v_t \in [0,1]\,, \quad s_t \in \{0,1\}\,. \tag{22}$$

Here, $s_t = 0$ means that the plant dies at the end of period $[t, t+1[$, while $s_t = 1$ corresponds to survival. The sequence $s_0, ..., s_{T-1}$ is i.i.d. with binomial distribution $\mathbb{P}(s_t = 1) = \beta$.

4.1 Dynamics of the plant

Open a file `plantI1.sci` and write a Scilab function `dyn_plant` representing the growth function, depending on the state x (the value of the parameter r will be given later).

```
function b=dyn_plant(x)
  // b = total biomass generated by the plant in one period
  //x = vegetative biomass
  b=r*x^{power}
endfunction
```

In the file `plantI1.sci`, write the fonction `dyn_plant_control`.

```
function b=dyn_plant_control(x,v,s)
  // Growth function, with control v and random term s
  // b = future vegetative biomass
  // x = vegetative biomass
  // v = fraction allocated to growth (control)
  // s = survival factor (s=1 survival, s=0 death)
  // if s=0, x transits towards 0
  // if s=1, x transits towards v*dyn_plant(x)
  if v < 0 | v > 1 then
    b='ERROR : control beyond bounds'
  else
    if s==0 then
      b=0
```

```
      //transition towards 0 if the survival factor is zero
    else
      b=v*dyn_plant(x)
    end
  end
  // the formula b=s.*v.*dyn_plant(x) has a problem?
endfunction
```

4.2 Costs functions

In the file `plantI1.sci`, write the following instantaneous and final costs functions.

```
// ateb instead of beta, since beta may be a predefined Scilab macro

function cost=offspring(state,control,time)
  cost=(1-control)*ateb^{time}*dyn_plant(state);
endfunction

function cost=final_cost_zero(state,time)
  cost=0;
endfunction
```

4.3 Comparison between strategies

Examples of strategies

In the file `plantI1.sci`, write the following Scilab macros:

- `strategie_D` (D for die) representing the strategy "to reproduce and to die": the output of `strategie_M` is 0;
- `strategie_R` (R for random) representing a uniform random decision: the output of `strategie_R` is a random real in $[0, 1]$;
- `strategie_G` (G for growth) representing the strategy "growing without reproducing except at the last decision period": the output of `strategie_G` is either 0 (to reproduce and die) or 1 (to grow without reproducing).

```
function v=strategy_D(t,x)
  v=0
endfunction

function v=strategy_R(t,x)
  v=rand()
endfunction
```

```
function v=strategy_G(t,x)
  if t < T then
    // t=1:T corresponds to t=0,...,T-1
    v=1
  else
    v=0
  end
endfunction
```

Scilab functions for trajectory evaluation

In the file `plantI1.sci`, write the following Scilab function `traj_feedback` with arguments

- an initial state,
- a Scilab function `controlled_dynamics` (similar to `dyn_plant_control`),
- a Scilab function `feedback` (similar to `strategy_X`),
- a random vector (two-dimensional: survival alea and resources alea; this latter alea is of no use at this stage)

and with outputs the corresponding state and control trajectories after feedback.

```
function [x,v]=traj_feedback(initial_state,controlled_dynamics,feedback,alea)
  // initial_state
  // controlled_dynamics(x,v,s) : returns a state
  // feedback(t,x) : returns a control
  // alea : sequence of 0 or 1, with length horizon
  // x : state trajectory
  // v : control trajectory
  horizon=size(alea,2);
  x=initial_state
  v=[]
  for t=1:horizon do
    // t=0,...,T-1
    v=[v,feedback(x($),t)]
    // x(£) is the last value of vector x
    // v has dimension horizon=T
    x=[x,controlled_dynamics(x($),v($),alea(t))]
    // x has dimension 1+horizon=1+T
  end
endfunction
```

In the file `plantI1.sci`, write the following Scilab function `total_cost` with arguments

- `trajectory`, a state trajectory, that is a vector with dimension $(1, T+1)$
- `control`, a control trajectory, that is a vector with dimension $(1, T)$
- a Scilab function `inst_cost`,
- a Scilab function `fin_cost`,

and with output the value of the additive criterion along state and control trajectories.

```
function cost=total_cost(trajectory,control,inst_cost,final_cost)
  // trajectory is a vector of dimension 1+horizon,
  // control is a vector of control of dimension horizon
  // inst_cost(state,control,time) is a function
  // final_cost(state,time) is a function
  // cost is a vector of dimension 1+horizon, comprising the sequence of
  // cumulated sums of instantaneous cost (and of final cost)

  horizon=prod(size(control))

  if 1+horizon <> prod(size(trajectory)) then
    cost='ERROR : dim(trajectory) different from 1+dim(control)'
  else
    cost=[]
    for t=1:horizon do
      // i.e. t=0,...,T-1
      cost=[cost,cost($)+inst_cost(trajectory(t),control(t),t)]
      // cumulated sums of instantaneous cost
    end
    cost=[cost,cost($)+final_cost(trajectory(1+horizon),1+horizon)]
  end
endfunction
```

Linear growth function

Open a file `plantI1.sce`. In this file, we will ask to load the macros in the file `plantI1.sci` and we will change the values of the following parameters.

```
// exec('plantI1.sce')
// parameters
power=1;//  gamma
ateb=0.9,// survival probability
r=1.2;
```

```
T=10;
x0=1;
// ATTENTION. The control is mathematically indexed by t=0,...,T-1.
// However, it is indexed by 1:T=1:horizon in Scilab
// The state is indexed by 1:(T+1)=1:(1+horizon) in Scilab
```

Question 8 *Compute state and control trajectories in closed loop with the different strategies* ***strategy_X****. Use for this Scilab function* ***traj_feedback****.*

Compute corresponding fitness. Use for this Scilab function ***total_cost****.*

Draw state, control and fitness trajectories thanks to Scilab macro ***plot2d2****.*

Write these instructions in file ***plantI1.sce****, and execute them by* ***exec('plantI1.sce')****.*

```
exec('plantI1.sci');
strategy=list();
strategy(1)=strategy_D;
strategy(2)=strategy_R;
strategy(3)=strategy_G;

correspondance=list();
correspondance(1)=string("death");
correspondance(2)=string("random");
correspondance(3)=string("growth");

// In what follows, we adopt the terminology
// of the arguments of function traj_feedback
initial_state=1;
dynamics=dyn_plant_control;
alea=ceil(ateb-rand(1,T));
// T independsent realizations of a binomial law B(ateb,1)
inst_cost=offspring;
final_cost=final_cost_zero;

for i=1:3 do
  [b,v]=traj_feedback(initial_state,dynamics,strategy(i),alea);
  f=total_cost(b,v,inst_cost,final_cost);

  xset("window",3*(i-1));xbasc();plot2d2(1:T+1,f,rect = [0,0,T+2,max(f)+1]);
  xtitle("strategy "+correspondance(i)+" : fitness");

  xset("window",3*(i-1)+1);xbasc();plot2d2(1:T,v,rect = [0,0,T+1,max(v)+1]);
  xtitle("strategy "+correspondance(i)+" : control");

  xset("window",3*(i-1)+2);xbasc();plot2d2(1:T+1,b,rect = [0,0,T+2,max(b)+1]);
```

```
  xtitle("strategy "+correspondance(i)+" : state");

  printf("the total fitness for strategy "+correspondance(i)+" is : %"+" f\n",f($))

  halt();
  xdel((3*(i-1)):(3*(i-1)+2));
  // deletes the windows
end
```

Question 9 *Compare total fitness for the different strategies.*

Strictly concave growth function with moderate slope

Question 10 *Give analytical expressions of x_+ and x_- defined at equations (18) and (19). Write in Scilab the corresponding formulas* `xp` *and* `xm`*.*

```
// parameters
power=0.5;
xp=(ateb*r*power)^{1/(1-power)};
xm=(xp/r)^{1/power};
```

Question 11 *In file* `plantI1.sci`*, write a Scilab function* `strategy_O` *(* `_O` *for optimal), with arguments a state x and a time t, and with output following feedback rule:*

1. *if the vegetative biomass is less than x_-, do no reproduce;*
2. *if the vegetative biomass is greater than or equal to x_-, then the future vegetative biomass will be x_+.*

Check, with the above theoretical results, that this strategy is optimal.

```
function v=strategy_O(t,x)
  if x < xm then
    v=1
  else
    v=xp/dyn_plant(x)
  end
endfunction
```

Question 12 *Do the same questions as above with the following strategy.*

```
strategy(4)=strategy_O;
correspondance(4)=string("optimal");
```

Question 13 *What do you observe when γ varies in $[0,1]$?*

```
P=0.1:0.2:0.9;
for j=1:prod(size(P)) do
  power=P(j);
  xp=(ateb*r*power)^{1/(1-power)};
  xm=(xp/r)^{1/power};
  x0=xm/10;
  // initial biomass less than threshold xm
  printf("gamma=%"+" f\n",P(j));
  for i=1:4 do
    [y,v]=traj_feedback(x0,dynamics,strategy(i),alea);
    f=total_cost(y,v,inst_cost,final_cost);
    printf("the total fitness for strategy "+correspondance(i)+" is : %"+" f\n",f($))
  end
end
```

5 Numerical Evaluation of Optimal Strategies in Constant Environment (PW Scilab)

We take

$$0 < \gamma < 1 . \tag{23}$$

5.1 Parameters

Copy the following parameters in a file `plantI2.sce`.

```
// exec('plantI2.sce')
T=10;
ateb=0.9;
r=1.2;
power=0.5;
```

5.2 State, control and cost discretization

Copy the following code in file `plantI2.sce`. It provides the state and control discretizations, as well as instantaneous cost and final cost. See *Programmation dynamique, macros generales*.

```
state=[0:(xm/6):(1.2*xp)];
control=0:0.05:1;

offspring=list();
discount=cumprod([1,ateb*ones(1,T-1)]);
```

```
for l=1:prod(size(control)) do
  offspring(l)=(1-control(l))*(r*(state')^{power})*discount;
  // offspring(l) is a matrix
end

final_cost_zero=zeros(state');
```

Note that Scilab functions defined in Section 4 are now redefined.

5.3 Discretization of the dynamics. Transition matrices

Get the Scilab functions `predec_succes`, `egal` and `discretisation` from *Programmation dynamique, macros generales* and copy them in the file `plantI2.sci`, as well as the following function.

```
function b=dyn_plant_control(x,v)
  //b = total biomasse
  //x = vegetative biomass
  //v = control
  b=v*r*x^{power}
endfunction
```

Copy the following code in file `plantI2.sce`.

```
exec('plantI2.sci')

controlled_dynamics=dyn_plant_control;

transition_matrix=discretisation(state,control,controlled_dynamics);
```

Question 14 *Check that the list of matrices* `matrix_transition` *indeed consists of transition matrices, that is with nonnegative coefficients summing to 1 on each row.*

```
cardinal_control=size(transition_matrix);
for l=1:cardinal_control do
  sum(transition_matrix(l),"c")'
  mini(transition_matrix(l))
  maxi(transition_matrix(l))
  halt();
end
```

5.4 Numerical resolution by dynamic programming

Get the Scilab functions `Bell_stoch` and `trajopt` from *Programmation dynamique, macros generales* and copy them in file `plantI2.sci`

Copy the following code in file `plantI2.sce`.

```
instant_cost=offspring;
final_cost=final_cost_zero;

[value,feedback]=Bell_stoch(transition_matrix,instant_cost,final_cost,1);
// solves the dynamic programming equation

initial_state=grand(1,1,'uin',2,prod(size(state)));
// corresponds to original state  >= state(2)
z=trajopt(transition_matrix,feedback,instant_cost,final_cost,initial_state);
// computation of optimal trajectories (indexes)
zz=list();
zz(1)=state(z(1));
zz(2)=control(z(2));
zz(3)=z(3);
// optimal trajectories

xset("window",1);xbasc();plot2d2(1:prod(size(zz(1))),zz(1));xtitle("taille")
xset("window",2);xbasc();plot2d2(1:prod(size(zz(2))),zz(2));xtitle("control")
xset("window",3);xbasc();plot2d2(1:prod(size(zz(3))),zz(3));xtitle("fitness")
// drawing of optimal trajectories
```

Question 15 *Study the trajectories, and compare them with those obtained above.*

6 Scilab Code

plantI1.sci plantI1.sce plantI2.sci plantI2.sce

References

[1] S. Amir and D. Cohen. Optimal reproductive efforts and the timing of reproduction of annual plants in randomly varying environments. *Journal of Theoretical Biology*, 147:17–42, 1990.