Simulation of dynamical systems in discrete time
Ecological and environmental examples

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We consider the discrete time dynamics system in $\mathbb{R}^n$:

$$x(t+1) = f(t, x(t)), \quad \text{for } t = t_0, .., T-1, \tag{1}$$

starting from initial state $x_0 \in \mathbb{R}^n$ at time t_0

$$x(t_0) = x_0. \tag{2}$$

Knowing function f and (t_0, x_0), we are able to compute the whole sequence $x(t_0), x(t_0 + 1), \ldots, x(T)$ solution of the problem.

1 Population dynamics in ecology

Consider the population dynamics

$$x(t+1) = \frac{Rx(t)}{1 + Sx(t)}, \; t_0 = 1, \; T = 100, \; x(t_0) = x_0 = 1,$$

with parameters $R = 1.2$ and $S = 0.02$.

```
t0=1;x0=1;
x(t0)=x0;T=100;R=1.2;S=0.02;

function y=f(t,x)
  y=R*x ./(1+S*x)
```

```
endfunction

for t=t0:1:T do
  x(t+1)=f(t,x(t));
end;
plot2d(t0:T+1,x(t0:T+1))
```

Question 1 *Change the population dynamics with $f(t,x) = (ax)^{0.5}$ where $a = 10$. Compare the behavior of the solutions $x(t)$.*

2 Carbon cycle

Consider the carbon cycle

$$M(t+1) = M(t) + \alpha E_{\text{BAU}}(t)(1-a) - \delta(M(t) - M_{-\infty}),$$

starting from initial conditions $M_0 = 354$ (ppm) at year $t_0 = 1990$ with time horizon $T = 100$. The parameters are $\alpha = 0.471$, preindustrial concentration $M_{-\infty} = 280$, removal rate $\delta = 0.01$. It is assumed that the "business as usual" CO_2 emissions path is

$$E_{\text{BAU}}(t) = E_{\text{BAU}}(t_0)(1+g)^{t-t_0} \text{ (GtC)}$$

with the emissions growth set to $g = 1\%$ and initial emissions to $E_{\text{BAU}}(1990) = 7.2$ (GtC).

```
t0=1990;T=2100;M_0=354;
alpha=0.471;M_infty=280;delta=1/120;sigma=0.519;
E_BAU=7.2;g=0.01
a=0;

function y=EBAU(t)
  y=E_BAU*(1+g)^(t-t0);
endfunction

function y=f(t,M)
  y=M+alpha*EBAU(t)*(1-a)-delta*(M-M_infty);
endfunction

x(t0)=M_0;
for t=t0:1:T do
  x(t+1)=f(t,x(t));
end;
plot2d(t0:T+1,x(t0:T+1),rect = [t0,0,T+1,1000])
```

Question 2 *Change the mitigation rate a ($a \in [0,1]$) to compare the behavior of the concentrations profile $M(t)$.*

3 An ecosystem

We consider two populations $x_1(t), x_2(t)$ interacting within a trophic web. The dynamics based on a Lotka-Volterra form is characterized by

$$\left\{ \begin{array}{lcl} x_1(t+1) & = & x_1(t)(1 + r_1 - q_1 x_2(t)) \\ x_2(t+1) & = & x_2(t)(1 - d_2 + q_2 x_1(t)) \end{array} \right. \tag{3}$$

where $r_1 > 0$ is the intrinsic growth of prey while $d_2 > 0$ is the intrinsic decrease of predator. Parameters $q_1 > 0, q_2 > 0$ are related to the catchability and efficiency of trophic relations.

```
t0=1;x0=[1;1];
T=100;
r_1=0.1;d_2=0.1;q_1=0.1;q_2=0.2;

function y=f(t,x)
  x_1=x(1);x_2=x(2);
  y_1=x_1 .*(1+r_1-q_1*x_2);
  y_2=x_2 .*(1-d_2+q_2*x_1);
  y=[y_1;y_2];
endfunction

x=zeros(2,T+1);
x(:,t0)=x0;
for t=t0:1:T do
  x(:,t+1)=f(t,x(:,t));
end;
plot2d(t0:T+1,x(:,t0:T+1)')
```

Question 3 *Change parameters r, d or q to compare the behavior of the populations $(x_1(t), x_2(t))$.*