Equations with random coefficients: Convergence to deterministic or stochastic limits and theory of correctors

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Mardi 6 juillet 2010 à 14h00, salle de réunion (B413)

Equations with small scale structures abound in applied sciences. Such structures often cannot be modeled at the microscopic level and thus require that one understand their macroscopic influence. I will consider the situation of partial differential equations with random, highly oscillatory, potentials. One is then interested in the behavior of the solutions to that equation as the frequency of oscillations in the micro-structure tends to infinity. Depending on spatial dimension and the decorrelation properties of the random potential, I will show that the limit is the solution to either a deterministic, homogenized (effective medium) equation or a stochastic equation with multiplicative noise. More precisely, there is a critical spatial dimension above which we observe convergence to a deterministic solution and below which we observe convergence to a stochastic solution. In the former case, a theory of correctors to homogenization allows one to asymptotically capture the randomness in the solution to the equation with the small scale structure. Once properly rescaled, this corrector is shown to solve a stochastic equation with additive noise.