A mathematical homogenized model for the mechanical behavior of the lungs' parenchyma

Paul Cazeaux, LJLL

October 10th, 2PM

In this talk, we present a homogenized mathematical model for the deformation of the human lung tissue, called the lung parenchyma, during the respiration process. The parenchyma is a foam-like elastic material containing millions of air-filled alveoli connected by a tree-shaped network of airways. In this model, the parenchyma is governed by the linearized elasticity equations and the air movement in the tree by the Poiseuille law in each airway. The geometric arrangement of the alveoli is assumed to be periodic with a small period $\epsilon > 0$. We use the two-scale convergence theory to study the asymptotic behavior as $\epsilon$ goes to zero. The effect of the network of airways is described by a nonlocal operator and we propose a simple geometrical setting for which we show that this operator converges. We identify in the limit the equations modeling the homogenized behavior under an abstract convergence condition on this nonlocal operator. We derive some mechanical properties of the limit material by studying the homogenized equations: the limit model is nonlocal both in space and time if the parenchyma material is considered compressible, but only in space if it is incompressible. Finally, we propose a numerical method to solve the homogenized equations and we study numerically a few properties of the homogenized parenchyma model.