Hamilton-Jacobi equations with discontinuous source terms

Nao Hamamuki

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We study the initial-value problem for a Hamilton-Jacobi equation whose Hamiltonian is discontinuous with respect to space variables.

Our motivation comes from a model describing the two dimensional nucleation in crystal growth phenomena. This growth is started by external supply of crystal molecules for a flat face. Such a source of supply is called a step source. The associated equation is nonlinear and has a semicontinuous source term, but viscosity solutions, which are powerful tools to solve such kind of nonlinear equations, are not unique because of the discontinuity. To overcome this issue we introduce a new notion of viscosity solutions and prove that the initial-value problem admits a unique global-in-time solution for any bounded uniformly continuous initial data.

We also give a representation formula of the solution as a value function of the optimal control problem with a semicontinuous running cost function.