CAN THERE BE AN EXCESSIVE MATHEMATIZATION OF THE WORLD?

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I will address the question on what philosophical basis and under what circumstances can one say that the mathematization of the world is excessive, goes too far. I think that there is no reason to leave this issue to researchers from humanities who see these excesses as encroachments on their fields or to philosophers who are often unaware of what constitutes mathematics and talk as if we had not need to know mathematics to understand the issues. I think instead it is for us, especially applied mathematicians, to clearly see the origin of this problem and draw epistemic lessons.

A. Participation of mathematics to knowledge: some historical elements and first remarks.

Since the beginnings of civilization mathematics was associated with most forms of knowledge. Few are still areas that have not had their influence. From this long and branched history we extract some hallmarks.

I. The Baconian program served by mathematics. It is in *Il Saggiatore* (The Assayer) in 1623 that Galileo posits that the universe is written in mathematical language, this αποφτεγμα as it is called, will lay the foundations of all Western science. This clarifies the program of Francis Bacon that man has a Promethean perspective, because it is subject to God to share in his power. It can conquer, dominate and transform nature. Galileo tells us how he can know. In fact later in his work - like Alexander Koyré has clearly shown - Galileo proceeds essentially by thought experiments following mathematical reasoning and not by experiments to collect data for subsequent modeling.

He believes that mathematics is a sufficient footprint of the essence of God in nature so that it confesses its secrets simply by geometric and algebraic deductions. Over a century later, Kant placed in the center of his philosophy the explicit idea that mathematics although not depending on sensory experiment (*a priori* judgments) nevertheless teach about the world (synthetic judgments). Subsequently mathematics gradually leaves the philosophical throne of *synthetic a priori judgments*, but without ever losing the prestige of a natural fertility. After the bifurcation between the two cultures in the early 19th century with on one side mathematics taking a modern and rigorous turn in the writings of Gauss, Cauchy and Bolzano, and on another side the philosophy, Hegel's *Logic* having nothing of mathematics, the emergence of non-Euclidean geometries and crises in the foundations will give rise to a plurality of views about mathematics and its role in the development of scientific knowledge. At the end of the 19th and 20th centuries with the development of physics that becomes the heart of epistemology, mathematics is, with variations depending on the authors, considered discipline servant of the natural sciences, this is called their ancillary role.[1]

II. The emergence of mathematical economics. Sociology, introduced by Auguste Comte, takes a non-mathematical way except using statistics in particular by Durkheim. Then it will receive its own methodological bases with Max Weber in the early 20th century. The economy, however, is mathematized as soon as mid 19th century with Jules Dupuit and Augustin Cournot without really using statistics. How the maths have been accepted in this social science?

[1] With the notable exception of Bachelard who attributes to them a heuristic function for physics itself.

Let us follow the approach of Jules Dupuit (1804-1865). Civil engineer he realizes that it can be done better than setting a single price of tolls on a bridge. For double reason that for some users it is too expensive while others would pay a toll even higher. He is the inventor of what today is called market segmentation. Having a good mathematical training he has the intuition that with a single price is not recovered all the integral of a curve that would quantify the willingness to pay, but only that of a truncated curve. We see perfectly this idea of integral in his articles.

Observe immediately that this "willingness to pay" is a poorly defined concept. It depends on many factors, the weather, time, seasons, and a thousand social and economic causes. It seems impossible to measure. A collection of experiments of traffic according to a toll level would not provide a curve but a cloud of points. It also depends on other toll crossings. Also whether users may consult among themselves and sell their rights of way etc.

At the end of the Restoration and the beginning of the French Second Empire, the concept is being debated under the terms of *utility*. Dupuit follows the idea that the mathematical phenomenon that it has uncovered *will help to clarify the concept*. He postulates the existence of this quantity as a property of the thing exchanged and its price, which is divided according to the benefits of seller-manufacturer and the consumer. "*Political economy*, he wrote [as opposed to the social economy], *to measure the utility of an object must take the sacrifice that each consumer would be willing to do to get it*" and he took the still famous example of a bridge: "[the utility of a toll bridge] *always divides into two main parts 1) the lost utility that corresponds to the passages that would have occurred if the toll was abolished and who does not take place with the current level 2) the utility produced by the crossings executed. This utility splits itself into two parts: a) utility for the producer or utility of the toll b) consumer utility, or excess value of the service on the price it costs*."[2]

Dupuit explains: "*There is the fine, the very fine, super fine, extra fine, which, though out of the same barrel and showing no difference other than the superlative of the label, are sold at prices very different*" and this changes the optimization of public taxes: "*So when the bridge is done and that the state shall establish a tariff, it has no regard more to production costs, it imposes less a heavy cart which wears out more the bridge than a suspended car. Why two different prices for the same service? It is that the poor do not attach to the advantage of passing the same price as the rich, and that raising the tariff would only the prevent it from passing*." "*The goal is always the same, he says, is to charge for the service rendered, not what it costs, but what the buyer considers*." [3]

Dupuit fully realizes that, by the thought experiments that define it, his notion of utility is difficult to be measured. He acknowledges that it is abstract. "*It may be objected that the calculation of which we gave the formula is based on data that no statistics can provide, thus we will never able to express precisely the utility provided by a machine, road, any work* ..." But he argues the famous argument, which will be repeated extensively by neo-classical and until today, that economics is only an approximation, the argument where arise the whole ambiguity of the passage of descriptive to normative and to the performativity of discourse. This will open economics to all imaginable mathematical refinements.

Dupuit starts from a mathematical property and uses it to account for the psychological, it is interesting to compare his approach with that of Condorcet, who proposed at the end of the preceding century a different kind of mathematization of the social.

The project of Condorcet, high level mathematician, is to apply the probability calculus to understand how spreads and propagates the "reason to believe", a concept somewhat similar to that of utility but based on the truth or falsity judgments. He pushes this program that introduced him to this great discovery of the "paradox of the vote of an assembly". But he did not think that it would be possible to go until computing the behavior of the people.

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[2] J. Dupuit *Annales des Ponts et Chaussées* 1849.

[3] *Ibid*.

"*On the use of language of geometry. The amount of universal commodity, that of a particular commodity can be approximated by numbers, but the urge to buy and sell cannot be object of any calculation, and yet the price change depends on this moral quantity, which itself depends on the opinions and passions. It's a great idea to try to submit everything to calculation, but see the greatest mathematicians of Europe, the D'Alembert and Lagrange. Well they want movement of three bodies which attract each other: they assume that these bodies are masses without extension, or very slightly different body of a sphere, and this issue, although limited by one hundred conditions facilitating it, has occupied them for twenty years and still now. The effect of forces acting on the head of the dullest shopkeeper is much more difficult to calculate.* "[4]

Condorcet's approach starts from the psychological, the reason to believe, and attempts a mathematization of sociality by the probability calculus. His epistemology is an extension of that of Laplace: we can not determine everything, principles, laws of forces and their way of acting, only is relevant the calculation of probability. It is an approach to *a priori* limitation of science. Condorcet needs to explicit all assumptions, independence or correlation of opinions etc., before doing calculations.

Dupuit in contrast, can immediately perform calculations, he does some ones in his articles indeed, he builds concepts that interpret price curves (assumed to be obtained), his concepts require very strong assumptions of independence, but it leaves explanation of the assumptions to future improvements.

These features: independence of agents presented as approximation, approach from prices and quantities to concepts, then, in the 19th century, production function, problem solving by local differentiation, will be the backbone of the neo-classical theory with Stanley Jevons, Carl Menger, Léon Walras (general equilibrium), von Böhm-Bawerk, Vilfredo Pareto (theory of optimum), Irving Fisher, etc.. creating a language evocative and highly flexible still used now.

III. Advanced mathematization of finance. It is a very recent phenomenon, which I recounted the history somewhere else and that you also know well. I will simply explain how an apparently very clever mathematization of risk, helped divert practitioners of safe behaviors and facilitated the emergence of the subprime crisis.[5]

The crisis occurred at a time when finance is very mathematized. This situation is the result of the "Black-Scholes revolution". The work of Bachelier and modelling by Brownian motion have been rediscovered and developments of stochastic calculus after the Second World War in particular due to K. Ito (1915-2008) provide a mathematical language (that of semi-martingales) where the non-arbitrage principle may be expressed under broad assumptions that may be suitable for operational cases. Hence methods for pricing (pricing) and hedging (hedging) options provided by partial differential equations. The simplest case is when volatility is constant, but it is clear to everyone that these methods are largely *perfectible*, epistemologically essential point.

This resulted in three historical phenomena - the development of derivatives markets in the U.S. first, then Japan and Europe - a transformation of professional profiles in banks and call of new academic skills in mathematics - an enhanced political role of finance that was felt during the construction of Europe and then in the globalization movement.

From the core of the hedging of options (European or American) on stocks and currencies, the mathematical formalization then spread to more sensitive issues: the rate models. It is the bond market and the term structure of interest rates. Models of Cox-Ingersoll-Ross and Heath-Jarrow-Morton allow to apply there the non-arbitrage principle. But here the theory can involve infinite-dimensional models that are forced to be simplified and calibrated on the current data. These models relate the behaviour of agents in five, ten or twenty years and are therefore highly uncertain, their uncertainty is addressed in the language of probability theory.

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[4] Letter to P. Verri 1773.

[5] For more details cf. N. Bouleau "Finance et opinion" *Esprit* nov. 1998 and "Malaise dans la finance, malaise dans la mathématisation" *Esprit* fév. 2009, p37-50.

But the most ambitious level in the mathematization goes even beyond and deals with securitization of receivables and risk measures. Putting risk into the market is *a priori* a good idea. This is in line to not put all your eggs in one basket. But this assumes that players (banks, insurance companies) are able to assess the risks.

There also a mathematical innovation is to be mentioned. It has been remarked that to estimate the risk of a portfolio of contingent claims the classical method known as "value at risk" based on a criterion of the form (level of losses, probability of this level) presented some logic deffects. And it has been shown that any criterion satisfying the desired consistency was of a certain mathematical form called "coherent measure of risk". Let us emphasize that these tools allow calculations on complex portfolios assuming known probability of rare events, i.e. the tails of probability distributions which have great influence on the results. So these are methods that yield quantification from the unknown.

The marketing of credit risk is a situation where financial institutions have mathematical tools to estimate risks on reassembled folders for the purpose of exchanging them and improving the situation of everyone given its own utility function and its aversion to risk. It has often been stressed in the comments of the crisis that the new tools of these markets especially CDO and CDS (credit default swaps) did not encourage operators to exercise caution. That's right, the change in behaviour with respect to the risk of an agent protected by an insurance who is appointed by the Anglo-Saxons as the "moral hazard ", surely had a role in the rise of the soufflé of the crisis. But equally important is the fact that it was wrong in thinking that the risk was "in the folder". The risk is interpretative in nature and as "the beauty of the Parthenon is not found in the dust of the Parthenon" these mathematical tools do not see the global economical interpretations related to the decline in U.S. household savings etc.[6]

IV. The quantification of uncertainty is a deletion of meaning. On the epistemic point of view it should be stressed that fundamental point. That is *the significance* of the event that makes the risk. The probabilistic representation of risk is typically a couple of mathematical quantities, 1° a probability law that governs the states that can arise, 2° a random variable, i.e., a function that maps each state to the damage costs (algebraically counted if benefits). This representation by a couple of quantities is a mathematical scheme both too simple and too ideal for thinking risk. Too ideal because we are almost never in a situation where this model is well informed. We do not know the tails of probability distributions because they concern rare events where data are insufficient. We do not know what correlations occur to assess the damage and we do not have a full description of what can happen. Moreover, the model is simplistic because it removes the reasons that make us interested in the events as if their translation into costs could be done automatically and objectively.

The true purpose of risk analysis is to move forward with a little foresight in organizing facts and social practices. It may be the risk that a child be knocked down while crossing the street, the risk that the air of Paris poisons its inhabitants, such as bankruptcy of a bank propagates to others, etc.. The intellectual operation of probabilizing a situation is fundamentally an erasure of meaning. It is largely a problem for all matters concerning human behaviour. Risk analysis is necessarily *understanding* of interpretations.

It is the meaning of the event that produces the risk. As an example, suppose a particular type of cancer is counted with a certain proportion in the Swiss population, this proportion will be used to estimate the risk. If it happens that we discover that most people with this cancer had consumed cannabis say twenty years ago, then all cannabis users become potential patients, the risk is much higher, the meaning of the event has changed. Reducing risk to a probability distribution of monetary sums amounts to trusting mathematization as approximation, as if it would describe a physical reality, but it addresses meaning whose subjectivity permeates all social relations of the

[6] *Ibid* for the concrete articulations of this phenomenon.

agents. This epistemological point is extremely important. They are interpretations, thus meaning, which is replaced by a number.

We have seen significant improvements made recently to finance especially with the so-called coherent measures of risk. *All these methods of decision under uncertainty have the innate defect to suppose the interpretative process to be closed*. Now it is permanently emerging. Once a new reading is made, it creates new risks that are perceived only by those who understand it. If in 2006, nobody sees the growth of house prices and the decline of household savings in the United States as a phenomenon open to several interpretations, the corresponding risk is not perceived. Mathematization of risk conceals these difficulties behind assumptions about the tails of probability distributions. It is not enough to say that they are poorly known. They are by nature provisional and changeable according to the interpretative knowledge that agents provide from their understanding of economic phenomena.

V. In liberal economy, any quantification opens a possible extension of the market. The examples are numerous. The latest, which affect you directly in the life of your labs, is the quantification of research work. Until late last century, it was thought that the quality of the researchers was made idiosyncratic talent that could only assess experienced researchers in the daily activities of the teams. The implementation of all the known machinery with indices of publications and journals notoriety has deeply upset the working relationship of the profession. I do not dwell. The result was the emergence of an international market for students, teachers and researchers, universities are facing a new logic where their budget determines what level they can afford these athletes in mind.

Another example is biodiversity which is more serious in its consequences in the long term. Mathematization here is based on sharing species in two categories. On the one hand *the remarkable biodiversity* comprising those species considered by the official as threatened, they are calculated for the cost of maintenance as it happens for historical monuments. On the other hand *the ordinary biodiversity* including other species for which is calculated *the ecological service* they provide, from prokaryotes (bacteria) to eukaryotes (higher species) by standard methods of cost-benefit analysis. It is then possible to buy and sell any part of nature or to exchange it against goods or services already quantified by the economy.

B. When and how is there excessive mathematization?

It is now appropriate to examine the specific type of inefficiency and discomfort that induces a diagnosis of excessive mathematization.

VI. We realize this ex post. The case of the recent financial crisis is quite illustrative in this regard. As the crisis has not happened - except for some non-Orthodox observers as there are always - every agent, every financial institution believes that its interest is to estimate the risk of their complex portfolios of term products such as credit derivatives by the methods best suited to the very nature of these mathematized products. Coherent measures of risk are making assumptions on the tails of laws but can handle multiple scenarios. The weak point is that they omit scenarios based on global interpretations where each portfolio can not be estimated by considering the others remain *non varietur*.

Once triggered the crisis and after the upheavals that result, what happens is the result of political forces: on one hand a strong current of opinion emerges to adopt control measures to finance in order to avoid future crises or at least limit the damage, on the other hand most practitioners of the financial system consider it simply suffices to take into account the interpretation that had been neglected, i.e. here, to improve the global readings of risky situations by strengthening for instance the role of rating agencies. For the latter they are now told, and they take in account the neglected facts (resistance to "stress" of the various institutions, etc.). For the public opinion it is business as before with the same tools and similar defects.[7]

VII. Calculations conceal ignorance. This is blatant about financial risks. Because we do not know really quantify counterparty risks, or those related to market liquidity, much less those who are due to human error or to a legal change, very precise calculations are mixed with rough estimates expecting they will have no sensitive impact on the outcome. Applying sophisticated calculations, such as coherent measures of risk, to complex portfolios supposes that the risks are expressed perfectly in the ontology of objects considered at the outset. In other words it adds a second level: existence of ignorance is ignored. This affects the market (organized or OTC) of credits and their derivatives. Thanks to the market, portfolios are acquiring a value, everyone trusts the other calculations that are no better. This instability may be called "methodological moral hazard" which is the belief that mathematics is able to capture new interpretations if the calculations are done by everyone. This kind of instability is more severe than conventional asset markets and their options because the timelines are much longer (tens of years instead of tens of months) and punishment of economic reality much slower.

VIII. The role of mathematics ancillary discipline is confused with that of the served discipline. The previous idea can be generalized to all situations of mathematized knowledge. Let us take the case of physics. Certainly, improving the mathematics used by physicists renders services to physics, there is there a real fertility which has been particularly emphasized by Gaston Bachelard. But it works with the same interpretations of the served science. We are in the syntactic part of normal science in Kuhn's sense. Although Bachelard, with his usual talent, shows that mathematics can suggest questions for physicists, it is impossible with this only way to really inject new interpretations of phenomena occurring in the domain of the master discipline. Mathematization is an essential component of the phenomenon of scientific crisis such as Thomas Kuhn has described.

IX. That a theoretical representation be perfectible does not mean it is the only way to deal with reality and does not guarantee that it can consider the whole reality. By theoretical representation I mean a semi-artificial language using mathematics as in physics or modelling. The point is fundamental, perfectibility gives the illusion of completeness. The case of Ptolemy's geocentric planetary system here is enlightening. The excess of mathematization lies in cycles and hypocycles that can be added at will. the system was improved by Tycho Brahe and is infinitely perfectible. It appears as excess only after the new reading done by Copernicus. Its only flaw is to be unable to make place for this new interpretation. Yet this new interpretation is much less precise at the stage where the Copernican model involves heliocentric circles. But the matter is astronomy and not plane geometry, the new reading acquires legitimacy by the fact that it could have been, too, a starting point for improvements. It equally carries in it possible improvements. Galileo can not depart from this new interpretation because he recognized in Jupiter and its satellites a Copernican system. Nevertheless, at that pre-Newtonian time with only a *kinematic* description of phenomena, he has no compelling argument against the geocentric system. He was accused during his trial to be based on "beliefs" that are not in the sacred texts. It is in the register of an interpretation against another interpretation which has been addressed by Augustin Cournot. The position of Cardinal Robert Bellarmine is that faith has the monopoly of beliefs, and that science must remain a means of describing facts allowed in the work of God.

X. There is confusion between creativity of the representation and creativity of the world. Inside a system of thought, especially perfectible, no reason to escape may be detected. This is related to

[7] It is impossible to predict the next crises, but we can assume that they will revolve around the lack of consideration of limits. Boundaries, the finiteness of the world, of resources, of raw materials, of agricultural land, etc. are scotomised by economic thinking. Upward expectations of scarcity in a random context can create unpredictable instabilities.

Quine's remarks on the ontological commitment and on the almost impossibility to talk about what we ignore or deny existence. Quine emphasizes our strong tendency to speak of objects and to think about objects both in ordinary language and in physical or economic theories where agents and objects are subjected to certain relationships. "It is hard to say how else there is to talk, not because our objectifying pattern is an invariable trait of human nature, but because we are bound to adapt any alien pattern to our own in the very process of understanding or translating the alien sentences." Quine also considers the ontological conflict to clarify them. The novelty of the famous article "On What There Is" is to propose a definition of ontological commitment which in principle applies quite generally. Now, these fine arguments inspired by mathematical logic and based on the use of logical quantifiers are quite abstract and do not focus on the emergence of new objects.

A more concrete historical example is very illuminating. I mean the abandonment of the Pythagorean scale. The octave, fifth and other basic musical intervals correspond initially to the shares of a vibrating string into simple fractions, one-half to one octave, two-thirds for the fifth, three fourths of the fourth, etc. This is strictly a mathematical harmony that is truly perceived by the ear through sound frequencies. It turns out that when moving from fifth to fifth by iterating the operation to take the 2/3, we find that twelve fifths are about seven octaves. Hence, plotting this share on the original octave yields the definition of the twelve intervals of the Pythagorean scale. It is approached since 12 fifths are not exactly 7 octaves, but it is very close to the mathematics of vibrating strings, which is the natural (and scientific) basis of sounds. It took more than twenty centuries to abandon this scale and its improvements and to adopt the so-called "well-tempered" scale which gives exactly the same role to all intervals. The instruments built on the tempered scale give no preference to any tone, but they do not respect fully the laws of vibrating strings. It is the creativity of the musicians who won on the mathematics of music. The victory is in fact not total, by the problem of sharp harmonics which are heard as dissonance, etc. But the point that must be emphasized is that we have left out the ideal world of mathematics on behalf of a world of practice.

C. Why normal science and jolts of revolutions? Why orthodox economics and crises?

One has the impression that things are progressing as tectonic plates, in jolts. What is it due? How to implement a production of knowledge that goes beyond the Kuhnian epistemology?

XI. As thought by Kuhn, normal science is very close to the Popperian vision. Only the modalities of its functioning are seen with a more social emphasis on paradigms as shared understandings of scientific communities. The real difference with Popper is that the disorder that precedes a crisis is more complex than simply encountering a decisive experiment that could refute the theory, there are attempts to negotiate with the contours of interpretations. Usually, the plasticity of the paradigms allows to accept new facts or events in the theory, Kuhn takes the example of a child learning to distinguish ducks, swans and geese in a zoo, his father playing the role experimental verdict. He stresses the importance of categories slightly fuzzy whose vagueness is not mathematically quantified. But in certain historical situations, the various ways of arranging things lead to too artificial choices (properties of the ether, for example) and legitimize more radical interpretative changes.

XII. But most mathematization situations are not Popperian. Economic theories, or more peculiar models, are not likely to be refuted by any observations of facts. The social world is changing and does not appear twice successively. More specialized models that have predictive ambitions are probabilistic and cannot be falsified by a single trajectory. More generally, mathematizations useful for studying changes in the environment (pollution, climate change) are always open to several competing models, each based on a different perspective (extrapolation from ice cores or CO_2 emissions), each perfectible if new data are available. The simplest generic example is that of modelling the flow of a river for flood forecasting. Families of models based on Gaussian ARMA processes 1° of the water depth, 2° of the flow rate, 3 ° of the logarithm of the depth, 4° the logarithm of flow, are each infinitely perfectible if new measured data are available but do not predict the same probabilities of crossing a threshold. This does not mean that these models are useless, far from it. It just shows that it is not because reality is plural that it is not scientific. In fact, for one kind of phenomenon, the data are always finite in number and by a finite number of points can be passed either polynomials or combinations of Gaussians or combinations of real exponentials or trigonometric functions etc. If you think about the immense field opened to knowledge thanks to modelling, we quickly became convinced that it is the Popperian theory which are the exception. For a theory to be Popperian must have only a fixed number of parameters all numerically fixed. Difficult to name a few besides gravitation and some physical theories. Probabilistic theories are never in this case because an infinite number of events is needed to determine a probability distribution.[8]

So, this remark also applies to normal science in the sense of Kuhn. It is an extremely restrictive view of knowledge. Let us be more precise.

XIII. It is the monism required at each step that makes the jolts. From where comes the new interpretation that is characteristic of a scientific revolution? It can only arise from differences in the disciplinary community. The jolts come from the absolute will that the community live with only one truth. Now this is a particular vision of knowledge and social organization of science. If we consider instead that the "reality" is also and above all people, groups, with their powers, their habits, their psychology, and their means of interacting with their environment, it is understandable that the only way to capture or at least to take account of innovation in the world is to give place to the instances where new representations are constructed : users' associations, professional groups, consulting experts, victims of contingencies, etc. As Funtowicz and Ravetz have well analysed, this route is that of a knowledge *of better quality*, more reliable and where we can have more confidence.[9]

It is a pluralistic knowledge, this does not mean relativistic. Capital nuance. Specifically, as soon as one requires a certain level of rigor and consistency, different approaches are few in number. As the major political ideas concern a limited number of parties in multiparty parliamentary systems. To say that out of the monism of unique truth one falls into relativism is the coarse argument of dominant representations, that jolts of crises refute periodically.

Nevertheless, if the implementation of that pluralistic knowledge is on track in some areas such as climate change or protecting sensitive areas (despite bad joints with political power, which is not new), it presents rather peculiar difficulties for the economy. With globalization, knowledge concerning the economic exchanges have a strong tendency to monism. Presumably, however, that the growing environmental problems will lead to greater tolerance in the implementation of specific economic experiments (local exchange systems, fair trade, etc.) and their maintenance as a condition of a better accompanying of natural equilibriums.

D. Interpretative pluralism is not destructive of knowledge, rather it is a type of knowledge of better quality.

We now propose to examine more thoroughly the features of that better quality and what role mathematics can play. This will necessitate a step back on science as it is currently most often thought and practiced. Beyond the concept of "confined research" introduced by Michel Callon, it

[8] It is not surprising in this respect that Chapter VIII of the *Logic of Scientific Discovery* on probability is the least convincing. Popper failed to see that a probability theory without sigma-additivity is of little use and his posture towards Kolmogorov is that of a philistine.

[9] S. O. Funtovicz and J. R. Ravetz "Three Types of Risk Assessment and the Emergence of Post-Normal Science" in *Social Theory of Risk*, Sh. Krimsky and D. Golding eds, Preager 1992.

appears that what is at stake is the *conquering* character of Baconian program and the *masculine virtues* connected with it.

We will use for convenience the term challenge-science to describe the vision that was until recently shared by most scientists to see knowledge as *a challenge to nature*. It causes nature to single combat. The loyalty of the game is to respect the assumptions that will govern the rules of protocol of the experiment. This includes Popperian science and Kuhn's normal science. In fact this is very old. In this regard, the induction principle advocated by many philosophers and scientists to account for knowledge is similar. Simply, Popper proposes a induction articulated on a theory. Instead of accepting the thesis that knowledge has its philosophical essence in the ability to read a regular pattern and to continue it - an idea championed simultaneously (1843) by John Stuart Mill and Augustin Cournot who finely analyses it - thus drawing from a large number of results or a large number of circumstances a law candidate for assessment, Popper reinforces the criterion by requiring that we move from observed facts to a representation with the dress of a theory, that is to say, based on a mathematical syntax like mechanics formulated by Lagrange or Hamilton. Historically, it is indisputable that during the whole period where industrialization had not yet too complexified the technology, science was practiced with few experimentations and many challenges also turned to colleagues as well as to nature. The discoveries at the time of Pascal, Fermat and Mersenne father are often announced as enigmas whose answer is only known by the finder thrown to the sagacity of his contemporaries.[10]

In the early 21st century, a new awareness, unique in the evolution, is happening. The indefinite continuation of the growth is impossible, and if the limit is not reached, the current pace is so destructive that it must be drastically curbed. It is less clear that using the challenge-science vis-à-vis the environment by new technical devices and a progressive mathematization to calculate the economical optimums with cost-benefit analysis in the context of democracy and liberal economy can overcome the global challenges: arable land, species, climate change, pollution of soil and water, etc.. New options for production and consumption (economy of functionality) and on the democratic structures (new bicameral system) are probably essential. But more fundamentally, we must also consider the question of what kind of knowledge. The epistemological question of how producing knowledge also arises.

XIV. Which logical status may possess the new knowledge? Is there still any sort of "room" for something else? What are the characteristics of knowledges that are not falsifiable theories? Are there some? They would eventually be forgotten as they are innumerable. Included in this field are all *useful discoveries* that form the symmetric logical category of refutable hypotheses. Most of the knowledge on the mineral, vegetable and animal, and a lot of technical learnings are of this type.

Belongs to this class most of the chemistry that has long been considered a pre-scientific state vis-à-vis the physics. The great chemist Henry Le Chatelier in the early twentieth century says: "These two sciences have a similar purpose, they study both phenomena resulting in transformations of energy, i.e. mechanical, calorific, electrical or chemical power. Teaching of physics refers only to the laws of natural phenomena: laws of Mariotte, Gay-Lussac, Ohm, Joule, Descartes, Carnot, etc.. [...] Chemistry, however, is only an indefinite list of small particular facts [...] the materials thus accumulated will be very useful for the subsequent establishment of science but yet they do not constitute it in any way".[11] Why such a disgrace? Is it justified in terms of rendered services?

Also, most of the medical and environmental knowledge. Long before Popper, about medicine Claude Bernard wrote "you can do in science two kinds of discoveries, some are predicted by theory, they suppose two conditions: a very advanced science, e.g., physics, and simplicity of the phenomena. The others are unexpected: they appear unexpectedly in the experiment, not as

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[10] See Koyré, *ibid*, on the fact that Galileo never experienced the stone falling from the mast of a moving boat, and on the enigma with which he announced to his contemporaries his discovery of the phases of Venus.

[11] H. Le Chatelier, *Leçons sur le carbone, la combustion, les lois chimiques*, préface, Paris, 1908.

corollaries of the theory and devoted to confirming it, but always outside of it and therefore contrary to it. "[12]

More generally, not belonging to the challenge-science are all the knowledge that tells us how the world *is*, what features make it as we have it, and not another one that follows the same laws. This is not inconsistent with general knowledge in Aristotle style, but these innumerable and fortuitous data, that reflect what life and history have made, are quite important to support the nature and the social. Besides, without them the challenge-science is nothing. Computers can help us to store them but they do not reduce to ratings or coordinates, they are interpretative like the new paradigms that Kuhnian revolutions bring. We must therefore accept that some and others be complementary, they are plural on the same object as reports of different styles emphasizing different features.

XV. A knowledge whose social function is not prediction but care. We must make a place for stories, testimonies, for what makes the current understanding of the world in its diversity. They found the uses and the values that give meaning to representations including the scientific ones.

About mathematics, there is no reason to exclude it, we need it here too. But symbols may be used more freely than in axiomatized theories. It is perfectly legitimate to reveal a phenomenon, to represent a trend or a natural evolution using existing scientific languages from the established sciences or from engineering which are semi-artificial languages with partial mathematization. For managing natural equilibriums of life and for working on the collective decisions of social groups, it is necessary to allow various representations and even different rationalities to coexist. Use of mathematics as thought patterns, by the simple value of linguistic symbols and combinations thereof, is useful and desirable, they are not reserved to write the truth of challenge-science.

XVI. The main tool of a better-quality science is contradictory criticism thanks to modelling. The models are able first to take into account the particularities of situations and to apply them assessed knowledge and secondly to provide an interpretation of the complexity of interest, thanks to the ordinary language which forms an internal binder and an external context.

In order they be not considered as low level or amateur challenge-science, it is essential that models be always thought of as a facet of a plurality. First, they must be subjected to the same requirement for the validation by data as usually requested by scientists. Validation is not an assessment of truth, but simply an unlikelihood elimination. Secondly, they must be recognized as a social expression hence a language from an agent (group, association, company, territorial entity, etc.) to an audience to contribute to a decision and therefore lent to criticism by other models. Let us say that knowledge is no longer formed exclusively by a struggle between theory and nature but by a contest between models. This process obviously requires a specific organization, such as challenge-science requires careful experimental protocols. The "rules" are not currently codified, the experiments are underway at international level for the IPCC and in the public debates, citizen juries etc., in a kind of applied living epistemology in new development.

To criticize a model is difficult. The quantified arguments are linked together, everything is bound. This is a huge task to lift out of all the implicit assumptions of a model. Even knowing that every model is by some aspects arbitrary, we cannot apply this freedom in practice. When discussing on a single model, we remain in the ruts of the thinking. The best way is to build another model *ex nihilo*. Then the options appear clearer.

To construct another model, the dualities introduced by the philosophy of science are significant, they allow for a dialectic setting with the occurrence of what may be called co-truths. Let us give a few examples.

Discrete / continuous. Much of the economics may be made without individualizing agents nor goods, some scholars consider illuminating to recover the global laws from a micro-economic

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[12] Cl. Bernard *Leçons de physiologie expérimentale appliquée à la médecine faites au collège de France*, Paris, 1885.

individual rationality. In the case of the traffic, we consider, depending on the stakes, either hydrological models or models with individualized vehicles. Sometimes it is thought that the discretization, spatial or temporal, simplifies the problems, that the recurrence rules are more elementary than differential equations and that the finite element programs bring the partial differential equations to the simple algebra. But often the opposite, the discrete probabilities are sometimes intractable and some algorithms (such as Kalman), are best understood in continuous time.

Descriptive / explanatory. In 1970, two American authors, G. E. P. Box and G. M. Jenkins transpose methods invented by Wiener for signal processing into predicting economy. From annual series treated regardless of their economic significance, they obtained sometimes better predictions. This is the fundamental duality which we pointed out in this article. In the history of science, it often occurs in successive periods. The purely descriptive approach can be an advance when it releases from too pervasively present interpretations. While the explanations allow a reading to enlight situations other than those already encountered.[13]

Quantitative / qualitative. The philosophical work of René Thom has brilliantly illustrated that mathematics provide representation tools that go far beyond the quantitative. A huge field of natural phenomena can be addressed qualitatively through a language adapted to the evolution of forms.[14]

Deterministic / random. A large number of situations of modelling involves risks. The spontaneous tendency of modelers is to probabilize the uncertainties - we already talked about this tendency. This provides a very efficient syntax thanks to stochastic calculus that has been developed in the 20th century. But this, especially by the tails of laws, conceals ignorance. Uncertainty is sometimes better illustrated by some typical or extreme trajectories obtained from different scenarios.

Image / symbol. Let us take the example of dance. Dozens of writing systems have been developed by the choreographers to record ballets, based on a limited vocabulary of successive steps (system Feuillet 1700) or more elaborate to note the dancer's energy in each movement (system Laban 1927). The question arises in terms of modelling, with the usual constraints of relevance for the choreographer and dancers. *But is not this a false problem since film and video can provide us with an almost perfect image of the ballet* ? The image reproduces, it can provide the perfect illusion of reality, only it does not allow by itself choreographic creation. The writing systems of dance have the immense superiority to make possible noting a ballet that has never been danced.

Modelling criticism cannot result from recipies or an *a priori* classification. Especially, as we have emphasized, its relevance depends on the social group that proposes it. *The quality* of the plural knowledge thus produced comes especially from that it can bring a real along which challenge-science could pass without even seeing it. Applied in good conditions for open democracy, it is likely to show hidden effects, not identified risks, possibly unsuspected solutions. Challenged science instead, by the successive stages of its rockets, draws only one direction.

Conclusion: The problem is not that there is too much mathematics, but they are too exclusively used as a framework of theories that claim to univocal truth.

The propensity to mathematize more and more can occur in both the development of classical theoretical thinking and that of modelling, but especially if one assigns a value of absolute truth to the interpretative framework in which we are, so that syntactic developments will be seen as revealing reality. This occurs in modelling because the modelers tend to think that their models are reality. But faced with foreign models they will be forced to feel *the scope* of their approach. In contrast, in the case of a Popperian conception further mathematization can occur without any

[13] On this duality, see R. Thom, *Prédire n'est pas expliquer*, Flammarion, 1993.
[14] R. Thom, *Stabilité structurelle et morphogenèse*, Benjamin, 1972.

brake, until a jolt. This analysis of mathematization has indeed the advantage of opening the philosophy of knowledge to a new field of reflection. One steps back with respect to challenge-science considering it a very special modality of apprehending the world.

It is also ultimately a choice between what is important and what is not. A river basin for example, will remain for centuries, we are faced here with contradictory logics, politicians who want to develop jobs, farmers who want to irrigate, associations that want to respect the landscape, companies that want to do dams for electricity, etc. Neither the economic nor the democratic vote, can exceed the basic addition of selfishness. *Accompany* the scenes of natural life involves intermediate languages between native speech and falsifiable science, languages opposing without destroying themselves, which open by their plurality to *data interpretation* and *imagination of eventualities*.

About mathematical creativity, mathematicians must assume their libido: the pleasure of a mind game. This game does not need to be the skeleton of a unique large building of knowledge. The nonstandard analysis, predicate calculus of second order, etc.., the attempt to extend the fertility of a method off the beaten path is rewarded with the surprising findings collected.

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