Examples of

sensitivity analysis of dynamical systems

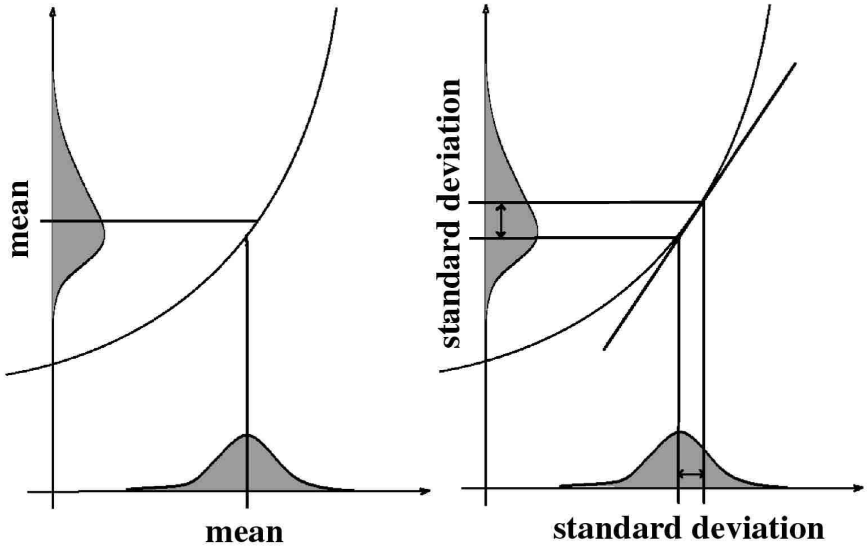
- **Magnitude of errors**

2. The propagation of errors

let us apply successively non-linear applications

quadratic bias

quadratic bias 2



We observe the following properties

- **The error doesn't remain centered : a bias appears**

- **The variances transmit with a first order differential calculus**

$$\sigma_{n+1}^2 = f_{n+1}'^2(x_n)\,\sigma_n^2$$

- **The biases and the variances keep (except special case) the same order of magnitude**

- **The biases follow a second order differential calculus involving the variances**

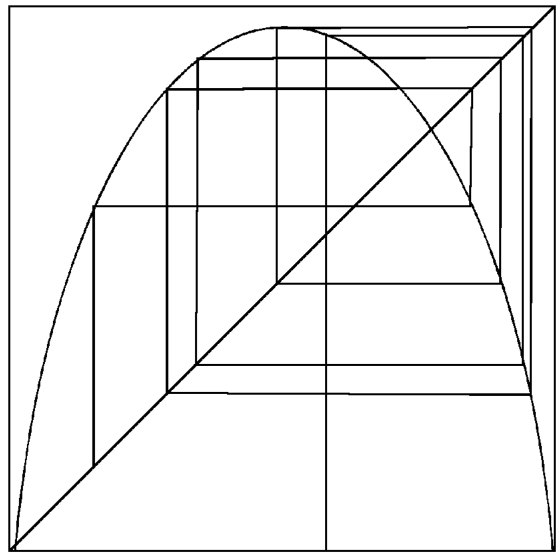
$$\mathrm{bias}_{n+1} = f'_{n+1}(x_n)\mathrm{bias}_n + \frac{1}{2}f''_{n+1}(x_n)\sigma_n^2$$

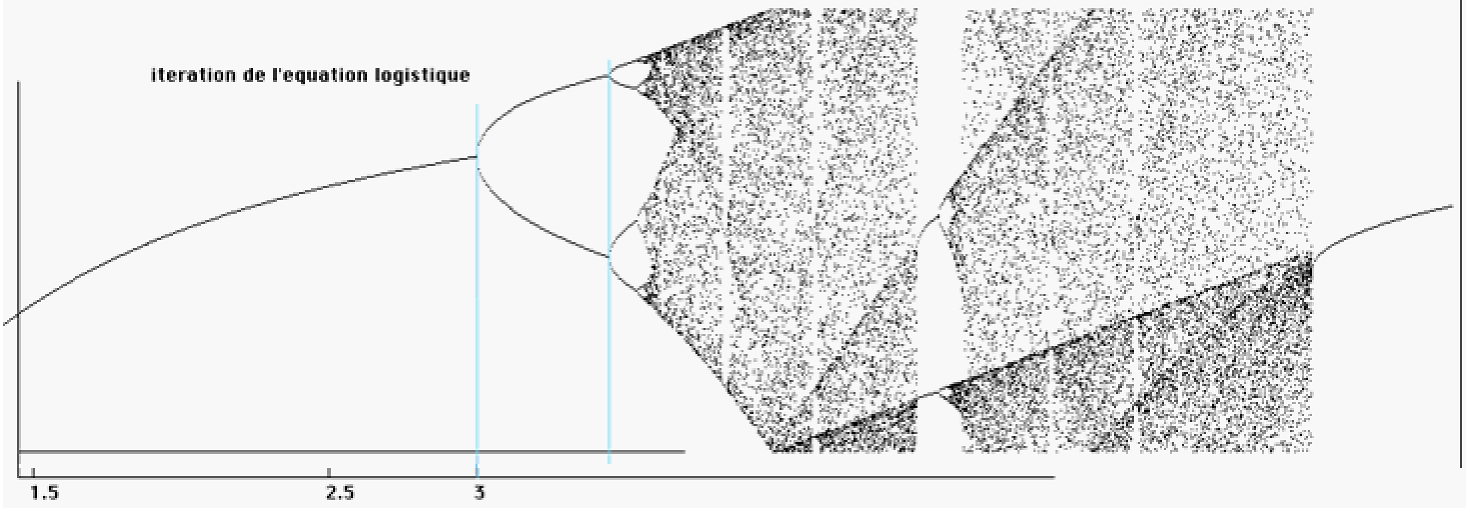
Calcul de sensibilité sur l'itération de l'équation logistique

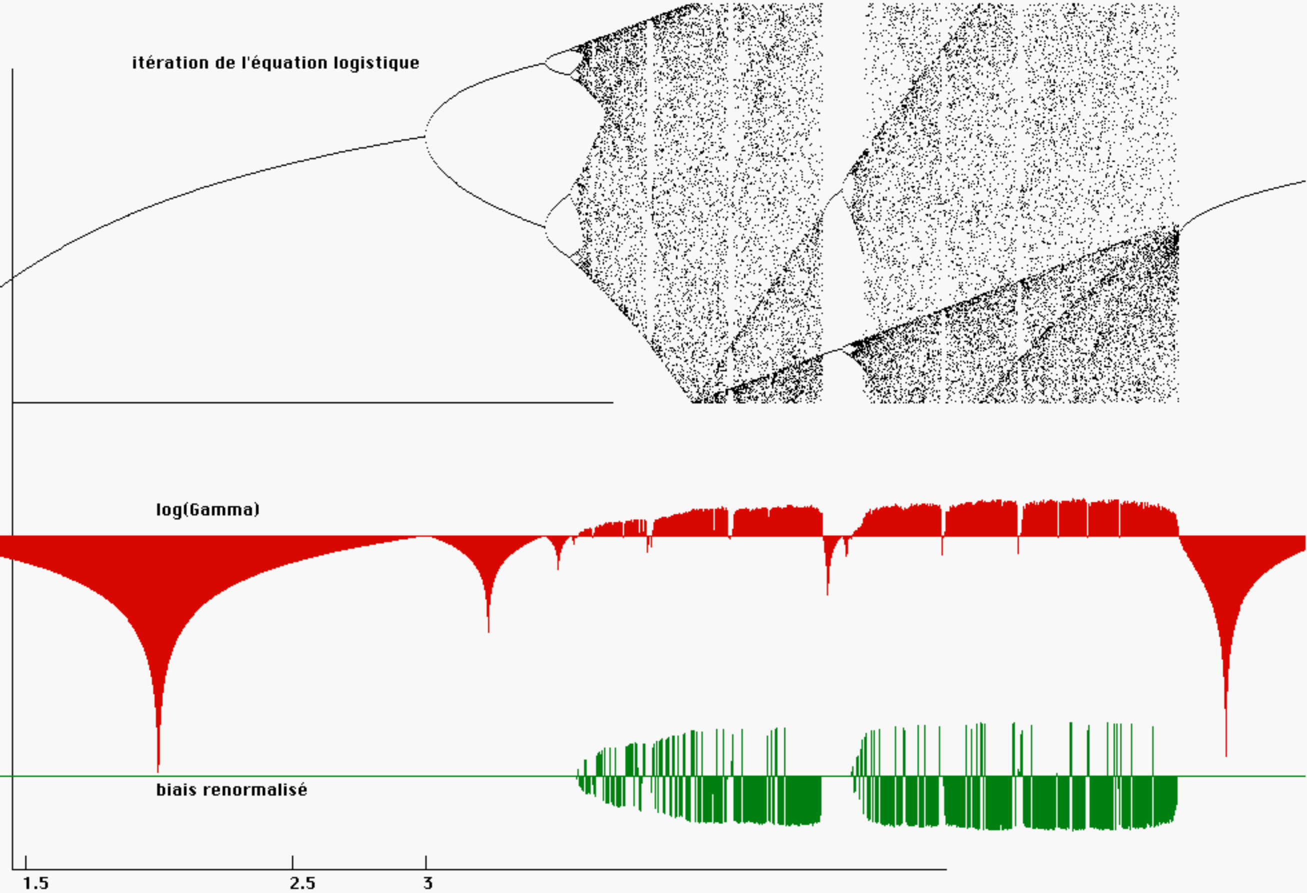
$$\boxed{x_{n+1} = a\, x_n(1-x_n)}$$

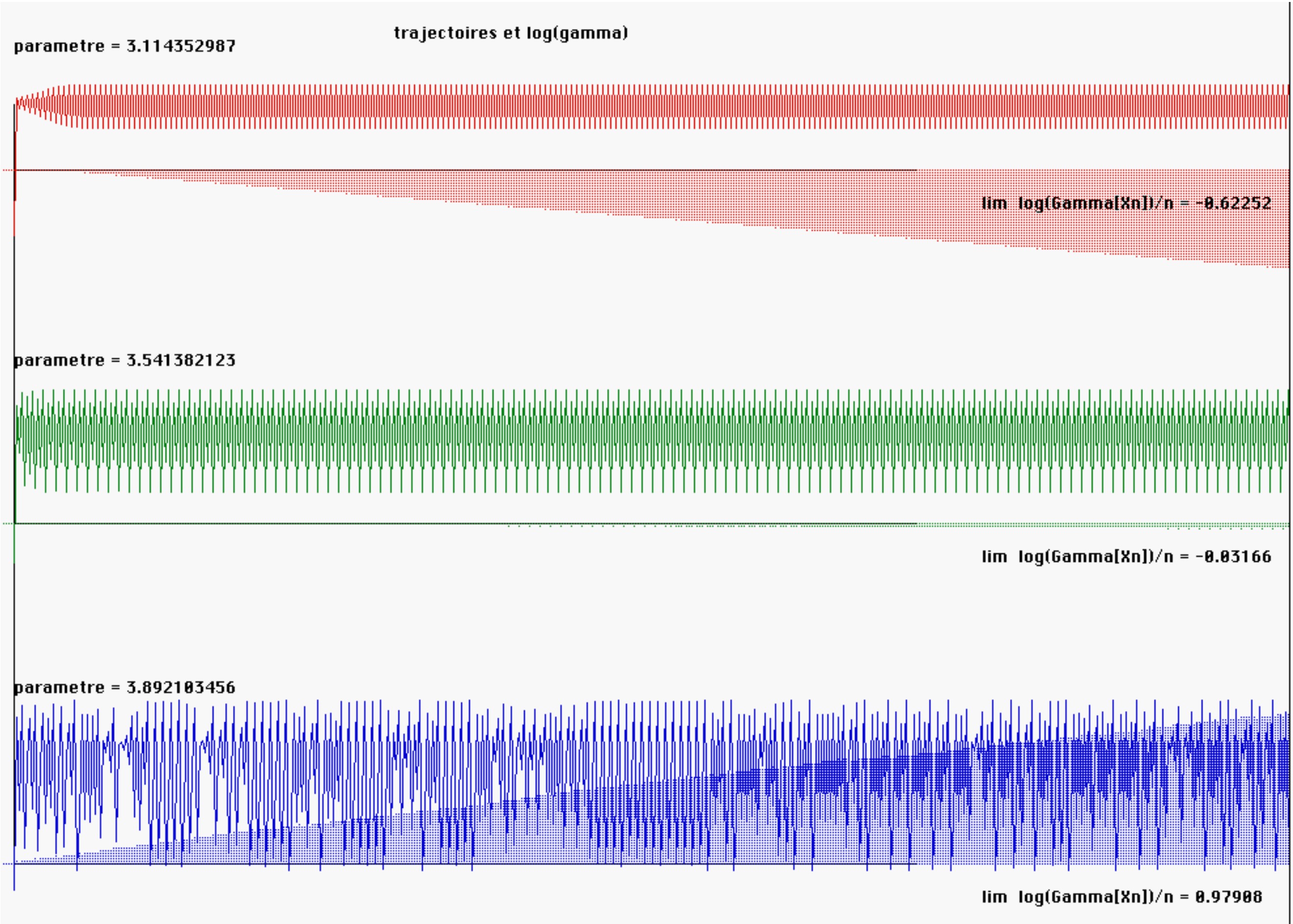
$$\Gamma[x_{n+1}] = a^2\,(1-2x_n)^2\,\Gamma[x_n]$$

$$\mathrm{A}[x_{n+1}] = a\,(1-2x_n)\mathrm{A}[x_n] - a\,\Gamma[x_n]$$




iteration de l'equation logistique
1.5
2.5
3


itération de l'équation logistique
log(Gamma)
biais renormalisé
1.5
2.5
3


trajectoires et log(gamma)
parametre = 3.114352987
lim log(Gamma[Xn])/n = -0.62252
parametre = 3.541382123
lim log(Gamma[Xn])/n = -0.03166
parametre = 3.892103456
lim log(Gamma[Xn])/n = 0.97908


trajectoires et biais
parametre = 3.114352987
biais
lim log(abs(A[Xn]))/n = -0.30644
parametre = 3.541382123
biais
lim log(abs(A[Xn]))/n = -0.00906
parametre = 3.912103456
lim log(abs(A[Xn]))/n = 0.95368
biais

sensibilité d'un système linéaire par morceaux

Système dynamique de Lorenz

lancer Lorenz

Lorenz par paquets

sensibilité aux données et ellipse

sensibilité aux paramètres

$$x'=a(y-x)$$

$$y'=bx-y-xz$$

$$z'=xy-cz$$

c'est-à-dire

$$x=x_o+\int_o^t a(y-x)\,dt$$

$$y=y_o+\int_o^t (bx-y-xz)\,dt$$

$$z=z_o+\int_o^t (xy-cz)\,dt$$

Etude de la sensibilité en fonction du point de départ