Algorithms for train scheduling on a single line

Laurent DAUDET$^1$

PhD advisor: Frédéric MEUNIER$^1$

$^1$CERMICS, Centre d’Enseignement et de Recherche en Mathématiques et Calcul Scientifique, ENPC

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The context

Scientific chair between
- École Nationale des Ponts et Chaussées
- Eurotunnel
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Questions?
- How to increase the global capacity with current means?
- How to improve the quality of service?
The tunnel under the Channel

- Line length: 50 km
- From Coquelles (France) to Folkestone (England)
- A tunnel for each direction (A) and a service tunnel (B)
  - High-Speed-Trains (Eurostar) 160 km/h
  - Freight trains 100-120 km/h
  - Passenger shuttles (PAX) 140 km/h
  - Freight shuttles (HGV) 140 km/h
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1. General context

2. One-hour schedules maximizing HGV shuttles

3. Joint scheduling and pricing problem

4. Minimizing the waiting time for a one-way shuttle service
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What are the goals?

- Confirm optimality of current schedules.
- Compute schedules with new instances.
- “Price” the constraints for future investments and negotiations.
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The problem

Objective

Compute one-hour schedule with maximum number of HGV shuttles.

Constraints

- Fixed number of Eurostars, freight trains, and PAX shuttles.
- Security.
- Other constraints.

Output

- \( d = \{ d^{Eur}_j, d^{Fr}_j, d^{PAX}_j, d^{HGV}_j \} \): scheduled departure times of all trains.
- \( d^A_j \): \( j \)th departure time of train of type \( A \).
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Some other constraints

- **Commercial agreements**: Eurostars “equally” distributed in the period. → Eurostars grouped by pairs with departure times at $d$ and $d + 30 \text{ min}$.  
  
- Loading platforms: impossible to load three HGV or PAX shuttles at the same time.  
  → at most 2 HGV shuttles in any 12-minute time-window.  
  → at most 2 PAX shuttles in any 12-minute time-window.  
  
- Discretization: departure times on full minutes.
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Mathematical model

where \( X = \) set of constraints

→ can be expressed with linear constraints

(only non immediate constraint: security headway [Serafini and Ukovich, 1989]).

⇒ Mixed Integer Linear Program

Solved by commercial solver CPLEX.
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One-hour schedules maximizing HGV shuttles

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- \( d = \{ d_{j}^{E}, d_{j}^{F}, d_{j}^{P}, d_{j}^{H} \} \): scheduled departure times of all trains.
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A current schedule

**Instance**: 4 Eurostars, 5 PAX shuttles, and 1 freight train.
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⇒ **Maximum 4 HGV shuttles in the schedule**
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Some schedule improvements

Parameters

- $T$: length of the period.
- $L$: 12-minute time-window.
- $\eta$: full minute discretization.
- $C_{\text{Eur}}$: 30-minute gap between grouped Eurostars.
Some schedule improvements

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**Constraint relaxed**

<table>
<thead>
<tr>
<th>Instance</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length cyclic period</strong></td>
<td>( T: 1 \text{ h} \rightarrow 4 \text{ h} )</td>
</tr>
<tr>
<td><strong>Loading platforms</strong></td>
<td>( L: 12 \text{ min} \rightarrow 0 \text{ min} )</td>
</tr>
<tr>
<td><strong>Full minute discretization</strong></td>
<td>( \eta: 1 \text{ min} \rightarrow 1 \text{ s} )</td>
</tr>
<tr>
<td><strong>Agreements with Eurostar</strong></td>
<td>( C^{\text{Eur}}: 30 \text{ min} \rightarrow [27 \text{ min}-33 \text{ min}] )</td>
</tr>
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Some other problems

- Compute one-hour schedules with minimum delays.
- Compute one-hour schedules with maximum number of HGV shuttles and minimum delays.

→ Stochastic Optimization, Sample Average Approximation.
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General context

One-hour schedules maximizing HGV shuttles

Joint scheduling and pricing problem

Minimizing the waiting time for a one-way shuttle service
Why such a problem?

- Departures and prices computed jointly in airline companies.

→ Increase of customers' satisfaction and company's revenue.

- Same objective for rail transportation.

- Toy problem to challenge this idea.
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- $Q$ customers want to purchase tickets for this trip.
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→ Buy tickets that satisfy them the most, or leave without purchasing.

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<tr>
<th>Departure</th>
<th>Arrival</th>
<th>Price</th>
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<tbody>
<tr>
<td>17h29</td>
<td>19h26</td>
<td>87,00 €</td>
</tr>
<tr>
<td>17h53</td>
<td>19h56</td>
<td>97,00 €</td>
</tr>
<tr>
<td>18h41</td>
<td>20h44</td>
<td>78,00 €</td>
</tr>
<tr>
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- $Q$ customers want to purchase tickets for this trip.
- Buy tickets that satisfy them the most, or leave without purchasing.

Objective

Maximize the revenue of the company.
**Each customer \( i \)**

- has a preferred departure time: random variable \( \chi_i \)
- belongs to economic class \( b_i \) (e.g. business, tourist, low-cost, ...)
- "value of time" \( v_{b_i} \) for economic class \( b_i \)
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We assume
- $v_1 \leq v_2 \leq \cdots$
- Customers of class 1 make their choice first, then 2, ...
The problem (2/2)

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Discrete choice model
- Each customer $i$ and product $j$ (departure $d_j$ at price $p_j$)
- Random utility $U_{ij}(d, p)$ representing satisfaction.
The model (1/2)

- Denote by $\xi$ vector representing uncertainty.
- Revenue of company $\rightarrow R(d, p, \xi)$ where $\xi$ has been revealed.
- Easy to compute (simulation, Linear Programming).
- Objective function $\rightarrow f(d, p) = \mathbb{E}[R(d, p, \xi)]$.
- Remark: $f(d, p)$ well defined (for all $(d, p) \in X$, $R(d, p, \cdot)$ measurable and $\mathbb{E}[R(d, p, \xi)] < \infty$) and $\text{Var}[R(d, p, \xi)] < \infty$. 
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The model (2/2)

Mathematical model

$$\max_{(d, p) \in X} f(d, p) = \mathbb{E}[R(d, p, \xi)]$$

where $R(d, p, \xi)$ is revenue and $X$ is set of constraints.
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Compute $d$ and $p$ without knowing $\xi$!
Sample Average Approximation

- \((\xi_1, \ldots, \xi_\Omega)\) of \(\Omega\) independent and identically distributed realizations
  \(\Rightarrow\) \(\xi_\omega\) is not random variable!
- We approximate objective function \(f(d, p) = \mathbb{E}[R(d, p, \xi)]\) by
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We denote by

- \(v^* = \max_{(d, p) \in X} f(d, p)\)
- \(\hat{v}_\Omega = \max_{(d, p) \in X} \hat{f}_\Omega(d, p)\)
SAA properties

Proposition, Shapiro et al., 2009

We have

(i) \( \mathbb{E}[\hat{f}_\Omega(d, p)] = f(d, p) \), for all \((d, p) \in X\),

(ii) \( \hat{f}_\Omega(d, p) \) converges to \( f(d, p) \) w.p. 1, for all \((d, p) \in X\),

(iii) \( \mathbb{E}[\hat{v}_\Omega] \geq v^* \), and

(iv) \( \hat{v}_\Omega \) converges to \( v^* \) w.p. 1.

- \( \mathbb{E}[\hat{v}_\Omega] \) is an upper bound on \( v^* \) (iii).
- With the Central Limit Theorem, we can compute \((1 - \alpha)\)-confidence interval.
- For any solution \((\hat{d}, \hat{p}) \in X, f(\hat{d}, \hat{p}) \leq v^* \) \( (= \max_{(d, p) \in X} f(d, p) ) \)
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→ With (i) and the Central Limit Theorem, we can compute \((1 - \alpha)\)-confidence interval.
A first heuristic: a sequential heuristic

→ Try to mimic natural way of scheduling and then fixing prices.

1. Compute departure times $d$ with optimization problem maximizing utilities $U_{ij}$ with prices $p = 0$. 
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$$\max_{d \in X_d} \sum_{i,j} \mathbb{E} [U_{ij}(d, 0)]$$

2. Compute prices $p$ with previous two-stage recourse program (with fixed departure times $\tilde{d}$).

$$\max_{p \in X_p} \hat{f}_\Omega(\tilde{d}, p)$$
A second heuristic: a Gauss-Seidel heuristic

1. Initialize \((d, p)\) to some \((d^0, p^0)\).
2. Let \(m \in \mathbb{Z}_+\) and \(J_1 \cup J_2 \cup \cdots \cup J_m\) be a partition of \(\{1, \ldots, S\}\).
3. Generate sequence of feasible solutions \((d^k, p^k)\):
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(d^{k+1}_{J_2}, p^{k+1}_{J_2}) \in \arg \max_{d, p} f((d^k_{J_1}, d, d^k_{J_3}, \ldots, d^k_{J_m}), (p^k_{J_1}, p, p^k_{J_3}, \ldots, p^k_{J_m}))
\]
A second heuristic: a Gauss-Seidel heuristic

1. Initialize \((d, p)\) to some \((d^0, p^0)\).
2. Let \(m \in \mathbb{Z}_+\) and \(J_1 \cup J_2 \cup \cdots \cup J_m\) be a partition of \(\{1, \ldots, S\}\).
3. Generate sequence of feasible solutions \((d^k, p^k)\):

\[
(d_{J_1}^{k+1}, p_{J_1}^{k+1}) \in \arg\max_{d,p} f((d, d_{J_2}^k, \ldots, d_{J_m}^k), (p, p_{J_2}^k, \ldots, p_{J_m}^k))
\]

\[
(d_{J_2}^{k+1}, p_{J_2}^{k+1}) \in \arg\max_{d,p} f((d_{J_1}^{k+1}, d, d_{J_3}^k, \ldots, d_{J_m}^k), (p_{J_1}^{k+1}, p, p_{J_3}^k, \ldots, p_{J_m}^k))
\]

\[
\vdots
\]

\[
(d_{J_m}^{k+1}, p_{J_m}^{k+1}) \in \arg\max_{d,p} f((d_{J_1}^{k+1}, \ldots, d_{J_{m-1}}^{k+1}, d), (p_{J_1}^{k+1}, \ldots, p_{J_{m-1}}^{k+1}, p))
\]
The results

Academic instance:
- Realistic distribution for preferred departure times.
- Gumbel distribution for random part of utilities.

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<td>S</td>
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Other heuristic: Lagrangian relaxation
⇒ does not improve Frontal SAA for big instances.
Conclusion

- Joint scheduling and pricing: higher revenues.
- Upper bounds: not very precise...
1. General context

2. One-hour schedules maximizing HGV shuttles

3. Joint scheduling and pricing problem

4. Minimizing the waiting time for a one-way shuttle service
Why such a problem?

- Trucks arrive continuously on terminals.
- Huge waiting lines during peak hours.
- Design schedules to decrease congestion (waiting times).
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- One-way trip.
- Infinitesimal users arriving continuously (demand known in advance).
- Company wants to schedule $S$ shuttles of capacity $C$.
- Peculiar loading process.
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Minimizing the waiting time for a one-way shuttle service

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The loading process

$D$ vs $t$
The loading process
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Minimizing the waiting time for a one-way shuttle service

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Objectives

- Minimize maximum waiting time of users $\Rightarrow$ Problem $P_{\text{max}}$
- Minimize average waiting time of users $\Rightarrow$ Problem $P_{\text{ave}}$

$\rightarrow$ Waiting time: time between arrival time on terminal and departure of shuttle.
The model (1/3)

Variables

- $d_j$: departure time of the $j$th shuttle.
- $y_j$: cumulative loads for shuttles 1 to $j$.

Parameters

- $S$: number of shuttles.
- $C$: capacity.
- $\nu$: loading rate (loading $x$ users takes a time $\nu x$).
- $T$: time horizon.
- $D: [0, T] \rightarrow \mathbb{R}_+$: cumulative demand known a priori (oracle).
- We assume that $D(\cdot)$ is upper semicontinuous.
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The model (2/3)

Arrival time of user $y \rightarrow$ function $\tau(\cdot)$, pseudo-inverses of $D(\cdot)$:

$$\tau(y) = \inf \{ t \in [0, T] : D(t) \geq y \}.$$

Objective function

$$g^{\text{ave}}(d, y) = \frac{1}{D(T)} \sum_j \int_{y_{j-1}}^{y_j} (d_j - \tau(y)) \, dy$$
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Similar function $g^{\text{max}}(d, y)$ for maximum waiting time.
Minimizing the waiting time for a one-way shuttle service

The model (3/3)

\[
\begin{align*}
\text{Min}_{d, y} & \quad g(d, y) \\
\text{s.t.} & \quad y_j - y_{j-1} \leq C \\
& \quad y_{j-1} \leq y_j \\
& \quad d_{j-1} \leq d_j \\
& \quad y_S = D(T) \\
& \quad d_j \geq \tau(y_j) + \nu(y_j - y_{j-1}) \\
& \quad y_0 = 0.
\end{align*}
\]
When demand is a step function

**Theorem**

Assume that $D(\cdot)$ is a step function defined with $K$ discontinuities, and that $\nu = 0$. There is an algorithm computing an optimal solution of $P^{\text{ave}}$ in $O(K^2 S)$. 
Minimizing the waiting time for a one-way shuttle service

Piecewise constant demand

Precision on \( \nu = 0 \)
Precision on $\nu = 0$
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An algorithm for $P^{\text{ave}}$ and $C = \infty$

- Shortest path in $S$ arcs minimizing sum on arcs in directed acyclic graph with $K + 1$ vertices
  \[ \Rightarrow \text{complexity } O(K^2S). \]
- $C < \infty$, idem in graph with $O(K)$ vertices.
An algorithm for $P^{\text{ave}}$ and $C = \infty$

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- $C < \infty$, idem in graph with $O(K)$ vertices.
An approximation scheme for $P^{\text{ave}}$

Theorem

Suppose that $D(\cdot)$ admits right derivatives everywhere (denoted $D'_+(t)$) and $\inf_{t \in [0, T)} D'_+(t)$ is positive. Then, for any positive integer $M$, a feasible solution of value

$$\text{SOL} \leq \text{OPT} + O\left(\frac{S^2}{M}\right)$$

can be computed in $O(SM^3)$.
Some elements of proof (1/2)
Lemma

There exists a collection of problems \((P^n)\), with parameter \(\eta\), providing:

(i) lower bound \(LB^n\) of \(OPT\).
(ii) upper bound \(UB^n\) of \(OPT\).
(iii) \(\lim_{\eta \to 0} UB^n - LB^n = 0\).
Some elements of proof (1/2)

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→ \(\eta\) depends on \(M\).

→ Problems \((P^n)\): Demand is discretized with step \(\eta\)
  ⇒ Shortest path in directed acircuitoic graph.
Elements of proof (2/2)
Existence of optimal solution of $P^{\text{ave}}$?

- $\tau(\cdot)$ lower semicontinuous $\implies$ set of constraints is closed.
  $\implies$ set of constraints is compact!
- $g^{\text{ave}}(d, y)$ continuous.
Existence of optimal solution of $P^{\text{ave}}$?

$$\text{Min}_{d,y} \quad g^{\text{ave}}(d, y) = \frac{1}{D(T)} \sum_j \int_{y_{j-1}}^{y_j} (d_j - \tau(y)) \, dy$$

s.t. $\quad y_j - y_{j-1} \leq C$

$\quad y_{j-1} \leq y_j$

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$\quad y_S = D(T)$

$\quad d_j \geq \tau(y_j) + \nu(y_j - y_{j-1})$

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Minimizing the waiting time for a one-way shuttle service

Approximation scheme for $P^\text{ave}$

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**Exact algorithm**: $O(K^2 S)$

**Approx. algorithm**: $O(S M^3)$
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Laurent DAUDET
PhD Defense
December 22nd, 2017
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Latex code for the table:

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               & $p_{\text{max}}$ & $p_{\text{ave}}$ \\
\hline
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Minimizing the waiting time for a one-way shuttle service

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</tr>
</thead>
<tbody>
<tr>
<td><strong>Exact algorithm</strong></td>
<td>$O(K^2S)$</td>
<td>$O(K^2S)$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td><strong>Approx. algorithm</strong></td>
<td>$O\left(S\log\frac{S}{\varepsilon}\right)$</td>
<td>$O\left(SM^3\right)$</td>
<td>Approx. algorithm</td>
<td>$O\left(\beta^3S M^{2S+1}\right)$</td>
</tr>
<tr>
<td><strong>Closed-form expression</strong></td>
<td>$O(1)$</td>
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</tr>
<tr>
<td><strong>Exact algorithm</strong></td>
<td>$O(1)$</td>
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<td>Exact algorithm</td>
<td>$O(S')$</td>
</tr>
</tbody>
</table>
## Table of results

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{max}}$</th>
<th>$P_{\text{ave}}$</th>
<th>$P_{\text{max return}}$</th>
<th>$P_{\text{ave return}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact algorithm</td>
<td>$O(K^2 S')$</td>
<td>$O(K^2 S')$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Approx. algorithm</td>
<td>$O(S \log \frac{S}{\epsilon})$</td>
<td>$O(S M^3)$</td>
<td>$O(\beta^3 S M^{2S+1})$</td>
<td></td>
</tr>
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<td>$O(1)$</td>
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<td>$O(1)$</td>
<td>$O(S')$</td>
</tr>
</tbody>
</table>

- $K$: number of stops
- $S$: number of passengers
- $\epsilon$: precision
- $\beta$: parameter
- $M$: number of time slots
Conjecture

There exists an optimal solution of $P_{\text{ave}}^{\text{return}}$. 
A conjecture for $P_{ave}$ return

**Conjecture**

There exists an optimal solution of $P_{ave}$ return.

→ If such conjecture true, similar theorem than for $P_{max}$ return.
General conclusion

- Three main problems:
  - Operational scheduling problem with maximum number of shuttles.
  - Prospective scheduling and pricing problem with maximum revenue.
  - Theoretical scheduling problem with minimum waiting time.

- Various methods:
  - Mixed Integer Linear Programming.
  - Stochastic Optimization and Sample Average Approximation.
  - Lagrangian relaxation.
  - Heuristics.
  - Exact algorithms and approximation schemes.
Thank you for your attention.