Measures of biodiversity and conservation policies

On which index to base trade-offs?

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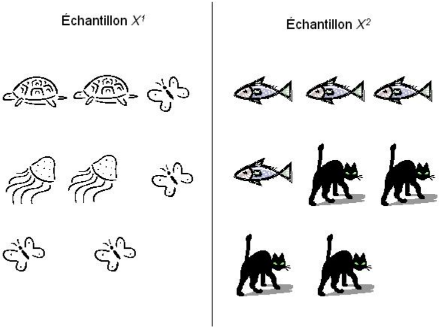
Introduction

- Partial survey on biodiversity
- Do we control the properties of its measurement instruments? –> more or less
- Are the various indices equivalent? –> no.
- Is their use easy? –> no, mainly for practical reasons.
- Take-home message : interest of the axiomatic approach. Axioms $\Leftrightarrow$ index of diversity.

Plan

Which sample to preserve?

Consider two samples: $X^1 = \{T_1, T_2, M_1, M_2, P_1, P_2, P_3, P_4\}$ and $X^2 = \{S_1, S_2, S_3, S_4, C_1, C_2, C_3, C_4\}$, one of which must be sacrificed.



Let us note

- $X^1 \succeq X^2$ for « sample X^1 is at least as diversified as sample X^2 »
- $X^1 \succ X^2$ for « sample X^1 is strictly more diversified than sample X^2 »
- $X^1 \sim X^2$ for « sample X^1 offers exactly the same biodiversity as sample X^2 »

The notion of species

Notion of *dichotomic similarity* : two individuals i and i' are either equivalent ($i \; E \; i'$), or different ($i \; \overline{E} \; i'$)
Allows to divide the set of individuals into five equivalence classes:

$$\begin{aligned} T &= \{T_1, T_2\}, \; M = \{M_1, M_2\}, \; P = \{P_1, P_2, P_3, P_4\}, \\ S &= \{S_1, S_2, S_3, S_4\}, \; C = \{C_1, C_2, C_3, C_4\}, \end{aligned}$$

and to define the partitions S^1 and S^2 de X^1 et X^2, based on similarity:

$$S^1 = \{T, M, P\} \text{ and } S^2 = \{S, C\}.$$

Notations : $S^0 = S^1 \cup S^2$, and $X^0 = X^1 \cup X^2$.

The cardinal criterion

Pattanaik and Xu (2000a) : the cardinal criterion $n^j = Card(S^j)$ is the only reflexive and transitive $\succeq$ which satisfies the following three axioms:

- A1 **Indifference between singletons**: Whatever the individuals i and $i' \in X^0$, $\{i\} \sim \{i'\}$.
- A2 **Monotony with respect to the addition of a species to a singleton** : Whatever the distinct individuals i and i' taken in X^0, $i\ E\ i' \Rightarrow \{i, i'\} \sim \{i\}$, and $i\ \overline{E}\ i' \Rightarrow \{i, i'\} \succ \{i\}$.
- A3 **Independance** : For all subsets $A, B \in S^0$, and whatever $i \in X^0 \backslash (A \cup B)$, if $[i\ E\ a$ and $i\ E\ b$ for some $a \in A$ and some $b \in B]$ or $\left[i\ \overline{E}\ z \text{ for all } z \in A \cup B\right]$, then $A \succ B$ iff $A \cup \{i\} \succ B \cup \{i\}$.

The cardinal criterion : remarks

1. The ranking is sensitive to the definition of species. Impact on the ranking if all individual are different species?
2. The ranking is not sensitive to the relative abundance of species. Impact on the ranking if 10,000 cats are added to sample 2?

Combining the notion of uniformity and richness

- let a_A^j be the absolute abundance of species A in sample j
- and let $p_A^j = \frac{a_A^j}{\sum\limits_{A \in S^j} a_i^j}$ be its relative abundance

The expression:

$$D_\alpha\left(n^j, p^j\right) = \left[\sum_{A \in S^j} \left(p_i^j\right)^\alpha\right]^{\frac{1}{1-\alpha}} .$$

gives a family of indices, configured by parameter α, whose logarithmic transformation is called *generalized entropy*, or *entropy of order* α (Rényi, 1961, Hill, 1973).

Entropy: particular cases

- the cardinal criterion: $D_0\left(n^j, p^j\right) = n^j$
- The Berger-Parker index, with $\alpha = +\infty$,

$$D_{+\infty}\left(n^j, p^j\right) = 1/\max\left(p_A^j\right).$$

According to this index, $X^1 \sim X^2$, since

$$D_{+\infty}\left(n^1, p^1\right) = D_{+\infty}\left(n^2, p^2\right) = 2.$$

Entropy indices: particular cases (continued)

- Simpson index with $\alpha = 2$,

$$D_2\left(n^j, p^j\right) = 1/\sum_{A \in S^j}\left(p_A^j\right)^2.$$

This index declares $X^1 \succ X^2$, with respective values $D_2\left(n^1, p^1\right) \simeq 2,66$ and $D_2\left(n^2, p^2\right) = 2$.

- the Shannon-Wiener measure, from information theory, with $\alpha \to 1$,

$$D_1\left(n^j, p^j\right) = \exp\left(-\sum_{A \in S^j} p_i^j \ln p_i^j\right),$$

$D_1\left(n^1, p^1\right) \simeq 2,83$. $D_1\left(n^2, p^2\right) \simeq 2$.

Entopy indices : axiomatic (Shorroks, 1984)

Any ranking of samples obtained via index D_α, avec $\alpha \neq 0$, is logically equivalent to a ranking that satisfies the 4 following axioms:

- A4 **Symmetry** : if sample $\tilde{S}$ is obtained from a permutation of species proportions in sample S, then $\tilde{S} \sim S$.
- A5 **Scale Invariance** : for any number $t > 0$, if $\tilde{S}$ is obtained form S by multiplying the population of each species by t, then $\tilde{S} \sim S$.

Entropy Indices : axiomatic continued (Shorroks, 1984)

- A6 **Abundance equalization** : Consider two samples X^1 et X^2 composed of the same species. If they are distributed in same proportions in the two samples, except for two species such that:

$$a_h^1 = a_h^2 + \delta < a_k^2 - \delta = a_k^1$$

avec $\delta > 0$, alors $X^1 \succ X^2$.

- A7 **Decomposable Representability** : there exists a numeric function $D_\alpha(.,.)$ that represents the ranking $\succeq$ and such that, for any pair of samples X^1, X^2 :

$$\begin{aligned} D_\alpha\left(n^1+n^2,(p^1,p^2)\right) &= D_\alpha\left(n^1,p^1\right)+D_\alpha\left(n^2,p^2\right) \\ &\quad +D_\alpha\left(n^1+n^2,(\bar{p}^1,\bar{p}^2)\right) \end{aligned}$$

Entropy indices : remarks

- This family abide by the axiom of indifference with respect of singletons, but do not necessarily respect the monotony axiom and the independance axiom.
- Common limitation : indices not sensible to dissimilarities that may exist among species. A sample with 4 bees and 4 ants would be judged equivalent to a sample with 4 bees and 4 lions by any of these indices.

Weitzman index (1992)

- A dissimilarity measure is a distance function $d : X^j \times X^j \rightarrow R^+$, which satisfies the two following properties:

  1. symmetry: $d(i, i') = d(i', i)\,,\ \forall i, i' \in X^j$,
  2. $d(i, i') = 0$, iff $i\ E\ i'$.

- Assume that:

  - in sample 1 : $d(T, P) = a > d(T, M) = b > d(M, P) = c$,
  - in sample 2 : $d(S, C) = d > a + c$.

- The distance between a singleton i'' and a set, here the non empty subset $\mathbf{x}^j$ of X^j, is:

$$\delta\left(i'', \mathbf{x}^j\right) = \min_{h \in \mathbf{x}^j} d\left(i'', h\right)\ , \quad \forall i'' \in X^j \backslash \mathbf{x}^j, \forall \mathbf{x}^j \sqsubseteq X^j\ .$$

Weitzman index(1992) (continued)

$$D_W\left(X^j\right) = \max_{i \in X^j}\left\{D_W\left(X^j - i\right) + \delta\left(i, X^j - \{i\}\right)\right\} \ .$$

For our two samples, under the normalization $D^0 = 0$

$$D_W\left(X^1\right) = a + c < D_W(X^2) = d \ .$$

Important observation: this index reverses the previous rankings!

Weitzman's index (1992) : properties

- A8 **Monotonicity with respect to species** If an individual i is added to a sample X^j, then

$$D_W\left(X^j \cup i\right) \geq D_W\left(X^j\right) + \delta\left(i, X^j\right).$$

- A9 **Existence of a link** For any sample X^j made of at least two species, there exists at least a species $l(X^j) \in X^j$, called "link", which satisfies

$$D_W\left(X^j\right) = D_W\left(X^j - \{l\}\right) + \delta\left(l, X^j - \{l\}\right).$$

- A10 **Monotonicity with respect to distances**

Weitzman's index (1992) : calculator

In practice, computations are very complicated and time consuming.
A "calculator" is available on the net:
http://www1.montpellier.inra.fr/lameta/biodiv/login.php

Contact me in case of need!

Rao's quadratic entropy (1982)

Makes use of information on frequencies and on dissimilarities:

$$D_R(X) = \sum_{i \in X} \sum_{j \in X} p_i * p_j * d_{ij} \ .$$

Measure of the **expected dissimiarity** between two species randomly drawn in X. No axiomatic characterization?

Rao's index (1982) (calculus)

$$
\begin{aligned}
D_R\left(X^1\right) &= 2 * \frac{1}{4} * \frac{1}{2} * d(T,P) \\
&\quad +2 * \frac{1}{4} * \frac{1}{4} * d(T,M) \\
&\quad +2 * \frac{1}{4} * \frac{1}{2} * d(M,P) \\
&= \frac{1}{4} * (a+c) + \frac{1}{8} * b \ .
\end{aligned}
$$

$$
\begin{aligned}
D_R\left(X^2\right) &= 2 * \frac{1}{2} * \frac{1}{2} * d(S,C) \\
&= \frac{1}{2} * d \ .
\end{aligned}
$$

Rao's index (1982) (calculus continued)

Therefore

$$D_R\left(X^2\right) > D_R\left(X^1\right)$$

iff

$$\frac{1}{2} * d > \frac{1}{4} * (a + c) + \frac{1}{8} * b\,.$$

Since we have assumed that $d > a + c$, this inequality is possible only if $d(T, M) = b$ is not too large.

Synthesis on indices

- Formal description $\Omega^j = \{E, n^j, p^j, d()\}$ of a sample j
- Diversity index $D : \Omega^j \to R$, synthetizes pieces of information.
- a protection policy may favor biodiversity according to a particular index, and be harmful according to another index
- Indices do not all abide by the same principles. Some are based on a fonctional point of view, other obey a patrimonial logic.
- Because of the growing concern for values such as freedom and well-being, the interest tends to shift from diversity towards ecosystems services (cf MEA).

Conservation policies: the Noah's ark metaphor

Let $P_i \in [0,1]$ stand for the survival probability of species $i = P, T$.
4 possible states of the world:

1. P and T survive, with proba $P_1 * P_2$,
2. only P survives, with proba $P_1 * (1 - P_2)$,
3. only T survives, with proba $(1 - P_1) * P_2$,
4. P and T disappear, with proba $(1 - P_1) * (1 - P_2)$.

Conservation policy : a tool that varies survival probabilities

$$\underline{P_i} \leq P_i \leq \bar{P}_i, \forall i = 1, 2, \quad (1)$$

where $\underline{P_i}$ if no protection, and $\bar{P}_i$ in case of maximal protection.

Noah's ark metaphor

C_i cost associated to $\bar{P}_i$
B total budget

$$\sum C_i \frac{P_i - \bar{P}_i}{\bar{P}_i - \underline{P_i}} = B \tag{2}$$

Noah's utility: addition of expected existence value

$$U(P_1, P_2) = P_1 U_1 + P_2 U_2,$$

and expected diversity of all the species $W(P_1, P_2)$

$$\begin{aligned} W(P_1, P_2) \quad = \quad & P_1 * P_2 * D_W(\{s_1, s_2\}) \\ & + P_1 * (1 - P_2) * D_W(\{s_1\}) \\ & + (1 - P_1) * P_2 * D_W(\{s_2\}) \\ & + (1 - P_1) * (1 - P_2) * D_W(\varnothing). \end{aligned}$$

Noah's ark metaphor

Noah's problem :

$$\max_{P_1,P_2} W(P_1,P_2) + U(P_1,P_2) \tag{3}$$

subject to (1) et (2).
Expected marginal divesity

$$\begin{aligned} \frac{\partial}{\partial P_i} W\left(P_1,P_2\right) &= P_j * \left[D_W\left(\{s_i,s_j\}\right) - D_W\left(\{s_j\}\right)\right] \\ &\quad + (1-P_j) * D_W\left(\{s_i\}\right) \;> 0 \end{aligned} \tag{4}$$

The diversity function is strictly increasing: 'corner' solutions .
Is this 'corner' property robust?...

Noah's ark metaphor

Another contribution of Weitzman : the optimal solution can be approximated by a simple rule. Let species be ranked according to the values:

$$R_i = (D_i + U_i)(\frac{\Delta P_i}{C_i}) \tag{5}$$

with

$$\Delta P_i = \bar{P}_i - \underline{P_i} \ ,$$

and species are embarked until budget is exhausted

More to read I

S. Aulong, K. Erdlenbruch et C. Figuières,
Un tour d'horizon des critères d'évaluation de la diversité biologique.
Economie Publique, 2006.

S. Aulong, K. Erdlenbruch et C. Figuières,
Les critères d'évaluation de la biodiversité : propriétés et difficultés d'usage.
Inra Sciences Sociales, 2008, n° 4-5. (See also the english version, on the same page)

More to read II

van der Heide, C. Martijn, van den Bergh, Jeroen C.J.M., van Ierland, Ekko C.,

Extending Weitzman's economic ranking of biodiversity protection: combining ecological and genetic considerations,

Ecological Economics, 2005, 55(2), 218-223.

Thanks for your attention!