Differential Games in the Economics and Management of Pollution: A Tutorial

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References

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Agenda

1. Introduction to the Domain
2. Introduction to Differential Games
3. Policy Control Instruments
4. Transbounday Pollution
5. Macroeconomics Problems (not covered)
6. Concluding Remarks

Introduction: Defining the Domain

- Environmental and resource economics are concerned with the economic aspects of the utilization of:
  - natural renewable resources (forests, fisheries),
  - natural exhaustible resources (oil, coal, minerals), and
  - environmental resources (soil, water, air).
- Focus on *pollution*, a major environmental issue.
- Pollution is a by-product of extraction of resources, production, heating, transportation, etc.
- Abatement of pollution requires equipment and money.
- Key words: externality, ownership (privately owned resource vs. open access).

Introduction: Three Important Characteristics

Interdependence: Actions of an economic agent affect the welfare (payoff, utility) of the agent, and the welfare of other agents.

1. International transbounday pollution, downstream pollution, markets for tradeable pollution permits.
2. Environmental interdependence is related to environmental externalities.

Time. Environmental problems are intrinsically dynamic...

1. Consumption patterns, habits, technologies, etc. cannot be changed overnight.
2. Damage is often caused by accumulation of pollution (and by the flow).

Introduction: Three Important Characteristics

Strategic and *forward-looking behavior* on the part of the agents (firms, communities, nations) who take actions that affect the environment.

1. Different agents, different objectives, different course of actions.
2. Agents act strategically and take into account the present and future consequences of their own actions and those of other agents.

Introduction: What Does it Take?

- Dynamic (state-space) games have been of considerable value to represent **time**, **strategic behavior** and **interdependencies.**
- State variables:
  - describe the main features of a dynamic system at any instant of time,
  - summarize all relevant consequences of the past history of the game.
- Time can be continuous or discrete.
- Opportunity to account for:
  - flow pollution damage effects (as in static models), and
  - stock pollution damage effects.

- Differential games are offsprings of game theory and optimal control.
- Initiated by R. Isaacs at the Rand Corporation in the late 1950s and early 1960s.
- Initial focal points: military applications and zero-sum games.
- Now, applications are found in many areas, e.g., in management science (operations management, marketing, finance), economics (industrial organization, macro, resource, environmental economics, etc.), biology, ecology, military, etc.
- Textbooks: Başar and Olsder (1982, 1995), Petrosjan (1993), Dockner et al. (2000), Jørgensen and Zaccour (2004), Engwerda (2005), Yeung and Petrosjan (2005), Haurie, Krawczyk and Zaccour (2012).

Elements of a differential game

A deterministic differential game (DG) played on a time interval $[t^0, T]$ involves the following elements:

- A set of players $M = \{1, \ldots, m\}$;
- For each player $j \in M$, a vector of controls $\mathbf{u}_j(t) \in U_j \subseteq \mathbb{R}^{m_j}$, where U_j is the set of admissible control values for Player j;
- A vector of state variables $\mathbf{x}(t) \in X \subseteq \mathbb{R}^n$, where X is the set of admissible states. The evolution of the state variables is governed by a system of differential equations, called the state equations:

$$\dot{\mathbf{x}}(t) = \frac{d\mathbf{x}}{dt}(t) = f\left(\mathbf{x}(t), \underline{\mathbf{u}}(t), t\right), \quad \mathbf{x}(t^0) = \mathbf{x}^0, \tag{1}$$

where $\underline{\mathbf{u}}(t) \triangleq (\mathbf{u}_1(t), \ldots, \mathbf{u}_m(t))$;

Elements of a differential game (cont'd)

- A payoff for Player $j, j \in M$,

$$J_j \triangleq \int_{t^0}^{T} g_j(\mathbf{x}(t), \underline{\mathbf{u}}(t), t)\, dt + S_j(\mathbf{x}(T)) \tag{2}$$

  where function g_j is Player j's instantaneous payoff and function S_j is his terminal payoff;
- An information structure, i.e., information available to Player j when he selects $\mathbf{u}_j(t)$ at t;
- A strategy set Γ_j, where a strategy $\gamma_j \in \Gamma_j$ is a decision rule that defines the control $\mathbf{u}_j(t) \in U_j$ as a function of the information available at time t.

Elements of a differential game (cont'd)

Assumption: All feasible state trajectories remain in the interior of the set of admissible states X.

Assumption: Functions f and g are continuously differentiable in $\mathbf{x}, \underline{\mathbf{u}}$ and t. The S_j functions are continuously differentiable in $\mathbf{x}$.

Control set: $\mathbf{u}_j(t) \in U_j$, with U_j set of admissible controls (or control set).

- Control set could be:
  - Time-invariant and independent of the state;
  - Depend on the *position of the game* $(t, \mathbf{x}(t))$, i.e., $\mathbf{u}_j(t) \in U_j(t, \mathbf{x}(t)))$.
  - Depend also on controls of other players (coupled constraints).

Elements of a differential game (cont'd)

Information structure:

Open loop: players base their decision only on time and an initial condition;

Feedback or Markovian: players use the *position of the game* $(t, \mathbf{x}(t))$ as information basis;

Non-Markovian: players use history when choosing their strategies.

Elements of a differential game (cont'd)

Strategies:

Open-loop strategy: selects the control action according to a decision rule μ_j, which is a function of the initial state $\mathbf{x}^0$:
$\mathbf{u}_j(t) = \mu_j(\mathbf{x}^0, t)$.

As $\mathbf{x}^0$ is fixed, no need to distinguish between $\mathbf{u}_j(t)$ and $\mu_j(\mathbf{x}^0, t)$. Player commits to a fixed time path for his control.

Markovian strategy: selects the control action according to a feedback rule $\mathbf{u}_j(t) = \sigma_j\left(t, \mathbf{x}(t)\right)$. Player j's reaction to any position of the system is predetermined.

- The decision rule σ_j can be, e.g., linear or quadratic function of $\mathbf{x}$ with coefficients depending on t.
- It also can be a nonsmooth function of x and t (e.g., bang-bang controls). Complicated problem....

Elements of a differential game (cont'd)

State equations (*system dynamics*, *evolution equations* or *equations of motion):*

$$\dot{\mathbf{x}}(t) = \frac{d\mathbf{x}}{dt}(t) = f\left(\mathbf{x}(t), \underline{\mathbf{u}}(t), t\right), \quad \mathbf{x}(t^0) = \mathbf{x}^0,$$

- State vector's rate of change depends on $t, \mathbf{x}(t)$ and $\underline{\mathbf{u}}(t)$.
- OL strategies are piecewise continuous in time. A unique trajectory will be generated from $\mathbf{x}^0$.
- For feedback strategies, we make the following simplifying assumption:

Assumption For every admissible strategy vector $\underline{\sigma} = (\sigma_j : j \in M)$, the DE $\dot{\mathbf{x}}(t)$ admit a unique solution, i.e., a unique state trajectory, which is an absolutely continuous function of t.

Assumption met when: (i) $f(\mathbf{x}(t), \underline{\mathbf{u}}(t))$ is continuous in t for each x and $\mathbf{u}_j, j \in M$; (ii) $f\left(\mathbf{x}(t), \mathbf{u}(t), t\right)$ is uniformly Lipschitz in x, $\mathbf{u}_1, \ldots, \mathbf{u}_m$; and (iii) $\sigma_j(t, \mathbf{x})$ is continuous in t for each $\mathbf{x}$ and uniformly Lipschitz in $\mathbf{x}$.

Time horizon:

- T can be finite or infinite;
- T can be prespecified or endogenous (as in, e.g., pursuit-evasion games and patent-race games).

Nash Equilibrium: The definition

- Normal form representation: Set of players' admissible strategies; payoffs expressed as functions of strategies rather than actions.
- Assume that Player $j, j \in M$, maximizes a stream of discounted gains, that is,

$$J_j \triangleq \int_{t^0}^{T} e^{-\rho_j t} g_j(\mathbf{x}(t), \underline{\mathbf{u}}(t), t)\, dt + e^{-\rho_j T} S_j(\mathbf{x}(T)), \tag{3}$$

where ρ_j is the discount rate satisfying $\rho_j \geq 0$.

Nash Equilibrium: The definition (cont'd)

Open-loop Nash equilibrium.
The payoff functions with the state equations and initial data $(t^0, \mathbf{x}^0)$ define the normal form of an OL differential game:

$$\underline{\mathbf{u}}(\cdot) = (\mathbf{u}_1(\cdot), \ldots, \mathbf{u}_j(\cdot), \ldots, \mathbf{u}_m(\cdot)) \mapsto J_j(t^0, \mathbf{x}^0; \underline{\mathbf{u}}(\cdot)), \quad j \in M. \tag{4}$$

Definition

The control m-tuple $\underline{\mathbf{u}}^*(\cdot) = (\mathbf{u}_1^*(\cdot), \ldots, \mathbf{u}_m^*(\cdot))$ is an **open-loop Nash equilibrium** (OLNE) at $(t^0, \mathbf{x}^0)$ if the following holds:

$$J_j(t^0, \mathbf{x}^0; \underline{\mathbf{u}}^*(\cdot)) \geq J_j(t^0, \mathbf{x}^0; [\mathbf{u}_j(\cdot), \underline{\mathbf{u}}^*_{-j}(\cdot)]), \quad \forall \mathbf{u}_j(\cdot), j \in M,$$

where $\mathbf{u}_j(\cdot)$ is any admissible control of Player j and $[\mathbf{u}_j(\cdot), \underline{\mathbf{u}}^*_{-j}(\cdot)]$ is the m-vector of controls obtained by replacing the j-th component in $\underline{\mathbf{u}}^*(\cdot)$ by $\mathbf{u}_j(\cdot)$.

Nash Equilibrium: The definition (cont'd)

Open-loop Nash equilibrium.
Player j solves the optimal-control problem

$$\max_{\mathbf{u}_j(\cdot)} \left\{ \int_{t^0}^{T} e^{-\rho_j t} g_j \left(\mathbf{x}(t), [\mathbf{u}_j(t), \underline{\mathbf{u}}^*_{-j}(t)], t \right) dt + e^{-\rho_j T} S_j(\mathbf{x}(T)) \right\},$$

subject to the state equations

$$\dot{\mathbf{x}}(t) = \frac{d\mathbf{x}}{dt}(t) = f\left(\mathbf{x}(t), [\mathbf{u}_j(t), \underline{\mathbf{u}}^*_{-j}(t)], t \right), \quad \mathbf{x}(t^0) = \mathbf{x}^0. \qquad (5)$$

Nash Equilibrium: The definition (cont'd)

Markovian (feedback)-Nash equilibrium.
Players use feedback strategies $\underline{\sigma}(t, \mathbf{x}) = (\sigma_j(t, \mathbf{x}) : j \in M)$. The normal form of the game, at $(t^0, \mathbf{x}^0)$ is defined by

$$\begin{aligned} J_j(t^0, \mathbf{x}^0; \underline{\sigma}) &= \int_{t^0}^{T} e^{-\rho_j t} g_j\left(\underline{\sigma}(t, \mathbf{x}), t\right) dt + e^{-\rho_j T} S_j(\mathbf{x}(T)), \\ \dot{\mathbf{x}}(t) &= \frac{d\mathbf{x}}{dt}(t) = f\left(\mathbf{x}(t), \underline{\sigma}(t, \mathbf{x}(t)), t\right), \quad \mathbf{x}(t^0) = \mathbf{x}^0. \end{aligned}$$

Define the $(m-1)-$vector

$$\sigma_{-j}(t, \mathbf{x}(t)) \triangleq (\sigma_1(t, \mathbf{x}(t)), \ldots, \sigma_{j-1}(t, \mathbf{x}(t)), \sigma_{j+1}(t, \mathbf{x}(t)), \ldots, \sigma_m(t, \mathbf{x}(t)))$$

Nash Equilibrium: The definition (cont'd)

Markovian (feedback)-Nash equilibrium.

Definition

The feedback m-tuple $\underline{\sigma}^*(\cdot) = (\sigma_1^*(\cdot), \ldots, \sigma_m^*(\cdot))$ is a **feedback or Markovian-Nash equilibrium** (MNE) on $[0, T] \times X$ if for each $(t^0, \mathbf{x}^0)$ in $[0, T] \times X$, the following holds:

$$J_j(t^0, \mathbf{x}^0; \underline{\sigma}^*(\cdot)) \geq J_j(t^0, \mathbf{x}^0; [\sigma_j(\cdot), \boldsymbol{\sigma}^*_{-j}(\cdot)];), \quad \forall \sigma_j(\cdot), j \in M,$$

where $\sigma_j(\cdot)$ is any admissible feedback law for Player j and $[\sigma_j(\cdot), \boldsymbol{\sigma}^*_{-j}(\cdot)]$ is the m-vector of controls obtained by replacing the j-th component in $\boldsymbol{\sigma}^*(\cdot)$ by $\sigma_j(\cdot)$.

Markovian (feedback)-Nash equilibrium.
In other words, $\mathbf{u}_j^*(t) \equiv \sigma_j^*(t, \mathbf{x}^*(t))$, where $\mathbf{x}^*(\cdot)$ is generated by $\underline{\sigma}^*$ from $(t^0, \mathbf{x}^0)$, solves the optimal-control problem

$$
\begin{aligned}
& \max_{\mathbf{u}_j(\cdot)} \left\{ \int_{t^0}^{T} e^{-\rho_j t} g_j \left(\mathbf{x}(t), \left[\mathbf{u}_j(t), \sigma_{-j}^*(t, \mathbf{x}(t)) \right], t \right) dt \right. \\
& \left. + e^{-\rho_j T} S_j(\mathbf{x}(T)), \right\} \\
\text{s.t.} \quad \dot{\mathbf{x}}(t) &= f\left(\mathbf{x}(t), \left[\mathbf{u}_j(t), \sigma_{-j}^*(t, \mathbf{x}(t)) \right], t\right), \ \mathbf{x}(t^0) = \mathbf{x}^0.
\end{aligned}
$$

Pollution Control Instruments

Two broad groups.

- *Command-and-control* instruments, e.g.,prohibition of specific inputs, processes, and technologies, discharge permits, emission standards and quotas, technological specifications for the handling of pollutants.
- *Market-based instruments,* e.g., tradeable emission permits, emission charges and other tax schemes, subsidies, and liability law provisions.

New direction?

- *Environmental information disclosure to the public.*

Pollution Control Instruments: A Map of Contributions

- Taxes
  - Optimal Intertemporal Taxation Schemes
  - Nonpoint Source Pollution
- Standards and Taxes
- Subsidies
- Tradeable Emission Permits
  - Other Kyoto Protocol Instruments (joint implementation)
- Assessment of Policy Instrument Effects

Pollution Control Instruments: Taxation Example

Example

Benchekroun and Long (1998).
n identical firms producing a homogeneous good
Constant unit cost of production, c,
Production rate =emissions:$q_i(t) = e_i(t)$, industry output
$Q(t) = \sum_{i=1}^{n} q_i(t)$
Market price $P(t) = P(Q(t))$ such that $P'(Q) < 0$, $P(0) > c$.
Stock of pollution $S(t)$. Dynamics

$$\frac{dS}{dt}(t) = Q(t) - \delta S(t),$$

Pollution Control Instruments: Taxation Example

Example

Firms are free to choose their output rates, but they must pay taxes

$$T_i = T(q_i(t), S(t)).$$

Function T is the same for all firms: Equal firms are treated equally.
$T(\cdot,\cdot)$ could, for instance, be linear in the production rate: $T_i = f(S)q_i$.
The profit function of firm i is its long-run, discounted profit:

$$\pi_i = \int_0^\infty e^{-rt}\left\{P(Q(t)) - c - f(S(t))\right\} q_i(t)dt.$$

Each firm knows that its current production will add to the future stock of pollution and thus affect its future tax payments.
At time $t = 0$ the government announces the per unit tax rule $f(S(\cdot))$
Design a tax that induces firms to choose production paths that are socially optimal.

Optimal Intertemporal Taxation Schemes

Focus on optimal taxation schemes, i.e., find intertemporal pollution taxation schemes that sustain Pareto efficient outcomes.

- Benchekroun and Long (1998):
  - Infinite horizon differential game
  - Time-independent output tax rule such that firms will choose socially optimal production and pollution paths.
  - Firms play a noncooperative differential game with open-loop or feedback strategies.
  - Which game is played affects the choice of parameters in the tax rule.
  - In any case there exists a time-independent tax (per unit of output) rate that depends on the current level of the pollution stock.

Optimal Intertemporal Taxation Schemes

- Hoel (1992, 1993):
  - Finite horizon difference game.
  - Neglecting taxes, the social optimum is determined and a noncooperative game is played with open-loop and feedback strategies.
  - A time-dependent emission tax is introduced, being the same for all countries.
  - Tax providing the Pareto optimum is the same for feedback and open-loop equilibria.
- Karp and Livernois (1994):
  - Level of an emissions tax that is needed to achieve a *target level* of pollution.
  - Linear mechanism that adjusts the tax when aggregate emissions deviate from the target level.

Optimal Intertemporal Taxation Schemes

- A fair part of the literature on the taxation of pollution is concerned with *carbon taxes*.
- Dynamic bilateral interaction between producers and consumers, producers and government, a *resource-exporting cartel* (typically: OPEC) and a *coalition of resource-importing countries, etc.*
- Examples: Martin et al. (1993), Wirl (1994a, 1994b, 1994c, 1995, 2007b), Wirl and Dockner (1995), Tahvonen (1996), Rubio and Escriche (2001), Liski and Tahvonen (2004).

Nonpoint Source Pollution

- Problems and Issues
  - Source and volume of individual emissions are not observable by regulators.
  - Regulator can measure the ambient pollution at specific points
  - Problems of monitoring and measurement due to informational asymmetries between dischargers and regulators
  - Standard regulatory instruments are useless as incentives to make dischargers adopt socially preferable policies.
- (Short) survey in Xepapadeas (2002)

Nonpoint Source Pollution

- Challenge: Find policy instruments that may be appropriate for nonpoint source pollution problems.
- Two broad categories of instruments:
  - *Tax schemes* based on observed ambient pollution
  - *input-based schemes* that tax observable, polluting inputs.
- Shortle et al. (1998)
  - Research issues that arise in the analysis of input taxes and ambient taxes.
  - Theoretical and empirical issues in the choice between the two tax approaches (in static setting).

Nonpoint Source Pollution

- Xepapadeas (1992):
  - A tax per unit of deviation between the socially desirable and the observed ambient pollution levels.
  - Pollution level converges to the socially desirable steady state.
  - The size of the tax depends on the strategic behavior of the firms (higher for feedback strategies)
  - Data requirements could be formidable.
- Karp (2005):
  - An *ambient tax:* unit tax based on the *aggregate* level of pollution
  - An ambient tax can sustain a social optimum, given that firms recognize that their decisions affect the aggregate emission level.
  - A surprise: even if the regulator has perfect information about firms' emissions, they might get a higher payoff if the regulator acted as if it were unable to observe individual emissions!

Standards and Taxes

- Pollution taxes (*incentives*) vs. standards (*mandatory* requirements)
- Helfand (2002) provides a summary of factors affecting the choice of a tax versus a standard.
- Dynamic game: focus on productive and/or abatement *capital accumulation.*
- Question is whether taxes will lead to more investment than standards?

Standards and Taxes

Example

Feenstra, Kort, de Zeeuw (2001). Two firms. Environmental policy: Tax or emission standard. Output: $q_i = q_i(e_i(t), K_i(t))$. Revenue and dynamics:

$$R_i = R_i \left[q_i(e_i, K_i), q_j(e_j, K_j) \right], \quad \frac{dK_i}{dt}(t) = I_i(t) - a_i K_i(t),$$

Investment convex costs $C(I_i)$. Price of polluting input $p(t)$. Profit is

$$\pi_i = \int_0^\infty e^{-rt} \left\{ R_i - p(t) e_i(t) - C(I_i(t) \right\} dt.$$

Tax: $\tau^i(t)$ or emission standard constraint: $e_i(t) \leq \bar{e}_i(t)$

Open loop: Taxes lead to more investment (Ulph (1992), Feenstra et al. (1996)). Feedback: No clear-cut result (Feenstra et al. (2001)).

Subsidies

- Subsidies are offered as incentives to take environmentally favorable actions
- R&D investments to develop cleaner production technologies (e.g., Katsoulacos and Xepapadeas (1996))
- Polluting firms to adopt existing, cleaner technologies (e.g.,Krawczyk and Zaccour (1996, 1999))
- Countries or regions to reduce the rate of deforestation of their land (Fredj, Martín-Herrán, Zaccour (2004, 2006))

Subsidies

- Fredj, Martín-Herrán, Zaccour (2004, 2006):
  - Two-player differential game: *North* and *South*
  - South faces a trade-off between the exploitation of its forests (timber production) and agricultural activities.
  - *North* max tropical forest, *South* its welfare
  - Benchmark: *South* ignores *North* and solves a dynamic optimization problem.
  - Strategic scenario: *North* is leader who offers *South* a subsidy if it reduces deforestation rate.
  - Fredj, Martín-Herrán, Zaccour (2004): A subsidy that depends on the deforestation rate
  - If *North* has a sufficient budget, it can slow down deforestation
  - Fredj, Martín-Herrán, Zaccour (2006):An incentive such that it is in the best interest of the *South* to follow in the short run the sustainable exploitation path of its forest

Transboundary Pollution Problems

- More than one independent jurisdiction.
- TPP includes:
  - Unidirectional or downstream pollution problems
  - International and global pollution problems
- Basic element of TPP: absence of a transnational institution that can impose an environmental policy.

Transbounday Pollution Problems

The papers dealing with TPP can be regrouped under three headings:

1. *Noncooperative and cooperative solutions.* Determination and comparison of the two solutions.
2. *International environmental agreements (IEA).* Determination of stable environmental agreements (noncooperative and cooperative game approaches).
3. *Empirical dynamic games of TPP.* Model's parameters estimated with real (or realistic) data.

TPP: Noncooperative and Cooperative Solutions

Example

Van der Ploeg and de Zeeuw (1992).
N identical countries. $Y_i(t)$ is the production that generates pollution which accumulates over time according to the dynamics:

$$\frac{dS}{dt}(t) = \frac{\alpha}{N}\sum_{i=1}^{N} Y_i(t) - \delta S(t), \qquad S(0) = S_0,$$

Country i derives profits from production, measured by a concave function $B(Y_i)$, and incurs a damage cost $D(S)$ due to pollution; $D'(S) > 0$, $D''(S) \geq 0$. Player i (i.e., the government of country i) maximizes the welfare function

$$W_i = \int_0^\infty e^{-rt}\left(B(Y_i(t)) - D(S(t))\right) dt.$$

TPP: Noncooperative and Cooperative Solutions

- Two scenarios in van der Ploeg and de Zeeuw (1992)
- International coordination of emissions, i.e., max joint payoff $\sum_{i=1}^{N} W_i$.
- Noncooperative game (open-loop and feedback Nash equilibria).
- Major (by now classical) conclusions:
  - Production and emission levels under international coordination are lower than in the noncooperative case
  - Open-loop equilibrium leads to lower production and emissions levels than in the feedback equilibrium.

TPP: Noncooperative and Cooperative Solutions

- Long (1992): two-player transboundary differential game.
  - Each player aims at maximizing a stream of welfare which depends positively on consumption and negatively on the stock of pollution.
  - Three solutions: joint max of welfare, OLNE and OLSE.
  - Interesting result: Stackelberg Steady state is higher than Nash counterpart.
- Dockner and Long (1993) reconsider the model of Long (1992) and analyze feedback Nash equilibria along with the cooperative solution.
  - Linear-quadratic DG, linear strategies, ...
  - **Scoop:** Existence of nonlinear feedback strategies and show that if the players have a low discount rate, then the cooperative solution can be sustained as an equilibrium outcome with these non-linear strategies.

TPP: Noncooperative and Cooperative Solutions

- Rubio and Casino (2002) show that for the Dockner and Long result to hold, it is necessary that the initial value of the stock of pollution be higher than the cooperative solution stock.
- Wirl (2007a, 2008): Multiple equilibria and uncertainty. Increasing uncertainty reduces pollution and irreversibility of pollution accentuates this reduction.
- Xepapadeas (1995): *Technical progress* in production processes. Characterization of different equilibria and discussion of how R&D subsidies and emissions taxes can lead to the implementation of a first-best outcome.
- Yanase (2005): Each player's objective function depends on **all** players' emissions.
- Literature is dominated by LQDG. Kossioris et al. (2008) is an exception. Numerical method to derive non-linear feedback Nash equilibria. Application to a shallow lake pollution game.

TPP: Noncooperative Games with Coupling Constraints

- Haurie and Zaccour (1995):
  - n players who face a *common constraint*
  - Coordination by taxes or emissions permits
  - Rosen (1965) was the first to deal with the problem of determining coupled-constraint Nash equilibria.
  - Constraint: $(e_1, \ldots, e_n) \in E \subset E_1 \times \ldots \times E_n$. Example: $\sum_{i=1}^{n} e_i \leq \bar{e}$
  - $\pi_i (e_1, \ldots, e_n)$ be the payoff of player i. A coupled equilibrium is defined as an n-tuple $(e_1^*, \ldots, e_n^*) \in E$ such that

$$\pi_i (e_1^*, \ldots, e_n^*) \geq \pi_i (e_1^*, \ldots, e_i, \ldots, e_n^*) \quad \forall e_i \in E_i$$
$$(e_1^*, \ldots e_i, \ldots e_n^*) \in E, \quad i \in \{1, \ldots, n\}.$$

  - If E is defined by a set of inequality constraints, there is for each player i a Karush-Kuhn-Tucker multiplier λ^i associated with the constraint.

TPP: Noncooperative Games with Coupling Constraints

- Rosen used the term "normalized equilibrium" for a coupled equilibrium in which multipliers satisfy

$$\lambda^i = \frac{\lambda^0}{r_i}, i = 1, \ldots, n,$$

  where $\lambda^0 \geq 0$ is a given vector and $r_i > 0, i = 1, \ldots, n$, are given weights.
- Under a concavity condition, Rosen showed that there exists a unique normalized equilibrium associated with each positive weighting vector $r = (r_1, ..., r_n)$.
- In the simple context of a coupling constraint $\sum_{i=1}^{n} e_i \leq \bar{e}$, the vector r is an indication of how the regulator has distributed the burden of satisfying the constraints (e.g., the taxation of emissions) among the players.

TPP: Noncooperative Games with Coupling Constraints

- Examples of applications in dynamic games: regional issues (Haurie and Krawczyk (1997),Krawczyk (2005)), post-Kyoto agreement (Drouet et al. (2008)), greenhouse gas emissions abatement (Bahn and Haurie (2008)), green certificates in electricity (Nasiri and Zaccour (2010)).
- Computation of normalized equilibrium: Krawczyk (2007), Krawczyk and Uryasev (2000).
- Back to Rosen' s multipliers:

$$\lambda^i = \frac{\lambda^0}{r_i}, i = 1, \ldots, n.$$

  - Tidball and Zaccour (2005) show, in a static setting, that a Pareto solution can be attained by a suitable choice of these weights.
  - Tidball and Zaccour (2009) show that the result does not carry over to a dynamic game.

International Environmental Agreements (IEA)

The literature has adopted one of two approaches:

1. **A noncooperative game:**
   - IEA is inherently voluntary. No transnational institution that can enforce such agreements
   - Find mechanisms that, when implemented, will lead to the largest possible stable coalition
2. **A cooperative game:**
   - Coordination of emissions of all countries leads to the best environmental and economical outcomes
   - Find a suitable allocation the joint burden (intertemporal stability,fairness, etc.).

IEA: Noncooperative Game Approach

- Starting point: d'Aspremont et al. (1983) stability of a cartel:
    - *Internal stability*:no member has an interest in leaving the agreement;
    - *External stability*: no non-member wishes to join the agreement.
- Two-stage game formalism:
    - First stage countries decide to join the IEA or not (membership game).
    - Second stage countries decide their emissions.
- General result: IEA is hard to achieve (very small coalition).
- This theoretical result contrasts with the high level of participation observed in some real-life agreements, e.g., the Montreal Protocol.

IEA: Noncooperative Game Approach

- To explain this discrepancy, a series of different elements have been incorporated in models:
  - Stackelberg leadership (Barrett (1994), Diamantoudi and Sartzetakis (2006), Rubio and Ulph (2006))
  - Transfers (Carraro and Siniscalco (1993), Hoel and Schneider (1997))
  - Reputation effects (Hoel and Schneider (1997), Jeppesen and Andersen (1998), Cabon-Dhersin and Ramani (2006)
  - Issue linkages (Botteon and Carraro (1998), Le Breton and Soubeyran (1997), Barrett (1997), Katsoulacos (1997), Carraro and Siniscalco (1997), Mohr and Thomas (1998), Kempfert (2005))
  - Punishments (Barrett (1997, 2003))
  - Regional agreements rather than global agreements (Asheim et al. (2006))
  - Other references: Carraro and Siniscalco (1998), Endres (2004), Finus (2001, 2008), and Wagner (2001).

IEA: Noncooperative Game Approach

- Breton, Sbragia, Zaccour (2010) criticized the static literature on two grounds:
  - Transboundary environmental damage is mainly related to the accumulation of pollution, rather than the flow of emissions.
  - No room for countries to reconsider their participation in an IEA .
- Rubio and Casino (2005):
  - Dynamic game. Decision whether to join the agreement is taken once and for all and signatories and non-signatories select their emission strategies in an infinite-horizon differential game.
  - Numerical results show that the stable size of an agreement is two.
  - When requiring a minimum number of signatories for the agreement to be in force, a stable agreement has precisely the same size as in the minimum clause.

IEA: Noncooperative Game Approach

- Rubio and Ulph (2007):
  - Correct for the once-for-all membership decision.
  - Discrete-time dynamic game setting, n symmetric players in each period solve an emission and membership games.
  - Due to symmetry, only the number (and not identity of) signatories is determined.
  - Signatories are randomly selected such that the stability concept is equality of welfare of signatories and non-signatories.
  - There exists a unique steady state of the pollution stock and a corresponding steady state size of a stable IEA.
  - Number of signatories in a stable agreement is a non-increasing function of the pollution stock.

IEA: Noncooperative Game Approach

- De Zeeuw (2008):
  - Extension of a *farsighted stable agreement* to a dynamic setup.
  - Impact of joining or leaving an agreement on other players.
  - Static game literature: farsightedness may lead to both small and large stable coalitions.
  - This result extends to a dynamic game only if the costs of emissions are sufficiently small compared to abatement costs.

IEA: Noncooperative Game Approach

- Breton, Sbragia, Zaccour (2010):
  - Symmetric discrete-time game and adopt a *replicator dynamics.*
  - The group that achieves a better result is joined by a fraction of new players.
  - The adjustment speed reflects the “psychological” or “physical” cost of changing behavior.
  - Process ends in an IEA which is stable over time and at the steady-state pollution stock.
  - The evolution of players’ welfare over time depends not only on the dynamics of emissions and pollution, but also on the evolution of the composition of the different groups.
- Follow up paper: Bahn, Breton, Sbragia, Zaccour (2009):
  - Calibration of the model’s parameters using the MERGE climate policy assessment model and provide numerical illustrations.

IEA: Cooperative Game Approach

Set I of n players; $K \subseteq I$ a coalition; $v(K) : \mathcal{P}(I) \rightarrow \Re$

ALGORITHM

1. Compute $v(I)$: joint maximization, international agreement
2. Compute all values of $v(K)$ (assumption on behavior of $I \backslash K$)
3. Adopt a solution concept: Core, Shapley value, Nucleolus, etc.
4. Deal with sustainability of agreement over time

Sustainability Issue:

- Cooperative Equilibrium Approach.
  - The idea is make a cooperative solution *an equilibrium* of an associated noncooperative game (*self-enforcing*).
  - *Trigger strategies*. A trigger strategy conditions a player's action on the history of actions and letting players use such strategies, a cooperative solution can be made an equilibrium (Applications in DG include Dockner et al. (2000). Cesar (1994), Tolwinski et al. (1986), Haurie and Pohjola (1987), Kaitala and Pohjola (1988), Haurie et al. (1993)).
  - *Incentive strategies.* An incentive strategy of a player conditions the player's action on the action of the other player (Ehtamo and Hämäläinen (1986, 1989, 1993), Jørgensen and Zaccour (2001b), Breton et al. (2008), Martín-Herrán and Zaccour (2005, 2009))

IEA: Cooperative Game Approach

- Dynamic Rationality.
  - Cooperative solution is *time consistent* if at no instant of time, no player or group of players wishes, along the cooperative state trajectory, to defect on the agreement. (Petrosjan (1993, 1997), Petrosjan and Zakharov (1997), Jørgensen and Zaccour (2002), Jørgensen et al. (2003, 2005), Yeung (2007), Yeung and Petrosjan (2006, 2008), Zaccour (2008)).
  - Cooperative solution is *agreeable* if at no instant of time, no player or group of players wishes, along any state trajectory, to defect on the agreement. (Kaitala and Pohjola (1990)).

IEA: Cooperative Game Approach

- Time consistency or agreeability can be achieved by using *side payments*.
  - Jørgensen and Zaccour (2001a) decompose over time of the total side payment as determined by the Nash bargaining solution to sustain cooperation in a downstream pollution game.
  - Germain et al. (2003), Jørgensen (2009) use the core solution and apply a particular side payment scheme to ensure individual and coalitional rationality throughout the game.
  - Petrosjan and Zaccour (2003) decompose over time the total side payment as determined by Shapley value to sustain cooperation among n players.

Concluding Remarks: Representation of Interactions

- Trade: Taking trade relationships into account is necessary in a meaningful analysis of questions such as the economic and environmental sustainability of growth in an increasingly globalized world.
- Strategic interactions: Analysis of open-loop and feedback strategies. Open-loop strategies have been considered "less satisfying" but:
  - Open-loop strategies represent behavior of agents who can and will precommit to their future actions, or agents who are unable to observe the evolution of pollution stocks in real time.
  - Open loop leads to lower pollution....
- Intergenerational interactions: The issue of sustainable development is intimately related to interactions between different generations of players.

Concluding Remarks: Extension of Models

Sustainable Agreements. Research is needed to better understand the formation of large and dynamically stable IEAs, taking into consideration factors such as heterogeneity of players, uncertainty in climate systems, linkage of multiple negotiation themes (environment, trade, R&D transfers) and the presence of regional agreements.

Population Growth. The two main components of population growth, net natural variations and migration are ignored in most of the economic-environmental models formulated as dynamic games.

R&D and Technological Progress. A more detailed description of the effects of investments, knowledge flows and learning-by-doing effects on the dynamics of the knowledge stock should provide better insights into the effects of climate policy and the conditions for the stability of IEAs on climate change.

Concluding Remarks: Model Components

The assumptions made in dynamic game models of pollution are often remarkably simple. They reflect the modeler's choice which could be based on rather diverse considerations.

- Simple description of pollution: flows and stocks and their dynamics are often highly stylized, e.g.,
  - a single stock of pollutant,
  - pollution dynamics are time invariant,
  - impacts of emissions are homogenous over time,
  - pollution dynamics are deterministic,
  - decay of pollution stocks assumed constant.

- Simple description of decision makers:
  - North vs. South,
  - government vs. industrial sector,
  - upstream country vs. downstream country,
  - one social planner,
  - decision makers are homogenous with respect to discount rates, cost functions, damage functions, and so forth.
- Benchmark: Social planner...Joint optimization (weights?).
- Time horizon routinely chosen as infinity:
  - Decision makers may have different time horizons.
  - Intergenerational effects: need for overlapping generations models.
- Open-loop and feedback strategies
  - Other sophisticated history-dependent ones are also relevant.

Concluding Remarks: Larger Variety of Tools

Differential games literature has predominantly used an analytical approach.
S. Jørgensen, G. Zaccour, "Developments in Differential Game Theory and Numerical Methods: Economic and Management Applications", *Computational Management Science*, Vol. 4, 2, 159-182, 2007.

Piecewise Deterministic Systems

H^∞ - Optimal Control Theory.

Impulsively Controlled Systems.

Stochastic Hybrid Models.

Nonlinear Dynamics.

Viability (Sustainability) + Strategic Interactions