Optimization, sustainable indicators and application to the design of a recovery program for overexploited fish species

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Wednesday 6, February 14h00-17h00

1 Introduction

The objective of this lecture is to show how a certain modeling approach is used for dealing with problems related to the management of natural resources with focus in two particular questions: how to compute trade-off between different kinds of objectives (environmental and production)?; and how to define a *good state* (of the exploited resource)?. Concerning the second question, we propose a method for design a recovering (or restoring) strategy (in order to attain a good state). Some examples of Chilean fisheries will be presented.

Even if it is not mentioned explicitely, the concepts behind this lecture are borrowed from the *viability theory* ([1, 3, 6, 8, 4, 9, 5, 12]).

The purpose of this document, is just to show the framework and the kind of problems that we will study during the lecture.

2 Preliminaries

The general framework will be controlled dynamical systems in discrete time (see [3]):

$$\begin{cases} x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 \ \text{ given} \end{cases} \tag{1}$$

where $x(t) \in \mathbb{X}$ is the state variable, $u(t) \in \mathbb{U}$ the decision (action or control) and $g : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{X}$ is the dynamics.

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Constraints on the trajectory. Often, it is desirable that trajectories, of states and decisions, satisfy some constraints expressed in terms of inequalities. In a general framework, we write

$$(x(t), u(t)) \in \mathbb{D}_\theta := \{(x, u) \in \mathbb{X} \times \mathbb{U} \;:\; I_i(x, u) \geq \theta_i, \quad i = 1, \ldots, n\},$$

where, for $i = 1, \ldots, n$, the functions $I_i : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$ represent indicators and $\theta = (\theta_1, \ldots, \theta_n)$ is the vector where the i-component is the associated threshold to the constraint I_i .

Thus, the controlled dynamical systems with constraints that we will study is

$$\begin{cases} x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0 + 1, \ldots \\ x(t_0) = x_0 \;\; \text{given} \\ (x(t), u(t)) \in \mathbb{D}_\theta \qquad t = t_0, t_0 + 1, \ldots \end{cases} \tag{2}$$

The natural questions is: **when there exists a trajectory of controls $u(t_0), u(t_0+1), \ldots$ such that the system (2) is feasible?**

Of course the answer depends on the initial condition x_0 and also depends on the constraints, parametrized by the vector of threshold θ.

Formal answers to that question will be presented in:

- Monday 18, February 9h00-12h00: course by Luc Doyen. **Viability**.
- Thursday 21, February 14h00-17h00: Tutorial by Eladio Ocaña. **Applications of viability theory in fisheries management**.

Let us formulate the above question in a different way: **Given an initial state x_0, for what vectors of thresholds $\theta = (\theta_1, \ldots, \theta_n)$ the system (2) is feasible?**

This question is equivalent to compute the following set :

$$\mathcal{S}(x_0) \equiv \left\{ \theta = (\theta_1, \ldots, \theta_n) \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \exists\, (u(t_0), u(t_0+1), \ldots) \text{ and} \\ (x(t_0), x(t_0+1), \ldots) \\ \text{satisfying } x(t_0) = x_0 \\ x(t+1) = g\big(x(t), u(t)\big) \\ \forall\; t = t_0, t_0+1, \ldots \text{ and} \\ I_i\big(x(t), u(t)\big) \geq \theta_i \quad \forall\; i = 1, \ldots, n \end{array} \right\} . \tag{3}$$

Intuitively, if the initial condition x_0 is *good* the set $\mathcal{S}(x_0)$ should be large and if we *manage well* the resource, the set $\mathcal{S}(x(t))$ will be larger at each period of time t.

Of course to compute $\mathcal{S}(x_0)$ is hard but, under some assumptions, we know how to do it.

3 Maximin as a sustainable indicator

For a given vector of thresholds $\theta = (\theta_1, \dots, \theta_n)$, consider the set of restrictions for the trajectories of the states and decisions:

$$\mathbb{D}_\theta = \{(x,u) \in \mathbb{X} \times \mathbb{U} \;:\; I_i(x,u) \geq \theta_i, \quad i = 1, \dots, n\}.$$

Suppose that the last indicator I_n is associated to some kind of (instantaneous) profit, and the others indicators $I_1, I_2, \dots, I_{n-1}$ are related to environmental constraints.

A natural questions is: **Given environmental thresholds $\theta_1, \theta_2, \dots, \theta_{n-1}$ associated to environmental constraints $I_1, I_2, \dots, I_{n-1}$, starting from the initial condition x_0, what is the maximal minimal constraint θ_n that can be satisfied?**

Mathematically, to answer this question is equivalent to solve the following Maximin problem (see Monday 4, February 9h00-12h00: course by Michel De Lara. **Optimality and sustainability**):

$$P(x_0, \theta_1, \theta_2, \dots, \theta_{n-1}) \begin{cases} \max\limits_{u(\cdot)} \min\limits_{t \geq t_0} I_n(x(t), u(t)) \\ \text{subject to:} \\ x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0 + 1, \dots \\ x(t_0) = x_0 \\ I_i(x(t), u(t)) \geq \theta_i \qquad i = 1, \dots, n-1; \; t = t_0, t_0 + 1, \dots \end{cases}$$

Thus, for a given initial condition x_0 and environmental thresholds $\theta_1, \theta_2, \dots, \theta_{n-1}$, we will denote by $\mathtt{Maximin}_{\theta_1, \theta_2, \dots, \theta_{n-1}}(x_0)$ the maximal profit that we can obtain as minimum starting from x_0 satisfying environmental thresholds $\theta_1, \theta_2, \dots, \theta_{n-1}$ (for every period t).

3.1 Use of `Maximin` as indicator of recovery situation

Assume the vector state $x(t)$ represents the status of one or several renewable natural resources. The instantaneous profit obtained from the exploitation of $x(t)$ is given by the indicator $I_n(x(t), u(t))$. If the *society*, the government or stakeholders involved in the harvesting define a minimum level `M` of profit, in order to do not have *problems* (over-investments, unemployment, strikes), given an initial condition x_0 and environmental thresholds $\theta_1, \theta_2, \ldots, \theta_{n-1}$, we have the following situations:

- If $\texttt{Maximin}_{\theta_1,\theta_2,\ldots,\theta_{n-1}}(x_0) \geq \texttt{M}$ then, it is possible to manage the resource in order to satisfy the environmental constraints and at each period, to get at least M as profit.
- If $\texttt{Maximin}_{\theta_1,\theta_2,\ldots,\theta_{n-1}}(x_0) < \texttt{M}$, it is not possible to satisfy, in a sustainable way (for each period of time), the required minimal profit. In this case, we say that the resource is over-exploited in the sense that the current condition is not so *good* for satisfying the established minimal requirement M.

In the second case the idea is, starting from x_0, to recover the state in order to, at some time T, to have

$$\texttt{Maximin}_{\theta_1,\theta_2,\ldots,\theta_{n-1}}(x(T)) \geq \texttt{M}.$$

This procedure of course has a cost, so the idea is to deal with in an optimal way.

4 Design of optimal recovery programs

Given an initial condition x_0, environmental thresholds $\theta_1, \theta_2, \ldots, \theta_{n-1}$, and a required minimal profit M, we propose the following optimization problem:

$$P_T(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1}, \texttt{M}) \begin{cases} \displaystyle\min_{u(t_0),\ldots,u(T-1)} \sum_{t=t_0}^{T-1} \rho^{t-t_0} \texttt{C}(I_n(x(t), u(t)); \texttt{M}) \\ \text{subject to:} \\ x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 \\ I_i(x(t), u(t)) \geq \theta_i \qquad i = 1, \ldots, n-1; \ t = t_0, t_0+1, \ldots \\ \texttt{Maximin}_{\theta_1,\theta_2,\ldots,\theta_{n-1}}(x(T)) \geq \texttt{M} \end{cases}$$

where T is the time for recovery the state and the function $\texttt{C}(I_n(x(t), u(t)); \texttt{M})$ represents the cost of have an instantaneous profit $I_n(x(t), u(t))$ lower than the requirement M during the recovery program. For example:

$$\texttt{C}(I_n; \texttt{M}) = \max\{\texttt{M} - I_n; 0\}.$$

Associated to this problem, we will present the implementation developed in [13] (not yet online).

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