Optimization and sustainable indicators

Application to the design of a recovery program for overexploited fish species

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Mathematics of Bio-Economics (MABIES) - IHP

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The social problem of overexploited fisheries

Low catch quotas

- Over investment
- Unemployment
- and ....

The social problem of overexploited fisheries

Low catch quotas

Valparaíso, Chile (2011)

The social problem of overexploited fisheries

Low catch quotas

Boulogne-sur-Mer, France (2009)

The social problem of overexploited fisheries

Low catch quotas

Boulogne-sur-Mer, France (2009)

The social problem of overexploited fisheries

Low catch quotas

What is a viable quota?

How to take into account the social expectation of a quota assignment?

How to minimize the social impact produced by a recovery program for an overexploited fishery?

Contents

Controlled dynamical systems

Discrete time

$$x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0 + 1, \ldots$$

$x(t_0) = x_0$ given (e.g. the current state of the resource)

where

- $x(t) \in \mathbb{X} \subset \mathbb{R}^N$ is the state variable
- $u(t) \in \mathbb{U} \subset \mathbb{R}^M$ is the decision (action or control)
- $g : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{X}$ is the dynamics (maps a state and a control into the state of the next period)

Examples: Dynamics

Controlled dynamical systems in discrete time

$$x(t+1) = f(x(t)) - u(t) \qquad t = t_0, t_0+1, \ldots$$

$$x(t_0) = x_0$$

where

- $x(t) \in \mathbb{X} \subset \mathbb{R}$ is the total biomass of a renewable resource
- $u(t) \in \mathbb{U} \subset \mathbb{R}$ is the level of harvesting
- $f : \mathbb{X} \longrightarrow \mathbb{X}$ is the biological growth function

Examples: Dynamics

Controlled dynamical systems in discrete time

$$x(t+1) = f(x(t)) - qx^{\alpha}(t)u^{\beta}(t) \quad t = t_0, t_0+1, \ldots$$

$$x(t_0) = x_0$$

where

- $x(t) \in \mathbb{X} \subset \mathbb{R}$ is the total biomass of a renewable resource
- $u(t) \in \mathbb{U} \subset \mathbb{R}$ is the harvesting effort
- $f : \mathbb{X} \longrightarrow \mathbb{X}$ is the biological growth function

Examples: Dynamics

Controlled dynamical systems in discrete time

$$x(t+1) = Ax(t) + Bu(t) \quad t = t_0, t_0+1, \dots$$

$$x(t_0) = x_0$$

where

- $x(t) = \begin{pmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{pmatrix} \in \mathbb{X} \subset \mathbb{R}^N$

  represents abundances of one or more species structured (age, maturity, size, weight)

- $u(t) \in \mathbb{U} \subset \mathbb{R}^M$ is a vector of harvesting efforts (one ore more technology, selectivity)

Examples: Dynamics

Controlled dynamical systems in discrete time

$$x(t+1) = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \cdots & \gamma_N \\ \alpha_1 & 0 & 0 & \cdots & 0 \\ 0 & \alpha_2 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \alpha_{N-1} & \alpha_N \end{bmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -1 \end{pmatrix} u(t)$$

where

- $x(t) = \begin{pmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{pmatrix} \in \mathbb{X} \subset \mathbb{R}^N \quad u(t) \in \mathbb{U} \subset \mathbb{R}$

Examples: Dynamics

Controlled dynamical systems in discrete time

$$x(t+1) = L(u(t))\, x(t) + b(x(t)) \quad t = t_0, t_0+1, \ldots$$

$$x(t_0) = x_0$$

where

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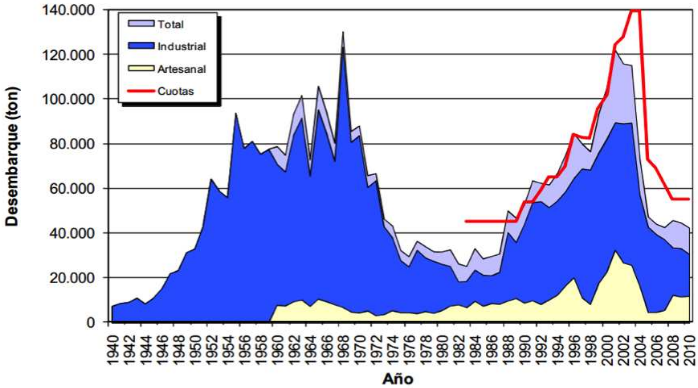
Indicators - Observations

- Usually, it is very difficult to observe the state $x(t)$ variable at each period

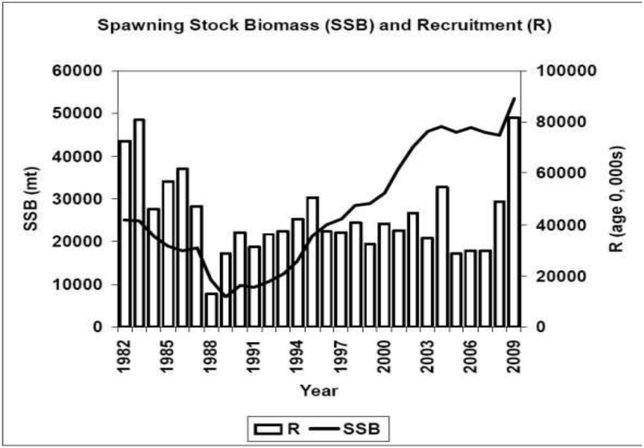
- In place to observe the state, one follows some indicators that are functions of the state and of the decisions

Indicators - Observations



Landings of Hake (Chile)

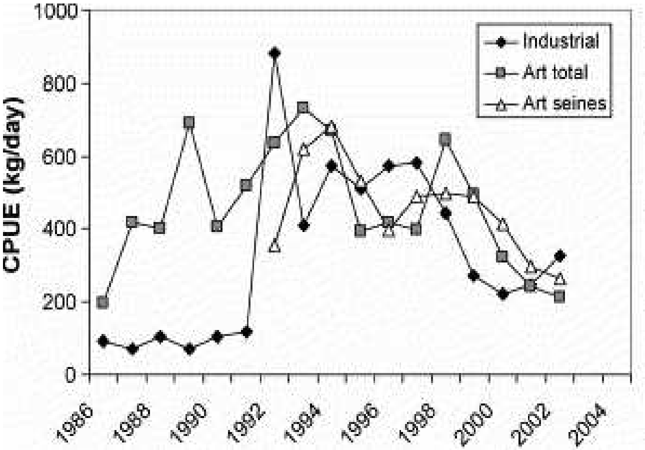
Indicators - Observations



Spawning stock biomass and recruitments of Sea Bass (US)

`http://www.fishingunited.com/`

Indicators - Observations



Various CPUE time series available for mackerel scad (Cape Verde) [1]

[1] Kim A. Stobberup, Karim Erzini, Assessing mackerel scad, Decapterus macarellus, in Cape Verde: Using a Bayesian approach to biomass dynamic modelling in a data-limited situation, Fisheries Research 2006.

Indicators - Observations

If the state of the resource is x and we apply a control (decision) u, we can observe the following n quantities

$$I_1(x,u),\ I_2(x,u), \dots, I_n(x,u)$$

where $I_i : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$ for $i = 1, 2, \dots, n$

We can also use functions I_i in order to represent some relation between state x and control u

Examples: Indicators

Controlled dynamical systems in discrete time

$$x(t+1) = f(x(t)) - qx^{\alpha}(t)u^{\beta}(t) \quad t = t_0, t_0+1, \dots$$

$$x(t_0) = x_0$$

Indicators

- $Y(x,u) = qx^{\alpha}u^{\beta}$: catches
- $CPUE(x,u) = \frac{Y(x,u)}{u}$: catch by unit of effort
- $B(x,u) = x$: biomass
- $U(x,u) = pY(x,u) - cu$: instantaneous utility

Examples: Indicators

Controlled dynamical systems in discrete time

$$x(t+1) = Ax(t) + Bu(t) \quad t = t_0, t_0 + 1, \ldots$$

$$x(t_0) = x_0$$

Indicators

- $U(x,u)$: instantaneous profit
- $M(x,u) = x_j$: a particular component of the state (e.g. age)

Main example: Dynamics

Age-structured fish stock population model

$$
\begin{aligned}
x_1(t+1) &= \varphi(SSB(x(t))) \\
x_2(t+1) &= x_1(t)\exp(-M - u(t)s_1) \\
x_3(t+1) &= x_2(t)\exp(-M - u(t)s_2) \\
&\vdots \\
x_{N-1}(t+1) &= x_{N-2}(t)\exp(-M - u(t)s_{N-2}) \\
x_N(t+1) &= x_{N-1}(t)\exp(-M - u(t)s_{N-1}) + x_N(t)\exp(-M - u(t)s_N)
\end{aligned}
$$

- $x_j(t)$: abundance (number of individuals) at age j
- $SSB(x(t))$: spawning stock biomass at period t
- M: natural mortality
- $u(t)$: fishing effort
- s_j: selectivity pattern at age j

Main example: Dynamics

Age-structured fish stock population model

Stock-recruitment function (examples)

$$\varphi(SSB) = \frac{SSB}{\alpha + \beta\, SSB}$$

$$\varphi(SSB) = \alpha\, SSB\, \exp(-\beta\, SSB)$$

Main example: Dynamics

Age-structured fish stock population model

Stock-recruitment function

$$\varphi(SSB) = \frac{SSB}{\alpha + \beta \, SSB}$$

Main example: Indicators

Age-structured fish stock population model

Spawning-stock biomass

$$SSB(x,u) = \sum_{j=1}^{N} \gamma_j \, w_j \, x_j$$

where

- γ_j: fecundity pattern at age j
- w_j: average weight at age j

Yield (catch in weight)

$$Y(x,u) = \sum_{j=1}^{N} w_j \left(\frac{u \, s_j}{u \, s_j + M} \right) (1 - \exp(-M - u \, s_j)) \, x_j$$

where

- M: natural mortality
- $u(t)$: fishing effort
- s_j: selectivity pattern at age j

Main example: Indicators

Age-structured fish stock population model

Catch by unit of effort

$$CPUE(x,u) = \frac{Y(x,u)}{u}$$

Induced fishing mortality

$$F(x,u) = u \sum_{j=1}^{N} s_j$$

Contents

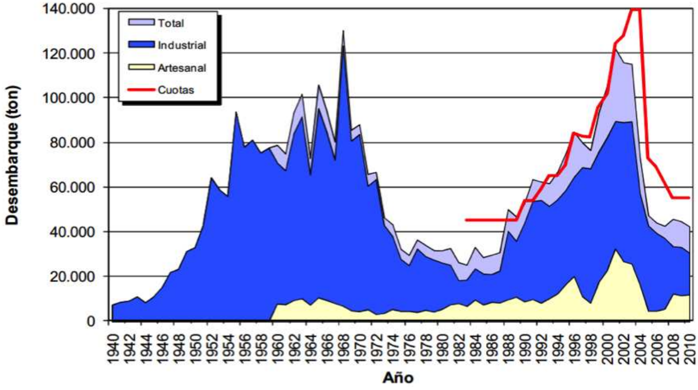
Constraints

in terms of indicators

- Often, it is requested that some (observable) quantities that depend on states (resource) and decisions (harvest effort), satisfy certain restrictions
- These requirements can be represented by observations or indicators satisfying inequalities during all the periods
- Relationships between states and controls (e.g. do not harvest more than available resources), can be expressed also as inequalities to be satisfied by indicators

Constraints

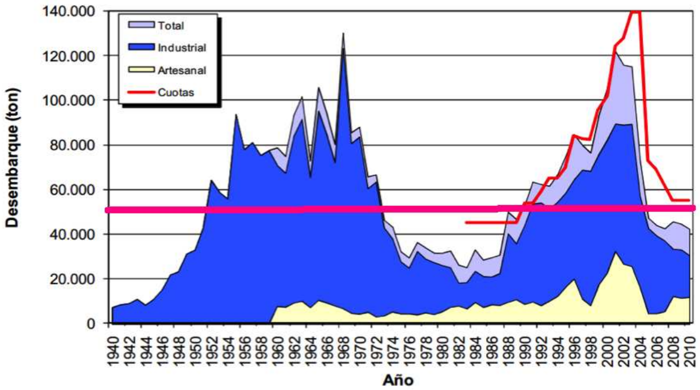
in terms of indicators



Landings of Hake (Chile)

Constraints

in terms of indicators



Landings of Hake (Chile)

Constraints

in terms of indicators

$$x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0+1, \ldots$$

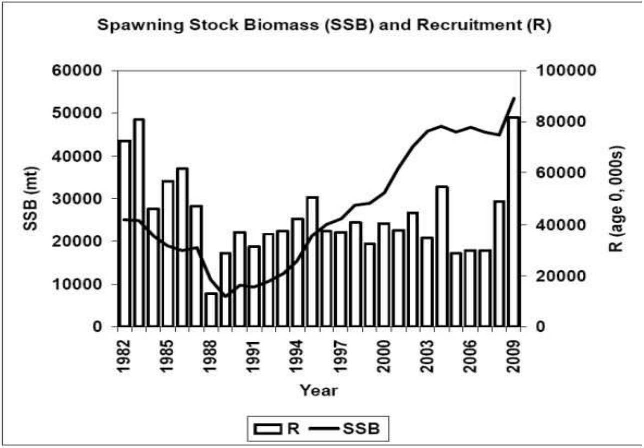
$$x(t_0) = x_0$$

Requirement for the yield

$$Y(x(t), u(t)) \geq y_{\text{min}} \qquad t = t_0, t_0+1, \ldots$$

Constraints

in terms of indicators



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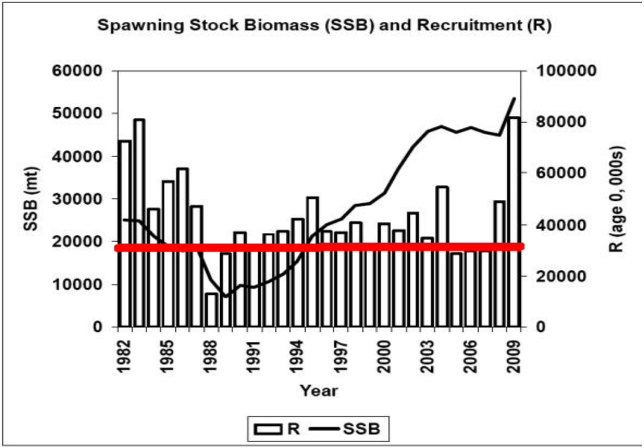

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$$x(t_0) = x_0$$

Requirement for the spawning-stock biomass

$$SSB(x(t), u(t)) \geq SSB_{\mathsf{min}} \qquad t = t_0, t_0+1, \ldots$$

Constraints

in terms of indicators

$$x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0 + 1, \ldots$$

$$x(t_0) = x_0$$

Requeriments

$$I_i(x(t), u(t)) \geq \theta_i \qquad t = t_0, t_0 + 1, \ldots; \quad i = 1, 2, \ldots, n$$

where

- $I_i : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$ is the i indicator (observation) function
- $\theta_i \in \mathbb{R}$ the threshold associated to the indicator i

Constraints

in terms of indicators

$$\begin{cases} x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0+1, \dots \\ x(t_0) = x_0 \\ I_i(x(t), u(t)) \geq \theta_i \qquad t = t_0, t_0+1, \dots; \quad i = 1, 2, \dots, n \end{cases}$$

Is it possible to manage the above system?

The above system has a solution?

There exists a sequence of controls $u(t_0), u(t_0+1), \dots$ for the above system?

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Viability theory

- **Viability**: Monday 18, February 9h00-12h00, course by Luc Doyen
- **Applications of viability theory in fisheries management**: Thursday 21, February 14h00-17h00, tutorial by Eladio Ocaña

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Viability theory

Viability kernel

$$\begin{cases} x(t+1) = g(x(t), u(t)) & t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 & \\ I_i(x(t), u(t)) \geq \theta_i & t = t_0, t_0+1, \ldots; \quad i = 1, 2, \ldots, n \end{cases}$$

The viability kernel associated to the thresholds $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ is the set of initial states x_0 for which the above system has solution

$$\mathbb{V}(\theta) \overset{\text{def}}{=} \left\{ x_0 \in \mathbb{X} \;\middle|\; \begin{array}{l} \exists\ (u(t_0), u(t_0+1), \ldots) \text{ and} \\ (x(t_0), x(t_0+1), \ldots) \\ \text{satisfying } x(t_0) = x_0 \\ x(t+1) = g\big(x(t), u(t)\big) \\ \forall\ t = t_0, t_0+1, \ldots \text{ and} \\ I_i\big(x(t), u(t)\big) \geq \theta_i \quad \forall\ i = 1, \ldots, n \end{array} \right\}$$

Viability theory

Viable thresholds

$$\begin{cases} x(t+1) = g(x(t), u(t)) & t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 \\ I_i(x(t), u(t)) \geq \theta_i & t = t_0, t_0+1, \ldots; \quad i = 1, 2, \ldots, n \end{cases}$$

Given the initial state x_0 (the current state of the resources), for what vector of thresholds $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ the above system has solution?

$$\mathcal{S}(x_0) \stackrel{\text{def}}{=} \left\{ \theta = (\theta_1, \ldots, \theta_n) \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \exists\ (u(t_0), u(t_0+1), \ldots) \text{ and} \\ (x(t_0), x(t_0+1), \ldots) \\ \text{satisfying } x(t_0) = x_0 \\ x(t+1) = g\big(x(t), u(t)\big) \\ \forall\ t = t_0, t_0+1, \ldots \text{ and} \\ I_i\big(x(t), u(t)\big) \geq \theta_i \quad \forall\ i = 1, \ldots, n \end{array} \right\}$$

What is a viable quota?

Boulogne-sur-Mer, France (2009)

What is a viable quota?

Age-structured fish stock population model

$$\begin{cases} x(t+1) = g(x(t), u(t)) & t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 & \text{(current state of the resource)} \\ Y(x(t), u(t)) \geq y_{\min} & t = t_0, t_0+1, \ldots \end{cases}$$

A viable quota is a level of yield $y_{\min}$ such that the above system has a solution

What is a viable quota?

Age-structured fish stock population model

One can include a preservation requirement in terms of the spawning-stock biomass

$$\begin{cases} x(t+1) = g(x(t), u(t)) & t = t_0, t_0 + 1, \dots \\ x(t_0) = x_0 & \text{(current state of the resource)} \\ Y(x(t), u(t)) \geq y_{\min} & t = t_0, t_0 + 1, \dots \\ SSB(x(t), u(t)) \geq SSB_{\min} & t = t_0, t_0 + 1, \dots \end{cases}$$

What is a viable quota?

Age-structured fish stock population model

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Given the current state of the resource x_0, for what constraints $(y_{\min}, SSB_{\min})$ the above system has solution?

To determine $\mathbb{S}(x_0)$

What is a viable quota?

Age-structured fish stock population model

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Given thresholds $(y_{\min}, SSB_{\min})$

the above system has solution ?

To determine if $(y_{\min}, SSB_{\min}) \in \mathbb{S}(x_0)$

What is a viable quota?

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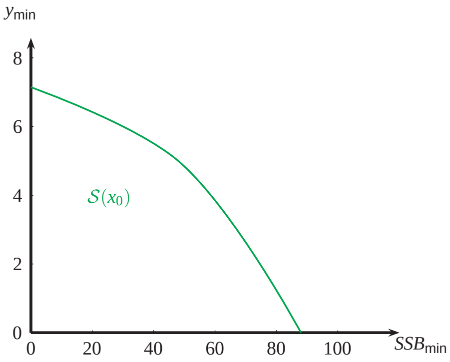
Given thresholds $(y_{\min}, SSB_{\min})$

the above system has solution ?

To determine if $(y_{\min}, SSB_{\min}) \in \mathbb{S}(x_0)$

$$\mathcal{S}(x_0) \stackrel{\text{def}}{=} \left\{ \theta = (y_{\min}, SSB_{\min}) \in \mathbb{R}^2 \;\middle|\; \begin{array}{l} \exists\ (u(t_0), u(t_0+1), \ldots) \\ \text{and} \quad (x(t_0), x(t_0+1), \ldots) \\ \text{satisfying} \ \ x(t_0) = x_0 \\ x(t+1) = g\big(x(t), u(t)\big) \\ \forall\ t = t_0, t_0+1, \ldots \ \text{ and} \\ Y(x(t), u(t)) \geq y_{\min} \\ SSB(x(t), u(t)) \geq SSB_{\min} \end{array} \right\}$$


$y_{\min}$
8
6
4
2
0
$\mathcal{S}(x_0)$
0
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40
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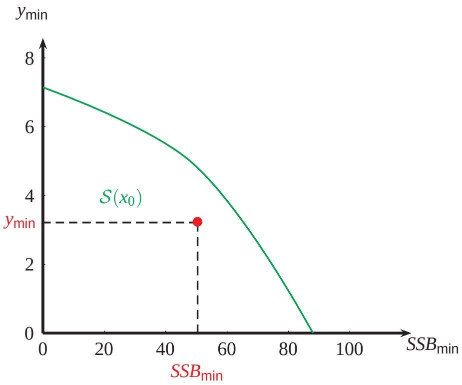
100
$SSB_{\min}$



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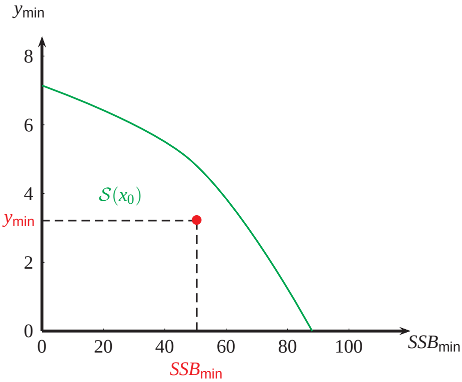
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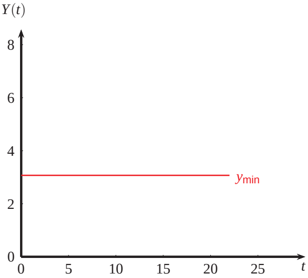
$Y(t)$
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$y_{\min}$
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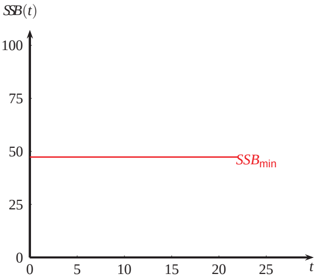
$SSB(t)$
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$SSB_{\min}$
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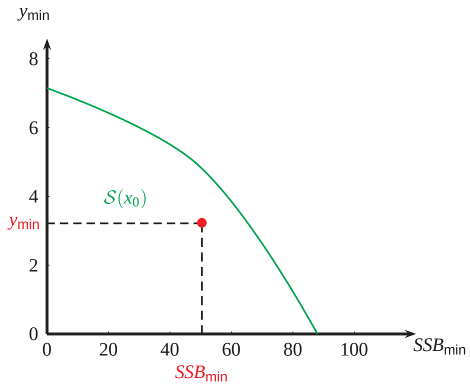
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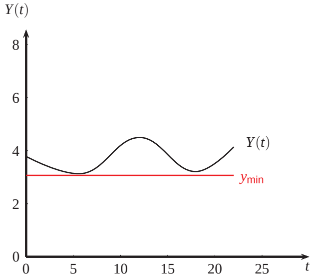
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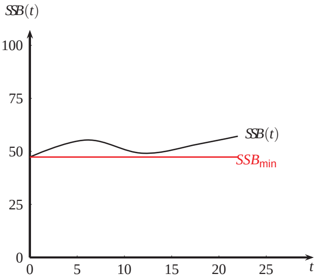
25
t

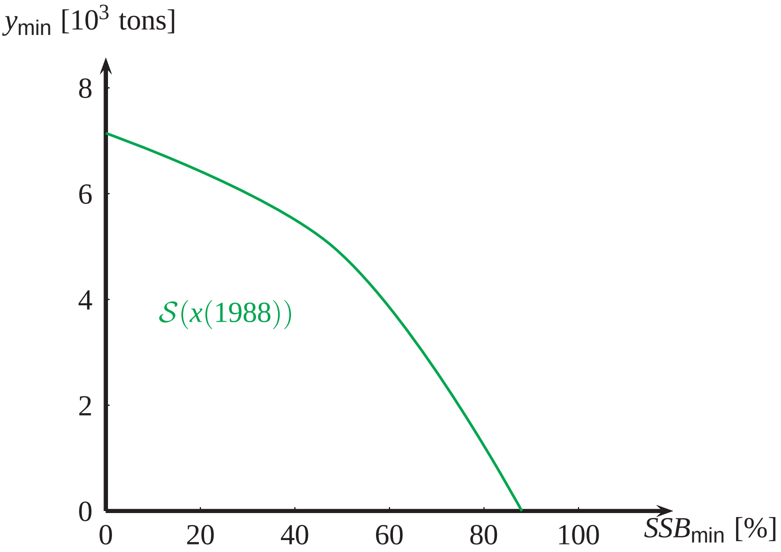

$SSB(t)$
100
75
50
25
0
$SSB(t)$
$SSB_{\min}$
0
5
10
15
20
25
t

Viable quotas
and environmental requirements

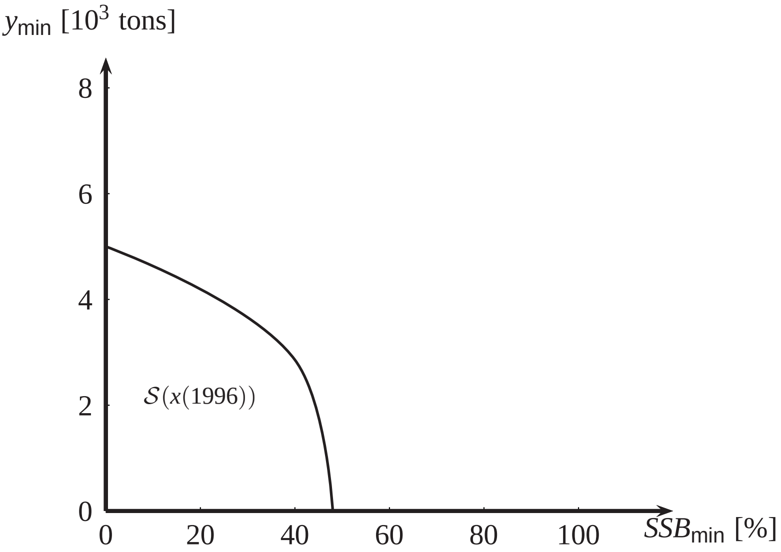
- To compute the set of thresholds that are sustainable $\mathcal{S}(x_0)$ may be very difficult
- But under some assumptions on the dynamics g and indicators I_i it is possible ......

Case study: Chilean sea bass

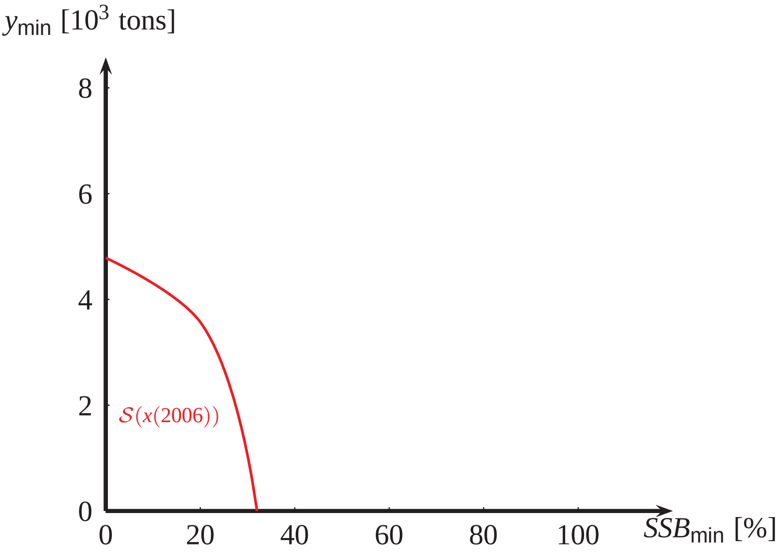
- Abundance at age (state): $x = (x_j)_{j=1,..,36}$
- Fishing effort multiplier (control): $u \in [u_{\min}, u_{\max}]$
- The most expensive white meat fish ($\approx 10^4$ [dollars/ton])
- Mean quota in the last five years (Chile): 2 500 – 3 000 tons



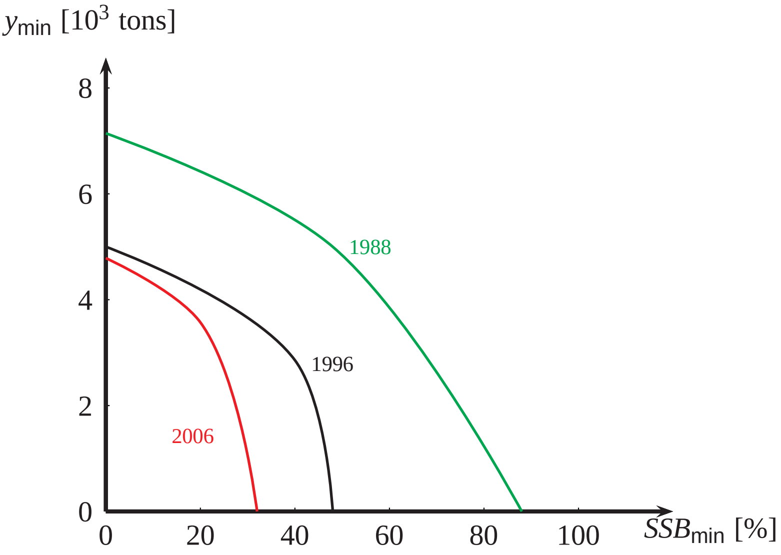
$y_{\min}$ [10^3 tons]
8
6
4
2
0
$\mathcal{S}(x(1988))$
0
20
40
60
80
100
$SSB_{\min}$ [%]



$y_{\min}$ [10^3 tons]
8
6
4
2
0
$\mathcal{S}(x(1996))$
0
20
40
60
80
100
$SSB_{\min}$ [%]



y_{min} [10^3 tons]
8
6
4
2
0
$\mathcal{S}(x(2006))$
0
20
40
60
80
100
SSB_{min} [%]



$y_{\min}$ [10^3 tons]
8
6
4
2
0
1988
1996
2006
0
20
40
60
80
100
$SSB_{\min}$ [%]

To recover an overexploited species

How to recover a fishery?

How to obtain *good* viable quotas?

- To define what is a *good* quota and a good environmental threshold: $(\overline{SSB}_{\mathrm{min}}, \bar{y}_{\mathrm{min}})$
- What means to be in presence of an overexploited fishery?

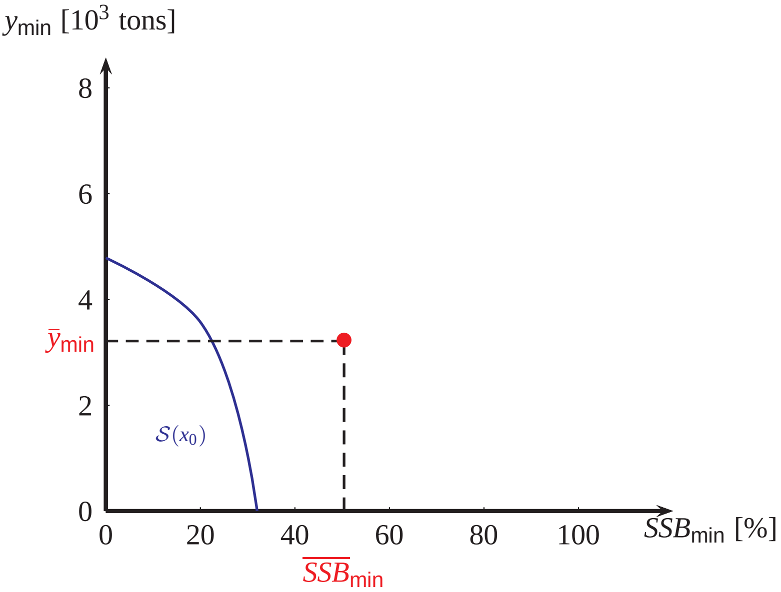
$$(\overline{SSB}_{\mathrm{min}}, \bar{y}_{\mathrm{min}}) \notin \mathcal{S}(x_0)$$

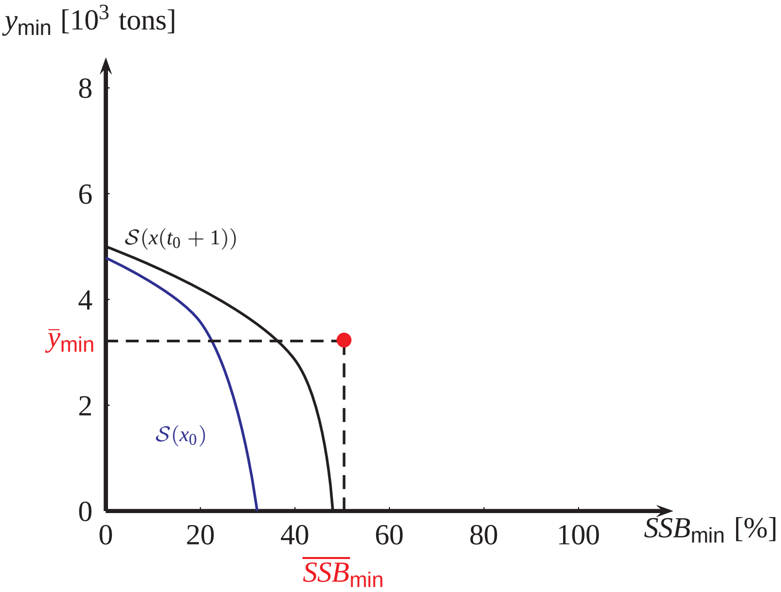
- Recovery program: From now (t_0), to manage the resource in order to have

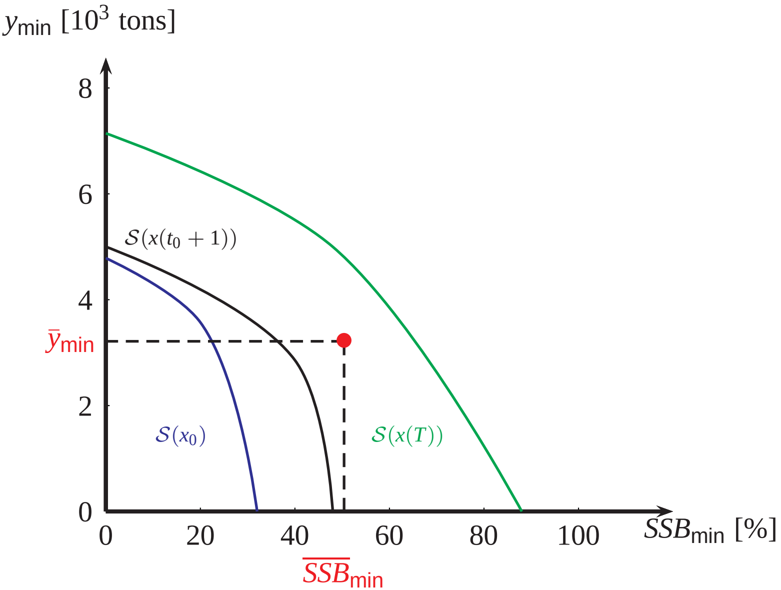
$$\mathcal{S}(x_0) \subset \mathcal{S}(x(t_0+1)) \subset \mathcal{S}(x(t_0+2)) \subset \ldots \subset \mathcal{S}(x(T))$$

such that

$$(\overline{SSB}_{\mathrm{min}}, \bar{y}_{\mathrm{min}}) \in \mathcal{S}(x(T))$$


$y_{\min}$ [10^3 tons]
8
6
4
$\bar{y}_{\min}$
2
0
$\mathcal{S}(x_0)$
0
20
40
60
80
100
$SSB_{\min}$ [%]
$\overline{SSB}_{\min}$


$y_{\min}$ [10^3 tons]
8
6
4
2
0
$\bar{y}_{\min}$
$\mathcal{S}(x(t_0+1))$
$\mathcal{S}(x_0)$
0
20
40
60
80
100
$SSB_{\min}$ [%]
$\overline{SSB}_{\min}$



$y_{\min}$ [10^3 tons]
8
6
4
$\bar{y}_{\min}$
2
0
0
20
40
60
80
100
$SSB_{\min}$ [%]
$\overline{SSB}_{\min}$
$\mathcal{S}(x(t_0+1))$
$\mathcal{S}(x_0)$
$\mathcal{S}(x(T))$

Contents

Maximal minimal production

$$\begin{cases} x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 \\ I_i(x(t), u(t)) \geq \theta_i \qquad t = t_0, t_0+1, \ldots; \quad i = 1, 2, \ldots, n \end{cases}$$

Let us suppose that the first $n-1$ indicators (with the associated thresholds)

$$I_1, \ I_2, \ldots, I_{n-1}$$

are related to environmental constraints (spawning-stock biomass, total biomass, ..) and the last indicator I_n is a productive observation (catches)

Given environmental thresholds

$$\theta_1, \theta_2, \ldots, \theta_{n-1}$$

associated to environmental constraints, starting from the initial condition x_0, what is the maximal minimal constraint θ_n that can be satisfied?

Maximal minimal production

$$
\begin{cases}
\texttt{Maximin}(x_0, \theta_1, \theta_2, \dots, \theta_{n-1}) \stackrel{\text{def}}{=} \max\limits_{u(\cdot)} \min\limits_{t \geq t_0} \; I_n(x(t), u(t)) \\
\text{subject to:} \\
x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0 + 1, \dots \\
x(t_0) = x_0 \\
I_i(x(t), u(t)) \geq \theta_i \qquad i = 1, \dots, n-1; \; t = t_0, t_0 + 1, \dots
\end{cases}
$$

$\texttt{Maximin}(x_0, \theta_1, \theta_2, \dots, \theta_{n-1})$ is the maximal level of production

(measured with I_n)

that we can obtain as minimum

satisfying the environmental thresholds $\theta_1, \theta_2, \dots, \theta_{n-1}$ for all periods

starting from x_0

Maximal minimal production

$$
\left\{
\begin{array}{l}
\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1}) \stackrel{\text{def}}{=} \max\limits_{u(\cdot)} \min\limits_{t \geq t_0} I_n(x(t), u(t)) \\
\text{subject to:} \\
x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0 + 1, \ldots \\
x(t_0) = x_0 \\
I_i(x(t), u(t)) \geq \theta_i \qquad i = 1, \ldots, n-1;\ t = t_0, t_0 + 1, \ldots
\end{array}
\right.
$$

How to compute $\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$??

Maximal minimal production

How to compute $\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$??

- Take all the sequences of controls $u(\cdot) = \{u(t_0),\ u(t_0+1), \ldots\}$ such that

$$\begin{cases} x(t+1) = g(x(t), u(t)) & t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 & \\ I_i(x(t), u(t)) \geq \theta_i & i = 1, \ldots, n-1;\ t = t_0, t_0+1, \ldots \end{cases}$$

- Compare

$$\min_{t \geq t_0} I_n(x(t), u(t))$$

- To choose $u(\cdot) = \{u(t_0),\ u(t_0+1), \ldots\}$ for which the above quantity is the maximum

Maximal minimal production

How to compute $\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$??

- Take all the sequences of controls $u(\cdot) = \{u(t_0),\ u(t_0+1), \ldots\}$

  such that

$$\begin{cases} x(t+1) = g(x(t), u(t)) & t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 & \\ I_i(x(t), u(t)) \geq \theta_i & i = 1, \ldots, n-1;\ t = t_0, t_0+1, \ldots \end{cases}$$

HOW?

In general it is not easy but, under some assumptions on the dynamics g and on the indicators

$$I_1,\ I_2, \ldots, I_{n-1},\ I_n$$

we know how to do it

Maximal minimal production

1. For a given production threshold θ_n we can define a sequence of controls $u_{\theta_n}(\cdot) = \{u_{\theta_n}(t_0),\ u_{\theta_n}(t_0+1), \ldots\}$

2. If $u_{\theta_n}(\cdot) = \{u_{\theta_n}(t_0),\ u_{\theta_n}(t_0+1), \ldots\}$ does not satisfy

$$\begin{cases} x(t+1) = g(x(t), u_{\theta_n}(t)) & t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 & \\ I_i(x(t), u_{\theta_n}(t)) \geq \theta_i & i = 1, \ldots, n-1, n;\ t = t_0, t_0+1, \ldots \end{cases}$$

   we can prove that for all other controls, the above system is not admissible. In this case, we repeat the Step 1 with a lower θ_n

3. If the above system is admissible for $u_{\theta_n}(\cdot)$ we repeat the Step 1 with a higher θ_n

4. This bisection algorithm leads to

$$\theta_n \ \rightarrow\ \texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$$

Maximal minimal production

Monotonicity

The dynamics

$$g : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{X}$$

$$(x, u) \longrightarrow g(x, u)$$

- is increasing with respect to the state x if :

$$x \leq \tilde{x} \quad \text{implies} \quad g(x, u) \leq g(\tilde{x}, u) \quad \text{for all} \quad u \in \mathbb{U}$$

- is decreasing with respect to the control u if :

$$u \leq \tilde{u} \quad \text{implies} \quad g(x, u) \geq g(x, \tilde{u}) \quad \text{for all} \quad x \in \mathbb{X}$$

Main example: Dynamics

Age-structured fish stock population model

$$
\begin{aligned}
x_1(t+1) &= \varphi(SSB(x(t))) \\
x_3(t+1) &= x_2(t)\exp(-M - u(t)s_2) \\
&\vdots \\
x_{N-1}(t+1) &= x_{N-2}(t)\exp(-M - u(t)s_{N-2}) \\
x_N(t+1) &= x_{N-1}(t)\exp(-M - u(t)s_{N-1}) + x_N(t)\exp(-M - u(t)s_N)
\end{aligned}
$$

- $x_j(t)$: abundance (number of individuals) at age j
- $SSB(x(t))$: spawning stock biomass at period t
- M: natural mortality
- $u(t)$: fishing effort
- s_j: selectivity pattern at age j

g is increasing with respect to the state variable x and decreasing with respect to the control variable u

Main example: Dynamics

Age-structured fish stock population model

Stock-recruitment function

$$\varphi(SSB) = \frac{SSB}{\alpha + \beta SSB}$$

Spawning-stock biomass

$$SSB(x) = \sum_{j=1}^{N} \gamma_j \, w_j \, x_j$$

where

- γ_j: fecundity pattern at age j
- w_j: average weight at age j

Maximal minimal production

Monotonicity

A function (as indicators)

$$I : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$$

$$(x, u) \longrightarrow I(x, u)$$

- is increasing with respect to the state x if :

$$x \leq \tilde{x} \quad \text{implies} \quad I(x, u) \leq I(\tilde{x}, u) \quad \text{for all } \ u \in \mathbb{U}$$

- is decreasing with respect to the control u if :

$$u \leq \tilde{u} \quad \text{implies} \quad I(x, u) \geq I(x, \tilde{u}) \quad \text{for all } \ x \in \mathbb{X}$$

Main example: Indicators

Age-structured fish stock population model

Spawning-stock biomass

$$SSB(x,u) = \sum_{j=1}^{N} \gamma_j \, w_j \, x_j$$

$SSB(x,u)$ in increasing with respect to the state x and decreasing with respect to the control u

Yield (catch in weight)

$$Y(x,u) = \sum_{j=1}^{N} w_j \left(\frac{u \, s_j}{u \, s_j + M} \right) \left(1 - \exp(-M - u \, s_j)\right) x_j$$

$Y(x,u)$ in increasing with respect to the state x and increasing with respect to the control u

Maximal minimal production

How to compute $\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$??

If

- g is increasing with respect to the state variable x and decreasing with respect to the control variable u
- All indicators $I_1, I_2, \ldots, I_{n-1}, I_n$ are increasing with respect to the state variable x
- The first $(n-1)$ indicators $I_1, I_2, \ldots, I_{n-1}$ are decreasing with respect to the control variable u
- The control $u \in \mathbb{U}$ is scalar

Then

- The bisection algorithm described previously leads to

$$\theta_n \rightarrow \texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$$

Maximal minimal production

Age-structured fish stock population model

How to compute $\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$??

Since

- g is increasing with respect to the state variable x and decreasing with respect to the control variable u
- The indicators $SSB(x, u)$ and $Y(x, u)$ are increasing with respect to the state variable x
- The indicator $SSB(x, u)$ is decreasing with respect to the control variable u
- The control $u \in \mathbb{U}$ is scalar

Then

- The bisection algorithm described previously leads to

$$\theta_n \rightarrow \texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$$

Maximal minimal production

Given the current state of the resource x_0 and environmental thresholds $\theta_1, \theta_2, \ldots, \theta_{n-1}$ and under some assumptions we can compute

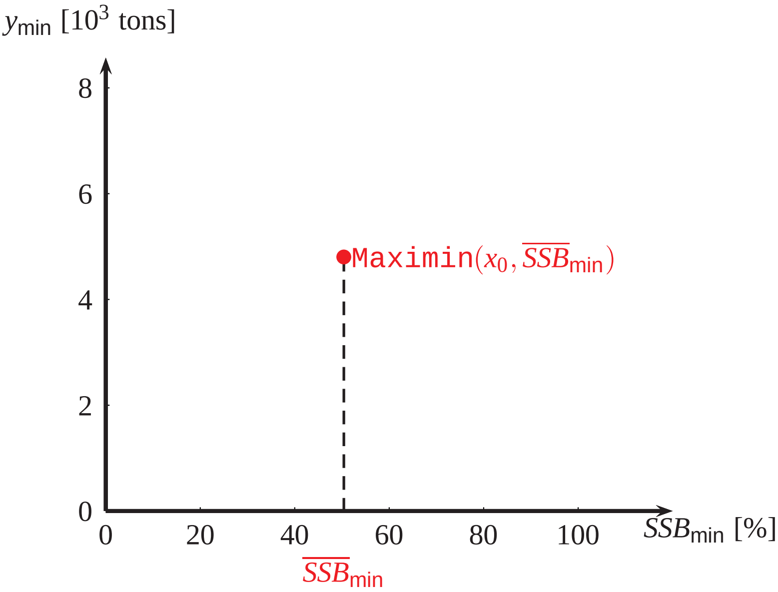
$$\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$$

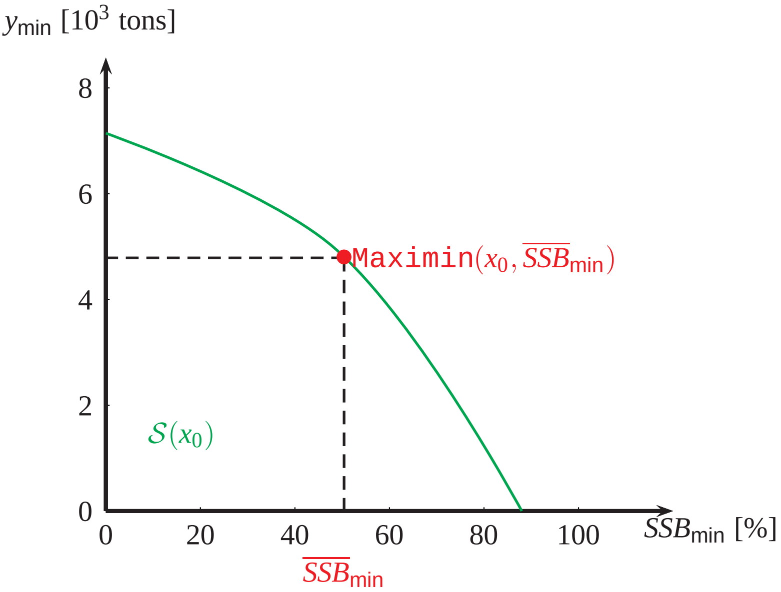
the maximal level of production (measured with I_n)

that we can obtain as minimum satisfying the environmental thresholds

$$\theta_1, \theta_2, \ldots, \theta_{n-1}$$

for all periods starting from x_0


y_{min} [10^3 tons]
8
6
4
2
0
Maximin$(x_0, \overline{SSB}_{\text{min}})$
0
20
40
60
80
100
SSB_{min} [%]
$\overline{SSB}_{\text{min}}$



$y_{\min}$ [10^3 tons]
8
6
4
2
0
0
20
40
60
80
100
$SSB_{\min}$ [%]
$\overline{SSB}_{\min}$
$\mathtt{Maximin}(x_0, \overline{SSB}_{\min})$
$\mathcal{S}(x_0)$

Contents

To recover an overexploited species

How to recover a fishery?

How to obtain *good* viable quotas?

- To define what is a *good* quota (social requirement) and a good environmental threshold:

$(\overline{SSB}_{\mathrm{min}}, \bar{y}_{\mathrm{min}}) =$ minimal thresholds for spawning-stock biomass and catches

- What means to be in presence of an overexploited fishery?

$$(\overline{SSB}_{\mathrm{min}}, \bar{y}_{\mathrm{min}}) \notin \mathcal{S}(x_0) \Leftrightarrow \bar{y}_{\mathrm{min}} > \mathtt{Maximin}(x_0, \overline{SSB}_{\mathrm{min}})$$

- Recovery program: to manage the resource in order to have

$$\bar{y}_{\mathrm{min}} \leq \mathtt{Maximin}(x(T), \overline{SSB}_{\mathrm{min}})$$

To recover an overexploited species

How to recover a fishery?

- Recovery program: to manage the resource in order to have

$$\bar{y}_{\text{min}} \leq \texttt{Maximin}(x(T), \overline{SSB}_{\text{min}})$$

- What about the (social) cost?? during $t = t_0, t_0 + 1, \ldots, T - 1$ where

$$\bar{y}_{\text{min}} > \texttt{Maximin}(x(t), \overline{SSB}_{\text{min}})$$

The social problem of overexploited fisheries

Low catch quotas

Valparaíso, Chile (2011)

Framework

Sustainability of constraints

`Maximin` as a sustainable indicator

Design of optimal recovery strategies

To recover an overexploited species

How to recover a fishery?

- For managing the resource in order to have

$$\bar{y}_{\min} \leq \ \texttt{Maximin}(x(T), \overline{SSB}_{\min})$$

during $t = t_0, t_0 + 1, \ldots, T - 1$ where

$$\bar{y}_{\min} > \ \texttt{Maximin}(x(t), \overline{SSB}_{\min})$$

we will need to have

$$Y(x(t), u(t)) < \ \bar{y}_{\min}$$

- We propose as a proxy of the cost the difference between

$$\bar{y}_{\min} \quad \text{and} \quad Y(x(t), u(t))$$

- This difference can be interpreted as a subsidy

To recover an overexploited species

How to recover a natural resource?

Given an initial condition x_0, environmental thresholds $\theta_1, \theta_2, \dots, \theta_{n-1}$, and a required minimal profit $\bar{\theta}_n$, we propose the following optimization problem:

$$
\left\{
\begin{array}{l}
\displaystyle \min_{u(t_0),\dots,u(T-1)} \sum_{t=t_0}^{T-1} \max\{\bar{\theta}_n - I_n(x(t),u(t))\ ;\ 0\} \\
\text{subject to:} \\
x(t+1) = g(x(t),u(t)) \qquad t = t_0, t_0+1, \dots \\
x(t_0) = x_0 \\
I_i(x(t),u(t)) \geq \theta_i \qquad i = 1,\dots,n-1;\ t = t_0, t_0+1, \dots \\
\texttt{Maximin}(x(T), \theta_1, \theta_2, \dots, \theta_{n-1}) \geq \bar{\theta}_n
\end{array}
\right.
$$

where T is the time for recovery

To recover an overexploited species

Age-structured fish stock population model

Given an initial condition x_0, the environmental thresholds $\overline{SSB}_{\min}$, and a required minimal level of catch $\bar{y}_{\min}$ (social requirement), we propose the following optimization problem:

$$
\begin{cases}
\displaystyle \min_{u(t_0),\ldots,u(T-1)} \sum_{t=t_0}^{T-1} \max\{\bar{y}_{\min} - Y(x(t),u(t))\ ;\ 0\} \\
\text{subject to:} \\
x(t+1) = g(x(t),u(t)) \qquad t = t_0, t_0+1, \ldots \\
x(t_0) = x_0 \\
SSB(x(t),u(t)) \geq \overline{SSB}_{\min} \qquad t = t_0, t_0+1, \ldots \\
\texttt{Maximin}(x(T), \overline{SSB}_{\min}) \geq y_{\min}
\end{cases}
$$

where T is the time for recovery

To recover an overexploited species

Age-structured fish stock population model

- We can try to solve the optimization problem using dynamical programming algorithms
- In the case of the study, we deal with a easier problem: constant catches during the recovery periods

To recover an overexploited species

Age-structured fish stock population model

Given an initial condition x_0, the environmental thresholds $\overline{SSB}_{\text{min}}$, and a required minimal level of catch $\bar{y}_{\text{min}}$ (social requirement), we deal with the following optimization problem:

$$\begin{cases} \min\limits_{u(t_0),\dots,u(T-1)} \sum\limits_{t=t_0}^{T-1} \max\{\bar{y}_{\text{min}} - \bar{Y}\ ;\ 0\} \\ \text{subject to:} \\ x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0+1, \dots \\ x(t_0) = x_0 \\ SSB(x(t), u(t)) \geq \overline{SSB}_{\text{min}} \qquad t = t_0, t_0+1, \dots \\ Y(x(t), u(t)) = \bar{Y} \\ \texttt{Maximin}(x(T), \overline{SSB}_{\text{min}}) \geq \bar{y}_{\text{min}} \end{cases}$$

where T is the time for recovery

Summary and challenges

$$\begin{cases} x(t+1) = g(x(t), u(t)) \qquad t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 \\ I_i(x(t), u(t)) \geq \theta_i \qquad i = 1, \ldots, n-1, n;\ t = t_0, t_0+1, \ldots \end{cases}$$

- Under some assumptions, we can compute:
  - $\mathcal{S}(x_0)$
  - $\texttt{Maximin}(x_0, \theta_1, \theta_2, \ldots, \theta_{n-1})$
- In this framework, one can establish an optimal recovery problem
- How to extend these approach for non-monotonic dynamics?

Project: *Recuperemos la Merluza*

Let's recover The Hake

http://www.recuperemoslamerluza.cl/