Sustainability criteria: between efficiency and equity

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Who I am

Senior Researcher at INRA (National Institute of Research in Agronomy), Department of Economics, Montpellier, France.

- Mathematician: Thesis from the National University of Rosario (Argentina) 1991: "Numerical resolution of the Hamilton-Jacobi-Bellman equations".
- Economist: Habilitation à diriger des recherches, University of Montpellier I (2005): "Environmental economics and strategic interactions".

Plan of the presentation

- Introduction
  1. Some words about sustainable development
  2. Usual and well-known criteria for intertemporal choice in economics
  3. Efficiency and Equity: a social and ethical dilemma

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- Introduction
  1. Some words about sustainable development
  2. Usual and well-known criteria for intertemporal choice in economics
  3. Efficiency and Equity: a social and ethical dilemma
- First part: Detailed study of the dilemma: axiomatic justification
  1. Binary relation
  2. Axioms and compatibility between axioms
  3. Social welfare function (that numerically represents a binary relation)
  4. Usual criteria and axioms and some characterisations

Deontological procedure: the normative position that judges the "success" of an action based on the action's adherence to a set of rules.

Plan of the presentation

- Second part: Two special criteria
  1. Mixed Bentham-Green Golden Rule (Chichilnisky) criterion
  2. Mixed Bentham-Rawls criterion

Sustainable Development

Vincent Martinet, "Economic Theory and Sustainable Development. What can we preserve for future generations? "(2012).

According to the Brundtland Report: sustainable development is development meeting the needs of the present without compromising the ability of future generations to meet their own needs.

Challenges:

- How to handle conflicting interests: environmental conservation - economic development
- How to insure intergenerational equity
- What should be preserved for future generations, in particular in terms of natural resources and environmental assets

Sustainable Development

Nobel laurate Robert Solow

"If sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the very long run. It is very important to understand what that something is: I think it has to be a generalized capacity to produce economic well-being."

Weak and Strong sustainability

Two different paradigms have been proposed to formalize sustainability

Weak sustainability
is based on traditional tools used in economics and resource economics: Nature has no more and no less value than man-made capital. There is a priori no need to conserve natural assets for future generations as long as one can substitute man-made capital for natural assets in value.
Kuznets curve: graphical representation of the hypothesis that as a country develops, there is a natural cycle of economic inequality driven by market forces which at first increases inequality, and then decreases it after a certain average income is attained.
Environmental Kuznets Curves is strongly contested.
Models of optimal growth

Weak and Strong sustainability

Strong sustainability is based on the definition of critical natural resources and ecosystems to be conserved. Natural capital and man-made capital are complement.

Models of optimal control with states and control constraints

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Difference

Weak sustainability: Manufactured capital of equal value can take the place of natural capital
Strong sustainability: The existing stock of natural capital must be maintained and enhanced because the functions it performs cannot be duplicated by manufactured capital

Usual criteria in economics

The discounted utility criterion (Bentham's criterion) is the MAIN criterion used for intertemporal choice in economics. It is questioned in terms of sustainability because it is only sensitive to what happens in the short run.

The simplest way to avoid discounting is to set the discount rate to zero this is the Ramsey undiscounted utilitarian criterion. It treats all the generations in the same way (with anonymity). Unfortunately this criterion is unable to rank all possible alternatives and it is very sensitive to the long-run behavior of path.

Usual sustainability criteria

The green Golden Rule defines the development path that maximizes the level of utility sustained in the long run. This criterion is only sensitive to the far future.

The maximin criterion (Rawls criterion) selects the development path for which the poorest generation is richer than the poorest generation along all other possible paths. This criterion guarantees that utility is maintained at the level of the minimum value.
The maximin looks at the worst that could happen under each action and then chooses the action with the largest payoff. "The best of the worst".

Infinite sequence of utilities $(u_1, u_2, ...)$

We want to compare sequences in order to be

- efficient: no one can be made better off without making at least one individual worse off.
- equal treatment for all generations: indifference of the "position" of the generation in time

Which we are going to prefer?

$$(1, 0, 1, 0....) \quad \text{or} \quad (0, 0, 1, 0, 1...).$$

Efficiency and Equity

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Dilemma

Intertemporal equity

Marc Fleurbaey, Philippe Michel, "Intertemporal equity and extension of Ramsey criterion" (Journal of Mathematical Economics, 2003)
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- To examine social orderings applied to infinite intergenerational consumption paths
- To propose a detailed study of the dilemma between efficiency and impartiality, and present an axiomatic justification of the traditional extensions of Ramsey's criterion.

Introduction

The theory of optimal growth mainly relies on a social objective with a discount factor:

$$\sum_{t=1}^{\infty} \beta^t u(c_t)$$

where c_t is the consumption of the generation living in t.
Reasons for introducing a discount factor β? Uncertainty about the future.

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What should the objective be when no uncertainty affects the existence of future generations?

Introduction

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Ramsey's criterion has nice properties of efficiency and impartiality. But the problem with this criterion is that it works well only on the subset of growth paths such that this series converges.

Framework

Consider a model with successive generations, each generation living exactly one period. Time is discrete, starting with period 1. $c = (c_1, c_2, ...)$, c_t, for $t \geq 1$ denotes the average consumption of generation t, X is the set of infinite stream of generational well-being.

The population size is assumed to be given and constant over time.

A typical binary relation will be denoted R , with symmetric and asymmetric parts denoted I and P ,
$c_{t:t'} = (c_t, c_{t+1}, ..., c_{t'})$, $c_{t:\infty} = (c_t, c_{t+1}, ...)$.
Given two sequences x and y ,

$$x \geq y \iff \forall t, \quad x_t \geq y_t$$

$$x > y \iff \forall t, \quad x_t \geq y_t, \quad x \neq y$$

$$x >> y \iff \forall t, \quad x_t > y_t$$

Framework

Given a binary relation R, a time T and a sequence c,

$$(x_1, ..., x_T) R_{c,T} (y_1, ..., y_T) \iff (x_1, ..., x_T, c_{T+1,:\infty}) R (y_1, ..., y_T, c_{T+1:\infty}).$$

- Transitivity:

$$\forall x, y, z, \quad xRy \quad \text{and} \quad yRz \Rightarrow xRz.$$

- Completeness:

$$\forall x, y, \quad xRy \quad \text{or} \quad yRx.$$

A quasi-ordering is a reflexive and transitive binary relation.
An ordering is a complete quasi-ordering.

Framework

Suppose there is a quasi-ordering R (e.g. based on the Ramsey criterion) which is unsatisfactory because it is not complete. How can one construct an ordering R' such that $R \subset R'$? The difficulty: R' may not preserve some desirable properties of R.

We can find a complete ordering

Although important, this result is not totally satisfactory since one would like to be able to construct the ordering R in an explicit way. The proof (Szpilrajn's lemma) makes use of the axiom of choice.

Efficiency

- weak Pareto : $\forall x, y, \quad x >> y \Rightarrow xPy$.
- strong Pareto : $\forall x, \quad x \geq y \Rightarrow xRy$ and $x > y \Rightarrow xPy$.

In a "strong Pareto" allocation, no one can be made better off without making at least one individual worse off. A change to a different allocation that makes at least one individual better off without making any other individual worse off is called a Pareto improvement. An allocation is defined as "Strong Pareto optimal" when no further Pareto improvements can be made.

A "weak Pareto" is an allocation for which there are no possible alternative allocations whose realization would cause every individual to gain. Thus an alternative allocation is considered to be a Pareto improvement only if the alternative allocation is strictly preferred by all individuals.

We will consider here that a relation that violates weak Pareto is unacceptable.
Some relations proposed in the literature do violate it. An example is the "medial limit" in Lauwers (1998). This criterion is based on special limits of

$$\frac{u(x_1) + ... + u(x_T)}{T}.$$

Impartiality between generations

In a fully certain framework all generations must receive the same attention from the social planner. This idea can be formulated with permutations: interchanging the consumptions of two generations should not change the social value of the consumption path.

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A diffculty with permutations is that in an infinite dimensional space, many different restrictions on permutations can be imagined to obtain a variety in the degree of impartiality one requires.

Impartiality between generations

- All permutations $\mathcal{P}$. Indifference to all permutations

$$\forall x, \forall \sigma \in \mathcal{P}, \quad x_{\sigma(t)} I x.$$

- Finite length permutations $\mathcal{P}_l$ (permutations such that the time distance between a generation and its permuted is bounded). Indifference to finite length permutations.
- Variable step permutations $\mathcal{P}_{vs}$ (permutations which only interchange finite disjoint subsets of contiguous generations).
- Fixed step permutations $\mathcal{P}_{fs}$ (permutations which only interchange disjoint subsets of k contiguous generations).
- Finite permutations $\mathcal{P}_f$ (permutations which only affect a finite number of generations).

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$$\mathcal{P}_f \subset \mathcal{P}_{fs} \subset \mathcal{P}_{vs} \subset \mathcal{P}_l \subset \mathcal{P}$$

Others conditions: Separability

It is very unlikely that individual intertemporal preferences are separable, but it seems reasonable that, concerning intergenerational justice, social preferences should be separable. It is usually admitted that a decision affecting only the present and future generations should not depend on the fixed fate of past generations.

Whether the growth rate was high or low in antiquity seems irrelevant to discussions about the desirable growth rate in the 21st century.

- Separable future: $\forall x, y, \forall t$,

$$(x_{1:t}, x_{t+1:\infty}) R (x_{1:t}, y_{t+1:\infty}) \iff (y_{1:t}, x_{t+1:\infty}) R (y_{1:t}, y_{t+1:\infty}).$$

Others conditions: Separability

One may actually argue that a decision concerning only generations from t on can be made as if present time (date 1) actually was in t . This gives the following, stronger condition:

- Independent future: $\forall x, y, \forall t$,

$$(x_{1:t}, x_{t+1:\infty})R(x_{1:t}, y_{t+1:\infty}) \iff (x_{t+1:\infty})R(y_{t+1:\infty}).$$

Two infinite sequences are ranked if all their finite subsequences are ranked in the same order.

- Limit ranking: $\forall x, y,$

$$[\exists c, \quad \forall T, (x_1, ..., x_T) R_{c,T} (y_1, ..., y_T)] \quad \Rightarrow \quad xRy.$$

- One may wish to strengthen this axiom by extending it to periodic comparisons of finite sequences:
  Limit ranking in fixed steps (periodic comparisons of finite sequences): $\forall x, y,$

$$[\exists c, \exists k \quad \forall T, (x_1, ..., x_{kT}) R_{c,kT} (y_1, ..., y_{kT})] \quad \Rightarrow \quad xRy.$$

Others conditions: Continuity

The choice of the topology with respect to which continuity is defined matters considerably and brings in arbitrariness.
Continuity with respect to some topologies conveys a substantial degree of myopia, especially when combined with efficiency conditions.

- Continuity in finite dimensional euclidean spaces:
  Finite continuity: $R_{c,T}$ is continuous over $\mathbb{R}^T$ for any c and T.
- The topology of absolute convergence: Continuity: $\forall x, y, x^n$

$$\left[\lim_{n\to\infty} \sum_{t=1}^{\infty} |x_t^n - x_t| = 0 \text{ and } \forall n,\ x^n R y (y R x^n)\right] \quad \Rightarrow \quad xRy(yRx).$$

- Strong continuity: $\forall x, y, x^n$

$$\left[\limsup_{n\to\infty} \sum_{t=1}^{\infty} |x_t^n - x_t| = 0 \text{ and } \forall n,\ x^n R y (y R x^n)\right] \quad \Rightarrow \quad xRy(yRx).$$

The efficiency-impartiality dilemma

Compatibilities between Pareto conditions and the various axioms of indifference to permutations. "NO" corresponds simply to dropping any requirement of indifference to permutations.

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Compatibilities between Pareto conditions and the various axioms of indifference to permutations. "NO" corresponds simply to dropping any requirement of indifference to permutations.

- Weak Pareto
  No - Finite - Fixed step - Variable step
- Weak Pareto + transitivity
  No - Finite - Fixed step
- Weak Pareto + transitivity + independent future
  No - Finite
- Weak Pareto + transitivity + independent future + limit ranking fixed steps
  No

The efficiency-impartiality dilemma

Weak Pareto, indifference to finite permutations, and strong continuity are incompatible.

Equal treatment of generations combined with sensitivity for the interests of each generation rules out explicitly defined preferences that can rank any pair of infinite well-being streams. Hence, either intergenerational social preferences must be incomplete or equal treatment and sensitivity must be compromised.

Most of the results of this literature fall into two categories:

- Impossibility results, showing that a given set of axioms is incompatible,
- Characterization results, establishing that a given set of axioms determines a particular class of social preferences.

1. If a social preference satisfies a given set of axioms, then it is in a particular class.
2. If a social preference is in a particular class, then it satisfies a given set of axioms.

Social welfare function

The social welfare function (SWF) is a function from the set of well-being streams (X), that numerically represents a reflexive, transitive and complete binary relation.

(SWF) is a mapping $W : X \to R$.

$$xRy \iff W(x) \geq W(y).$$

- W is monotone if

$$x \geq y \quad \Rightarrow \quad W(x) \geq W(y).$$

Social welfare function

Impartiality between generations also called anonymity when dealing with finite permutations: $W(x_\sigma) = W(x)$.

Welfare is sensitive to an increase in well-being for any generation and imposes equal treatment of all generations by requiring that the welfare is unchanged when the well-being levels of a finite number of generations are permuted.

Discounted Utilitarian criterion and Present-Future Conflicts

In the theory of economic growth and in the practical evaluation of economic policy with long-term effects it is common to apply the DU criterion. Discounted utilitarianism means that one infinite stream of well-beings is deemed better than another if and only if it generates a higher sum of utilities discounted by a constant discount $0 < \delta < 1$.

- Weak sensitivity:
  There exists x, y, z such that $(x_1, z_{2:\infty}) > (y_1, z_{2:\infty})$.
- Stationarity:

$$(z_1, x_{2:\infty})R(z_1, y_{2:\infty}) \iff (x_{2:\infty})R(y_{2:\infty}).$$

Strong Pareto implies monotonicity and weak sensitivity.
Stationarity and separable future implies time consistency: a plan is time consistent if the passage of time alone gives no reason to change it.

The following two statements are equivalent.

- A reflexive, transitive and complete binary relation satisfies Monotonicity, Weak sensitivity, Separable future, Separable present, Stationarity
- There exists W satisfying, for some nondecreasing and continuous utility function, $U : [0,1] \to \mathbb{R}$, with $U(0) < U(1)$ and $0 < \delta < 1$,

$$W(x) = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} U(x_t).$$

BUT

Discounted Utilitarianism is a dictatorship of the present.

NOW THE PROBLEM IS:

How define a "good" trade-off in present-future conflicts:

- Present generation makes a sacrifice for the infinite number of better-off future generations
- Infinite number of future generations makes a sacrifice for the benefit of the better-off present generation.

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No Dictatorship of the Present and the Future.

Chichilnisky's criterion "An axiomatic approach to sustainable development", (Social Choice and Welfare, 1996).

No Dictatorship of the Present and the Future.

- Axiom DP (dictatorship of the present): $\forall x, y, z, v$,

$$x > y, \quad \Longleftrightarrow \quad \exists T', \ / \ (x_{1:T}, z_{T+1:\infty}) > (y_{1:T}, v_{T+1:\infty}), \ \forall T \geq T'.$$

- Axiom NDP (no dictatorship of the present): Axiom DP does not hold.

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- Axiom DF (dictatorship of the future): $\forall x, y, z, v,$

$$x > y, \quad \Longleftrightarrow \quad \exists T', \ / \ (z_{1:T}, x_{T+1:\infty}) > (v_{1:T}, y_{T+1:\infty}), \ \forall T \geq T'.$$

- Axiom NDF (no dictatorship of the future): Axiom DF does not hold.

No Dictatorship of the Present and the Future. Comments

When axiom of no dictatorship of the present is satisfied, it is not only what happens before some finite time that matters, whereas when axiom of no dictatorship of the future is satisfied, it is not only what happens beyond some finite time that matters.

The axiom of no dictatorship of the present rules out DU criteria.

The axiom of no dictatorship of the future is implied by Strong Pareto.

No Dictatorship of the Present and the Future. Characterization

The following two statements are equivalent.

- A reflexive, transitive and complete binary relation satisfies, Strong Pareto, Strong continuity, Separable future, Separable present, and No Dictatorship of the Present.
- There exists W satisfying, for some sequence U_t of increasing and continuous utility functions, and some asymptotic part $\phi : X \to R$ which is an integral with respect to a purely finitely additive measure

$$W(x) = \sum_{t=1}^{\infty} U(x_t) + \phi(x).$$

No Dictatorship of the Present and the Future. Some comments

GOOD NEWS

- This characterization is based on Chichilnisky (1996, Theorem 2) By choosing, for each t

$$U_t(x_t) = \delta^{t-1} x_t, \text{ for some discount factor } 0 < \delta < 1,$$

$$\phi(x) = \lim \inf_{t \to \infty} x_t.$$

- The asymptotic part, $\phi(x)$ ensures that a sustainable preference is sensitive to what happens in the infinite future and thereby entails that the SWF is not a dictatorship of the present.

No Dictatorship of the Present and the Future. Some comments

BAD NEWS

- When applied to models of economic growth, there is a generic nonexistence problem, as welfare is increased by delaying the response to the interests of the infinite far future, whereas welfare is decreased if delay is infinite.
- It is not time consistent if social preferences are time invariant.

Some Asheim's paper conclusions

Axiomatic analyses of intergenerational equity and other systematic normative discussions of intergenerational distribution may promote normative reflection about intergenerational equity in society at large. And can give some answers to the following questions because we understand better the relations between axioms and social welfare functions.

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- Is axiomatic analysis of intergenerational equity relevant?
- Why does the present generation accumulate assets and conserve natural assets for the benefit of future generations? Generations overlap and members of each generation may be motivated to bequeath assets to their children, facilitating intergenerational transfers... People may assign intrinsic value to nature...
- What is a good intergenerational distribution?

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Can any of the axiomatized SWRs be implemented?

The key tool to prove Chichilnisky characterization

Let (S, Σ)denote the field of all subsets of a set S with the operations of unions and intersections of sets. A real valued, bounded additive set function on (S, Σ) is one which assigns a real value to each element of (S, Σ), and assigns the sum of the values to the union of two disjoint sets.

A real valued bounded additive set function is called countably additive if it assigns the countable sum of the values to a countable union of disjoint sets.

Probability measures on the real numbers are countably additive functions: Any sequence of positive real numbers a_i, $\sum_i a_i < \infty$ defines a countably additive measure on the integers Z.
$\mu(A) = \sum_{i \in A} a_i, \forall A \subset Z$.

The key tool to prove Chichilnisky characterization

A real valued bounded additive set function ϕ on (S, Σ) is called purely finitely additive if $\forall$ countable additive function v

$$v(A) \leq \phi(A) \quad \Rightarrow \quad v(A) = 0.$$

The representation theorem of Yosida and Hewitt

says that every non-negative, bounded, additive set function μ on (S, Σ), can be decomposed into the sum of a non-negative measure μ_1 and a purely finitely additive, non-negative set function μ_2.

$$\mu = \mu_1 + \mu_2.$$

$$W(x) = \sum_{t=1}^{\infty} U(x_t) + \phi(x).$$

The key tool to prove Chichilnisky characterization

Typical purely finitely additive set functions on the field of all subsets of the integers are

- the lim inf function on l_∞

$$\phi(x) = \liminf_{t \to \infty} x_t.$$

- The "long run averages" function

$$\phi(x) = \lim_{K,N \to \infty} \left(\frac{1}{K} \sum_{t=N}^{K+N} x_t \right).$$

Sustainable use of renewable resources. An example in continuous time of the use of different criteria

Graciela Chichilnisy, Geoffrey Heal, and Allessandro Vercelli "Sustainability : Dynamics and Uncertainty" (1998).

Optimal use patterns for renewable resources (fisheries, forests, soils, clean water, landscapes, capacities of ecosystems to assimilate and degrade wastes ...)
All of these have the capacity to renew themselves, but in addition all can be overused to the point where they are irreversibly damaged. Picking a time-path for the use of such resources is clearly important: indeed, it seems to lie at the heart of any concept of sustainable economic management.

Dynamics of the resource

As the resource is renewable, its dynamics are described by

$$\dot{s} = r(s) - c.$$

r is the growth rate of the resource, assumed to depend only on its current stock.

In general, r is a concave function which attains a maximum at a finite value of s, and declines thereafter.
In the field of population biology, $r(s)$ is often taken to be quadratic, in which case an unexploited population grows logistically.

The discounted utilitarian criterion

We maximize the discounted integral of utilities from consumption and from the existence of a stock

$$\max_c \int_0^\infty u(c,s)e^{-\delta t}dt, \quad \dot{s} = r(s) - c, \quad s(0) \text{ given.}$$

The Hamiltonean is:

$$H = u(c,s)e^{-\delta t} + \lambda e^{-\delta t}(r(s) - c).$$

FOC's conditions are

$$u_c(c,s) = \lambda.$$

$$\frac{d}{dt}(\lambda e^{-\delta t}) = -\frac{\partial H}{\partial s} = -[u_s(c,s)e^{-\delta t} + \lambda e^{-\delta t} r'(s)].$$

The discounted utilitarian criterion

For simplification in the example we consider

$$u(c, s) = u_1(c) + u_2(s).$$

In this case the solution of our problem is characterized by the following equations

$$u_1'(c(t)) = \lambda(t)$$
$$\dot{s}(t) = r(s(t)) - c(t)$$
$$\dot{\lambda}(t) - \delta\lambda(t) = u_2'(s(t) + \lambda(t)r'(s(t)).$$

The stationary solution.

At a stationary solution, by definition s is constant so that $r(s) = c$ and the shadow price λ is constant

The stationary solutions is:

$$r(s) = c$$

$$\frac{u_2'(s)}{u_1'(c)} = \delta - r'(s).$$

The first equation tells us that a stationary solution must lie on the curve on which consumption of the resource equals its renewal rate. The second gives us a relationship between the slope of an indifference curve in the $c - s$ plane and the slope of the renewal function at a stationary solution.

The discounted utilitarian criterion. The optimal solution.

By linearizing the system around the stationary solution, one can show that this solution is a saddle point and we can establish the following result:

Given any initial value of the stock s_0, there is a corresponding value of c_0 which will place the system on one of the stable branches leading to the stationary solution. The position of the stationary solution depends on the discount rate, and moves to higher values of the stationary stock as this decreases. As $\delta \to 0$, the stationary solution tends to a point satisfying $u_2'/u_1' = -r'$.

The discounted utilitarian criterion. The optimal solution.

Make a picture with the diagram of dynamics in the $c - s$ plane for the utilitarian solution

FOC's give necessary conditions for a path to be optimal.
Given concavity assumptions in $u(c, s)$ and $r(s)$, one can invoke standard arguments to show that these conditions are also sufficient.

The Green Golden Rule criterion. The optimal solution.

What configuration of the economy gives the maximum sustainable configuration of the economy: with sustainable values of consumption and of the stock? The solution of

$$\max_{c}[u_1(c) + u_2(s)], \quad \text{such that} \quad \dot{s} = r(s) - c = 0.$$

The Golden Rule satisfies

$$\frac{u_2'(s)}{u_1'(c)} = -r'(s), \quad r(s) = c.$$

At this point, the slope of an indifference curve equals that of the renewal function, so that the marginal rate of substitution between stock and flow equals the marginal rate of transformation along the curve $r(s)$.

The Green Golden Rule criterion. The optimal solution.

Note that this kind of criterion of optimality only determines the limiting behavior of the economy: it does not determine how the limit is approached.

This clearly is a weakness: of the many paths which approach the green golden rule, some will accumulate far more utility than others. One would like to know which of these is the best, or indeed whether there is such a best.

An interesting fact is that the green golden rule, and also for low enough discount rates the utilitarian solution, require stocks of the resource which are in excess of the maximum sustainable yield, which is the stock at which the maximum of $r(s)$ occurs.

This is important because only resource stocks in excess of that giving the maximum sustainable yield are stable under the natural population dynamics of the resource: they are ecologically stable.

Ecological Stability

Consider a fixed and constant depletion rate d, so that the resource dynamics is just

$$\dot{s(t)} = r(s(t)) - d.$$

For $d < max_s r(s)$, there are two values of s which give stationary solutions to this equation.

Only the stock to the right of the maximum sustainable yield is stable under the natural population adjustment process.

High discount rates and utility functions such that the stock of the resource is not an argument of the utility function, will give utilitarian optimal policies with stationary stocks below the maximum sustainable yield.

The maximin (Rawlsian) optimal solution

Remind that this criterion maximizes the poorest generation

$$\max_c \min_t U(c(t), s(t), \quad \dot{s} = r(s) - c.$$

- For an initial resource stock s_0 less than or equal to that associated with the green golden rule, the Rawlsian optimum is such that $c = r(s_0)$ for ever.
- For s_0 greater than the green golden rule stock, the green golden rule is a Rawlsian optimum.

The Chichilnisky criterion

$$\max_c \left[\int_0^\infty u(c,s) f(t) dt + \lim_{t \to \infty} u(c,s) \right], \quad \dot{s} = r(s) - c, \quad s(0) \text{ given.}$$

where $f(t)$ is a finite countably additive measure.

We can prove

- If $f(t) = e^{-\rho t}$, there is no solution to the overall optimization problems.
- There is a solution only if $f(t)$ takes a different, non-exponential form, implying a non-constant discount rate which tends asymptotically to zero.

Idea of the proof

- Consider the problem without the limiting part,
- Pick an initial value of c, say c_0, below the path leading to the saddle-point, and follow the path from c_0 satisfying the utilitarian necessary conditions,
- Follow this path until it leads to the resource stock corresponding the green golden rule, i.e. until a time t'
- At $t = t'$ increase consumption to the level corresponding to the green golden rule.

No existence for the Chichilnisky criterion for $f(t) = e^{-\rho t}$

Any such path will satisfy the necessary conditions for utilitarian optimality up to time t' and will lead to the green golden rule in finite time.

It will therefore attain a maximum of the term $\lim_t t, u(c, s)$ over feasible paths. However, the utility integral which constitutes the first part of the maximand can be improved by picking as lightly higher initial value co for consumption, again following the first order conditions for optimality and reaching the green golden rule slightly later than t'.

This does not detract from the second term in the maximand. By this process it will be possible to increase the integral term in the maximand without reducing the limiting term and thus to approximate the independent maximization of both terms in the maximand.

Existence for the Chichilnisky criterion for declining discount rate

We have noted before, that for the discounted utilitarian case, as the discount rate goes to zero, the stationary solution goes to the green golden rule.

We shall consider a modified objective function

$$\max_c \left[\int_0^\infty u(c,s)\Delta(t)dt + \lim_{t\to\infty} u(c,s) \right], \quad \dot{s} = r(s) - c, \quad s(0) \text{ given.}$$

where $\Delta(t)$ is the discount factor at time t, $\int \Delta(t)dt$ is finite, the discount rate $q(t)$ at time t is the proportional rate of change of the discount factor :

$$q(t) = -\frac{\dot{\Delta}(t)}{\Delta(t)},$$

and

$$\lim_{t\to\infty} q(t) = 0.$$

Existence for the Chichilnisky criterion for declining discount rate

The Chichilnisky criterion with declining discount rate as defined before has a solution

and it is the solution of the utilitarian criterion with the same declining discount rate.

Chichilnisky criterion is not time-consistent

Consider a solution to an intertemporal optimization problem which is computed today and is to be carried out over some future period of time starting today.
Suppose that the agent formulating it may at a future date recompute an optimal plan, using the same objective and the same constraints as initially but with initial conditions and starting date corresponding to those obtaining when the recomputation is done.
Then we say that the initial solution is time consistent if this leads the agent to continue with the implementation of the initial solution.
Another way of saying this is that a plan is time consistent if the passage of time alone gives no reason to change it.

The solution to the problem of optimal with a time-varying discount rate, is not time-consistent.

A satisfying use of Chichilnisky's criterion

Charles Figuières, Mabel Tidball, "Sustainable exploitation of a natural resource: a satisfying use of Chichilnisky's criterion " Econ Theory, 2012.

- Chichilnisky's criterion for sustainability has the merit to be, so far, the unique explicit, complete and continuous social welfare criterion that combines successfully the requirement of efficiency with an instrumental notion of intergenerational equity.
- But when applied in the context of renewable resources, and with a constant dis- count factor, there exists no exploitation path that maximizes this criterion.
- A way to cope with this problem: to restrict the set of admissible controls.

A restricted optimal solution

$$\max_{c_t} \left[\theta \sum_{t=0}^{\infty} \beta^t U(c_t, x_t) + (1-\theta) \lim_{t\to\infty} U(c_t, x_t) \right],$$

$$x_{t+1} = G(x_t - c_t), \quad x_0 \quad \text{given.}$$

Chichilnisky's criterion: To increase the initial and subsequent consumptions would increase the first part of the maximand without detracting to the second part. To postpone the switching date makes it possible to increase the first part while maintaining the value of the second part.

This can be seen as a lack of compactness in admissible consumptions, that could approach but not reach exactly upper bound levels.

A restricted admissible domain

- The possibility of non stationarity of controls creates the problem. This suggests to restrict the space of admissible controls to stationary controls.
- Varying the weight the problem becomes arbitrarily close to either the discounted utilitarian problem or the green golden rule problem, i.e. two problems for which optimal and stationary solutions generally exist. We want a restriction that encompasses both programs.

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- Varying the weight the problem becomes arbitrarily close to either the discounted utilitarian problem or the green golden rule problem, i.e. two problems for which optimal and stationary solutions generally exist. We want a restriction that encompasses both programs.

We propose the space of convex combinations between the optimal discounted utilitarian program and stationary programs leading to the green golden rule.

A restricted admissible domain: The idea

We assume some "usual" hypothesis:

- the transition function G is compact-valued, continuous, strictly increasing and concave.
- $F(x_t, x_{t+1}) = U(x_t - G^{-1}(x_{t+1}), x_t)$ is strictly concave.
- The Inada-like condition (This assumption rules out $c_t = 0$ as a possibility for the golden rule consumption).

A restricted admissible domain: The idea

We can prove:

- there exists a unique and continuous policy solution $c_t^{DU} = \phi_{DU}(x_t)$ for the discounted utilitarian problem.
- there exists a unique interior solution to the green golden rule problem and a linear (hence continuous) policy function $c_t^{GGR} = \phi_{GGR}(x_t)$ such that the economy converges towards the green golden rule.
- Any convex combination $c_t^{\gamma} = \gamma\phi_{DU}(x_t) + (1-\gamma)\phi_{GGR}(x_t)$ with $\gamma \in [0, 1]$ is also an admissible path.

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There exists a convex combination of the discounted utilitarianism and of the green golden rule, among the set of such combinations, that maximizes Chichilnisky's criterion.

Some remarks

- Assumptions are sufficient but not necessary.
- Simple and general approach to find restricted optimal solutions in concrete examples:
    1. compute the discounted utilitarian optimum and the green golden rule,
    2. find out the optimal convex combination between the two. Numerical procedures to carry on the second step can be very simple. For instance, one can try a finite number of values for γ, chosen on a pre-specified grid.
- the restricted optimal approach could be applied in more general contexts
- We can not guarantee time consistency (but we are going to have it in the following example).

The restricted solution: an example

$$U(c_t, x_t) = \ln c_t + \pi \ln x_t, \quad x_{t+1} = (x_t - c_t)^{\alpha}.$$

the associated discounted utilitarian problem gives the following linear feedback solution

$$c_t = \frac{1 - \alpha\beta}{1 + \pi\alpha\beta}\, x_t$$

the associated golden rule problem has a solution in a linear feedback form

$$x^{GGR} = \left[\frac{\alpha(1+\pi)}{1+\pi\alpha}\right]^{\frac{\alpha}{1-\alpha}}, \quad c^{GGR} = \frac{1-\alpha}{1+\pi\alpha}\, x.$$

The optimal linear feedback for Chichilnisky's criterion

Consider $c_t = \mu x_t$, we can easily compute the solution of the dynamic evolution

$$x_t = (1-\mu)^{\alpha+\alpha^2\dots+\alpha^t} x_0^{\alpha^t},$$

then we can compute explicitly

$$F(\mu) = \theta \sum_{t=0}^{\infty} \beta^t U(\mu x_t, x_t) + (1-\theta) \lim_{t\to\infty} U(\mu x_t, x_t),$$

and find the optimal μ. Moreover

$$\mu = \gamma \frac{1-\alpha\beta}{1+\pi\alpha\beta} + (1-\gamma)\frac{1-\alpha}{1+\pi\alpha},$$

for some γ, where γ is an increasing function of θ.

Properties of the optimal linear feedback solution

- The higher (resp. lower) θ, the closer the restricted solution to the discounted utilitarian control (resp. green golden rule).
- When $\theta \to 1$ and $\beta \to 1$, one can check that this is indeed the optimal extraction path according to Ramsey's criterion

$$\sum_{t=0}^{\infty} \left[U(c_t, x_t) - U(c^{GGR}, x^{GGR}) \right]$$

.

- When $\theta \to 0$ we obtain the Golden Rule solution
- The solution is time consistent

Another mixed criterion for intergenerational equity

Francisco Alvarez-Cuadrado, Ngo Van Long, "A mixed Bentham-Rawls criterion for intergenerational equity: Theory and implications" JEEM, 2009.

Welfare criterion that balances the need for development and the concern for the least advantaged generations. It is a weighted average of two terms: (a) the sum of discounted utilities and (b) the utility level of the least advantaged generation.

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- derive necessary conditions to characterize growth paths that satisfy this criterion,
- show that in some models with familiar dynamic specifications, an optimal path exists and displays appealing characteristics.

Bentham-Rawls criterion for intergenerational equity

This welfare criterion preserves the essence of Chichilnisky's criterion and at the same time yields an optimal path.

Bentham-Rawls criterion for intergenerational equity

This welfare criterion preserves the essence of Chichilnisky's criterion and at the same time yields an optimal path.

Chichilnisky did not obtain this criterion because she postulated an extra axiom "independence or linearity". This axiom says that if society is willing to trade an increase, say by d "utility units," in the utility of generation t_1 for a decrease of e "utility units," in the utility of generation t_2 , then this ratio d/e should remain the same regardless of the utility levels of the two generations. This criterion, by giving more weight to the least advantaged generation, does not satisfy this independence axiom.

$$W(u) = (1-\theta)\sum_{t=0}^{\infty}\beta^t u_t + \theta \inf\{u_0, u_1 \dots u_n \dots\}.$$

Another axiom: Non dictatorship of the least advantaged.
A welfare is said to display dictatorship of the least advantaged if for any u^i, u^j

$$W(u^i) > W(u^j) \iff \inf\left\{u_0^i, u_1^i \dots u_n^i \dots\right\} > \inf\left\{u_0^j, u_1^j \dots u_n^j \dots\right\}.$$

Non dictatorship of the future: because we have the first term

A proof for Non dictatorship of the present.

Infinite horizon optimization under the Bentham-Rawls criterion. The continuous time case

Suppose the time horizon is infinite and the rate of discount ρ is a positive constant. Then the social planner chooses $\bar{u}$ and $c(t)$ to maximize the mixed objective function:

$$\theta\bar{u}+(1-\theta)\int_{t=0}^{\infty} u(c,x)e^{-\rho t}dt \quad \text{such that} \quad \dot{x}=g(x,c), \quad u(c,x)\geq \bar{u}.$$

Roughly speaking

$$L=\theta\bar{u}\rho+(1-\theta)u(c,x)+\lambda g(x,c)+\mu(u(c,x)-\bar{u}).$$

Usual FOC's + optimality condition with respect to the control parameter $\bar{u}$

$$\int_{t=0}^{\infty}\frac{\partial L}{\partial \bar{u}}dt=0.$$

Optimal solution of the Bentham-Rawls criterion. An example

$$\theta\bar{u}+(1-\theta)\int_{t=0}^{\infty}u(c)e^{-\rho t}dt \quad \text{such that} \quad \dot{x}=G(x)-c, \;\; u(c)\geq\bar{u}.$$

Under the MBR criterion, does the optimal path approach a steady state that is somewhere between the DU stock level, x^{DU} , and the golden rule stock level, x^{GGR} ?

Optimal solution of the Bentham-Rawls criterion. An example

- If $x_0 > x^{DU}$, the optimal path consists of two phases. Phase I begins at $t = 0$ and ends at some finite t. During Phase I, the utility level and the resource stock are both falling. At time t, the pair (x, c) reaches and stays forever in a mixed Bentham-Rawls steady state pair (x_{mbr}, c_{mbr}) with $x^{DU} < x^{mbr} < x^{GGR}$.
- If $x_0 < x^{DU}$, the optimal path also consists of two phases. Phase I begins at $t = 0$ and ends at some finite t. During Phase I, utility is constant, which implies a time path of constant harvest rate and rising stock. In Phase II, the economy follows the standard utilitarian path approaching asymptotically the utilitarian steady state x^{DU}.

The Bentham-Rawls criterion. Some properties

Charles Figuières, Ngo Van Long, Mabel Tidball, " The rawlsian properties of the MBR criterion for intergenerational equity" Work in progress.

$$\theta\bar{u} + (1-\theta)\int_{t=0}^{\infty} u(c)e^{-\rho t}dt \quad \text{such that} \quad \dot{x} = g(x,c), \;\; u(c) \geq \bar{u}.$$

We extend the result of R. Hartl, "A simple proof of monotonicity of the state trajectories in autonomous control problems", JET, 1987.

Let $c^*(t)$ and $x^*(t)$ be a solution to the MBR problem. Assume that $c^*(t)$ is not constant and $x^*(t)$ is unique. Then $x^*(t)$ is a monotonous function.

The Bentham-Rawls criterion. Some properties

Can we say something about consumption?

Assumption stock x is a productive asset: For any pair of points in time (t_a, t_b), and any non-negative stock level a; let $c^*(t)$ be a feasible trajectory in the time interval $[t_a, t_b]$ such that

$$\dot{x} = g(x, c^*), \quad x(t_a) = a, \quad x(t) \geq 0, \forall t \in [t_a, t_b],$$

and let b be the resulting stock size at time t_b, then, for any $\epsilon > 0$, there exists a feasible path c_ϵ such that

$$c_\epsilon(t) \geq c^*(t), \quad \forall t \in [t_a, t_b],$$

and

$$\dot{x} = g(x, c_\epsilon), \quad x(t_a) = a + \epsilon, x(t_b) = b, \quad x(t) \geq 0, \forall t \in [t_a, t_b].$$

The Bentham-Rawls criterion. Some properties of optimal consumption

Let $c^*(t)$ and $x^*(t)$ be a solution to the MBR problem. Assume that $c^*(t)$ is not constant and $x^*(t)$ is unique. If the stock x is a productive asset then:

- If $x^*(t)$ is non-constant, and weakly-increasing over time, then the poorest generations cannot be at the end of the sequence,
- If $x^*(t)$ is non-constant, and weakly-decreasing over time, then the poorest generations cannot be at the beginning of the sequence.

The Bentham-Rawls criterion. Monotonicity of optimal consumption

Let $(x^*(t), c^*(t))$ be the unique solution starting from some x_0. Suppose $x^*(t)$ is non-constant and weakly increasing. Then there exists a finite time T such that after time T the solution $(x^*(t), c^*(t))$ is the solution of the utilitarian program

$$\max_c \int_{t=T}^{\infty} u(c)e^{-\rho t}dt, \quad \dot{x} = g(x,c), \quad x(T) = x^*(T),$$

in particular

$$c^*(t) > \underline{c}, \quad \forall t \geq T.$$

The Bentham-Rawls criterion. Monotonicity of optimal consumption

Under the same hypothesis , if $g(x, c)$ is concave in (x, c), with $g_c < 0$ and $g_{xc} \geq 0$ and $u(c)$ is concave, then after time T the time path $c^*(t)$ is weakly increasing.

Inequality among generations

Call $\overline{c} = \max_t \{c(t)\}$ and let

$$I(c(t)) = \overline{c} - \underline{c},$$

be an indicator of inequality.

Under the above hypothesis, let $c^{DU}(t)$ be the solution to the discounted utilitarian program. Suppose $x^{mbr}(t)$ is weakly increasing. Then

$$I(c^{mbr}(t)) < I(c^{DU}(t)).$$

Inequality among generations, is lower under the MBR criterion than under the discounted utilitarian criterion.

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$$I(c^{mbr}(t)) < I(c^{DU}(t)).$$

Inequality among generations, is lower under the MBR criterion than under the discounted utilitarian criterion.

These properties are proved in "continuous time".

DU, GGR, Maximin and Bentham-Rawls criteria. The discrete time case in a simple example

The Bentham-Rawls problem that we want to solve:

$$W^{mbr} = (1-\theta)\sum_{t=0}^{\infty} \beta^t c_t + \theta \underline{c}, \quad x_{t+1} = (x_t - c_t)^{\alpha}.$$

DU, GGR, Maximin and Bentham-Rawls criteria. The discrete time case in a simple example

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The solution of DU ($\theta = 0$), $x^{DU} = (\alpha\beta)^{1/(1-\alpha)}$:

$$c_t^{DU} = \begin{cases} 0 & \text{if } x_t < x^{DU} \\ x_t - x^{DU} & \text{if } x_t \geq x^{DU} \end{cases} \qquad x_t^{DU} = \begin{cases} (x_0)^{\alpha t} & \text{if } t = 0..t_0 \\ x^{DU} & \text{if } t > t_0 \end{cases}$$

t_0 such that $(x_0)^{\alpha t_0} > x^{DU}$.

The GGR problem

$$\max_{0 \le c \le x} c, \quad \text{such that,} \quad x = (x - c)^{\alpha}.$$

The solution

$$x^{GR} = \alpha^{\frac{\alpha}{1-\alpha}}, \quad c^{GR} = x^{GR} - (x^{GR})^{\frac{1}{\alpha}}.$$

The maximin problem is:

$$\max \bar{c}, \quad x_{t+1} = (x_t - c_t)^\alpha, x_0, \text{ given }, \quad x_t \geq 0, \quad c_t \geq \bar{c} \geq 0.$$

Note that boundary conditions ($\bar{c} = 0$ or $c_t = x_t$) imply $\bar{c} = 0$. We analyse interior solutions using

$$L = \bar{c} + \sum_{t=0}^{\infty} \omega_t (c_t - \bar{c}) + \sum_{t=0}^{\infty} \lambda_t ((x_t - c_t)^\alpha - x_{t+1});$$

The solution

- $x_0 > x^{GGR}$ then $c = c^{GGR}$ and the stock goes to x^{GGR};
- $x_0 < x^{GGR}$ then $x_t = x_0$ for all t.

The MBR problem when the economy starts "rich" ($x_0 \geq x^{DU}$)

The solution of MBR problem

- If $x^H \geq x_0 \geq x^{DU}$ then $x_t = x_0 \quad \forall t$.
  Moreover, when $\theta \to 1$, $x^H \to x^{GGR}$ and when $\theta \to 0$, $x^H \to x^{DU}$.

- If $x_0 > x^H$ then the policy made of one jump downward to x^H and then stay there for ever.

The MBR problem when the economy starts "poor" $(x_0 < x^{DU})$

The solution of MBR problem

- If $x_0 < x^{DU}$ then it is never optimal to put $x_t = x_0 \quad \forall t$.
- If $x_0 < x^{DU}$ there exist $t_1 = t_1(x_0)$ decreasing in x_0 such that

$$0 \leq \underline{c} = c_0 = \ldots c_{t_1 - 1} < c_{t_1} < c^{DU} = \ldots c_\infty.$$

The MBR problem. Conclusion

- Existence of optimal solution. In particular MBR verifies "usual" FOC's
- Maximin criterion does not result in an acceptable normative prescription for poor economies, since its focus on equality implies no saving, hence no growth, and does not permit to reach the threshold at which basic liberties can be realized. The pattern obtained under the MBR intergenerational welfare criterion gives an expansion in two stages, with an accumulation phase followed by a cruise phase. In the cruise phase, equality is justified again.

Other approaches

- Viability: The viability approach, contrary to optimisation approaches, designates not a single optimal paths, but all the paths that satisfy a set of ”sustainability constraints” at all time.

Other approaches

- Viability: The viability approach, contrary to optimisation approaches, designates not a single optimal paths, but all the paths that satisfy a set of "sustainability constraints" at all time.
- Rolling horizon procedure: Optimisation problem in finite time horizon T. We take the solution in the first period, we reactualise data and we restart the optimisation problem for finite horizon T.