

MABIES Program at IHP

Institut Henri Poincaré
quarterly thematic program MABIES
Mathematics of Bio-Economics
2013, January 7 - April 5

Michel DE LARA, LUC DOYEN



11 rue Pierre et Marie Curie
75005 Paris

Mathematics of Bio-Economics

January 7th - April 5th 2013

Organized by Michel De Lara and Luc Doyen

Sustainable Management of Renewable Resources

Biodiversity Scenarios Modelling

Mathematics, Ecology and Economics



CIRM Marseille S-14/04
Programme next trimester
(to be confirmed)
Stochastic control for
management of renewable
energies

Mathematics
Planet Earth 2013

Workshops

Mathematics and Ecological Economics 11-15/02
Risk and Learning in Biodiversity Management 4-8/03
Spatial Management of Biodiversity 25-29/03

Programme coordinated by the Centre Emile Borel of IHP

Registration is free however mandatory on <http://www.ihp.fr>

Participation of postdocs and Ph.D. students is strongly encouraged

Deadline for financial support: June 25th, 2012

For further informations, contact Claire Béranger mabies@ihp.fr

Scientific programme on <http://cermics.enpc.fr/~delara/MABIES/MABIES/>

It is sponsored also by: Fondation Sciences Mathématiques de Paris



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Dynamical Systems

Introduction & Bifurcations

Gerard Olivar

ABC Dynamics - PCI

<http://www.manizales.unal.edu.co/gta/abcdynamics>

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CeiBA Complejidad

Universidad Nacional de Colombia, Sede Manizales

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Simple model in Sustainable development

Resource dynamics (forest):

$$\frac{dS}{dt} = \rho \left(\frac{S}{k} - 1 \right) \left(1 - \frac{S}{K} \right) S - \alpha \beta LS$$

- **Natural growth**
- Resource profit

Population dynamics:

$$\frac{dL}{dt} = (\gamma \lambda (1 - \beta)^\delta L^{\delta-1} + \phi \alpha \beta S - \sigma) L$$

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● Resource profit

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● Rents

● Poverty threshold

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Phase portrait

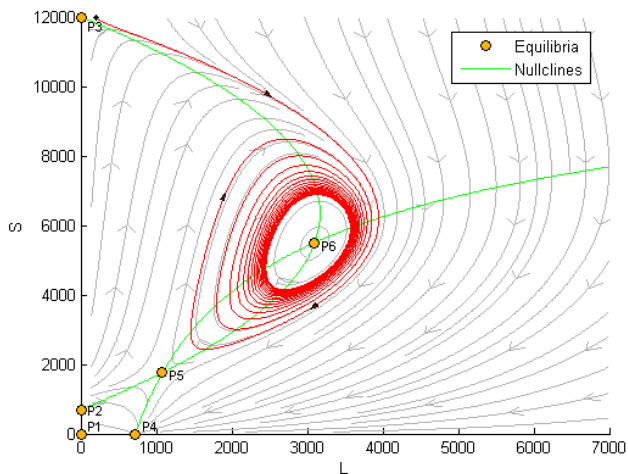


Figure : Equilibrium points and nullclines. P_4 is asymptotically stable while P_6 is unstable and P_5 is a saddle

Basins of attraction

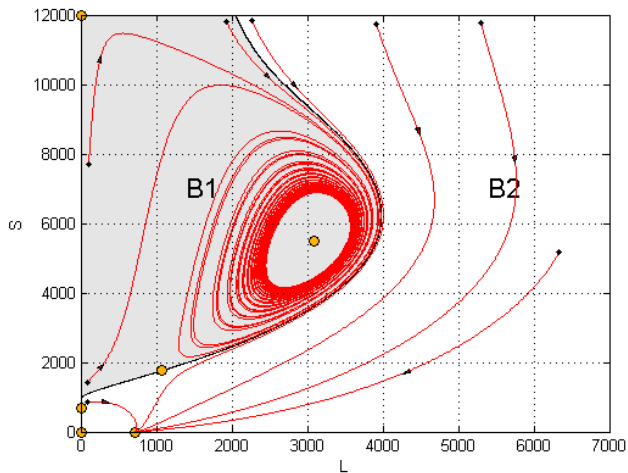
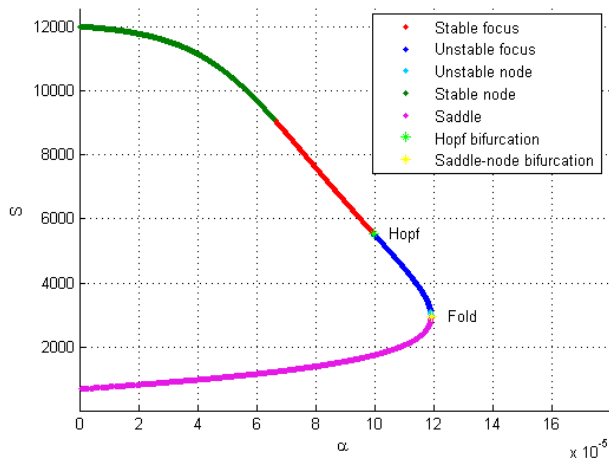
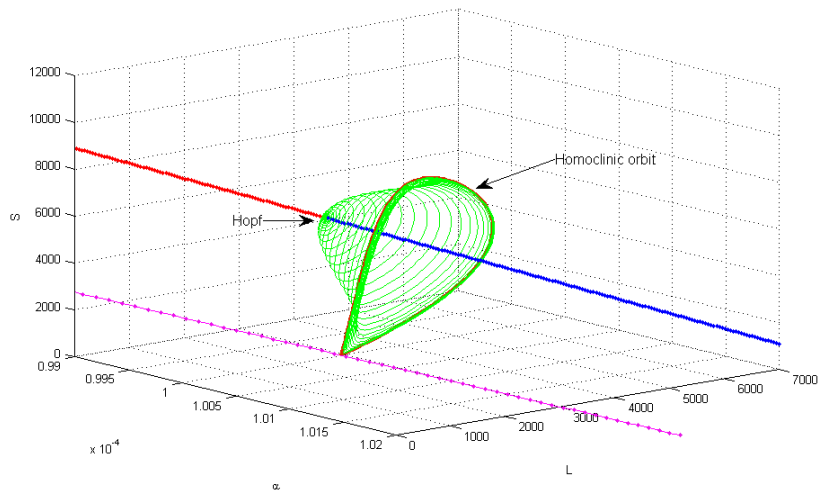


Figure : Basins

Continuation of Equilibria

Figure : Continuation with parameter α

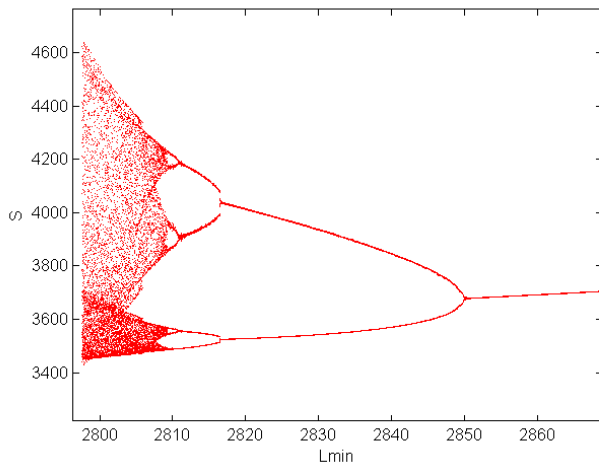
Hopf Bifurcation



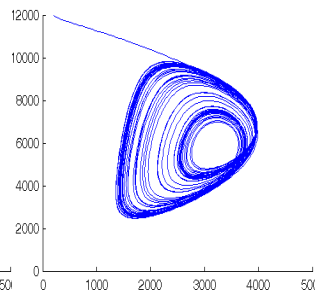
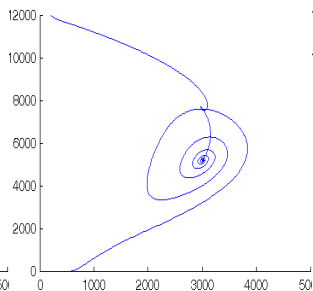
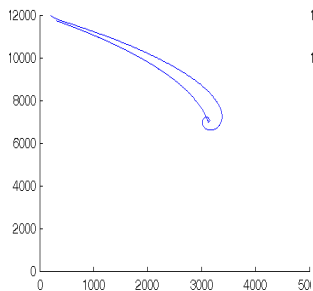
3D-model including technology

$$\frac{d\alpha}{dt} = k_L \alpha L^{\delta_2} \left[\frac{L - L_{\min}}{L_{\min}^2 + (L - L_{\min})^2} \right]$$

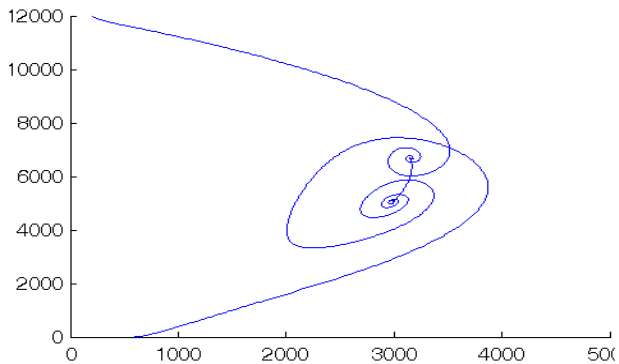
(Brute-force) Bifurcation diagram



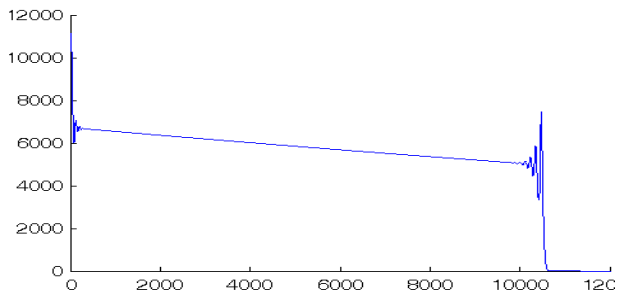
Different scenarios



Forest extinction I



Forest extinction II



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