Bioeconomics of stage/age structured population models.

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  - Background
  - Dynamic ptimization model

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  - Background
  - Dynamic optimization model

- 4. Connection stage structured models and biomass model harvesting

1. Introduction

- Often feeling that biomass (‘a fish is a fish’) models are missing something.

- The bioeconomic working-horse model (‘Clark’ model) clear predictions; price, cost and discount rent….

- But in management situations this is often not enough. Management plan different year classes of fish (or just mature and immature), hunting regulations related to size/age of the animals, etc. Questions of selectivity, spatial movement, etc.

- Also more details to really understand ‘what is going on’, and theoretical underpinning. Age/stage structured population models try to answer some of these questions

- What is age/stage structured population models?

- Demographic models where the actual animal/fish… populations are separated in different age/sex classes (but also more…)

- Given number of age/sex classes and life history
  - Natural mortality
  - Recruitment and recruitment function

- And… man-made mortality (fishing, harvesting, hunting…)

- Age structured models long history in biology/ecology. State of the art book Cashwell (2001). No economy, but some management cases. Also Getz and Haight (1989) where some economics/optimization is included

- In natural resource economics and fishery economics: Clark (1976) .
  - A chapter based on Beverton-Holt 1957 (fishing and natural mortality take place simultaneously over the whole year).
  - Also single harvesting cohort model
  - Multi cohort model with fixed (exogeneous) recruitment and non-selective harvesting (‘one fleet, one vessel…)
  - Based on the remarkable Hannesson (1975 paper. Pulse harvesting…
- Clark (1990) and Clark (2010) editions are more or less unchanged on this

- Others: Walters (1969); numerical model with endogenous recruitment. Max yield over a given time horizon (no costs included)

- Reed (1980) derives theoretical fishery MSY result. Marginal gain: harvest value (kg). Marginal loss: natural mortality (or survival rate). Come back to this

- In the 1990's and later: Several simulation/numerical models. But little theory

- More recently: Tahvonen (2009). 'Complete' dynamic fishery model with imperfect selectivity ('one fleet'). Theory behind pulse-harvesting.

- Own work:
  - Skonhoft, Quaas, Vestergaard (2012) : MSY and MEY theory (static)
  - Quaas, Requate, Skonhoft, Vestergaard (2013): Incentive theory and quotas dynamaic model
  - Naevdal, Olaussen and Skonhoft (2012): Theory dynamic model trophy hunting
  - Skonhoft and Olaussen (2011): MSY theory dynamic hunting model
  - Skonhoft (2008): Theory two stage livestock model

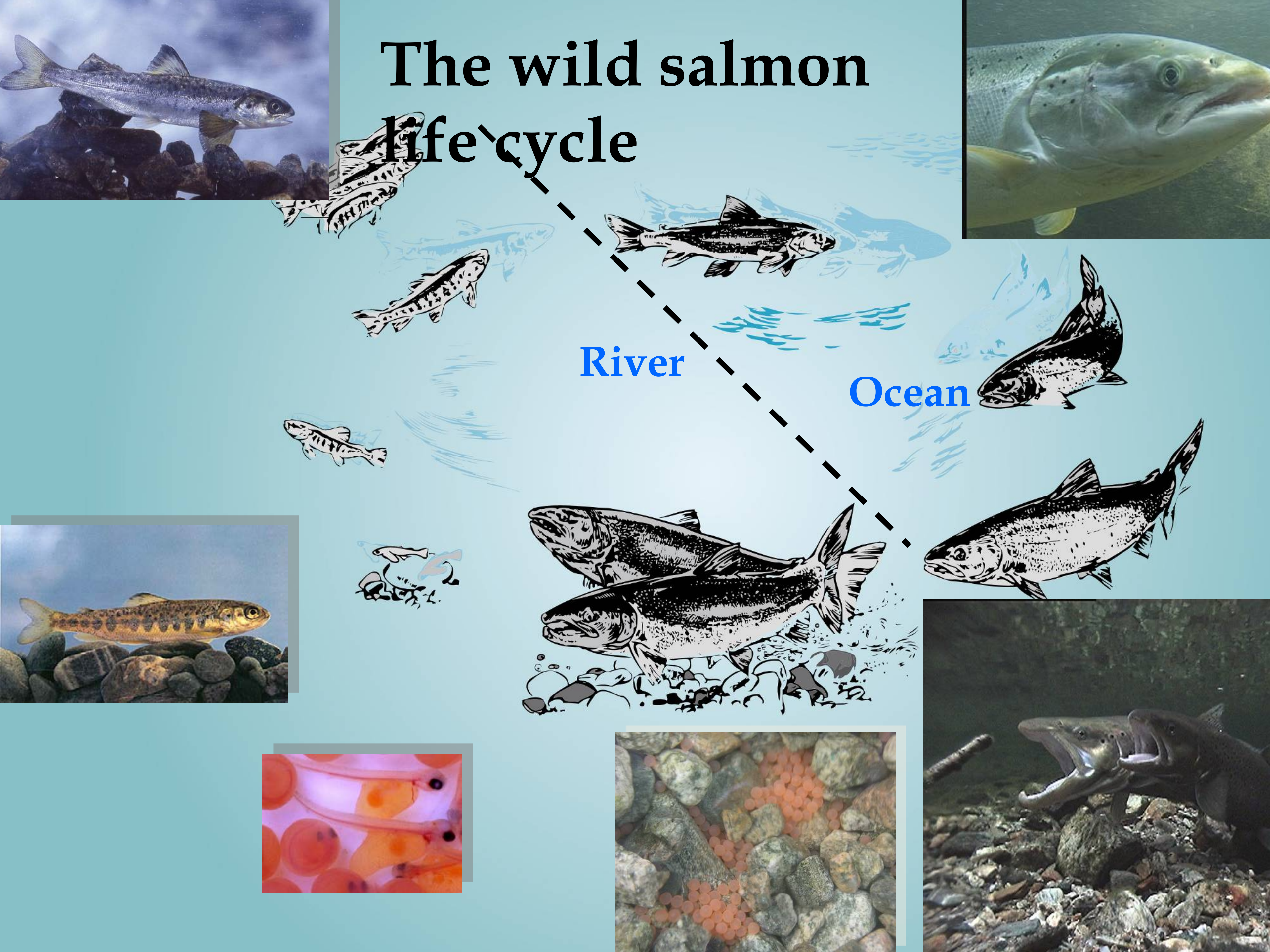
2. Example 1: Wild Atlantic salmon fishing

- Based on: Liu, Diserud, Hindar, Skonhoft (2012), Liu, Olaussen, Skonhoft (2011), Skonhoft (2012), Skonhoft and Gong (in prep)
- Start with giving some background for the Norwegian wild salmon fishery. History and management problems
- Then the conditions for optimal fishing. Basically thinking about a recreational fishery; far most important
- This is an age model, not stage model
- Always important: degree of selectivity in harvesting

2.1 Background/Overview

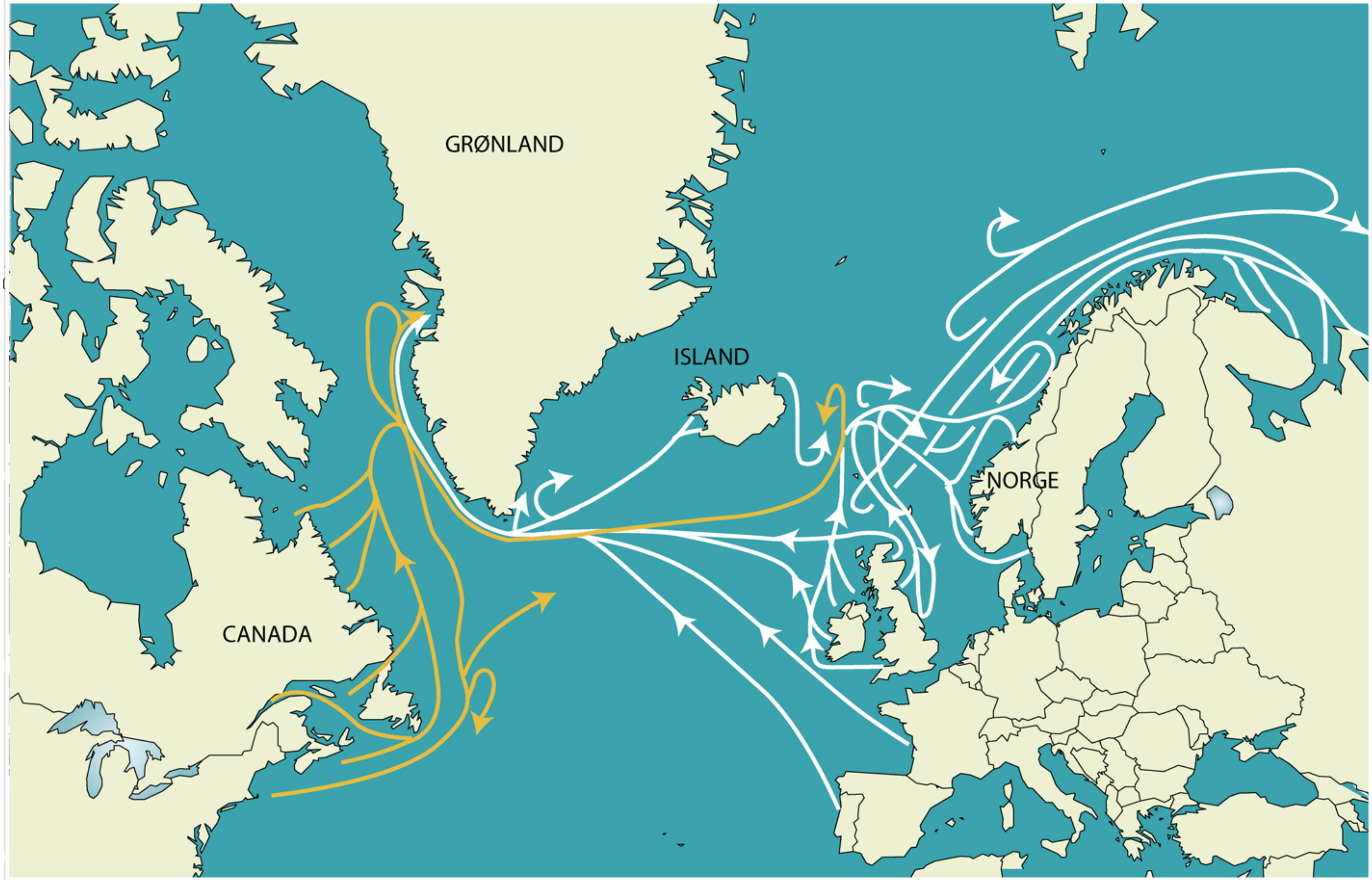
Back in the old days (1880’s)…english upper class with tweedsuits…

The wild salmon life cycle



- The life cycle:
- River (freshwater) spawning. The fish dies after spawning (<10% typically survive and spawn twice)
- Stay as egg, larvae and young in the river (2-3 years)
- ‘Smoltification’ and leaving river
- Staying in ocean/migration and feeding for 1-3 years
- Returning back to its **parent river** to spawn
- Exploitation/fishing only when returning back. River fishing (leisure) and fjord/inlets (semi commercial)

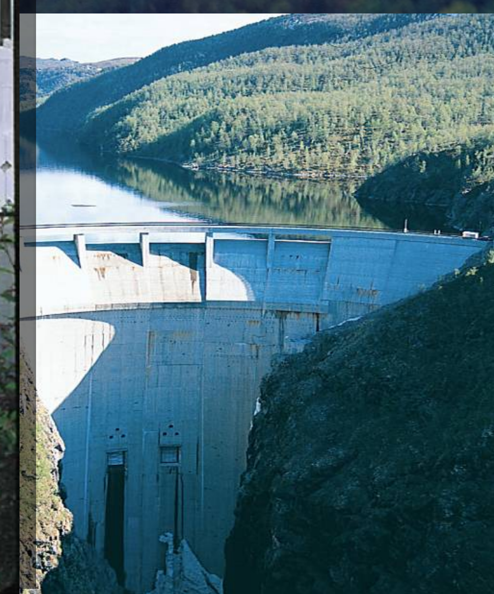
Migratory pattern



- Atlantic salmon threatened species

- Freshwater threats
- Saltwater threats

Freshwater threats



Freshwater threats

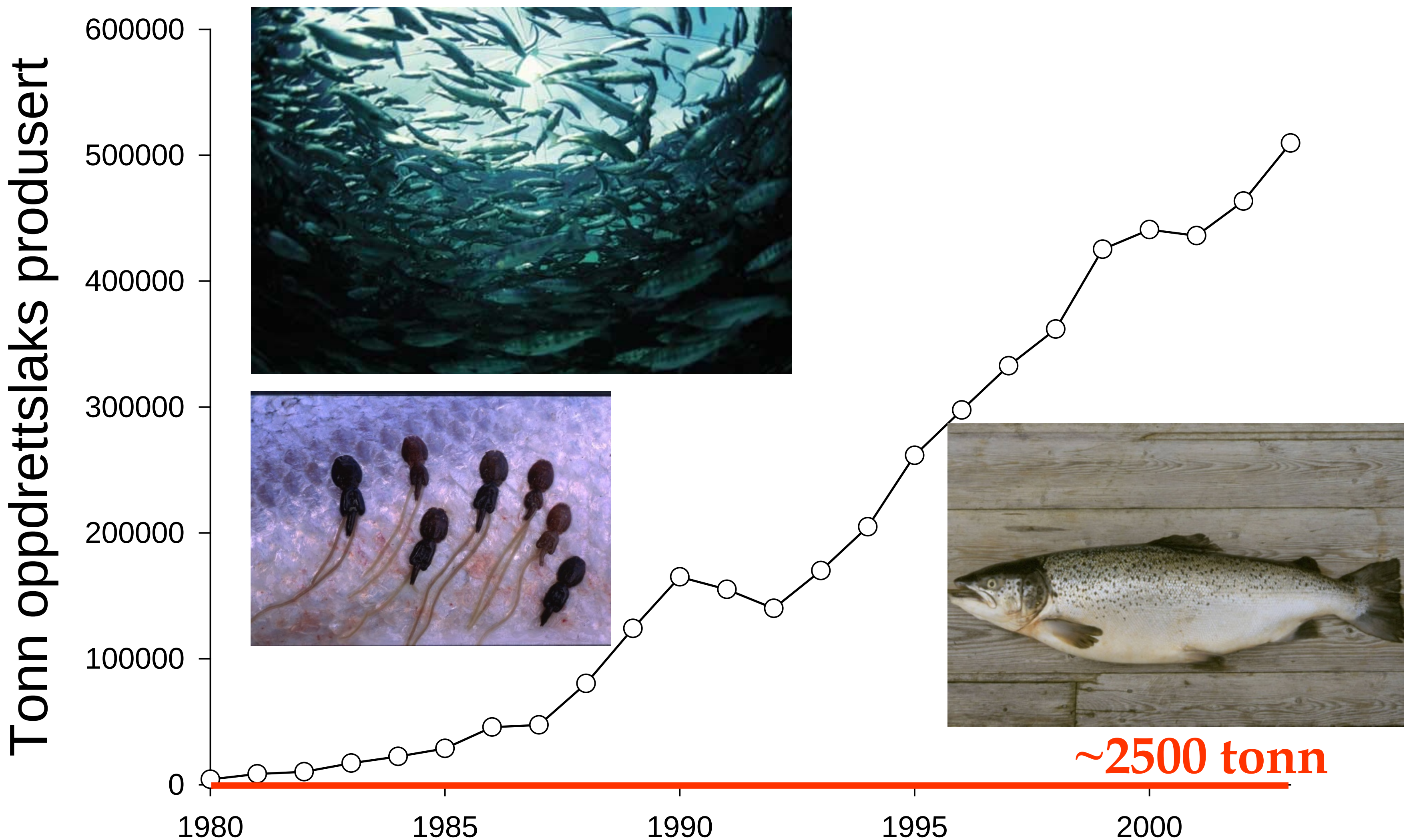
- *Gyrodactylus salaris* (parasit)
- Acid rain
- River water regulations
- Overfishing

..and ocean threats

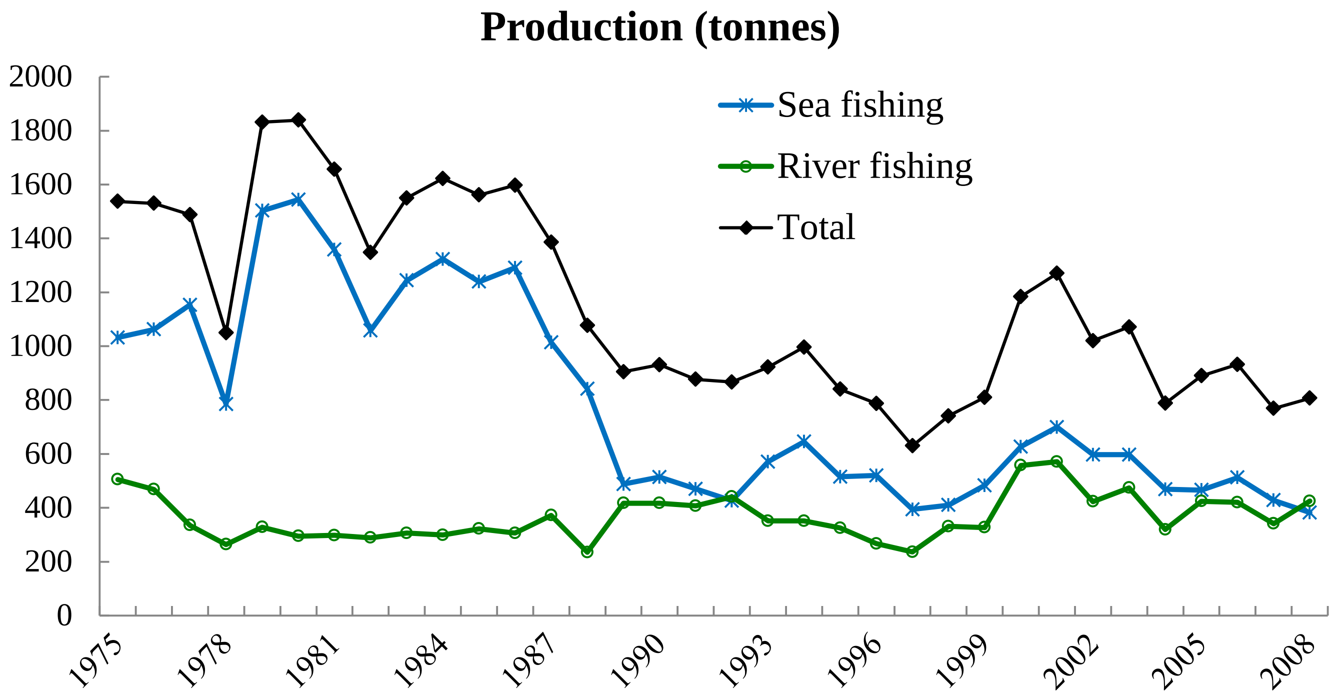
Ocean threats

- Escaped farmed salmon. Competing and interbreeding
- Salmon lice (transmitted from the farmed salmon industry)
- …and overfishing

Wild vs farmed salmon…a big number problem



Catches... in Norway



Threats and trends…

- Farmed samon – too much lice and too many escapees
- *Gyrodactylus salaris* –
- Acid rain and pollution – stable pattern
- River water regulations – stable pattern…
- Transport – opening of the Northeast passage
- Climate change – warming → new species
- The salmon fishery – more and more regulated, especially ocean fishery. Catch and release

2. 2 Salmon fishing and age structure

- Analyzing the economics of selective harvesting
- Constructing and analyzing an age-structured wild salmon dynamic model.
- Q: What is the optimal fishing composition: Which age classes should be fished, and what are the driving forces.
- The conservation issue is not considered here; wild salmon has an existence value. Only harvesting value taken into account

MODEL STRUCTURE

- Restricted number of age classes
- Three age classes immature plus recruitment
- Two adult/mature and harvestable classes; ‘young’ (‘small’) and ‘old’ (‘large’)
- Separtion exogeneous given; type of river
- These two harvestable year classes represent also the spawning population
- Higher fertility old than young
- Spawning: Density dependent (Beverton – Holt)
- Natural mortality fixed and density independent
- Dies after spawning. Special feature salmon and good approximation
- Basically think about recreational fishery (far highest value) with (net) fixed per fish values

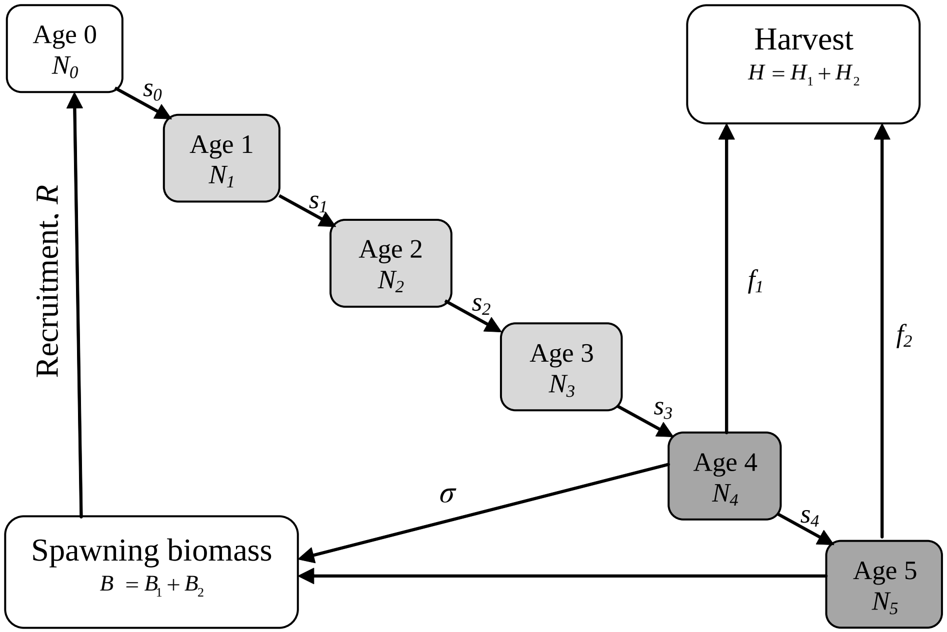


Figure 1. Schematic representation of the structure of a wild Atlantic salmon for a single cohort. Events shown are recruitment at age *0*, the following young salmon in freshwater habitats from age 1 to 3, and the returns of maturing parts of the stock and harvests at adult classes from age 4 to 5. N, age-specific salmon biomass in number of fish; *s*, age-specific survival rate; *f*, harvest rate; *H*, harvest; σ, the fraction of mature salmon at age class 4; R, recruitment; B, spawning biomass.

- Equations of motion:

$$N_{0,t} = R(B_t)$$

$$N_{a+1,t+1} = s_a N_{a,t} \qquad ; a = 0,1,2$$

$$N_{4,t+1} = s_3 N_{3,t} \sigma (1 - f_{4,t})$$

$$N_{5,t+1} = s_3 N_{3,t-1} (1 - \sigma) s_4 (1 - f_{5,t})$$

$$B_t = \gamma_4 N_{4,t} + \gamma_5 N_{5,t} = \gamma_4 s_3 N_{3,t-1} \sigma (1 - f_{4,t-1}) + \gamma_5 s_3 N_{3,t-2} (1 - \sigma) s_4 (1 - f_{5,t-1})$$

- The present-value maximum economic yield harvesting program:

$$\max_{f_{4,t},\, f_{5,t}} \quad \sum_{t=0}^{\infty} \left[p_4 \sigma N_{4,t} f_{4,t} + p_5 N_{5,t} f_{5,t} \right] \rho^t$$

- Subject to biological constraints and constraints harvesting fractions

- The Lagrangian of this problem

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \rho^t \{ & (p_4 \sigma N_{4,t} f_{4,t} + p_5 N_{5,t} f_{5,t}) - \mu_{1,t+1}\rho[N_{1,t+1} - R(\gamma_4 \sigma N_{4,t}(1 - f_{4,t}) \\
& + \gamma_5 N_{5,t}(1 - f_{5,t}))] - \mu_{2,t+1}\rho(N_{2,t+1} - s_1 N_{1,t}) - \mu_{3,t+1}\rho(N_{3,t+1} - s_2 N_{2,t}) \\
& - \mu_{4,t+1}\rho(N_{4,t+1} - s_3 N_{3,t}) - \mu_{5,t+1}\rho(N_{5,t+1} - s_4 N_{4,t}(1-\sigma)) \}
\end{aligned}
$$

(5)

;

,

and

(6)

;

,

- First order Kuhn-Tucker control conditions:

$$\partial L / \partial f_{4,t} = \rho^t \sigma N_{4,t} (p_4 - \gamma_4 R'(B_t) \rho \mu_{1,t+1}) \underset{\leq}{\overset{\geq}{-}} 0 \qquad 0 \leq f_{4,t} \leq 0$$

$$t = 0,1,2...$$

$$\partial L / \partial f_{5,t} = \rho^t N_{5,t} (p_5 - \gamma_5 R'(B_t) \rho \mu_{1,t+1}) \underset{\leq}{\overset{\geq}{-}} 0 \qquad 0 \leq f_{5,t} \leq 0$$

- Marginal value – cost relationship

- Marginal value: Price (Euro/fish)

- Marginal cost: Discounted shadow value of the reduced number of recruits (Euro/fish)

- Also portfolio conditions; dynamics of shadow prices. Not that clear interpretation

- But these conditions useful when analyszing the driving forces in the price – interest rate diagram (below)

- The control conditions:
  - Marginal gain: price
  - Marginal loss: loss due to spawning, discounted and shadow value

- These conditions different from Reed (1980), Tahvonen (2009). Skonhoft et al. (2012), Reason: In our salmon model, ths fish dies after spawning. In 'cod'models; the fish spawns and enters an older age class. Fertilty factors not included in control conditions

.,

and

, or ii)

and

.

- The various harvest options. Difference in price/fertility rate is the important factor
- Due to Kuhn-Tucker theorem seven options:

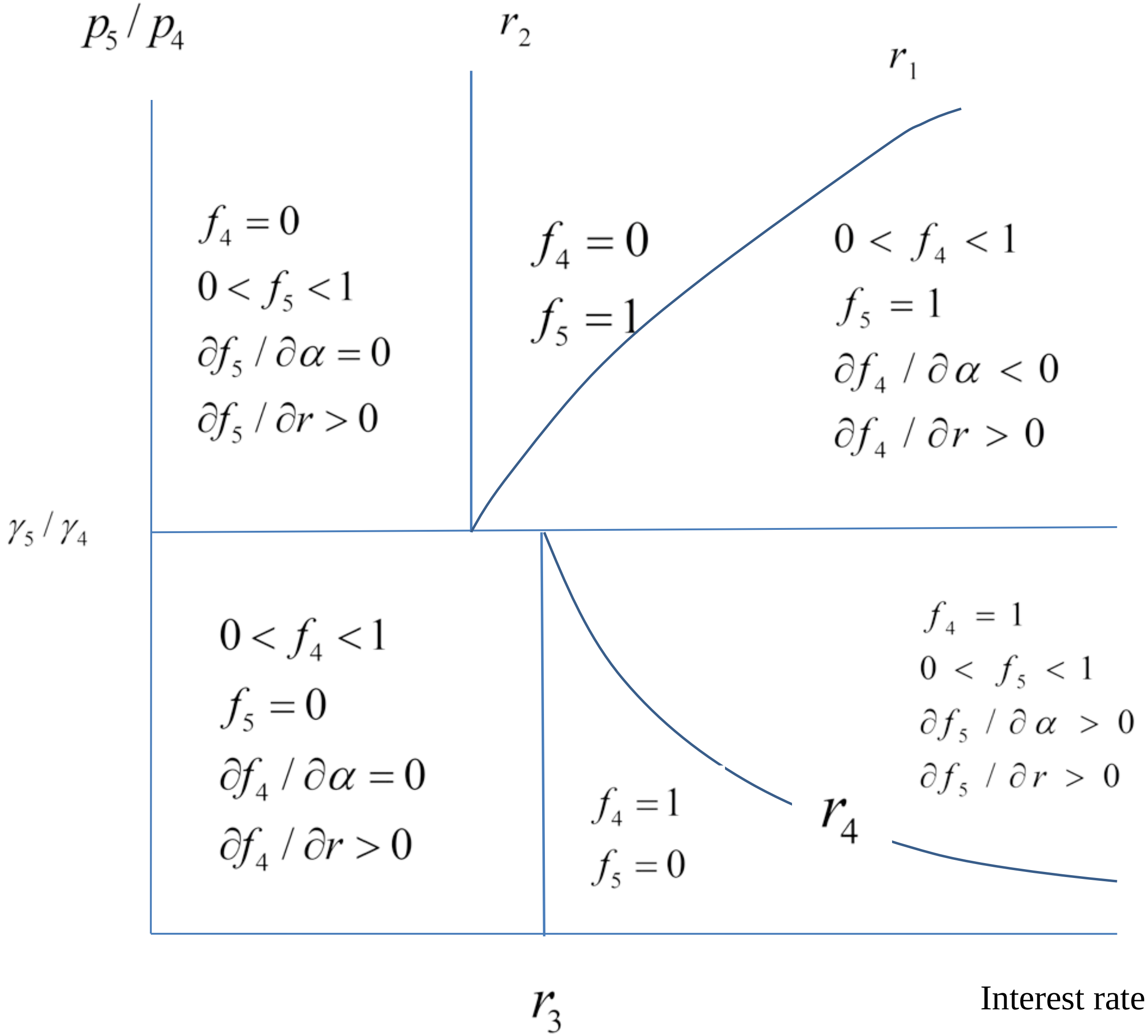
$$p_5/\gamma_5 > p_4/\gamma_4$$

$$f_{5,t} = 1 \qquad 0 < f_{4,t} < 1$$

$$f_{5,t} = 1 \qquad f_{4,t} = 0$$

$$0 < f_{5,t} < 1 \qquad f_{4,t} = 0$$

- Therfeore old mature class priority when price – fertilty ratio is higher for the young mature class
- Opposite. Young mature class fishing priority.
- The seventh option: Identical price – fertility rate. In the salmon fishery this will typically hold if fertility is perfectly related to weight. The optimality conditions are then satisfied by an infinite combinations of harvesting rates (intuition: Control conditions similar information; one degree of freedom…)
- But typically: Price – fertilty ratio is highest for the old mature class. Harvest of this age class should be priorized ,and higher harvest fraction


p_5 / p_4
r_2
r_1
$f_4 = 0$
$0 < f_5 < 1$
$\partial f_5 / \partial \alpha = 0$
$\partial f_5 / \partial r > 0$
$f_4 = 0$
$f_5 = 1$
$0 < f_4 < 1$
$f_5 = 1$
$\partial f_4 / \partial \alpha < 0$
$\partial f_4 / \partial r > 0$
γ_5 / γ_4
$0 < f_4 < 1$
$f_5 = 0$
$\partial f_4 / \partial \alpha = 0$
$\partial f_4 / \partial r > 0$
$f_4 = 1$
$f_5 = 0$
r_4
$f_4 = 1$
$0 < f_5 < 1$
$\partial f_5 / \partial \alpha > 0$
$\partial f_5 / \partial r > 0$
r_3
Interest rate

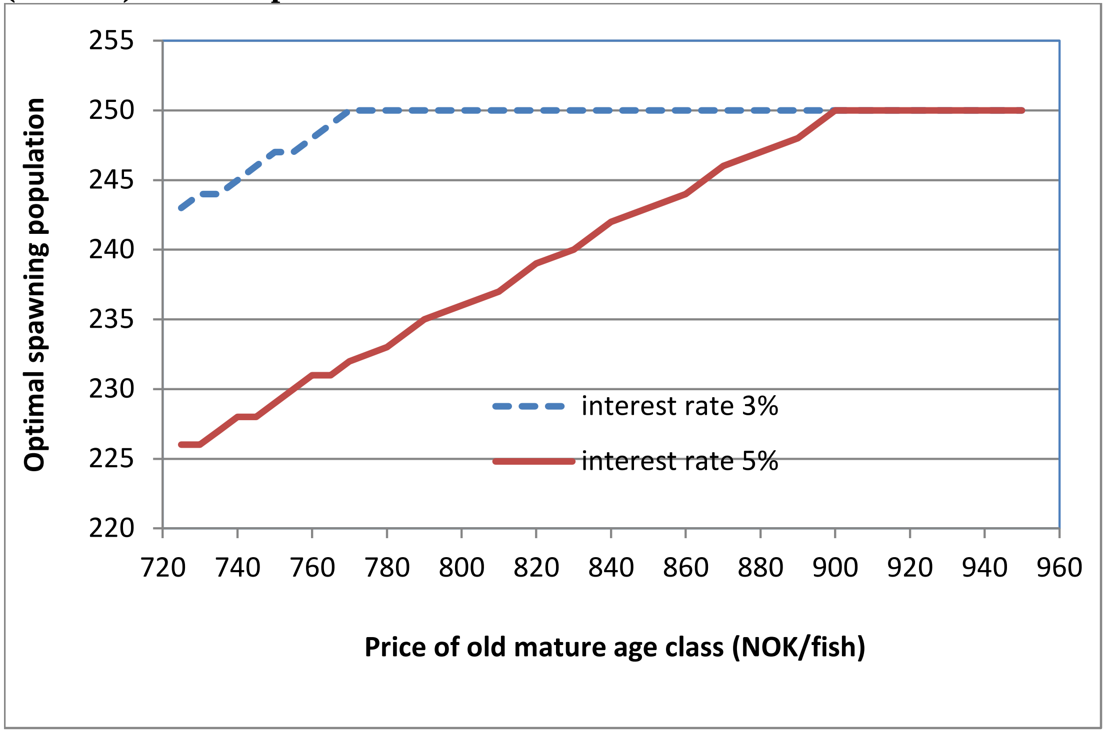
- Driving forces (in steady state):
  - Prices and discounting.

- Comparing biomass model (Clark model)
  - Discount rate works in a parallell manner; higher discount rate (alternative capital value of the fish) means more aggressive fishing
  - Price effect different; but no stock dependent cost included here

- Also numerical results and dynamics in Skonhoft and Gong (in prep).

- Dynamics: Smooth transitional dynamics

Figure 3. The optimal steady state spawning population (fecundity weighted sum in # of fish) associated with different interest rates and prices of the old mature age class. Baseline values (Table 1) all other parameters values



3. Example 2: Moose harvesting

3.1 Background/overview

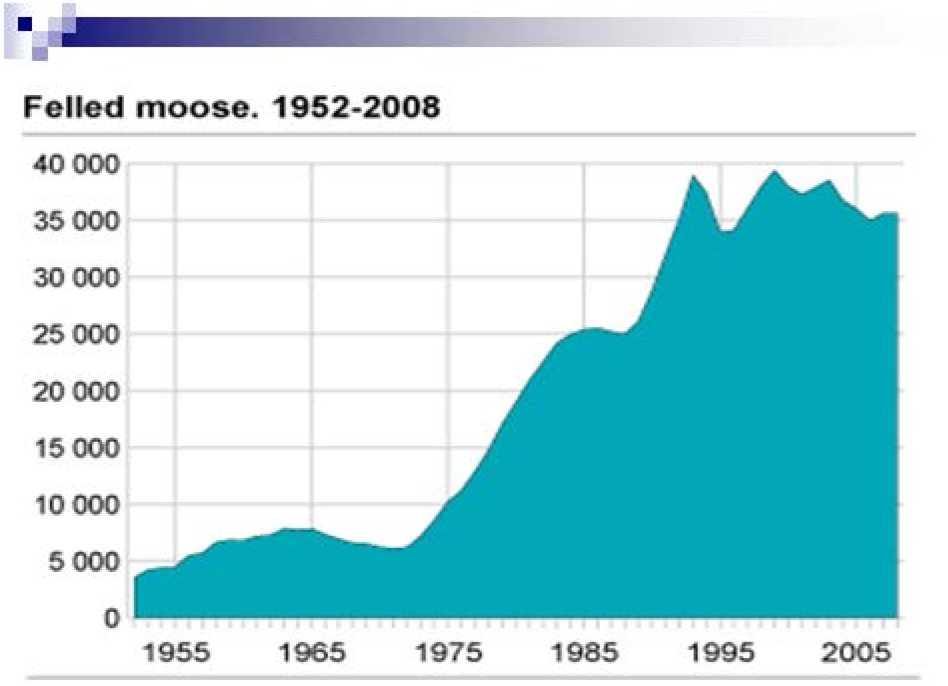
- Moose is the largest member of deer familiy found in North America, Russia, Europe
- Slaughter weight (about 55% of live weight): 170 kg for males, and 150 kg for females
- The most important game species in Scandinavia. In Sweden about 100.000 animals shot every year. In Norway 35000/40000

- Adult male…

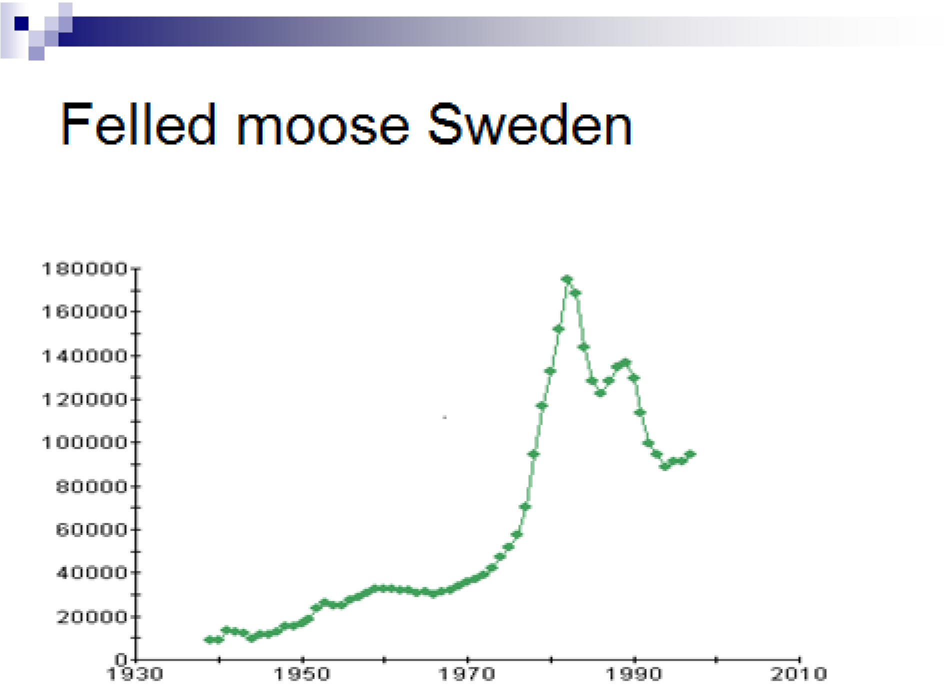
- And adult female…

- Easy to distinguish. Selective harvesting assumtion makes good sense

- Moose hunting Norway



- …and in Sweden



Increased harvest and increased populations:

1. Selective harvesting
   – Harvest more bulls, calves and yearlings, less females. But in Sweden overharvest in the 1980's

2. Forestry practice
   – From selective logging to clear cutting. More food available

- The moose harvest in Scandinavia takes place in September/October. A big event ('the big event') in many rural communities
- Hunting basically by the local people ('landowners and friends'), but also some commercial hunting (trophy)

- Positive value components:
  - Meat value (flow)
  - Recreational value among hunters
  - Tourist value (stock)
  - Intrinsic value (stock)

- Negative value componts
  - Browsing (forest) damage (considered as negative stock value)
  - Traffic incidents (also negative stock value)

- The hunting rights belongs to the landowners….externalities

3.2 Landowner hunting model

- Value categories included basic model
  - Meat value
  - Browsing damage
  - (but no traffic damage and intrinsic value)

- Four stages population model
  - Calves
  - Yearlings
  - Adult females
  - Adult males

- Notice now *stage* model; female and male adults

- Population stage model:

$$X_{c,t+1} = r(X_{f,t}, X_{m,t})X_{f,t}$$

$$X_{y,t+1} = s_c(1-h_{c,t})X_{c,t}$$

$$X_{f,t+1} = \psi s_y(1-h_{y,t})X_{y,t} + s(1-h_{f,t})X_{f,t}$$

$$X_{m,t+1} = (1-\psi)s_y(1-h_{y,t})X_{y,t} + s(1-h_{m,t})X_{m,t}$$

- Cost and benefits; meat value and browsing damage
- The sequences over the year:
  - Recruitment, hunting, damage, natural mortality (winter)

$$Q_t = p(w_c h_{c,t} X_{c,t} + w_y h_{y,t} X_{y,t} + w_f h_{f,t} X_{f,t} + w_m h_{m,t} X_{m,t})$$

$$D_t = d_c(1-h_{c,t})X_{c,t} + d_y(1-h_{y,t})X_{y,t} + d_f(1-h_{f,t})X_{f,t} + d_m(1-h_{m,t})X_{m,t}$$

- Linear functions are assumed. Browsing damage concave/convex. Linear compromise…

$$w_c < w_y < w_f < w_m$$

$$d_c < d_y < d_f = d_m$$

- Work in the same directions as harvesting reduce damage (browsing damage occurs after hunting)

- Formulation of the optimal program and the lagrange function. In the more general model included a 'cod of conduct' hunting constraint:

$$h_{f,t}X_{f,t} \leq h_{c,t}X_{c,t}$$

$$\begin{aligned}
L = \sum_{t=0}^{\infty} \rho^t \Big\{ & \Big[p\Big(w_c h_{c,t} r(X_{f,t}, X_{m,t}) X_{f,t} + w_y h_{y,t} X_{y,t} + w_f h_{f,t} X_{f,t} + w_m h_{m,t} X_{m,t} \Big) \Big] \\
& - [d_c(1-h_{c,t}) r(X_{f,t}, X_{m,t}) X_{f,t} + d_y(1-h_{y,t}) X_{y,t} + d_f(1-h_{f,t}) X_{f,t} + d_m(1-h_{m,t}) X_{m,t}] \\
& - \rho\eta_{t+1} \Big[X_{y,t+1} - s_c(1-h_{c,t}) r(X_{f,t}, X_{m,t}) X_{f,t} \Big] \\
& - \rho\lambda_{t+1} \Big[X_{f,t+1} - 0.5 s_y (1-h_{y,t}) X_{y,t} - s(1-h_{f,t}) X_{f,t} \Big] \\
& - \rho\mu_{t+1} \Big[X_{m,t+1} - 0.5 s_y (1-h_{y,t}) X_{y,t} - s(1-h_{m,t}) X_{m,t} \Big] - \rho\omega_{t+1} \Big[h_{f,t} X_{f,t} - h_{c,t} r(X_{f,t}, X_{m,t}) X_{f,t} \Big] \Big\}
\end{aligned}$$

;
,

;
,

- FOC control conditions

$$\frac{\partial L}{\partial h_{c,t}} = rX_{f,t}(pw_c + d_c - \rho\eta_{t+1}s_c + \rho\omega_{t+1}) \le 0$$

;
.

$$\frac{\partial L}{\partial h_{y,t}} = X_{y,t}(pw_y + d_y - \rho\lambda_{t+1}0.5s_y - \rho\mu_{t+1}0.5s_y) \le 0$$

$$0 \le h_{i,t} < 1$$

$$\frac{\partial L}{\partial h_{f,t}} = X_{f,t}(pw_f + d_f - \rho\lambda_{t+1}s - \rho\omega_{t+1}) \le 0$$

$$\frac{\partial L}{\partial h_{m,t}} = X_{m,t}(pw_m + d_m - \rho\mu_{t+1}s) \le 0$$

- Marginal gain: Harvesting value plus omitted browsing damage
- Marginal loss: Natural mortality (survival) discounted and evaluated by stock shadow values
- Different driving forces and more complicated than in wild salmon because yearlings are separated between two adult classes

- Analayszing the control and state conditions and using Kuhn-Tucker theorem we find (Olaussen and Skonhoft 2011):
- Harvesting yearlings and females, but not males, contradict FOC
- Harvesting yearlings and males, but not females, contradict FOC
- We find that harvest of yearlings is not an optimal option

- Then comparing harvest of calves and yearlings (may be seen as substitutes in the harvest)
-
- Assume postive calf harvest and zero yearling harvest. We then find (when code of conduct constraint neglected):

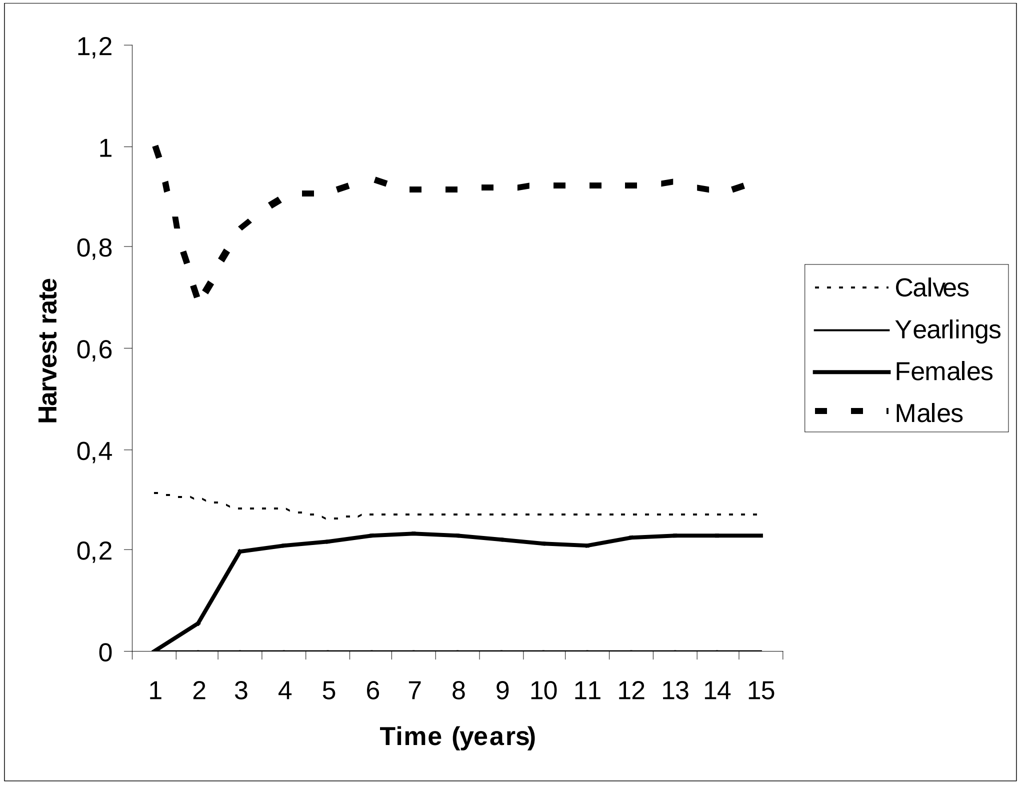
$$(pw_c + d_c)/s_c > \rho(pw_y + d_y)$$

- Again, see driving forces…
- Confronting data; also contradiction

- Because small differences in survival rate among the different stages:
  - First principle: Harvest the most valuable animals (meat value + omitted damage)

- But certain modifications: Recruitment (females), and also ‘cod of conduct’

Parameters	Description	Value	Reference/source
$\bar{r}$	maximum specific growth rate	1.15	Nilsen et al. (2005),
K	female stock level where density dependent factors dominates density independent factors	80,000 animal	Calibrated
a	male density recruitment factor	0.00024	Calibrated
b	density compensation parameter	2	Nilsen et al. (2005)
w_c	average weight calve	65 kg/animal	SSB (2004)
w_y	average weight young	135 kg/animal	SSB (2004)
w_f	average weight females	150 kg/animal	SSB (2004)
w_m	average weight male	170 kg/animal	SSB (2004)
s_c	natural survival rate calf	0.90	Nilsen et al. (2005)
s_y	natural survival rate young	0.95	Nilsen et al. (2005)
s	natural survival rate female and male	0.95	Nilsen et al. (2005)
p	meat price	75 NOK/kg	Storaas et al. (2001)
d_c	marginal browsing damage calf	250 NOK/calf	Larsen (2007)
d_y	marginal browsing cost yearling	500 NOK/yearling	Larsen (2007)
d_f	marginal browsing cost female	750 NOK/female	Larsen (2007)
d_m	marginal browsing cost male	750 NOK/male	Larsen (2007)
t_c	marginal traffic cost calf	800 NOK/calf	Solstad (2007)
t_y	marginal traffic cost yearling	1,700NOK/yearling	Solstad (2007)
t_f	marginal traffic cost female	1,900 NOK/female	Solstad (2007)
t_m	marginal traffic cost male	2,100 NOK/male	Solstad (2007)
δ	discount rate	0	


1,2
1
0,8
0,6
0,4
0,2
0
Harvest rate
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Time (years)
Calves
Yearlings
Females
Males

- Extension of this model in various directions:
  - Including external costs (traffic damage costs)
  - Trophy hunting: Demand for trophy hunting males, but meat value the other stages (Naevdal, Olaussen and Skonhoft 2011)
  - Seasonal migration over the year cycle; leaving the core area during winter bacuse of snow and food conditions. Basically not density dependent. Causes externalities beteween landowners
  -

4 Connection stage model and biomass harvesting

- Using the moose model to demonstrate connection with biomass model harvesting (can do similar for the wild salmon model…)

- Finding the moose sustainable yield function and MSY
- Two cases: i)Uniform harvesting, and ii) Zero young harvesting and similar adult harvesting

- Biological equilibrium (sustainability...)

$$X_{i,t+1} = X_{i,t} = X_i$$

- Case i) Uniform harvesting (identical harvest rates among all stages)

$$X_m = X_f$$

$$r(X_f, X_m) = [1 - s(1-h)] / 0.5 s_y s_c (1-h)^2$$

- Standing biomass (tonne)

$$B = (w_c X_c + w_y X_y + w_f X_f + w_m X_m)$$

$$B = w_c r(X_f, X_m) X_f + w_y s_c (1-h) r(X_f, X_m) X_f + w_f X_f + w_m X_m$$

$$Q = w_c r(X_f, X_m) X_f + w_y s_c r(X_f, X_m) X_f + w_f X_f + w_m X_m$$

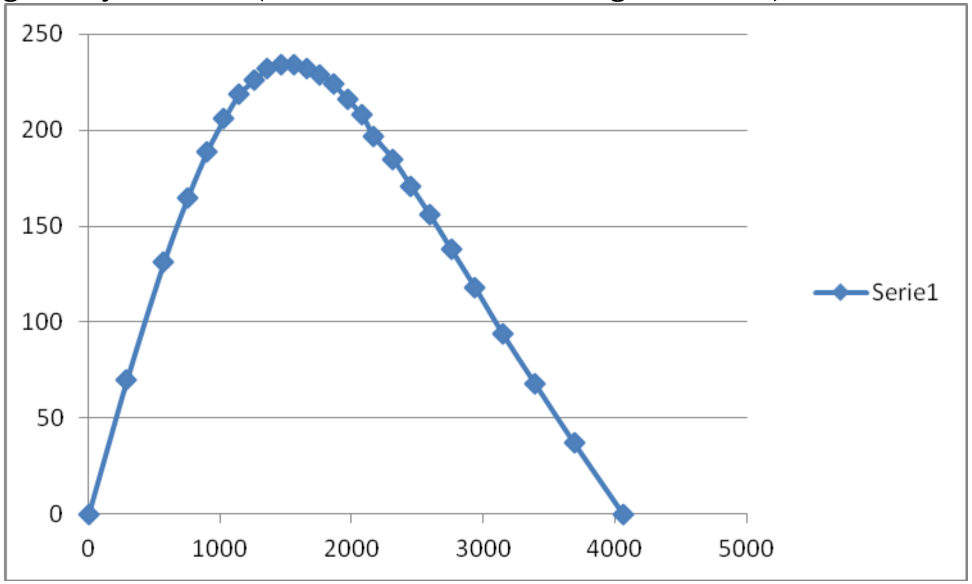
- Yield (tonne)

$$H = h(w_c X_c + w_y X_y + w_f X_f + w_m X_m)$$

Table A1

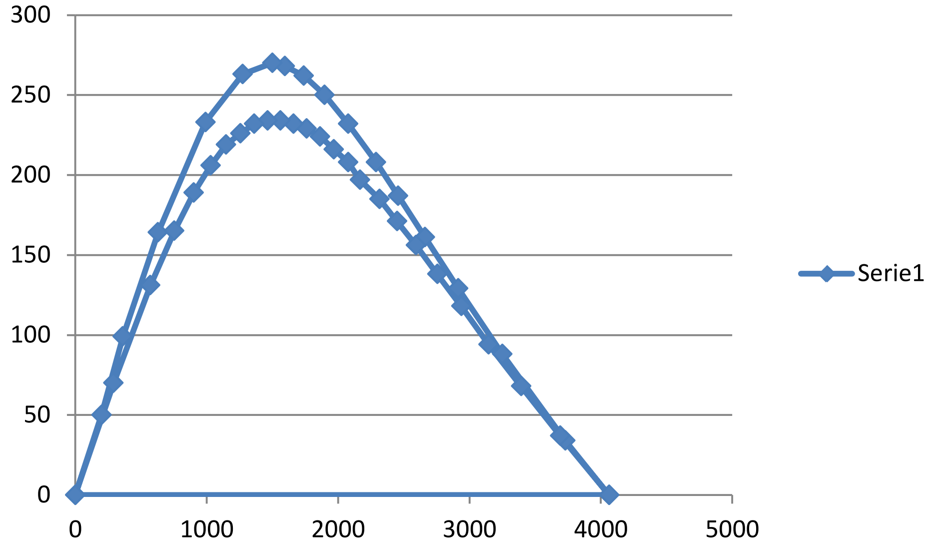
h	B	H	X_c	X_y	$X_m = X_f$
0	4063	0	1390	1251	11888
0,01	3691	37	1513	1348	10657
0,02	3393	68	1625	1433	9669
0,03	3147	94	1726	1507	8848
0,04	2938	118	1819	1572	8147
0,05	2756	138	1905	1629	7537
0,06	2594	156	1982	1676	6997
0,07	2449	171	2205	1717	6511
0,08	2315	185	2113	1750	6070
0,09	2167	197	2167	1775	5663
0,1	2077	208	2213	1792	5286
0,11	1968	216	2250	1802	4932
0,12	1863	224	2278	1804	4598
0,13	1761	229	2294	1796	4280
0,14	1661	232	2300	1781	3974
0,15	1562	234	2292	1753	3677
0,16	1463	234	2269	1715	3388
0,17	1362	232	2228	1664	3102
0,18	1258	226	2165	1598	2816
0,19	1148	219	2077	1514	2528
0,2	1031	206	1956	1409	2231
0,21	902	189	1793	1275	1917
0,22	754	165	1567	1100	1574
0,23	571	131	1240	859	1170
0,24	291	70	659	451	586
0,25	0	0	0	0	0

Figure 2. The sustainable yield – biomass relationship. Biomass B (tonne) at the horizontal axis and harvest H (tonne) at the vertical axis. The harvest fraction is given by $b = H / B$ (notice the different scaling of the axis)



- The shape; skew-logistic functional form
- MSY happens for a low harvesting rate, about h=0.16
- Thus h=0.16 is the uniform harvest rate giving MSY
- But can MSY be increased? From the stage model theory; harvest less of the of the toung and small!
- Now case ii) Zero calf and yearling harvest and similar female and male harvest rate

- The two cases



- Case i) niform MSY:
  - h=0.16, B=1480 (tonne), H=234 (tonne)
- Case ii) Zero calf and yearling harvest MSY:
  - h=0.25, B=1501, H=270
- Other sustainable yield curves and higher MSY?? Yes: From theory; harvest more males than females. Two forces: a) Male higher harvest value, b) Recruitment/fertility effect