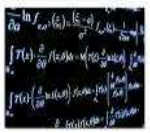
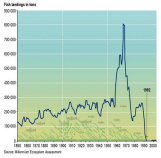
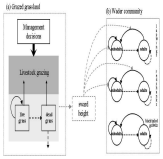
Mathematical Modeling for Environmental Economics and Sustainable Management

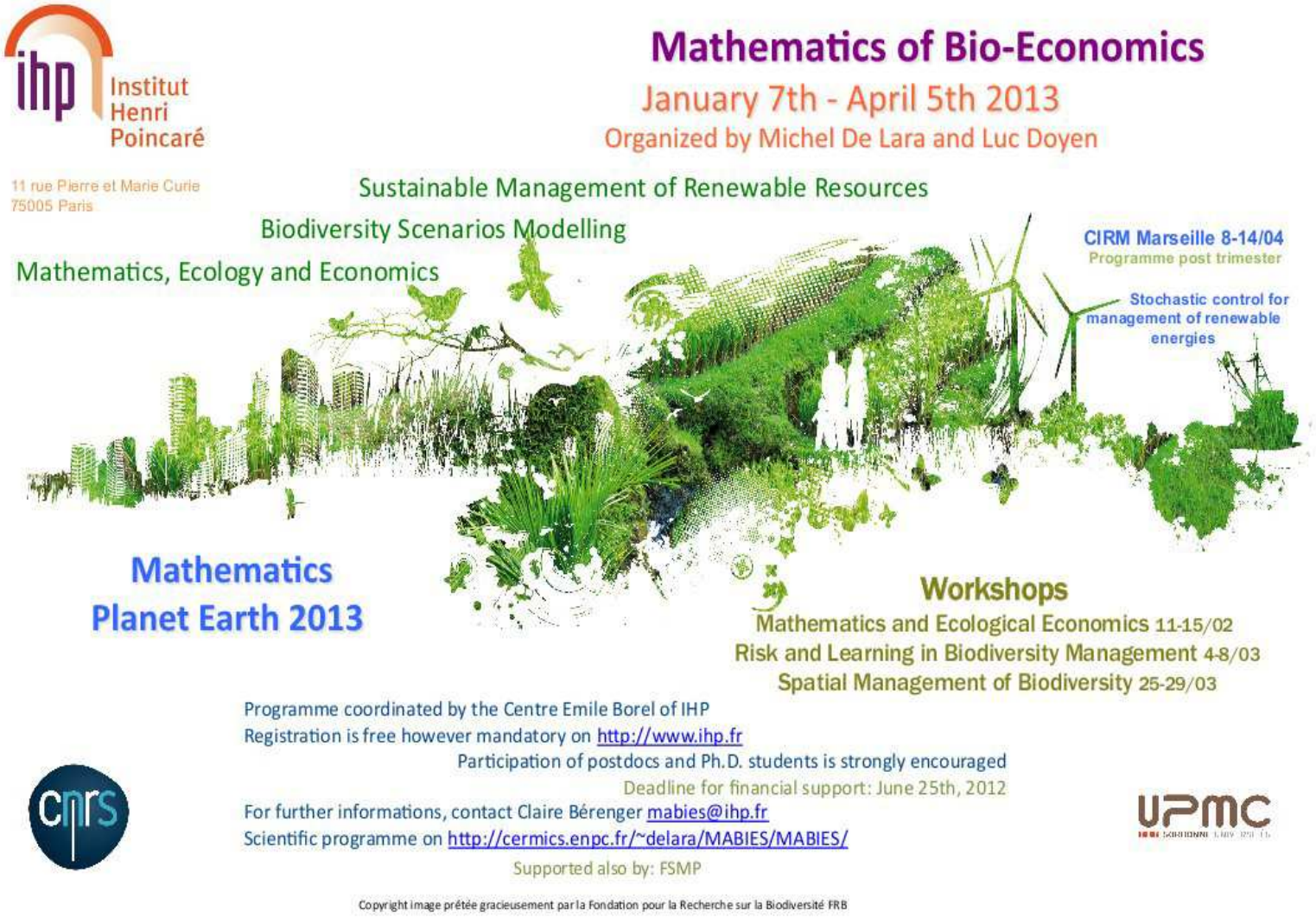
Outils mathématiques pour la décision en environnement
Master EDDEE

Luc Doyen,Michel De Lara




Trimester Mathematics of Bio-economics at IHP



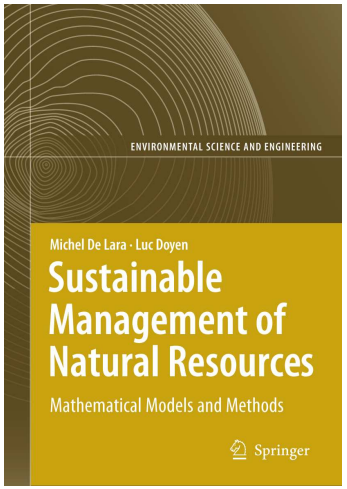
Program

- *Monday 7, January 10h00-13h00 :* **IHP :** Course Doyen-De Lara Introduction to decision modelling for sustainable management of natural resources
- *Monday 14, January 9h00-12h00 :* **ENGREF :** Computer Session De Lara-Doyen

  Introduction to the scientific software
- *Monday 21, January 9h00-12h00 :* **IHP** Course Doyen-De Lara Controlled dynamics and equilibria
- *Monday 28, January 9h00-12h00 :* **ENGREF :** Computer Session De Lara-Doyen Equilibria
- *Monday 4, February 9h00-12h00* : **IHP** Course Doyen-De Lara Optimality and sustainability
- *Monday 11, February 9h00-12h00 :* **ENGREF :** Computer Session De Lara-Doyen Optimality
- *Monday 18, February 9h00-12h00 :* **IHP** Course Doyen-De Lara Viability
- *Monday 25, February 9h00-12h00 :* **ENGREF :** Computer Session De Lara-Doyen Viability

Some References

- DeLara M.. & Doyen L., 2008, Sustainable Management of Natural Resources Mathematical Models and Methods, Springer.



- Clark C.W., (1976), *Mathematical Bio-economics : The Optimal Management of Renewable Resource*, J. Wiley & Sons, New York
- Heal G. (1998) Valuing the Future, Economic Theory and Sustainability. Columbia University Press, New York.
- Conrad J. M. 1999, *Resource Economics*, Cambridge University Press.

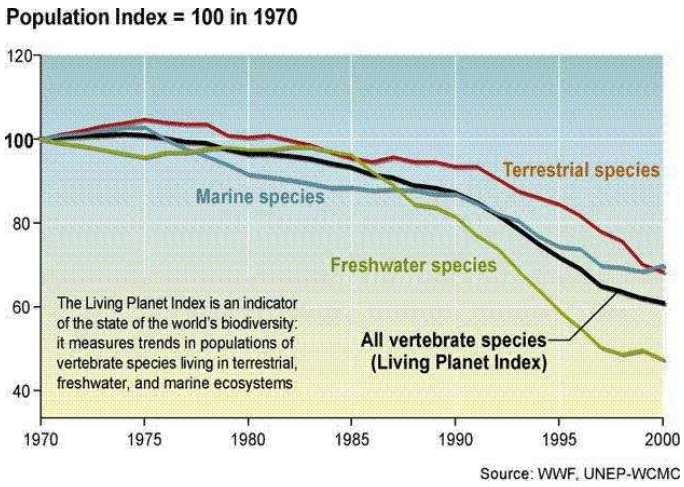
Outline

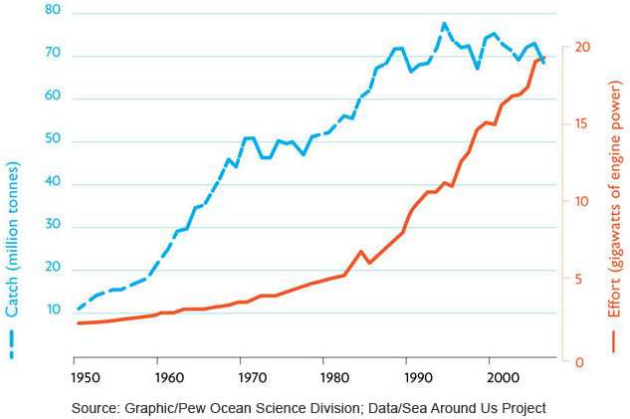
- General issues
- Deterministic models in discrete time
  - General framework ; Examples
  - Equilibrium and stability.
  - Viability.
  - Inter-temporal optimality.
- Models under uncertainty
  - Robust approach.
  - Stochastic approach.
- Partial information
- Strategic interactions

Environmental issues

- Management of exhaustible resources : oil, mining, ...
- Pollution management : ex : $C0_2$
- Management of renewable resources : Fisheries, wildlife, forest, agroecology

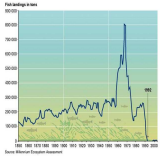
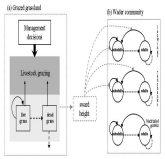
Sustainable management of biodiversity



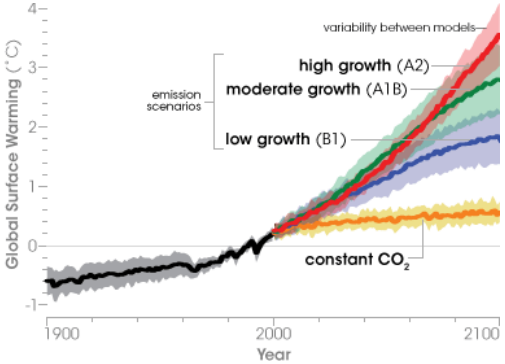


- Global changes in ecosystems
- Alarming trends : `MEA, IUCN, ...`
- Major concerns for ecosystem services
- $\Longrightarrow$ bio-economic risk
- Sustainable management



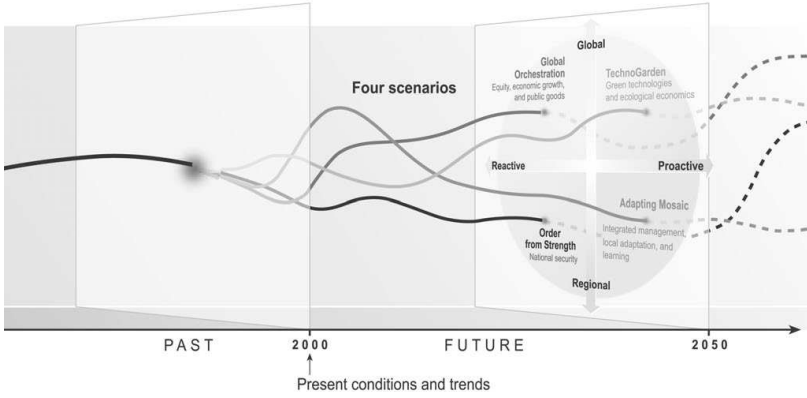





"From IPCC"


variability between models
high growth (A2)
moderate growth (A1B)
low growth (B1)
emission scenarios
constant CO_2
Global Surface Warming (°C)
4
3
2
1
0
-1
1900
2000
2100
Year

To

Biodiversity scenarios



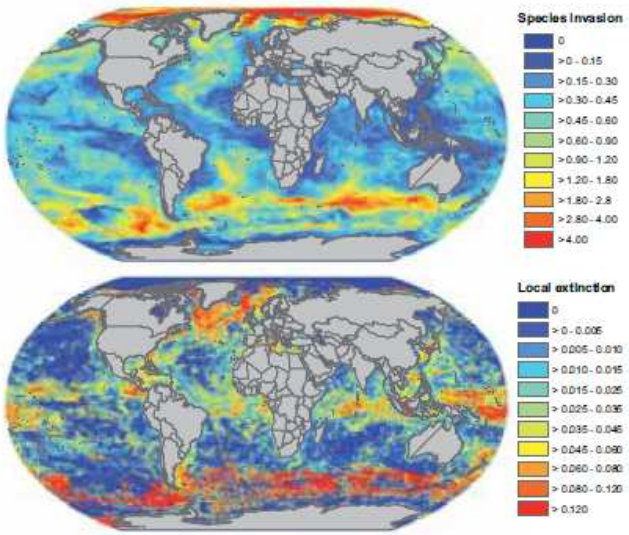
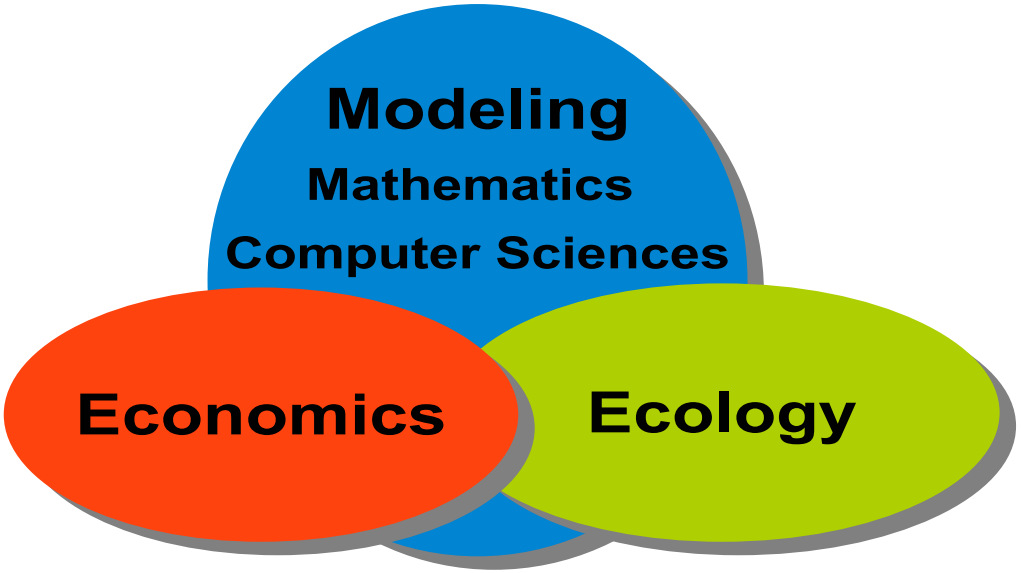


FIGURE 19 PROJECTED CHANGES IN MARINE BIODIVERSITY DUE TO CLIMATE CHANGE.

Biodiversity impact in 2050 under the IPCC SRES A1B scenario expressed in terms of: number of new species moving from other regions (top) and local extinction intensity (bottom). The projections are based on bioclimate envelope models for 1,066 species of fish and invertebrates. Source: redrawn from Cheung et al. 2009.

Bio-economic modeling



Bio-economic policies

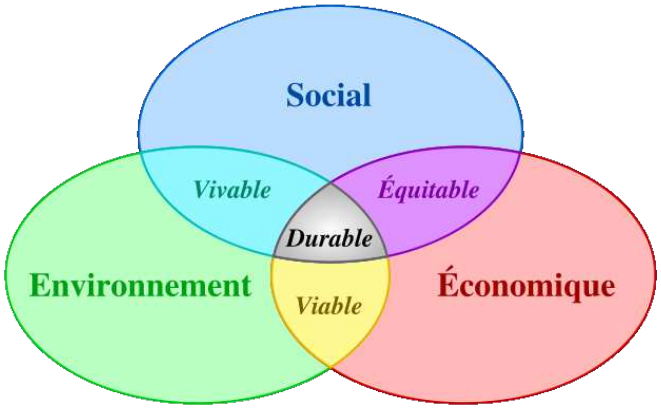
Three major components :

- Scientific knowledge :
  mechanisms, models, data.
- Shared intertemporal objectives :
  ex : Johannesburg 2002 : MSY by 2015
- Bio-economic instruments
  - standards : quotas, MPA, ...
  - monetary : landing taxes, ITQ, ...
  - informational : MSC,...

Important goals

- Sustainability
  - **Reconciliation**

    Economy–Social-Ecology
  - **Intergenerational equity**
  - **Intragenerational equity**
- Resilience
- Precaution



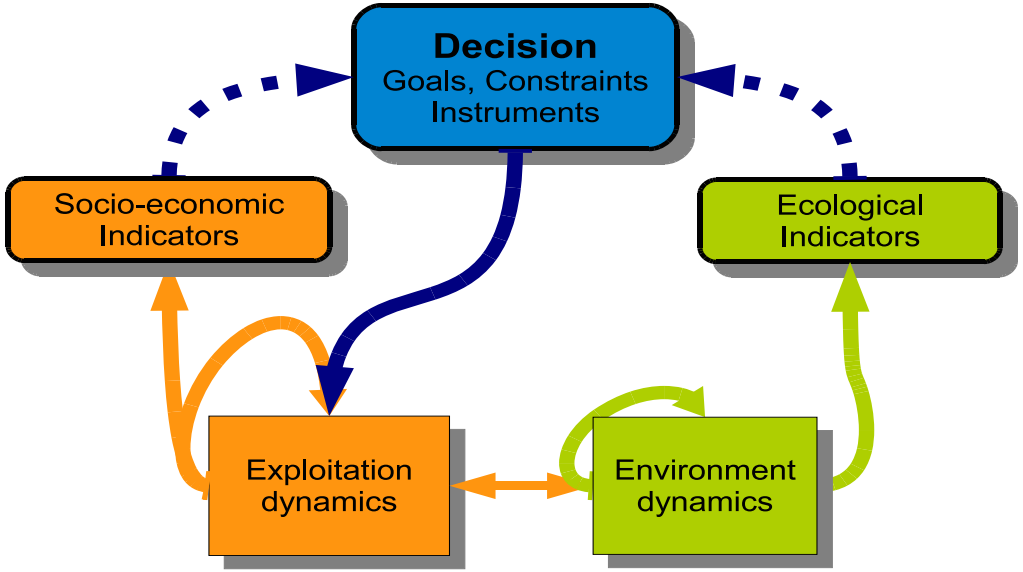
Modeling requirements

Systemic approach

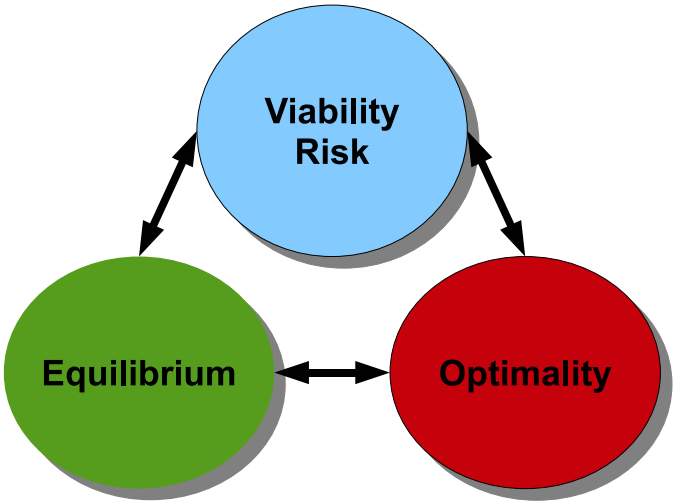
- Dynamics
- Uncertainties
- Decision

Evaluation

- Multi-criteria
  - Ecology
  - Socio-Economy
- Short and long term horizon



Decision approaches for sustainability



Equilibrium and stability :

- Steady state,
- MSY

  `Schaefer, 1954`

Intertemporal Optimality

- Present Value

  CB, CE

  `Clark, 1976`
- Maximin criteria
- ....

Constraints, viability and risk

- PVA

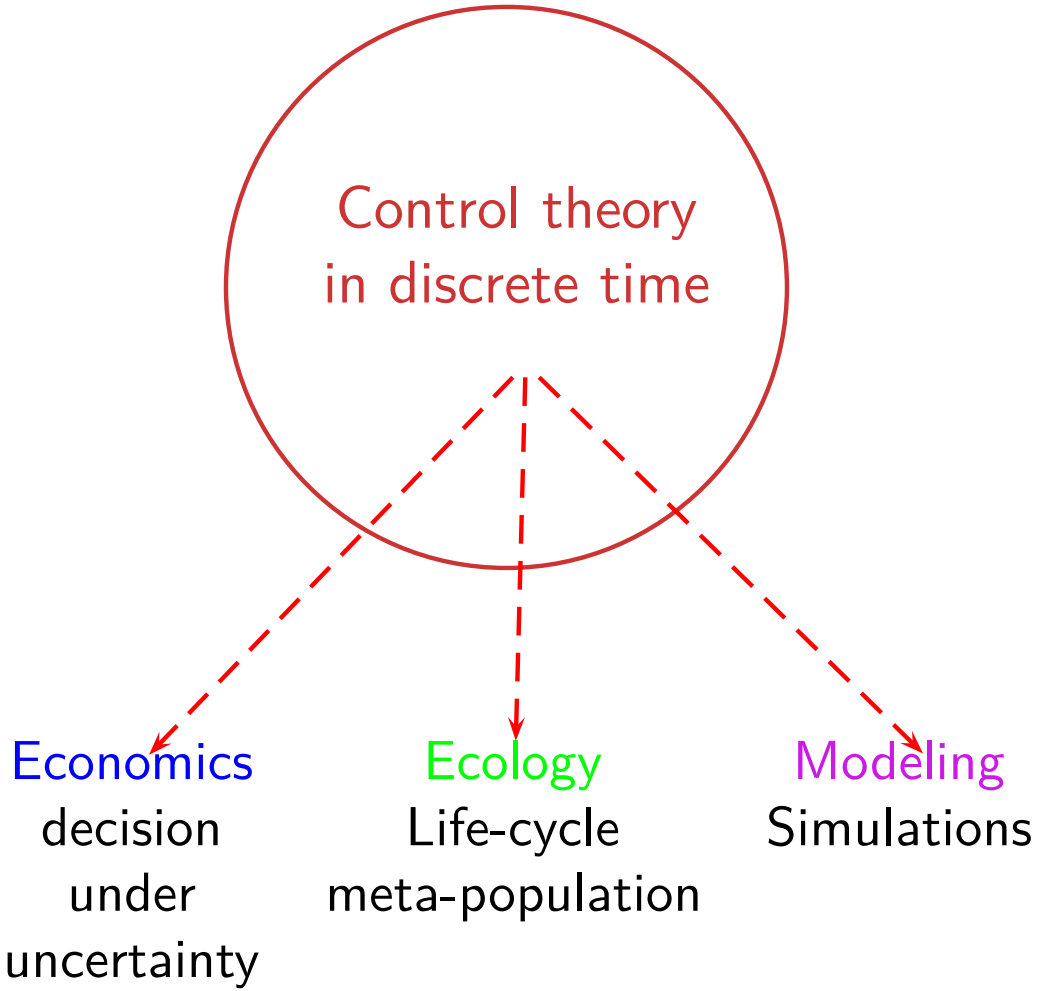
  `Beissinger, 2002`
- Viable control
- TWA
- SMS

  `Bishop, 1978`

Control theory

- Problem :
  Find **controls** to achieve various **goals** for **states** of a system
- Three main ingredients :
  - Controlled dynamics
  - Constraints
  - Criteria to optimize

Control theory in discrete time



Examples for natural resource management

Stylized models :

- Exploitation of an exhaustible resource
- Management of a renewable resource
- Mitigation policies for CO_2 emissions

Exploitation of an exhaustible resource

Management of a renewable resource

Mitigation policies for CO_2 emissions

A controlled dynamics in discrete time

- Difference equation :

$$\left\{ \; x_i(t+1) \;\; = F_i(t, x(t), u(t)), \quad t = 0, \ldots, T \qquad (1)\right.$$

where

  - the **state**, $x(t) = (x_1(t), \ldots, x_n(t)) \in \mathbb{X} = \mathbb{R}^n$;
  - the **control or decision**, $u(t) = (u_1(t), \ldots, u_n(t)) \in \mathbb{U} = \mathbb{R}^p$;
  - the **time Horizon** T

State constraints

- The admissible states

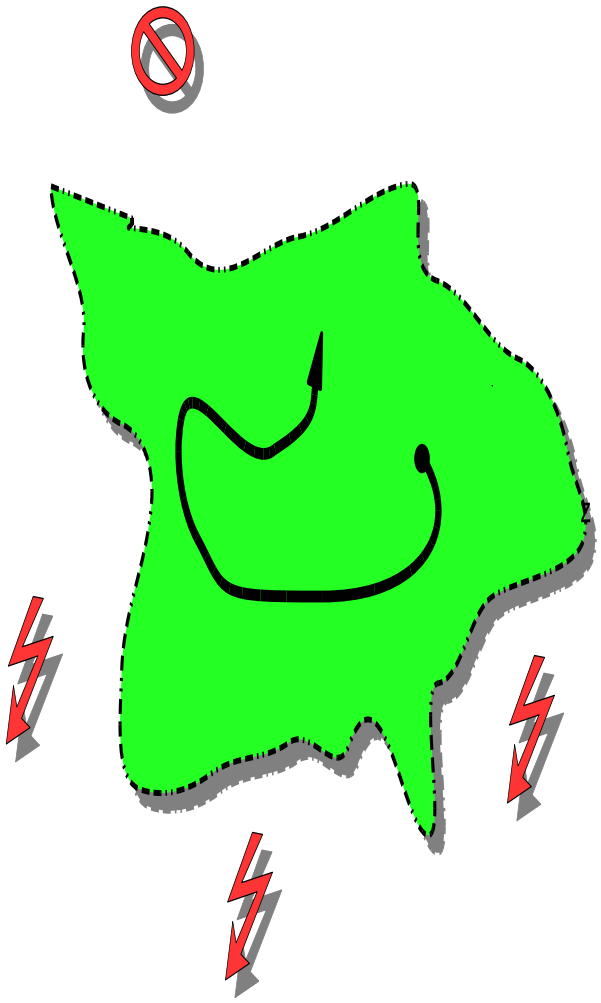
$$x(t) \in \mathbb{A}(t), \quad t = 0, \ldots, T-1\,, \qquad (2)$$

- generally specified under inequality form

$$\begin{cases} g_j(t, x(t)) \leq 0 \\ g_k(t, x(t)) = 0\,. \end{cases}$$

- In many examples, a constant set :

$$x(t) \in \mathbb{A}$$



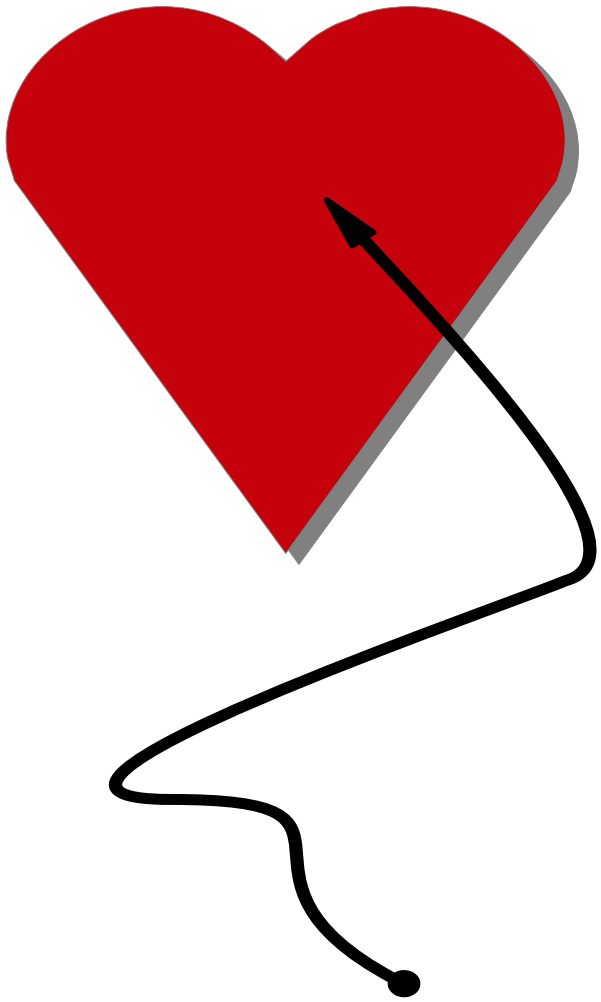
Final state constraints

- Final state reaches some **target** $\mathbb{C} \subset \mathbb{X}$:

$$x(T) \in \mathbb{C} \tag{3}$$

- Generally specified under inequality form

$$\begin{aligned} g_j(x(T)) &\leq 0 \\ g_k(x(T)) &= 0. \end{aligned}$$



Decision or control constraints

- The admissible decisions :

$$u(t) \in \mathbb{B}(t, x(t)), \quad t = 0, \ldots, T-1 \tag{4}$$

- generally specified under inequality form

$$\begin{cases} g_j(t, x(t), u(t)) \leq 0 \\ g_k(t, x(t), u(t)) = 0\,. \end{cases}$$

- Frequently, a constant set of feasible control

$$u(t) \in \mathbb{B}$$

The feasible decisions and sustainability

- The feasible strategy : $u(.) = (u(0), u(1), \ldots, u(T))$ such that
  - The constraints (3), (2) and (4)
  - where $x(.)$ with the dynamics (1)
- Feasibility set :

$$\mathcal{T}_{\mathrm{ad}} = \left\{ (x(.), u(.)) \;\middle|\; (1), (3), (2), (4) \text{ hold true} \right\}$$

- Viability or invariance approach
- A particular case : equilibria

Optimality

- An inter-temporal criterion

$$\pi\Big(x(0), x(1), \ldots, x(T), u(0), u(1), \ldots, u(T-1)\Big)$$

- The optimal control problem :

$$\pi(x^\star(\cdot), u^\star(\cdot)) = \max_{(x(.),u(.))\in\mathcal{T}_{\mathrm{ad}}} \pi(x(\cdot), u(\cdot)).$$

where

  - $u^\star(\cdot) = (u^\star(0), u^\star(1), \ldots)$ optimal decision sequence,
  - $x^\star(\cdot) = (x^\star(0), x^\star(1), \ldots)$ optimal state sequence.

Some criteria

- Present value

$$\pi(x(\cdot), u(\cdot)) = \sum_{t=0}^{T-1} \rho^t L(x(t), u(t))$$

- Maximin

$$\pi(x(\cdot), u(\cdot)) = \min_{t=0,\ldots,T-1} L(t, x(t), u(t)).$$

- Green

$$\pi(x(\cdot), u(\cdot)) = M(T, x(T))$$

- Chichilnisky

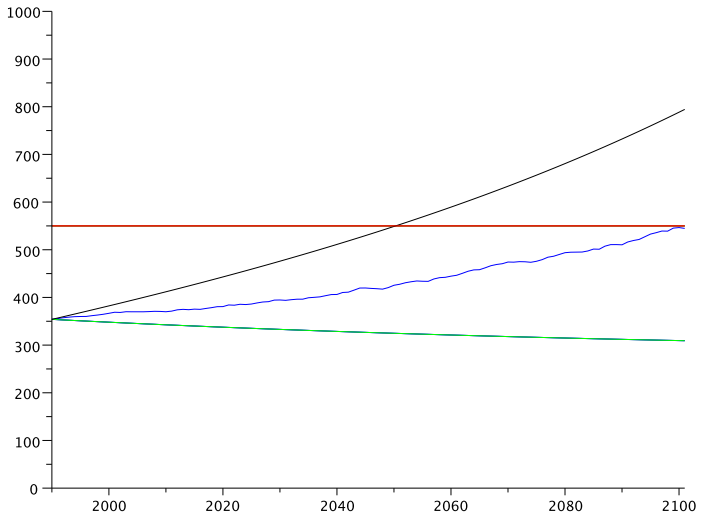
$$\pi(x, u) = \theta \sum_{t=0}^{T-1} L(t, x(t), u(t)) + (1-\theta) M(T, x(T))$$

A useful result

$$\min_{y \in A} f(y) = -\max_{y \in A} -f(y)$$

Simulations can help

- Given x_0, compute outputs of specific strategies
  - $u(t)$
  - $u(t, x)$
- Example : scenarios of mitigation policies



SCILAB Code

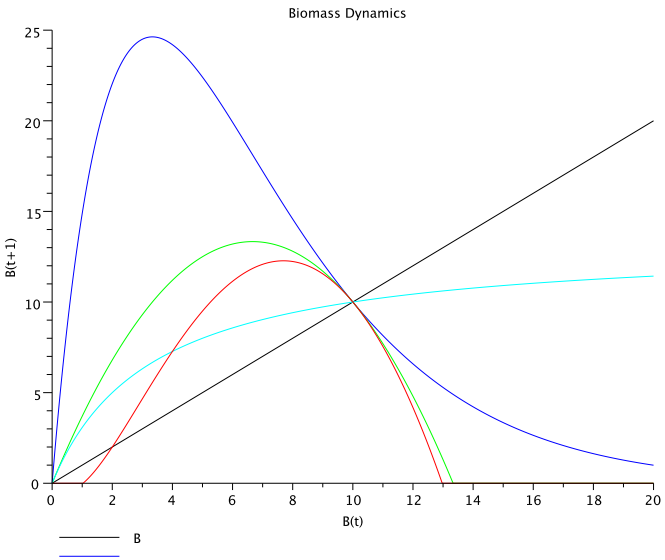
```
t0=1990 ; T=2100 ; M0=354 ;alp=0.471 ; Minf=280 ; d=1/120 ;
EBAU0=7.2 ;g=0.01 ;
//
function y=EBAU(t) ; y=EBAU0*(1+g)^(t-t0); endfunction
//
function y=f(t,M,ab) ; y=M+alp*EBAU(t)*(1-ab)-d*(M-Minf) ; endfunction
//
ab=zeros(1,T) ; // for instance
x(t0)=M0 ;
for (t=t0 :1 :T)
    x(t+1)=f(t,x(t),ab(t)) ;
end
//
plot2d(t0 :T+1,x(t0 :T+1),rect=[t0,0,T+1,1000])
```

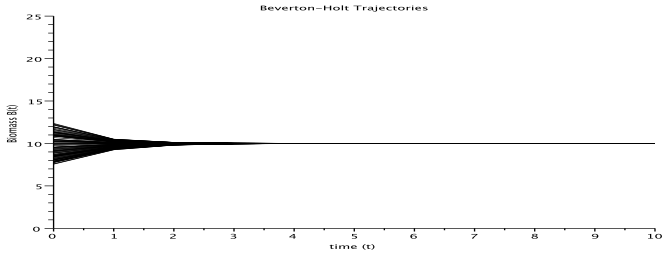
The sensitivity problem

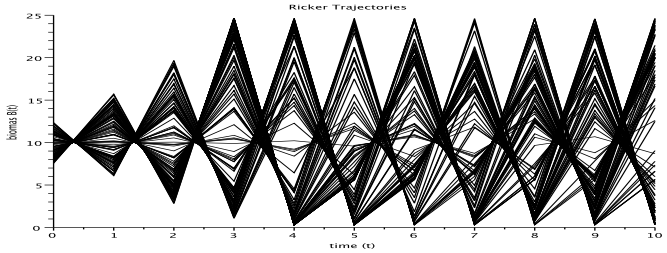
- Pb : The outputs $g(x(t), u(t))$, $L(x(t), u(t))$, π depend on :
  - the parameters of the model
  - the functional forms of the model
  - the initial conditions

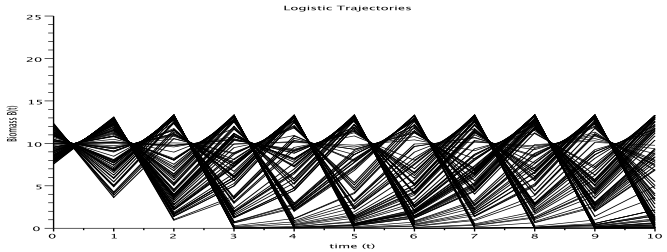
The sensitivity problem

Example : Different population dynamics with $r = 3$, $K = 10$









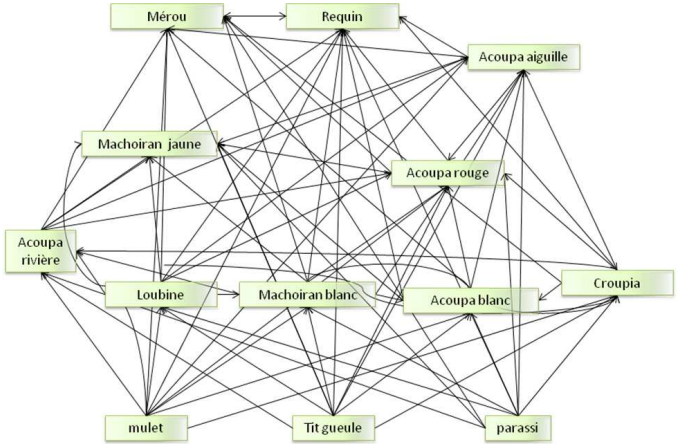
What about complexity ?

More complex models :

- Exploitation of an exhaustible resource with capital (DHS)
- A trophic web and sustainable use values
- An exploited metapopulation and PA management
- SSVPA Model

Exploitation of an exhaustible resource with capital (DHS)

A trophic web and sustainable use values



Simulations can help

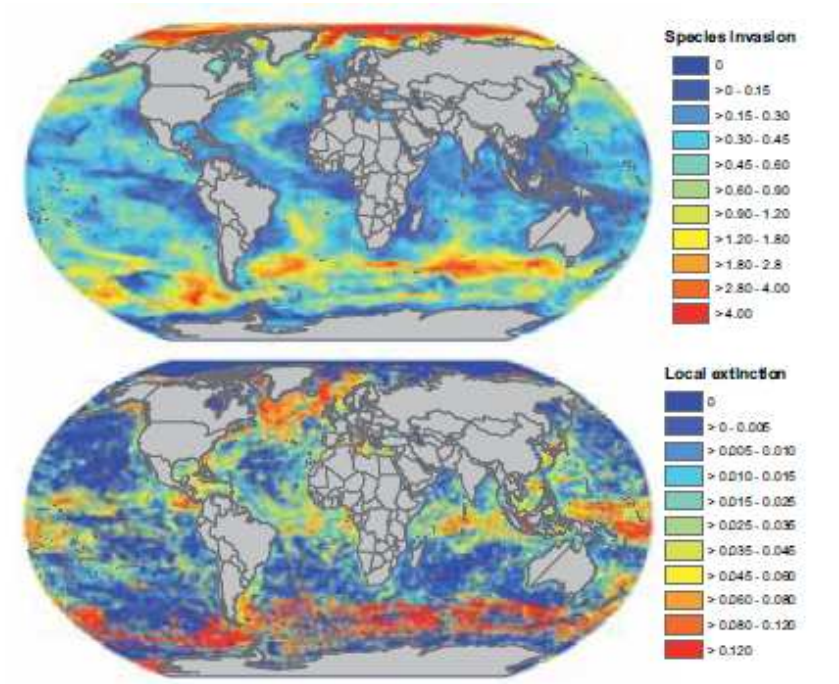
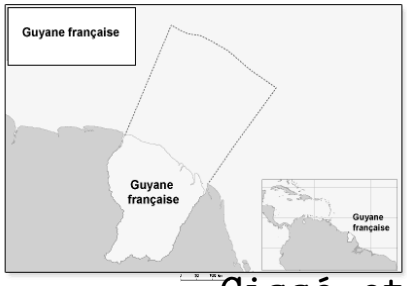


FIGURE 19 PROJECTED CHANGES IN MARINE BIODIVERSITY DUE TO CLIMATE CHANGE.

Biodiversity impact in 2050 under the IPCC SRES A1B scenario expressed in terms of: number of new species moving from other regions (top) and local extinction intensity (bottom). The projections are based on bioclimate envelope models for 1,066 species of fish and invertebrates. Source: redrawn from Cheung et al. 2009.

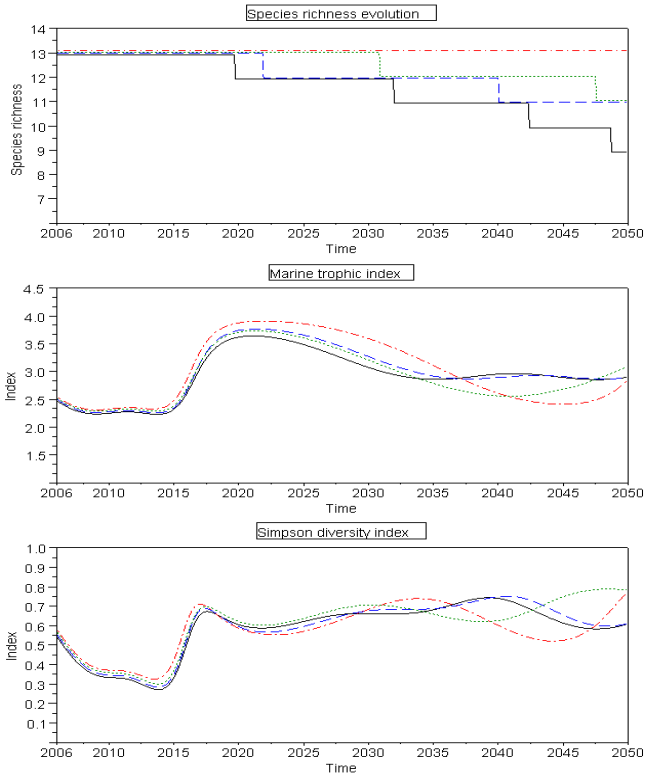
Cheung et al, Fish and Fisheries, 2009

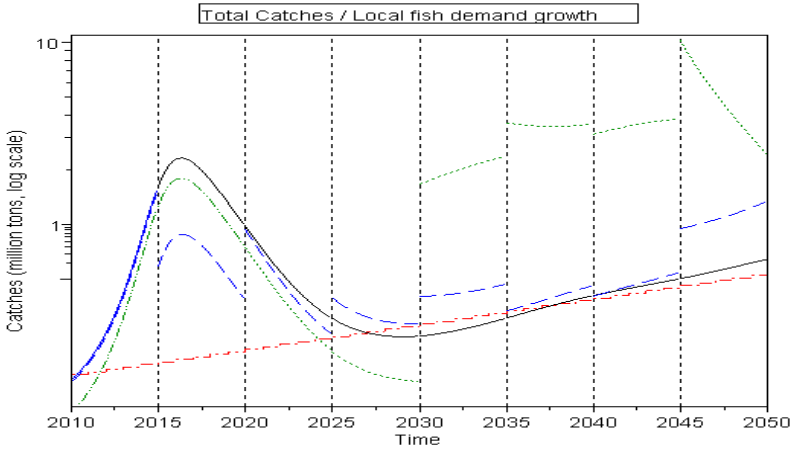
Simulations can help : Fisheries in French Guiana



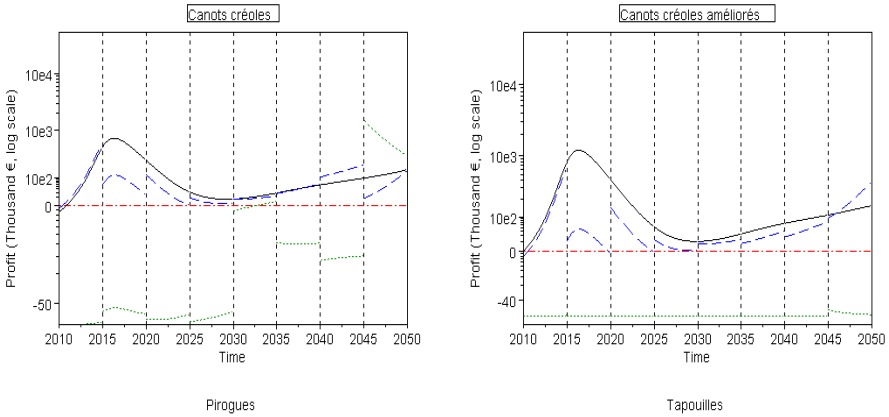
Cissé et al., Environment and Development Economics, 2013

Biodiversity



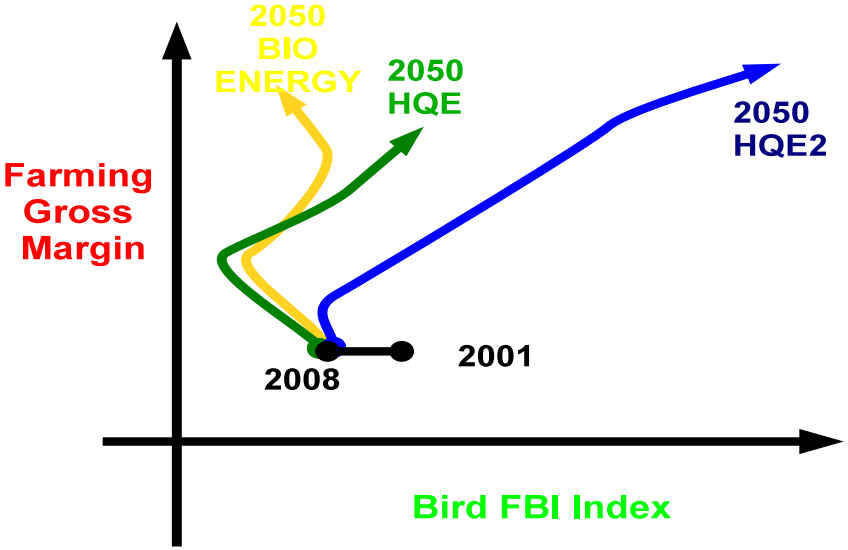


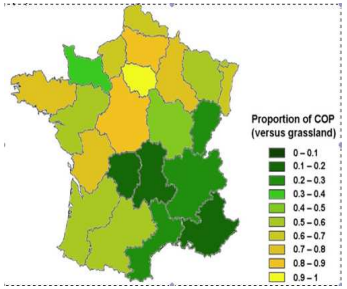
Seafood production



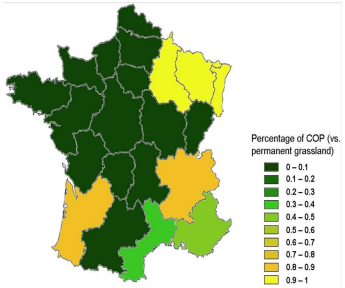
Simulations can help ! ! Farming land-use and birds

Mouysset et al., Ecological Economics, 2011

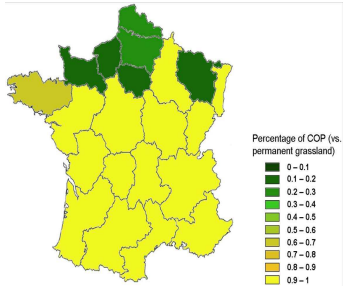




2008

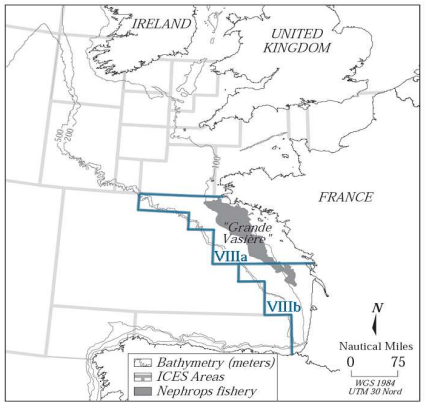


HQE2 2050



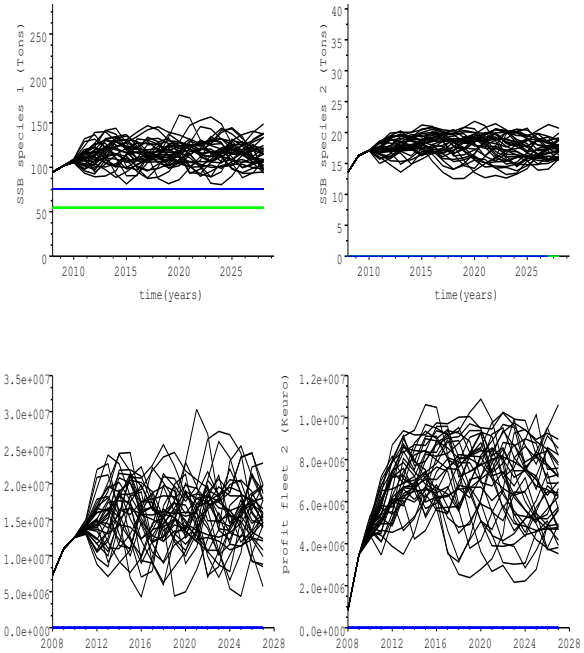
ENERGY 2050

Simulations can help ! ! Fisheries in Bay of Biscay

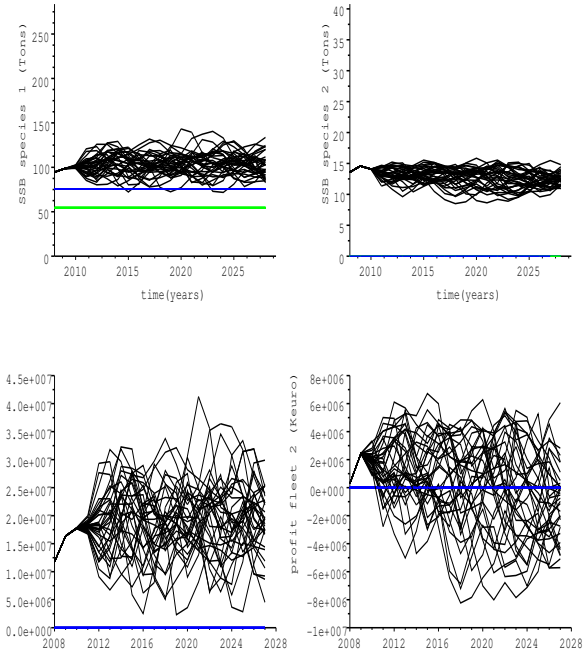


Doyen et al., Ecological Economics, 2012

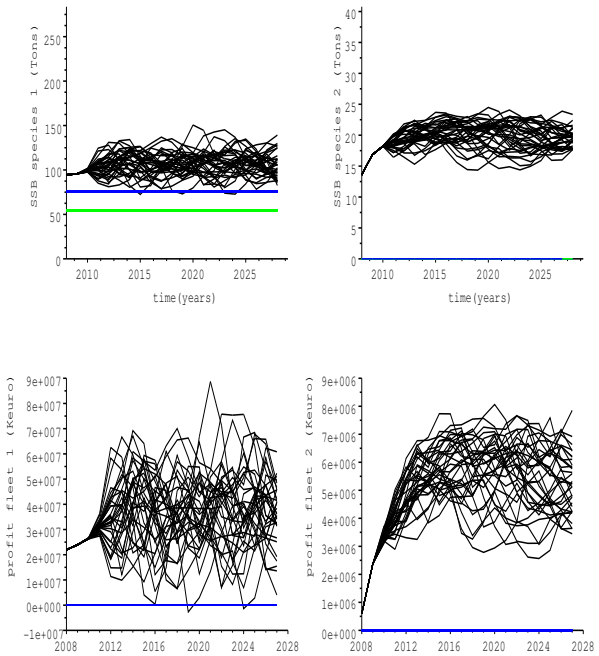
Co-viability



Status Quo 2008

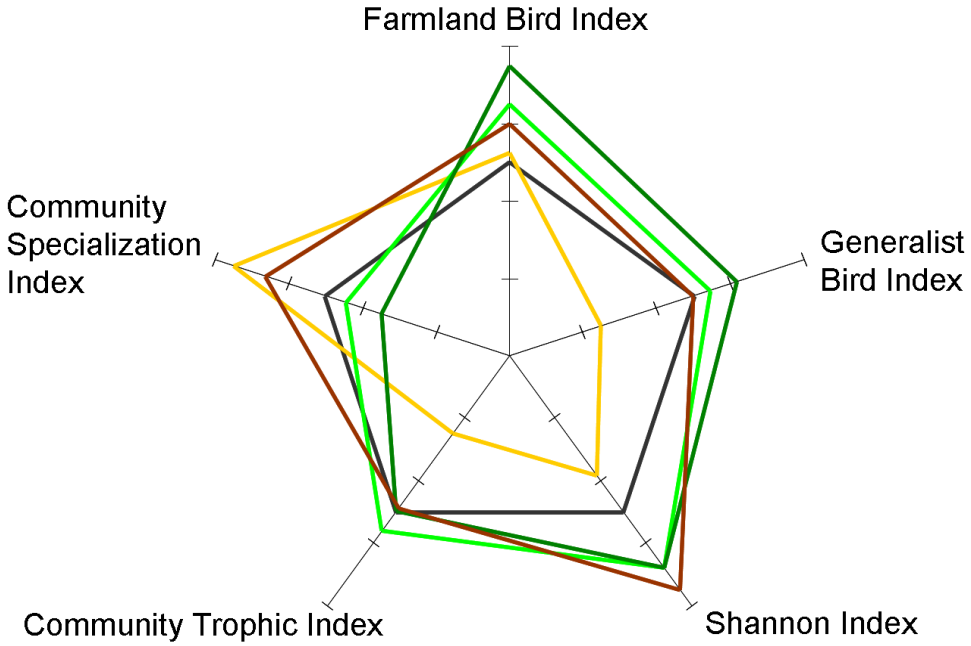


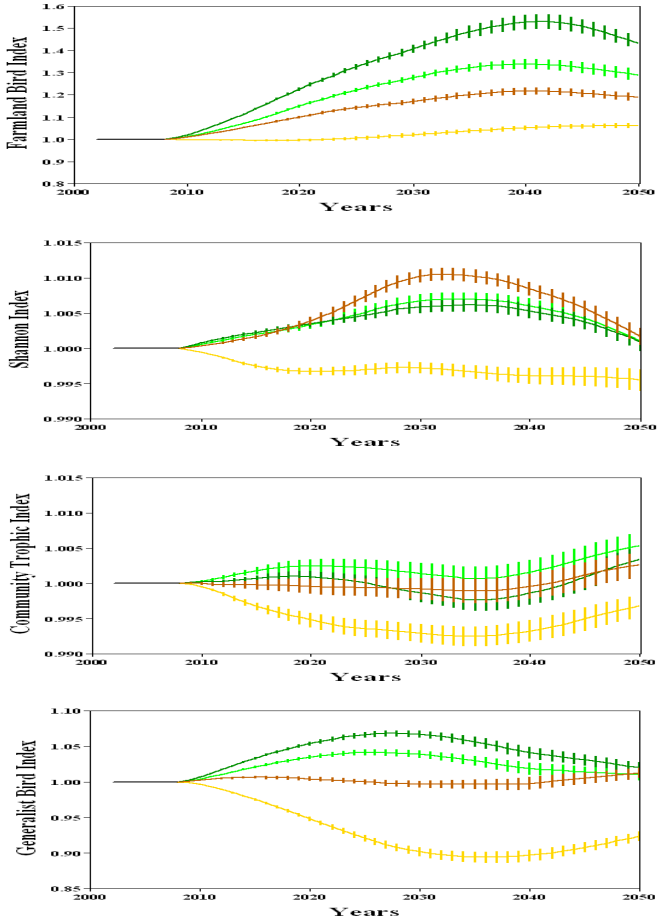
NPV scenario



But which synthetic indicators ? ? ?

Mouysset et al., Ecological indicators, 2011





Open loop or closed loop ?

- Open-loop : time-dependent sequences :

$$u : t \to u(t)\,.$$

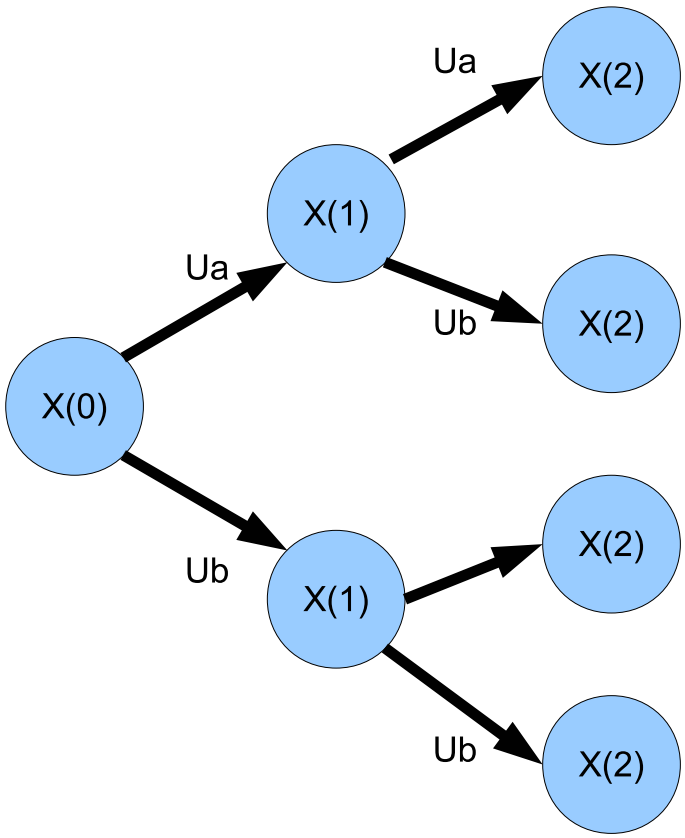
- Closed-loop $\approx$ feedback

$$u : (t, x) \to u(t, x)\,.$$

  - More robust
  - Dynamic programming methods.

Decision tree

- Finite countable decisions (ex $u \in \mathbb{U} = \{0, 1\}$)
- Given x_0
- Tree :
  - **nodes** = states $x(t)$
  - **edges** = decisions $u(t)$
- Evaluate the performance π on every path



The curse of the dimensionality

- A binary decision $u \in \{0, 1\}$ on horizon T
  $\Longrightarrow 2^T$ possible sequences $u(.)$
- On a computer :
  - **ram** : 8 GBytes $= 8(1\ 024)^3 = 2^{33}$ bytes
  - **a double-precision real** : 8 bytes $= 2^3$ bytes.

  $\Longrightarrow 2^{30}$ double-precision reals