Mathematical Modeling for Environmental Economics and Sustainable Management
Outils mathématiques pour la décision en environnement
Master EDDEE

Luc Doyen, Michel De Lara
Trimester Mathematics of Bio-economics at IHP

Mathematics of Bio-Economics
January 7th - April 5th 2013
Organized by Michel De Lara and Luc Doyen

Mathematics, Ecology and Economics
Sustainable Management of Renewable Resources
Biodiversity Scenarios Modelling

Mathematics Planet Earth 2013

Workshops
Mathematics and Ecological Economics 11-15/02
Risk and Learning in Biodiversity Management 4-8/03
Spatial Management of Biodiversity 25-29/03

Programme coordinated by the Centre Emile Borel of IHP
Registration is free however mandatory on http://www.ihp.fr
Participation of postdocs and Ph.D. students is strongly encouraged
Deadline for financial support: June 25th, 2012

For further informations, contact Claire Béranger mabies@ihp.fr
Scientific programme on http://cermics.enpc.fr/~delara/MABIES/MABIES/

Supported also by: FSMP

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Program

- **Monday 7, January 10h00-13h00**: IHP: Course Doyen-De Lara Introduction to decision modelling for sustainable management of natural resources
- **Monday 14, January 9h00-12h00**: ENGREF: Computer Session De Lara-Doyen Introduction to the scientific software
- **Monday 21, January 9h00-12h00**: IHP Course Doyen-De Lara Controlled dynamics and equilibria
- **Monday 28, January 9h00-12h00**: ENGREF: Computer Session De Lara-Doyen Equilibria
- **Monday 4, February 9h00-12h00**: IHP Course Doyen-De Lara Optimality and sustainability
- **Monday 11, February 9h00-12h00**: ENGREF: Computer Session De Lara-Doyen Optimality
- **Monday 18, February 9h00-12h00**: IHP Course Doyen-De Lara Viability
- **Monday 25, February 9h00-12h00**: ENGREF: Computer Session De Lara-Doyen Viability
Some References


Outline

- **General issues**
- **Deterministic models in discrete time**
  - General framework; Examples
  - Equilibrium and stability.
  - Viability.
  - Inter-temporal optimality.
- **Models under uncertainty**
  - Robust approach.
  - Stochastic approach.
- **Partial information**
- **Strategic interactions**
Environmental issues

- Management of exhaustible resources: oil, mining, ...
- Pollution management: ex: $\text{CO}_2$
- Management of renewable resources: Fisheries, wildlife, forest, agroecology
Sustainable management of biodiversity

- Global changes in ecosystems
- Alarming trends: MEA, IUCN, ...
- Major concerns for ecosystem services
  -.bio-economic risk
- Sustainable management

Population Index = 100 in 1970

- The Living Planet Index is an indicator of the state of the world's biodiversity. It measures trends in populations of vertebrate species living in terrestrial, freshwater, and marine ecosystems.

Catch (million tonnes)

- Effort (kilometers of engine power)

Source: WWF, UNEP-WCMC, Graphic/Pew Ocean Science Division, Data/Sea Around Us Project
"From IPCC"

To

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Biodiversity scenarios

SCÉNARIOS DE BIODIVERSITÉ: PROJECTIONS DES CHANGEMENTS DE LA BIODIVERSITÉ ET DES SERVICES ÉCOSYSTÉMIQUES POUR LE 21ᵉ SIÈCLE
Rapport technique pour les Perspectives mondiales de la diversité biologique 3

FIGURE 19: PROJECTED CHANGES IN MARINE BIODIVERSITY DUE TO CLIMATE CHANGE.
Biodiversity impact in 2050 under the IPCC SRES A1B scenario expressed in terms of: number of new species moving from other regions (top) and local extinction intensity (bottom). The projections are based on bioclimate envelope models for 1,066 species of fish and invertebrates. Source: redrawn from Cheung et al. 2009.
Bio-economic modeling

Modeling
Mathematics
Computer Sciences

Economics

Ecology
Bio-economic policies

Three major components:

- **Scientific knowledge**: mechanisms, models, data.
- **Shared intertemporal objectives**: ex: Johannesburg 2002: MSY by 2015
- **Bio-economic instruments**
  - standards: quotas, MPA, ...
  - monetary: landing taxes, ITQ, ...
  - informational: MSC,...
Important goals

- **Sustainability**
  - Reconciliation
    - Economy–Social-Ecology
  - Intergenerational equity
  - Intragenerational equity

- **Resilience**
- **Precaution**
Modeling requirements

Systemic approach

- Dynamics
- Uncertainties
- Decision

Evaluation

- Multi-criteria
  - Ecology
  - Socio-Economy
- Short and long term horizon

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Decision approaches for sustainability

Equilibrium and stability:
- Steady state
- MSY
  Schaefer, 1954

Intertemporal Optimality:
- Present Value
  CB, CE
  Clark, 1976
- Maximin criteria
- ....

Constraints, viability and risk:
- PVA
  Beissinger, 2002
- Viable control
- TWA
- SMS
  Bishop, 1978
Control theory

Problem:
Find **controls** to achieve various **goals** for **states** of a system

Three main ingredients:
- Controlled dynamics
- Constraints
- Criteria to optimize
Control theory in discrete time

- Economics: decision under uncertainty
- Ecology: life-cycle meta-population
- Modeling: simulations

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Examples for natural resource management

Stylized models:
- Exploitation of an exhaustible resource
- Management of a renewable resource
- Mitigation policies for CO$_2$ emissions
A controlled dynamics in discrete time

Difference equation:

\[
\begin{cases}
  x_i(t + 1) &= F_i(t, x(t), u(t)), \quad t = 0, \ldots, T \\
\end{cases} 
\] (1)

where

- the **state**, \( x(t) = (x_1(t), \ldots, x_n(t)) \in X = \mathbb{R}^n 
- the **control or decision**, \( u(t) = (u_1(t), \ldots, u_n(t)) \in U = \mathbb{R}^p 
- the **time Horizon** \( T \)
State constraints

- The admissible states

\[ x(t) \in \mathbb{A}(t), \quad t = 0, \ldots, T - 1, \quad (2) \]

- generally specified under inequality form

\[
\begin{cases}
    g_j(t, x(t)) \leq 0 \\
    g_k(t, x(t)) = 0.
\end{cases}
\]

- In many examples, a constant set:

\[ x(t) \in \mathbb{A} \]
Final state constraints

- Final state reaches some target $\mathbb{C} \subset \mathbb{X}$:

$$x(T) \in \mathbb{C}$$

(3)

- Generally specified under inequality form

$$g_j(x(T)) \leq 0$$
$$g_k(x(T)) = 0.$$
Decision or control constraints

- The admissible decisions:

\[ u(t) \in \mathbb{B}(t, x(t)), \quad t = 0, \ldots, T - 1 \quad (4) \]

- Generally specified under inequality form

\[
\begin{cases}
    g_j(t, x(t), u(t)) \leq 0 \\
    g_k(t, x(t), u(t)) = 0.
\end{cases}
\]

- Frequently, a constant set of feasible control

\[ u(t) \in \mathbb{B} \]
The feasible strategy: \( u(.) = (u(0), u(1), \ldots, u(T)) \) such that

- The constraints (3), (2) and (4)
- where \( x(.) \) with the dynamics (1)

Feasibility set:

\[
\mathcal{T}_{ad} = \left\{ (x(.) , u(.)) \left| (1), (3), (2), (4) \text{ hold true} \right. \right\}
\]

Viability or invariance approach

A particular case: equilibria
• An inter-temporal criterion

\[ \pi \left( x(0), x(1), \ldots, x(T), u(0), u(1), \ldots, u(T-1) \right) \]

• The optimal control problem:

\[ \pi(x^*(\cdot), u^*(\cdot)) = \max_{(x(\cdot), u(\cdot)) \in \mathcal{T}_{ad}} \pi(x(\cdot), u(\cdot)). \]

where

- \( u^*(\cdot) = (u^*(0), u^*(1), \ldots) \) optimal decision sequence,
- \( x^*(\cdot) = (x^*(0), x^*(1), \ldots) \) optimal state sequence.
Some criteria

- **Present value**
  \[
  \pi(x(\cdot), u(\cdot)) = \sum_{t=0}^{T-1} \rho^t L(x(t), u(t))
  \]

- **Maximin**
  \[
  \pi(x(\cdot), u(\cdot)) = \min_{t=0, \ldots, T-1} L(t, x(t), u(t)).
  \]

- **Green**
  \[
  \pi(x(\cdot), u(\cdot)) = M(T, x(T))
  \]

- **Chichilnisky**
  \[
  \pi(x, u) = \theta \sum_{t=0}^{T-1} L(t, x(t), u(t)) + (1 - \theta)M(T, x(T))
  \]
A useful result

\[
\min_{y \in A} f(y) = - \max_{y \in A} -f(y)
\]
Simulations can help

- Given $x_0$, compute outputs of specific strategies
  - $u(t)$
  - $u(t, x)$
- Example: scenarios of mitigation policies
t0=1990 ; T=2100 ; M0=354 ; alp=0.471 ; Minf=280 ; d=1/120 ; EBAU0=7.2 ; g=0.01 ;

//
function y=EBAU(t) ; y=EBAU0*(1+g)^(t-t0); endfunction

//
function y=f(t,M,ab) ; y=M+alp*EBAU(t)*(1-ab)-d*(M-Minf) ; endfunction

//
ab=zeros(1,T) ; // for instance
x(t0)=M0 ;
for (t=t0 :1 :T)
    x(t+1)=f(t,x(t),ab(t)) ;
end

//
plot2d(t0 :T+1,x(t0 :T+1),rect=[t0,0,T+1,1000])
The sensitivity problem

Pb: The outputs $g(x(t), u(t))$, $L(x(t), u(t))$, $\pi$ depend on:
- the parameters of the model
- the functional forms of the model
- the initial conditions
Example: Different population dynamics with $r = 3$, $K = 10$. 

The sensitivity problem
What about complexity?

More complex models:

- Exploitation of an exhaustible resource with capital (DHS)
- A trophic web and sustainable use values
- An exploited metapopulation and PA management
- SSVPA Model
A trophic web and sustainable use values
Simulations can help Cheung et al, Fish and Fisheries, 2009

**FIGURE 19** PROJECTED CHANGES IN MARINE BIODIVERSITY DUE TO CLIMATE CHANGE.

Biodiversity impact in 2050 under the IPCC SRES A1B scenario expressed in terms of: number of new species moving from other regions (top) and local extinction intensity (bottom). The projections are based on bioclimate envelope models for 1,066 species of fish and invertebrates. Source: redrawn from Cheung et al. 2009.
Simulations can help: Fisheries in French Guiana

Cissé et al., Environment and Development Economics, 2013

Biodiversity

Seafood production

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Modeling for the Sustainable Management
Simulations can help!! Farming land-use and birds

Mouysset et al., Ecological Economics, 2011

Farming Gross Margin

Bird FBI Index
Simulations can help!! Fisheries in Bay of Biscay

Doyen et al., Ecological Economics, 2012

Co-viability

Status Quo 2008

NPV scenario
But which synthetic indicators???

Mouysset et al., Ecological indicators, 2011
Open loop or closed loop?

- **Open-loop**: time-dependent sequences:
  \[ u : t \rightarrow u(t). \]

- **Closed-loop** \(\approx\) feedback
  \[ u : (t, x) \rightarrow u(t, x). \]

  - More robust
  - Dynamic programming methods.
- Finite countable decisions
  (ex $u \in \mathbb{U} = \{0, 1\}$)
- Given $x_0$
- Tree:
  - nodes = states $x(t)$
  - edges = decisions $u(t)$
- Evaluate the performance $\pi$ on every path
A binary decision \( u \in \{0, 1\} \) on horizon \( T \)
\[ \implies 2^T \text{ possible sequences } u(.) \]

On a computer:
- \textbf{ram}: 8 GBytes = \( 8(1 \,024)^3 = 2^{33} \) bytes
- \textbf{a double-precision real}: 8 bytes = \( 2^3 \) bytes.
\[ \implies 2^{30} \text{ double-precision reals} \]