OGRE: Optimization, Games and Renewable Energy

Report
PGMO/IROE Umbrella Project OGRE 2016-1749H

Project leader: Michel De Lara, École des Ponts ParisTech

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Chapter 1

Introduction
(Michel De Lara)

In this section, we describe the PGMO/IROE umbrella project OGRE: Optimization, Games and Renewable Energy.

1.1 Context

The transformation of energy systems is accelerating. The last COP 21 in Paris, the new French law on energy transition are institutional drivers of such change. On the other hand, local initiatives are blossoming, with the drop in renewable energy costs and the impulse of decentralized actors (individuals, collectivities).

Managing an energy system with myriads of decentralized sources (wind, sun) and actors (individuals, collectivities) is becoming more and more challenging. Equating supply and demand at all scales is especially delicate with a large share of intermittent and highly variable renewable energies. ”Smart grids”, ”demand response”, ”flexibility”, ”storage” are presented as potential solutions, but the question remains: how?

For a large actor as EDF, the question is also to better identify how to play in this transformed field.

1.2 The contributions from PGMO projects

The PGMO has launched several projects which, directly or indirectly, touch the subject of the role of optimization in the new energy landscape:

- STOCHDEC, Décomposition/Coordination en commande optimale stochastique (PGMO/IROE project, leader Pierre Carpentier, EDF correspondent Anes Dallagi).
- SMARTDEC, Décomposition/Coordination pour les smart-grids (PGMO/IROE project, leader Pierre Carpentier, EDF correspondent Anes Dallagi). The initial goal of this three-years (thesis) project was to apply decomposition methods to smart grids. It has been reorganized at the beginning of 2016 in a post-doc dealing with the centralized-decentralized approach in the forthcoming energy management system.
- LASON, Latin America Stochastic Optimization Network, to build a network on stochastic optimization for energy, with Chilean and French researchers (PGMO/PRMO, leaders Bernardo Pagnoncelli and Michel De Lara).

- LASON2, on Centralized versus Decentralized Energy Management in a Stochastic Setting, following LASON (PGMO/IROE, leaders Bernardo Pagnoncelli and Michel De Lara).
- STORY, scientific network on Stochastic and Robust Optimization and Applications, with US and French researchers (PGMO/PRMO, leaders Laurent El Ghaoui and Michel De Lara).
- LORI, Logiciels pour l'Optimisation des Réseaux Intelligents (PGMO/IROE, leader Michel De Lara).
- PALON, Stochastics and optimization for markets with renewable energy (PALON) PGMO/PRMO, leaders Teemu Pennanen and Jean-Philippe Chancelier, one year.

Through these projects, there now exists a critical mass of researchers that have learned to work together on energy systems management:

- Pierre Carpentier, ENSTA ParisTech, UMA, France
- Rodrigo Carrasco, University Adolfo Ibanez, Chile
- Jean Philippe Chancelier, Ecole des Ponts, CERMICS, France
- Michel De Lara, Ecole des Ponts, CERMICS, France
- Laurent El Ghaoui, UC Berkeley, USA
- Tito Homem-de-Mello, University Adolfo Ibanez, Chile
- Hélène Le Cadre, ENSTA ParisTech, UMA, France
- Vincent Leclère, Ecole des Ponts, CERMICS, France
- Bernardo Pagnoncelli, University Adolfo Ibanez, Chile
- Teemu Pennanen, King's college London, UK

Together with EDF researchers:

- Olivier Beaude, EDF R&D OSIRIS, France
- Sandrine Charousset, EDF R&D OSIRIS, France
- Anes Dallagi, EDF, UK
- Arnaud Lenoir, EDF R&D OSIRIS, France
- Nadia Oudjane, EDF R&D OSIRIS, France
- Riadh Zorgati, EDF R&D OSIRIS, France

1.3 Our proposal of a PGMO/IROE umbrella project OGRE

It is now time to digest all the material produced by different PGMO projects. We propose OGRE: Optimization, Games and Renewable Energy, as an umbrella project that aims at covering the above projects, gathering the expertise produced and restitute it in an integrated way.

The OGRE report is made of five chapters.

- Chapter 2 presents how EDF perceives the new playground of energy systems, with its different actors; it underlines some new management problems that EDF faces.
- Chapter 3 provides concepts that are useful to enlighten the mathematical structure of the new management problems: agents, information, criteria; then, it delineates corresponding classes of optimization and of game problems. Daniel Kadnikov's PhD thesis at Cermics (supervision by Michel De Lara) has built upon this framework, and developed it.
- Chapter 4 chapter treats mechanism design within the above formalism.
- Chapter 5 is made of a series of case studies.

Chapter 2

Challenges for the New Energy Systems (Riadh Zorgati)

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2.1 The French electricity system: from the former fully-integrated context to a decentralized vision

To introduce the current need for identifying the challenges in decentralized management of the electrical system, let us start with a brief description of the former context, when it was no question of this approach. By doing so, the major determinants of the activity of a stakeholder of the energy system, responsible for generation / distribution and supply, will be also introduced.

2.1.1 The fundamentals of energy management

Managed in a centralized or a decentralized manner, the operation of the electrical system is subject to a few fundamental aspects. This paragraph will present these key points within the context of a fully integrated management when generation, distribution and supply are controlled by a unique stakeholder (e.g., EDF in France before 2002).

Supply-demand balance: a physical and economic necessity

The first mission of such a unique (centralized) operator is to ensure the supply-demand balance. By using both physical and financial means (supply), this operator must provide the electricity needed by its customers (demand) at each time instant. In addition with the service provided to the end electricity users, this balance must be preserved for many reasons related to the electrical network operations. Both quantities must be balanced at each instant due to the current difficulty of storing energy at reasonable cost. This must be anticipated at different time scales from the long-term, when planning the evolution of power generation capacities, to the short-term, when adjusting in real-time the generation planning of the different units and also eventually using different financial electricity contracts to adjust the balance in real-time.

On the demand-side, an inherent aspect is the capability of forecasting electricity consumption at different time horizons, and different geographic scales. In turn, various studies about energy management use stochastic models and tools to fit with this fundamental point: even at a global scale and a few minutes before real-time, the demand forecast is not perfect. A wide variety of methods have been proposed to address this problem Bunn (2000); Taylor and McSharry (2007); Hippert et al. (2001); a good introduction and comparative analysis of these existing methods can be found in Feinberg and Genethliou (2005). Identifying the main drivers of this demand is part of this research. In this direction, both physical and societal aspects must be taken into account. Concerning physical aspects, the most significant is the thermosensibility of demand. Taking France as an extreme example due to the extensive use of electric heating, a decrease of one degree during the winter induces an increase of 2400MW de Transport d'Électricité (2016). Other very intuitive physical factors impact the demand profile: natural lighting, wind, rain, etc. Concerning societal aspects, the economic situation is the more relevant but other very specific events have a significant impact on the demand (e.g, an electoral event or the world cup final, which significantly impact television audience).

On the supply-side, physical and financial means need to be distinguished. Consider first the physical production means. The physical supply is the electricity that can be generated by the units available at a given time. This supply mainly consists of thermal plants (nuclear, fossil fuel, coal, gas turbines), hydraulic power units, and renewable electricity units (mainly photovoltaic, wind energies). As mentioned later, the later units are becoming increasingly important in the electrical system. Each power unit is characterized by static and dynamic constraints (minimal running time, predefinite interval and levels of production, etc.), a cost structure, as well as a few constraints related to the interaction with its local ecosystem. In conjunction with the physical supply, financial means are available for the operator responsible for the supply-demand balance. Tariff options allow to indirectly control a part of the demand in order to get a better temporal distribution of the total load. For example, a producer/provider can contract with (at this time, generally major) customers on "shedding mechanisms". In these contracts, it is specified that high tariffs can be used on a few time-slots, in return for a reduced tariff during the usual time-slots. With such a tariff, customers have a strong incentive to consume during the usual time-slots. A producer can also exchange a quantity of electricity with other producers at a predefinite price for a given period ("over-the-counter"). Eventually, the electricity or fuel markets may be used to adjust the supply.

The weather and the market: conditioning fluctuations

Ensuring the supply-demand balance is not an easy task. One of the main reasons for that is the stochastic nature of the main determinants of both supply and demand. As a first determinant, the weather directly - and significantly - impacts both supply and demand. It conditions the need for heating, but also water-flows, and primary energy resources of renewable units. A huge amount of research work is dedicated to make the link between weather forecast methods and the variables of the electrical system which strongly rely on it. In particular, the prediction of renewable energy generation is a key topic Ernst et al. (2007); Soman et al. (2010). Furthermore, these studies have to be now conducted at different temporal and geographical scales. While at a global scale the forecast errors roughly cancel out, the ones at a local scale can be dramatic[1]. This contributes to the strategic reflection about the choice between a decentralized or centralized management. The outage of production plants is another crucial stochastic aspect. Even if correlated with the weather, the dependency between both should be further analyzed to get a proper forecast on the availibility of the different units. In addition with these physical uncertainties, the energy market fluctuations must also be taken into account. Many studies model the formation of energy market prices Chen et al. (2010); Gonzalez et al. (2005), introducing a stochastic component.

Protection of the environment: in the air

The balance of supply and demand must be ensured while taking into account a certain number of rules to protect the environment and to combat climate change. These rules are often based on European Union directives. For the electricity producers, it mainly consists in limiting (or reducing) the greenhouse gas emissions, adapting to the rapid development of renewable energies, and evaluating,

[1] In other words, while we face a variability issue at a global scale, this is an intermittency one at a local scale Burtin and Silva (2015).

on the demand-side, the modifications induced by the changed building regulation (High Environment Quality labels). In particular, the massive integration or renewable energies strengthen the need for a comprehensive reflection about the technical and economical feasibility - at a local and a global scale - of connecting these sources to the existing largely interconnected system. A very recent EDF contribution considered the possibility for the European electricity system to integrate 60% of renewable energy sources Burtin and Silva (2015).

Energy management in this context

Energy management consists in making decisions to optimally manage both physical and financial assets, while providing the services contracted with the customers and respecting current legislative and commercial constraints. More precisely, two coherent and complementary strategies must be defined: on the one hand, the (physical) planning of the production units; on the other hand, the positions on the electricity market and the management of gas assets. These strategic choices are done in order to minimize the total cost, which contains both a physical and a financial component. With the later financial part, the risk must also be taken into account. Furthermore, energy management decisions have to be taken at different horizons, from long-term investment choices, to very short-term operation planning for the production units.

2.1.2 The progressive separation of activities in the electricity system: a new context is emerging

With the construction of the European electricity markets, opening-up to competition, generation, transmission and distribution activities have been - functionnaly then legally - separated in France. This leads to the creation of both a Transmission Network Operator (TNO) Réseau de Transport d'Électricité (RTE), and a Distribution Network Operator (DNO), Électricité Réseau Distribution France (ERDF). These new players are respectively responsible of providing an equal access - in an objective and transparent manner - to the transmission and distribution networks. This allows all the new power system players to access to the services provided by the electrical network. This naturally leads to a decentralized framework, by distinguishing different players - producers, TNO, DNO, suppliers - with different objectives, information available, and possible actions. It induces a need for coordination, information exchange between the different entities involved. Designing good rules for this new "game" will condition the performance of the different services, whose management is now shared between various players. The legislator will have a key role in this context; he directly contributes to the design of electricity markets, and the definition of the rules for the new services proposed to the customers.

Let us now present the main ingredients of this new decentralized vision of energy management in the electricity system.

2.2 The new decentralized vision: main ingredients

2.2.1 The multiplicity of stakeholders in the electricity system

As mentioned above in §2.1.2, new players recently entered the electricity system. The objective and management of production, network management and electricity supply are now clearly distinguished. But there is more than these traditional stakeholders; with the sociological and economic trend of reducing or at least better understanding and controlling the electricity usages, many new players enter this system in the so-called smart grids, or smart cities. This comes with the progressive implication of the society into the question of environmental protection.

This stimulates everyone's involvement in this field, starting from the motivation to get informed about his own consumption. Reference Faruqui et al. (2010) indicates that the residential electricity consumption could be decreased by 7-14% only by sending a signal to the households when they are consuming a significant amount of power, or when the network is congested (or it is very costly to produce electricity). More than being informed, big electricity customers are already thinking of making their consumption profile flexible according to individual metrics (electricity bill), or even taking into account (local) constraints of the electricity system (energy losses, asset aging). As a first step in this research direction, Mathieu et al. (2011) proposes a tool for building energy managers in order to better understand their electricity consumption profile and implement Demand Response. Reference Rao et al. (2012) studies how Internet service providers with distributed data centers could optimally schedule their electricity consumption across time and space in order to reduce their electricity bill, while ensuring their core service. Specific electricity contracts are already dedicated to these big consumers, as explained in §2.1.1.

The new paradigm of Smart Grid proposes to enlarge the pool of flexible customers to all households Du and Lu (2011). By optimally scheduling its own load profile, each household can meet the objective of minimum electricity payment or maximum comfort Mohsenian-Rad et al. (2010). This modifies the traditional relation between electricity suppliers and their customers, leading naturally to a competitive framework, as presented hereafter. City operators are also envisioned to be part of this evolution by optimizing their electricity consumption profiles. Smart lighting Karlicek (2012), smart scheduling of electric bus charging profiles Ríos et al. (2014), coordination of the charging decisions of a public system of shared vehicles or a taxi fleet Akhavan-Hejazi et al. (2014) are a few examples of this attempt. By making their consumption flexible, all these new players will directly impact the aggregate electricity demand profile. Thus, all these new decision-makers should be integrated in the decentralized vision of the electricity system.

The multiplicity of stakeholders in the electricity system comes with the development of new generation means, which also naturally lead to a decentralized vision.

2.2.2 The new physical generation means: rapid growth of renewable energies

We will not provide many details in this section, whose topic has been already mentioned in §2.1.1. A good introduction can be found in Burtin and Silva (2015). We only remind here the main characteristics of renewable energies generation. First, its variable cost and polluting emission is (approximately) zero, which makes it particularly appealing in the current evolution. However, because it is still difficult to forecast Ernst et al. (2007), it leads to an intermittency problems at a local scale. There is a clear need for local management of this production, in coordination with the electricity network Thomson and Infield (2007); Siano et al. (2010). In most cases, local production is thought to be used locally to promote self-production and self-consumption Novel-Cattin et al. (2015). Note that, even if a societal conviction also pushes forward to this local management, the service provided is not completly local, in the sense that the support of the interconnected network is needed in the event of a punctual need.

In conjunction with these new physical supply means, the concept of virtual power plant Ruiz et al. (2009) is now emerging. It defines a system integrating several types of physical or virtual power sources, so as to give a reliable overall power supply. For example, it can consist of the aggregation of many renewable energy units in order to mitigate the significant intermittency issue faced by a unique unit. Forming a coalition Baeyens et al. (2013), renewable producers can exploit the reduced aggregate power output intermittency to submit less risky offers to an electricity market. A virtual power plant can also integrate flexible electricity users, e.g. with shedding mechanisms.

2.2.3 New electricity consumptions and equipments

Many new electricity appliances also lead to rethink the standard centralized vision of the electricity system. As mobile electricity consumption units, electric vehicles are a typical example of these new consumptions. Here, the problem consists in designing good electric vehicle charging policies, if possible in interaction with the constraints observed on the electricity system. When capable of reinjecting electricity to the grid (vehicle-to-grid), electric vehicles can also be considered as a local production unit. Affordable residential storage units[2] could help the development of local consumption units, without necessarily being connected to the global electricity network.

2.2.4 New information technologies and capabilities

The envisioned smart grids of the future Ipakchi and Albuyeh (2009) also include a variety of local measures on the electricity network and on the consumption units. By using smart meters[3], a detailed analysis of the residential consumption profiles will be possible as well as sending incentive prices to these customers. In particular, this allows designing local energy management strategies to coordinate the consumption decisions of a set of electricity users Mohsenian-Rad et al. (2010). These newly available measures can be coupled with the fast-growing potential of learning methods (for example, deep learning), applied on large sets of data (big data). In turn, this new component seems particularly appealing in the context of electricity networks.

2.2.5 A new social perception of the electricity system

To conclude the description of the decentralized paradigm emerging in the electricity system, let us remark that this comes with a new societal perception of the electricity system. By strongly supporting the development of renewable energies, and deeply questioning the model of a fully centralized and integrated management in the electricity system, the political power naturally proposes to adopt a decentralized vision. By locally managing a small production/consumption units, this may also give the impression to local actors to escape from complex global mechanisms, related to the interconnected network or electricity markets.

Note, however, that this tendency brings up a certain number of difficult issues. First, this tendency could seem contradictory with regards to some fundamental principles still upheld by the political power to guide the definition of a future electricity system. Currently, geographically distinguishing residential electricity tariffs comes not into question; tariff equalisation is still valid. This is contradictory with the strong development of local production units and the objective of locally consuming the power generated. Currently, it seems natural that the electricity network plays its insurance role; when usually stand-alone consumption units need electricity from the grid, they pay for it as would do each other standard customer. However, the electricity system was not conceived with this operational vision in mind, and neither were the current economic signals sent to the consumption units. In the decentralized context, two first relevant questions arises:

- with the current electricity network structure, what kind of signals should be sent to the local electricity consumption units to properly guide their electricity network use?
- what would be the new structure of the electricity distribution (and transmission) network if it was based on a local philosophy?

[2]The Tesla powerwall, https://www.teslamotors.com/powerwall, recently received a great attention.

[3]In France, the installation of Linky is ongoing; by 2021, 35 millions of these smart meters should have been replaced according to ERDF.

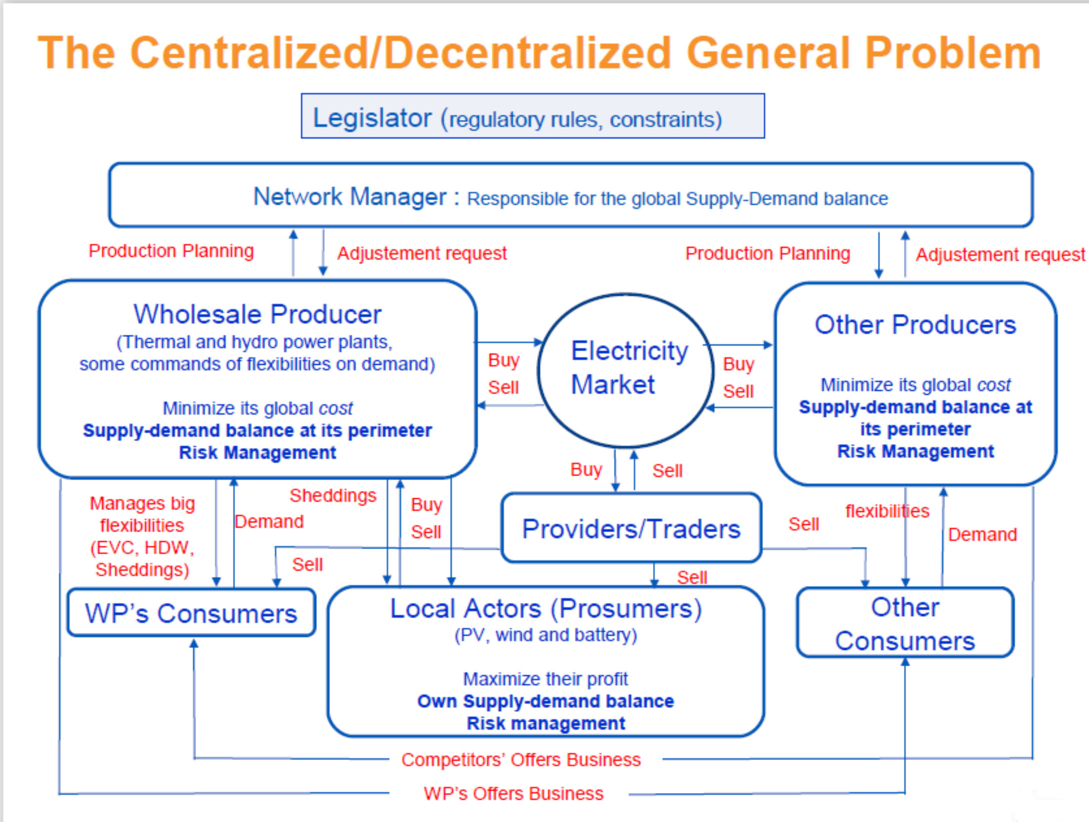

Figure 2.1: Overview of the centralized/decentralized general problem.

These questions are two first examples of very concrete analysis that may be conducted to think energy management in a decentralized vision. The next section proposes a schematic description of a few main questions arising in this context for a stakeholder like EDF.

2.3 Decentralized vision: a variety of problems

To conclude this part, Figure 2.1 illustrates the variety of problems in the electricity system that could be tackled in a decentralized context. Among the many suggested and directly related to the ingredients introduced in Sec. 2.2, let us introduce two particular problems.

The first is related to the joint optimization of production and demand. It is relevant for an electricity producer for which a set of flexible electricity consumers is available. Then, if this producer can directly manage demand flexibility (e.g., by shedding mechanisms as described previously), the framework obtained is the one a virtual power plant. With this innovative vision, production and demand decisions are jointly taken in order to minimize the production cost and the inconvenience of the consumer (which strongly depends on the type of appliances that take part of the flexibility mechanism). Observe that this problem can be formulated in a bilevel setting, where one problem (the one of the electricity customer) is embedded into another one (the one the producer). This structure is typical of many studies in a decentralized framework, with an operator of the network / production / markets at the upper-level and customers at the lower-level.

The second problem models the competitive interaction between different providers to share the market of residential electricity customers. Because the activity of electricity supply has been open to competition, this is the situation we currently face in France. Again, this can be interpreted in a model with two levels:

- at the upper-level, electricity suppliers propose electricity tariffs to the residential customers. Their choices result from an optimization problem, which essentially consists in maximizing their profit ;
- at the lower-level, residential customers choose the best electricity provider for them. This choice also results from an optimization problem in which the tariffs proposed by the suppliers is an input. Hence, the customers react to the decisions taken by the suppliers.

This model can be reinterpreted in many situations emerging in the electricity system, for example if an operator of the distribution network sends a signal to incite customers not to significantly increase energy losses and equipment aging. This model is studied in the case of electric vehicles as the flexible electricity consumption in Beaude et al. (2016).

Chapter 3

A Formal Presentation of Centralized/Decentralized Problems (Michel De Lara, Hélène Le Cadre, Benjamin Heymann)

Contents

3.1 One agent, one criterion (optimization)

- Optimization set $\mathbb{U}$ containing optimization variables $u \in \mathbb{U}$
- A criterion $J : \mathbb{U} \to \mathbb{R} \cup \{+\infty\}$
- Constraints of the form $u \in \mathbb{U}^{ad} \subset \mathbb{U}$

$$\inf_{u \in \mathbb{U}^{ad}} J(u)$$

3.1.1 Deterministic optimization

3.1.2 Deterministic multi-stage optimization

- A set $\{t_0, t_0+1, \dots, T\} \subset \mathbb{N}$ of discrete times t
- Control sets $\mathbb{U}_t$ containing control variable $u_t \in \mathbb{U}_t$, for $t = t_0, t_0+1, \dots, T$
- A criterion $J : \prod_{t=t_0}^{T} \mathbb{U}_t \to \mathbb{R} \cup \{+\infty\}$
- Constraints of the form $u = (u_{t_0}, \dots, u_T) \in \mathbb{U}^{ad} \subset \prod_{t=t_0}^{T} \mathbb{U}_t$

$$\inf_{(u_{t_0}, \dots, u_T) \in \mathbb{U}^{ad}} J(u_{t_0}, \dots, u_T)$$

3.1.3 One-stage optimization under uncertainty

What makes optimization under uncertainty specific

- Optimization set is made of random variables
- Criterion generally derives from a mathematical expectation,
  or from a risk measure

- Constraints
  - generally include measurability constraints,
    like the nonanticipativity constraints,
  - and may also include probability constraints, or robust constraints

Here are the ingredients for a general abstract optimization problem under uncertainty

- A set $\mathbb{U}$
- A set Ω of scenarios
- An optimization set $\mathbb{V} \subset \mathbb{U}^{\Omega}$ containing random variables $\mathbf{V} : \Omega \to \mathbb{U}$
- A criterion $J : \mathbb{V} \to \mathbb{R} \cup \{+\infty\}$
- Constraints of the form $\mathbf{V} \in \mathbb{V}^{ad} \subset \mathbb{V}$

$$\inf_{\mathbf{V} \in \mathbb{V}^{ad}} J(\mathbf{V})$$

Here is the most common framework for robust and stochastic optimization

- A set $\mathbb{U}$
- A set Ω of scenarios, or states of Nature,
  possibly equipped with a σ-algebra
- An optimization set $\mathbb{V} \subset \mathbb{U}^{\Omega}$ containing random variables $\mathbf{V} : \Omega \to \mathbb{U}$
- A risk measure $\mathbb{F} : \mathbb{V} \to \mathbb{R} \cup \{+\infty\}$
- A function $j : \mathbb{U} \times \Omega \to \mathbb{R} \cup \{+\infty\}$ (say, the "deterministic" criterion)
- Constraints of the form $\mathbf{V} \in \mathbb{V}^{ad} \subset \mathbb{V}$

$$\inf_{\mathbf{V} \in \mathbb{V}^{ad}} J(\mathbf{V}) = \mathbb{F}\big[j(\mathbf{V}(\cdot), \cdot)\big]$$

where the notation means that the risk measure $\mathbb{F}$
has for argument the random variable

$$j(\mathbf{V}(\cdot), \cdot) : \Omega \to \mathbb{R} \cup \{+\infty\} \ , \ \ \omega \mapsto j(\mathbf{V}(\omega), \omega)$$

Examples of classes of robust and stochastic optimization problems

- Stochastic optimization "à la" gradient stochastique
  - The risk measure $\mathbb{F}$ is a mathematical expectation $\mathbb{E}$
  - Measurability constraints make that random variables $\mathbf{V} \in \mathbb{V}^{ad}$ are constant, that is, are deterministic decision variables

$$\inf_{u \in \mathbb{U}^{ad}} \mathbb{E}_{\mathbb{P}}\big[j(u, \cdot)\big]$$

- Robust optimization
  - The risk measure $\mathbb{F}$ is the fear operator/worst case $\sup_{\omega\in\overline{\Omega}}$, where $\overline{\Omega} \subset \Omega$
  - Measurability constraints make that random variables $\mathbf{V} \in \mathbb{V}^{ad}$ are constant, that is, are deterministic decision variables

$$\inf_{u\in\mathbb{U}^{ad}} \sup_{\omega\in\overline{\Omega}} j(u,\cdot)$$

Examples

- A set $\mathbb{U}$
- A set Ω of scenarios
  Ω finite, $\Omega = \mathbb{N} \times \mathbb{W}^{\mathbb{N}}$ for discrete time stochastic processes
- An optimization set $\mathbb{V} \subset \mathbb{U}^{\Omega}$ containing random variables $\mathbf{V} : \Omega \to \mathbb{U}$
- A risk measure $\mathbb{F} : \mathbb{V} \to \mathbb{R} \cup \{+\infty\}$
  most often a mathematical expectation $\mathbb{E}$,
  but can be $\sup_{\omega\in\overline{\Omega}}$ in the robust case, with $\overline{\Omega} \subset \Omega$
- A function $j : \mathbb{U} \times \Omega \to \mathbb{R} \cup \{+\infty\}$
- Constraints of the form $\mathbf{V} \in \mathbb{V}^{ad} \subset \mathbb{V}$
  - Measurability constraints
  - Pointwise constraints,
    like probability constraints and robust constraints

Most common constraints in robust and stochastic optimization problems

- Measurability constraints

$$\mathbf{V} \in \text{linear subspace of } \mathbb{U}^{\Omega}$$

- Pointwise constraints, with $\mathbb{U}^{ad} : \Omega \rightrightarrows \mathbb{U}$
  - probability constraints

$$\mathbb{P}\big(\mathbf{V} \in \mathbb{U}^{ad}\big) \geq 1 - \epsilon$$

  - robust constraints

$$\mathbf{V}(\omega) \in \mathbb{U}^{ad}(\omega) \ , \ \ \forall \omega \in \overline{\Omega} \subset \Omega$$

Savage's minimal regret criterion... "Had I known" The regret performs an additive normalization of the function $j : \mathbb{U} \times \Omega \to \mathbb{R} \cup \{+\infty\}$

For $u \in \mathbb{U}$ and $\omega \in \Omega$, the regret is

$$r(u,\omega) = j(u,\omega) - \min_{u'\in\mathbb{U}} j(u',\omega)$$

Then, take any risk measure $\mathbb{F}$ and solve

$$\min_{\mathbf{V}\in\mathbb{V}^{ad}} \mathbb{F}\big[r(\mathbf{V},\cdot)\big] = \min_{\mathbf{V}\in\mathbb{V}^{ad}} \mathbb{F}\big[j(\mathbf{V}(\omega),\omega) - \min_{u\in\mathbb{U}} j(u,\omega)\big]$$

so that one can have minimal worse regret, minimal expected regret, etc.

3.2 One agent, multiple criteria (multi criteria optimization)

Here are the ingredients for a multi criteria optimization problem

- A set $\mathbb{U}$
- A finite set A of players or stakeholders
- A collection of criteria $J_a : \mathbb{U} \to \mathbb{R} \cup \{+\infty\}$, for $a \in A$

In multi criteria optimization, stakeholders $a \in A$ bargain over a common decision $u \in \mathbb{U}$

**In a multi criteria optimization problem,
a solution is a Pareto optimum** A decision $u^\flat \in \mathbb{U}$ is dominated by a decision $u^\sharp \in \mathbb{U}$ if

- all stake holders prefer $u^\sharp$ to $u^\flat$, that is,

$$J_a(u^\sharp) \geq J_a(u^\flat) \ , \ \ \forall a \in A$$

- at least one stake holder strictly prefers $u^\sharp$ to $u^\flat$, that is,

$$\exists a \in A \ , \ \ J_a(u^\sharp) > J_a(u^\flat)$$

A decision is a Pareto optimum if it is not dominated
by any other decision

3.3 An abstract model to highlight the role of information

Energy systems, which were controlled by a central agent, are becoming more and more decentralized with a multiplicity of local agents acting selfishly and having only partial access to the data and then to the information contained in these data.

Decentralized systems can take various forms: hierarchical, team based, coalition based, etc. In this section, we present a formal typology of such systems based on the Witsenhausen's intrinsic model Witsenhausen (1971, 1975a) which enables an abstract representation of the information available to each agent and of its relative influence on the other agents' decisions. We will also discuss the terminology to characterize decentralized systems.

We choose to restrict the presentation of this abstract information model to the most salient points that will be useful for the study of decentralized energy systems. A more detailed description can be found in Carpentier et al. (2015).

In § 3.3.1, we expose the so-called Witsenhausen's intrinsic model, with its high level of generality on how information is taken into account, and we examine the notions of solvability and of causality in § 3.3.2. In § 3.3.3, we provide a unified framework to define and study three binary relations between agents: i) precedence, ii) subsystem, and iii) information-memory relations. Equipped with these three binary relations between agents, we provide a synthetic overview of system typology in § 3.3.4, among which we distinguish the sequential and the partially nested ones.

3.3.1 Witsenhausen's intrinsic model

In this subsection, we focus on information representation and do consider neither costs nor constraints.

The Witsenhausen's intrinsic model Witsenhausen (1971, 1975a) consists of a finite set of agents, of a collection of decision sets, with a corresponding collection of σ-fields, and of a single sample set (universe) equipped with a σ-field, and representing uncertainties, or states of Nature. This model does not suppose any temporal ordering of decisions.

The extensive space of decisions and states of Nature

Let Ω be a measurable set, equipped with σ-field $\mathcal{F}$, which represents all uncertainties: any $\omega \in \Omega$ is called a *state of Nature*.

The *history space* is the product space

$$\mathbb{H} = \mathbb{U}_A \times \Omega = \prod_{b \in A} \mathbb{U}_b \times \Omega \ , \tag{3.1}$$

equipped with the product *history field*

$$\mathcal{H} = \mathcal{U}_A \otimes \mathcal{F} = \bigotimes_{b \in A} \mathcal{U}_b \otimes \mathcal{F} \ . \tag{3.2}$$

Example 1. *To illustrate the above concepts in the framework of energy systems, we consider an energy provider, called a. Its decision $u_a \in \mathbb{U}_a$ coincides with the unit price at which it supplies energy to its consumers and production level. It is therefore a vector. In case of a power plant, the energy provider dynamically optimizes its power production following a production schedule determined in day ahead; in case of a renewable producer, the energy provider dynamically optimizes the speed of its wind turbines depending on meteorological conditions. The state of Nature $\omega \in \Omega$ contains the meteorological conditions (whether the sun shines, the wind blows, etc.). It also captures the stochasticity associated with the consumer demand. Stochasticity might be caused by bias introduced in the day-ahead forecasts of the consumer demand (we will make this assumption in the rest of this chapter) or by privacy.*

We consider now a more general problem involving $card(A)$ agents. The history $h \in \mathbb{H}$ associated with this problem contains the product of the past decisions of all the energy providers $\prod_{a \in A} u_a$ i.e., the past prices and production levels and the state of Nature $\omega \in \Omega$.

For every subset $C \subset A$ of agents, we introduce the subfield

$$\mathcal{U}_C = \bigotimes_{b \in C} \mathcal{U}_b \otimes \bigotimes_{b \notin C} \{\emptyset, \mathbb{U}_b\} \otimes \mathcal{F} \subset \mathcal{U}_A \ , \tag{3.3}$$

and the subfield

$$\mathcal{D}_C = \mathcal{U}_C \otimes \{\emptyset, \Omega\} = \bigotimes_{b \in C} \mathcal{U}_b \otimes \bigotimes_{b \notin C} \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega\} \subset \mathcal{U}_A \otimes \mathcal{F} =\subset \mathcal{H} \tag{3.4}$$

which contains the *information provided by the decisions* of the agents in C.

Information fields

The *information field* of agent $a \in A$ is a σ-field:

$$\mathcal{I}_a \subset \mathcal{H} \ . \tag{3.5}$$

This representation means that the information of agent a may depend on the states of Nature and on all agents' decisions (including itself in case of self-information, as we will define below).

Example 2. *We replace this formal representation in the framework of the energy provider a already introduced in Example 1. Its information $\mathcal{I}_a$ depends on its prices and production level (in case of self-information), on the other providers' prices and production levels and on the state of Nature, capturing meteorological conditions and demand forecast error.*

Definition 3. *A* stochastic system *is a collection consisting of a finite set A of agents, states of Nature $(\Omega, \mathcal{F})$, decision sets, fields and information fields: $\{\mathbb{U}_a, \mathcal{U}_a, \mathcal{I}_a\}_{a\in A}$.*

Example 4. *To illustrate the use of information fields, we consider two energy providers a and b:*

- *The condition $\mathcal{I}_b \subset \mathcal{I}_a$ formally expresses that what provider b* knows *is known by provider a.*
- *The condition $\mathcal{D}_b \subset \mathcal{I}_a$ formally expresses that what* does *provider b is observed by provider a. In other words, provider a observes the price and the quantity of power produced by b.*

We define a partition field, or a π-field, as a collection of subsets of the universe Ω which is stable under arbitrary union and intersection (countable or not). Partition fields may be adequate to represent information Carpentier et al. (2015).

Consider $\mathcal{P}$ and $\mathcal{P}'$ two partition fields on Ω. The *greatest lower bound* of the partition fields $\mathcal{P}$ and $\mathcal{P}'$ is $\mathcal{P} \vee \mathcal{P}' = \mathcal{P} \cap \mathcal{P}'$, made of subsets of Ω which belong both to $\mathcal{P}$ and $\mathcal{P}'$. The *least upper bound* of the partition fields $\mathcal{P}$ and $\mathcal{P}'$ is $\mathcal{P} \wedge \mathcal{P}' = \pi\big(\mathcal{P} \cup \mathcal{P}'\big)$ the partition field generated by the subsets of Ω which belongs either to $\mathcal{P}$ or to $\mathcal{P}'$.

The set of σ-fields on Ω is a lattice, with the operators $\vee$ and $\wedge$ defined as for the partition fields Carpentier et al. (2015).

We define the *information $\mathcal{I}_C \subset \mathcal{H}$ of the subset $C \subset A$ of agents* by:

$$\mathcal{I}_C = \bigvee_{b\in C} \mathcal{I}_b \ . \tag{3.6}$$

With the above notations, we can express the property that the information of an agent cannot depend on its own decision.

Definition 5. *The* absence of self-information *is the property that:*

$$\mathcal{I}_a \subset \mathcal{U}_{A\setminus\{a\}} \otimes \mathcal{F} \ , \tag{3.7}$$

for all agent $a \in A$.

This means that the information of agent a may depend on the states of Nature and on all the other agents' decisions but not on its own decision. In other words, in case of absence of self-information, agent a's decision has no impact on its information.

Another difficult aspect of dynamic information patterns is that future information may be affected by past decisions. Such situations are called situations with *dual effect*, a terminology that tries to convey the idea that present decisions have two, very conflicting effects: directly contributing to optimizing the cost function on the one hand, modifying the informational constraints to which future decisions are subject, on the other hand Carpentier et al. (2015).

Pure and randomized strategies

A *pure strategy* for agent a is a measurable mapping

$$\gamma_a : (\mathbb{H}, \mathcal{H}) \to (\mathbb{U}_a, \mathcal{U}_a) \tag{3.8}$$

from histories to decisions.

By extension, a *randomized (also called mixed) strategy* for any agent a is a probability distribution over $\mathbb{U}_a$. Formally, it is a measurable mapping:

$$\gamma_a : (\mathbb{H}, \mathcal{H}) \to (\Delta(\mathbb{U}_a), \mathcal{F}_{\Delta(\mathbb{U}_a)}) \ . \tag{3.9}$$

where $\Delta(\mathbb{U}_a)$ denotes the set of all possible probability distributions over $\mathbb{U}_a$ for agent a and $\mathcal{F}_{\Delta(\mathbb{U}_a)}$ is the σ-field associated with $\Delta(\mathbb{U}_a)$.

Example 6. *To illustrate the notion of randomized strategy, we consider an agent a which needs to take a decision in $\mathbb{U}_a$ based on its observations and following a randomized strategy γ_a. In case where $\mathbb{U}_a$ is a discrete finite set, this means that agent a associates with each element in $\mathbb{U}_a$ a probability in $[0; 1]$ (the sum of the probabilities of all the elements in $\mathbb{U}_a$ sums up to 1). In case where $\mathbb{U}_a$ is a continuous set, this means that agent a defines a continuous probability distribution over $\mathbb{U}_a$. Then, agent a makes its choice by sampling an action in $\mathbb{U}_a$ according to the probability distribution that it has defined on $\mathbb{U}_a$.*

To emphasize the distinction from randomized strategies, the strategies in $\mathbb{U}_a$ are called *pure strategies*. A *randomized strategy profile* is any vector that specifies one randomized strategy for each agent; so the set of all randomized strategy profiles is $\prod_{a \in A} \Delta(\mathbb{U}_a)$.

Definition 7. *An* admissible pure strategy, *for agent a is a mapping $\gamma_a : (\mathbb{H}, \mathcal{H}) \to (\mathbb{U}_a, \mathcal{U}_a)$ which is measurable w.r.t. the information field $\mathcal{I}_a$ of agent a:*

$$\gamma_a^{-1}(\mathcal{U}_a) \subset \mathcal{I}_a \ . \tag{3.10}$$

Condition (3.10) expresses the property that the admissible strategy of agent a may only depend upon the information $\mathcal{I}_a$ available to it.

We denote the *set of admissible pure strategies* of agent a by:

$$\Gamma_a^{ad} = \left\{ \gamma_a : (\mathbb{H}, \mathcal{H}) \to (\mathbb{U}_a, \mathcal{U}_a) \ \middle|\ \gamma_a^{-1}(\mathcal{U}_a) \subset \mathcal{I}_a \right\} \tag{3.11}$$

and the set of admissible strategies of all agents is:

$$\Gamma_A^{ad} = \prod_{a \in A} \Gamma_a^{ad} \ . \tag{3.12}$$

Identically, to characterize the set of *admissible randomized strategies* of agent a, we introduce the set

$$\widetilde{\Gamma}_a^{ad} = \left\{ \gamma_a : (\mathbb{H}, \mathcal{H}) \to (\Delta(\mathbb{U}_a), \mathcal{F}_{\Delta(\mathbb{U}_a)}) \ \middle|\ \gamma_a^{-1}(\mathcal{F}_{\Delta(\mathbb{U}_a)}) \subset \mathcal{I}_a \right\} \ . \tag{3.13}$$

3.3.2 Solvability and causality

In the Witsenhausen's intrinsic model Witsenhausen (1971, 1975a), agents make decisions in an order which is not fixed in advance. Briefly speaking, solvability is the property that, for each state of Nature, the agents' decisions are uniquely determined by their strategies. In a causal system, agents are ordered, one playing after the other with available information depending only on agents acting earlier, but the order may depend upon the history.

Solvability

Consider a collection $\gamma = \{\gamma_a\}_{a \in A} \in \Gamma_A^{ad}$ of admissible policies. By (3.11) and (3.12), the policy $\gamma_a : \mathbb{U}_A \times \Omega \to \mathbb{U}_a$ of agent a is measurable w.r.t. the information field $\mathcal{I}_a$.

Thus, agent a makes a decision according to the information it has on the state of Nature ω and on all the decisions $\{u_b\}_{b \in A}$.

The problem is to find, for any $\omega \in \Omega$, solutions $u \in \mathbb{U}_A$ (depending upon ω) satisfying the implicit equations

$$u = \gamma(u, \omega) \;, \tag{3.14}$$

or, equivalently,

$$u_a = \gamma_a(\{u_b\}_{b \in A}, \omega) \;, \;\; \forall a \in A \;. \tag{3.15}$$

Existence and uniqueness of the solutions of (3.14) is related to information patterns. For instance, consider an information structure with two agents a and b, and displaying the absence of self-information. Assuming that $\mathcal{U}_a$ and $\mathcal{U}_b$ contain singletons, by (3.10), policies have the form: $\gamma_a(u, \omega) = \widetilde{\gamma}_a(u_b, \omega)$ and $\gamma_b(u, \omega) = \widetilde{\gamma}_b(u_a, \omega)$. Equation (3.14) is now $u_a = \widetilde{\gamma}_a(u_b, \omega)$ and $u_b = \widetilde{\gamma}_b(u_a, \omega)$, which may display zero solutions, one solution (solvability) or multiple solutions (undeterminacy).

Definition 8. *The* solvability property *holds true when, for any collection $\gamma \in \Gamma_A^{ad}$ of admissible pure strategies, and any state of Nature $\omega \in \Omega$, there exists one, and only one, decision $u \in \mathbb{U}_A$ satisfying (3.14). Denoting $M_\gamma(\omega)$ this unique $u \in \mathbb{U}_A$, we obtain a mapping: $M_\gamma : \Omega \to \mathbb{U}_A$. The* solvability/measurability *property holds true when, in addition, the mapping $M_\gamma : \Omega \to \mathbb{U}_A$ is measurable from $(\Omega, \mathcal{F})$ to $(\mathbb{U}_A, \mathcal{U}_A)$.*

Definition 9. *Suppose that the solvability property holds true. Thanks to the mapping M_γ, we define the* solution map $S_\gamma : \Omega \to \mathbb{H}$ *by*

$$S_\gamma(\omega) = \big(M_\gamma(\omega), \omega\big) \;, \;\; \forall \omega \in \Omega \;, \tag{3.16}$$

that is,

$$(u, \omega) = S_\gamma(\omega) \iff u = \gamma(u, \omega) \;, \;\; \forall (u, \omega) \in \mathbb{U}_A \times \Omega \;. \tag{3.17}$$

We include ω in the image of $S_\gamma(\omega)$ to map the universe Ω towards the history space $\mathbb{H}$ and to interpret $S_\gamma(\omega)$ as a state trajectory. Indeed, the mapping S_γ yields all the history generated by the state of Nature ω and by the admissible policy γ.

Causality

In a causal system, agents are ordered, one playing after the other with available information depending only on agents acting earlier, but the order may depend upon the history.

Let $\mathbb{O}$ denote the set of total orderings of agents in A, that is, injective mappings from $\{1, \ldots, n\}$ to A, where $n = \mathrm{card}(A)$. For $k \in \{1, \ldots, n\}$, let $\mathbb{O}_k$ denote the set of k-orderings, that is, injective mappings from $\{1, \ldots, k\}$ to A (thus $\mathbb{O} = \mathbb{O}_n$). There is a natural mapping ψ_k from $\mathbb{O}$ to $\mathbb{O}_k$, the restriction of any ordering of A to the domain set $\{1, \ldots, k\}$.

Define a *history-ordering* as a mapping $\varphi : \mathbb{H} \to \mathbb{O}$ from histories towards orderings: along each history $h \in \mathbb{H}$, the agents are ordered by $\varphi(h) \in \mathbb{O}$. With any history-ordering φ, any $k \in \{1, \ldots, n\}$ and k-ordering $\rho_k \in \mathbb{O}_k$, we associate the set $\mathbb{H}^{\varphi}_{k,\rho_k}$ which contains all the histories that would induce the same order for the agents having a rank smaller or equal to k:

$$\mathbb{H}^{\varphi}_{k,\rho_k} = \{h \in \mathbb{H} \mid \psi_k(\varphi(h)) = \rho_k\} \;. \tag{3.18}$$

Definition 10. *Witsenhausen (1971) A system is* causal *if there exists (at least one) history-ordering φ from $\mathbb{H}$ towards $\mathbb{O}$, with the property that for any $k \in \{1, \dots, n\}$ and $\rho_k \in \mathbb{O}_k$:*

$$\mathbb{H}^{\varphi}_{k,\rho_k} \cap G \in \mathcal{U}_{\{\rho_k(1),\dots,\rho_k(k-1)\}} \otimes \mathcal{F} \ , \ \ \forall G \in \mathcal{I}_{\rho_k(k)} \ . \tag{3.19}$$

In other words, when the first k agents are known and given by $(\rho_k(1), \dots, \rho_k(k))$, the information $\mathcal{I}_{\rho_k(k)}$ of the agent $\rho_k(k)$ with rank k depends at most on the decisions of agents $\rho_k(1)$, ..., $\rho_k(k-1)$ with rank stricly less than k.

Proposition 11. *Witsenhausen (1971) Causality implies (recursive) solvability with a measurable solution map.*

3.3.3 Binary relations between agents

We provide a unified framework to define and study three binary relations between agents Ho and Chu (1972, 1974); Witsenhausen (1975a).

The precedence relation $\mathfrak{P}$

The precedence binary relation identifies the agents whose decisions influence the observations of a given agent (cf. point 2 of Example 4).

For a given agent a, let us consider the set $\mathcal{P}_a \subset 2^A$ of subsets $C \subset A$ such that $\mathcal{I}_a \subset \mathcal{U}_C \otimes \mathcal{F}$. Any $C \in \mathcal{P}_a$ contains agents whose decisions may affect the information $\mathcal{I}_a$ available to agent a[1].

The set $\mathcal{P}_a$ is stable under intersection. This motivates the following definition.

Definition 12. *Carpentier et al. (2015) Let $\langle a \rangle_{\mathfrak{P}} \subset A$ be the intersection of subsets $C \subset A$ such that $\mathcal{I}_a \subset \mathcal{U}_C \otimes \mathcal{F}$. We define a* precedence *binary relation $\mathfrak{P}$ on A by:*

$$b \,\mathfrak{P}\, a \iff b \in \langle a \rangle_{\mathfrak{P}} \ , \tag{3.20}$$

and we say that b is a predecessor *of a.*

In other words, the decisions of any *predecessor* of an agent *affect* the information of this agent: any agent is influenced by its predecessors (when they exist, because $\langle a \rangle_{\mathfrak{P}}$ might be empty).

By construction, the subset of agents $\langle a \rangle_{\mathfrak{P}}$ is the smallest subset $C \subset A$ such that $\mathcal{I}_a \subset \mathcal{U}_C \otimes \mathcal{F}$. In other words, $\langle a \rangle_{\mathfrak{P}}$ is characterized by:

$$\mathcal{I}_a \subset \mathcal{U}_{\langle a \rangle_{\mathfrak{P}}} \otimes \mathcal{F} \text{ and } \Big(\mathcal{I}_a \subset \mathcal{U}_C \otimes \mathcal{F} \Rightarrow \langle a \rangle_{\mathfrak{P}} \subset C \Big) \tag{3.21}$$

Whenever $\langle a \rangle_{\mathfrak{P}} \neq \emptyset$, there is a potential for *signaling*, that is for information transmission. Indeed, any agent b in $\langle a \rangle_{\mathfrak{P}}$ influences the information $\mathcal{I}_a$ upon which agent a bases its decisions. Therefore, whenever agent b is a predecessor of agent a, the former can, by means of its decisions, send a signal to the latter. In case $\langle a \rangle_{\mathfrak{P}} = \emptyset$, the decisions of agent a depend, at most, on the state of Nature, and there is no room for signaling.

We introduce the following definitions:

Definition 13. *For any $C \subset A$, we introduce the following subsets of agents:*

$$\langle C \rangle_{\mathfrak{P}} = \bigcup_{b \in C} \langle b \rangle_{\mathfrak{P}} \ , \ \ \langle C \rangle^0_{\mathfrak{P}} = C \ \text{ and } \ \langle C \rangle^{n+1}_{\mathfrak{P}} = \Big\langle \langle C \rangle^n_{\mathfrak{P}} \Big\rangle_{\mathfrak{P}} \ , \ \ \forall n \in \mathbb{N} \tag{3.22}$$

When C is a singleton $\{a\}$, we denote $\langle a \rangle^n_{\mathfrak{P}}$ for $\langle \{a\} \rangle^n_{\mathfrak{P}}$.

[1]We note that $A \in \mathcal{P}_a$ because $\mathcal{I}_a \subset \mathcal{U}_A \otimes \mathcal{F}$.

The converse of the precedence relation $\mathfrak{P}$ is the *successor relation* $\mathfrak{P}^{-1}$, characterized by

$$b\,\mathfrak{P}^{-1}\,a \iff a\,\mathfrak{P}\,b \tag{3.23}$$

Quite naturally, b is a successor of a iff a is a predecessor of b.

The subsystem relation $\mathfrak{S}$

It is called subsystem relation in the Witsenhausen's formalism Witsenhausen (1975a). We prefer the terminology *subsystem* which makes the link with the concept in use in network economics and social network literature Massoulié et al. (2015). In the network economics literature, the problem of subsystem detection is generally replaced in the context of a graph whose nodes coincides with one agent. Following Kaufmann et al. Kaufmann et al. (2015), the commonly accepted definition of a subsystem is that "nodes tend to be more densely connected within a subsystem than with the rest of the graph". In our approach, we forget the graph structure and interpret the relations between the agents using binary relations which determine how information is passed on them.

Definition 14. *Witsenhausen (1975b) A nonempty subset C of agents in A is a* subsystem *if the information field $\mathfrak{I}_C$ defined in (3.6) at most depends on the decisions of the agents in C:*

$$\mathfrak{I}_C \subset \mathfrak{U}_C \otimes \mathfrak{F}$$

Thus, the information received by agents in C depends upon states of Nature and decisions of members of C only.

$b\,\mathfrak{S}\,a$ means that agent b belongs to the subsystem generated by agent a or, equivalently, that the subsystem generated by agent a contains that generated by agent b. Indeed, by the properties of a topological closure, we have that:

$$b\,\mathfrak{S}\,a \iff \overline{\{b\}} \subset \overline{\{a\}}\,, \quad \forall (a,b) \in A^2 \tag{3.24}$$

Notice also that a subset $C \subset A$ is a subsystem if, and only if, it coincides with the generated subsystem:

$$C \text{ is a subsystem} \iff C = \overline{C} \tag{3.25}$$

Proposition 15. *Witsenhausen (1975b) The subsystem relation $\mathfrak{S}$ is a pre-order, namely it is reflexive and transitive.*

The following Proposition 16 describes connections between the subsystem relation $\mathfrak{S}$ and the precedence binary relation $\mathfrak{P}$.

Proposition 16. *Carpentier et al. (2015) Let C be a subset of the set A of agents, and a be an agent in A.*

1. *A subset $C \subset A$ is a subsystem iff $\langle C\rangle_{\mathfrak{P}} \subset C$, that is, iff the predecessors of agents in C belong to C:*

$$C \text{ is a subsystem} \iff \overline{C} = C \iff \langle C\rangle_{\mathfrak{P}} \subset C \tag{3.26}$$

2. *For any $a \in A$, the subsystem generated by agent a is the union of a and of all its iterated predecessors (see Definition 13):*

$$\overline{\{a\}} = \bigcup_{n\in\mathbb{N}} \langle a\rangle_{\mathfrak{P}}^{n} \tag{3.27}$$

When there is a subsystem, the solution map displays the following co-cycle property.

Proposition 17. *We suppose that the stochastic system* $\{\mathbb{U}_a, \mathcal{U}_a, \mathcal{I}_a\}_{a \in A}$ *displays the solvability property. We consider a partition* $A = B \cup C$ *and, for any policy* $\gamma \in \Gamma_A^{ad}$*, we write*

$$\gamma = (\gamma_B, \gamma_C) \text{ where } \gamma_B : \mathbb{U}_B \times \mathbb{U}_C \times \Omega \to \mathbb{U}_B \ , \ \ \gamma_C : \mathbb{U}_B \times \mathbb{U}_C \times \Omega \to \mathbb{U}_C \ . \tag{3.28}$$

If B *is a subsystem, the policy* γ_B *can be identified with*

$$\gamma_B : \mathbb{U}_B \times \Omega \to \mathbb{U}_B \ , \tag{3.29}$$

and the solution map has the following (co-cycle) property

$$M_{(\gamma_B, \gamma_C)}(\omega) = \Big(M_{\gamma_B}(\omega), M_{\gamma_C\big(M_{\gamma_B}(\omega), \cdot\big)}(\omega) \Big) \ , \ \ \forall \omega \in \Omega \ , \tag{3.30}$$

in the sense that

$$M_{(\gamma_B, \gamma_C)}(\omega) = (u_B, u_C) \iff \begin{cases} u_B & = M_{\gamma_B}(\omega) \ , \\ u_C & = \gamma_C\big(u_B, u_C, \omega\big) \ . \end{cases} \tag{3.31}$$

The information-memory relation $\mathfrak{M}$

Definition 18. *Barty et al. (2006) With any agent* $a \in A$*, we associate the subset* $\langle a \rangle_{\mathfrak{M}}$ *of agents which pass on their information to* a*:*

$$\langle a \rangle_{\mathfrak{M}} = \{ b \in A \mid \mathcal{I}_b \subset \mathcal{I}_a \} \ . \tag{3.32}$$

We define an information memory *binary relation* $\mathfrak{M}$ *on* A *by:*

$$b \, \mathfrak{M} \, a \iff b \in \langle a \rangle_{\mathfrak{M}} \iff \mathcal{I}_b \subset \mathcal{I}_a \ , \ \ \forall (a, b) \in A^2 \ . \tag{3.33}$$

When $b \, \mathfrak{M} \, b$, we say that agent b information is *remembered by* or *passed on to* agent a, or that the information of agent b is embedded in the information of agent a. When agent b belongs to $\langle a \rangle_{\mathfrak{M}}$, the information available to b is also available to agent a.

By construction, the subset of agents $\langle a \rangle_{\mathfrak{M}}$ is the largest subset $C \subset A$ such that $\mathcal{I}_C \subset \mathcal{I}_a$. Therefore, we have that

$$C \subset \langle a \rangle_{\mathfrak{M}} \iff \mathcal{I}_C \subset \mathcal{I}_a \ , \tag{3.34}$$

and, in particular,

$$a \in \langle a \rangle_{\mathfrak{M}} \text{ and } \mathcal{I}_{\langle a \rangle_{\mathfrak{M}}} = \mathcal{I}_a \ . \tag{3.35}$$

Proposition 19. *Barty et al. (2006) The* information memory *relation* $\mathfrak{M}$ *is a pre-order, namely* $\mathfrak{M}$ *is reflexive and transitive.*

The following Proposition 20 describes connections between the information-memory relation $\mathfrak{M}$ and the precedence binary relation $\mathfrak{P}$. For any $C \subset A$, let us introduce the set of agents:

$$\langle C \rangle_{\mathfrak{M}} = \bigcup_{b \in C} \langle b \rangle_{\mathfrak{M}} \tag{3.36}$$

who pass on their information to at least one agent in C.

Proposition 20. *Barty et al. (2006) Let C and C' be two subsets of the set A of agents. We have that that the composition of the precedence relation and information-memory relation is included into the precedence relation. Formally, we write:*

$$\mathfrak{P}\mathfrak{M} \subset \mathfrak{P} \tag{3.37}$$

and that, if a set C of agents pass on their information to at least one agent in C', then the predecessors of C are predecessors of C':

$$C \subset \langle C' \rangle_{\mathfrak{M}} \Rightarrow \langle C \rangle_{\mathfrak{P}} \subset \langle C' \rangle_{\mathfrak{P}} \tag{3.38}$$

The decision-memory relation $\mathfrak{D}$

The following decision-memory relation is not, to our knowledge, found in the literature. It proves useful to express strictly classical and strictly quasi-classical systems in §3.3.4.

Definition 21. *With any agent $a \in A$, we associate*

$$\langle a \rangle_{\mathfrak{D}} = \{ b \in A \mid \mathcal{D}_b \subset \mathcal{I}_a \} \ , \tag{3.39}$$

the subset of agents b whose decision is passed on to a, where the decision subfield $\mathcal{D}_b$ is defined in (3.4).

We define a decision-memory *binary relation $\mathfrak{D}$ on A by*

$$b \, \mathfrak{D} \, a \iff b \in \langle a \rangle_{\mathfrak{D}} \iff \mathcal{D}_b \subset \mathcal{I}_a \ , \ \forall (a,b) \in A^2 \ . \tag{3.40}$$

When $b \, \mathfrak{D} \, a$, we say that agent b decision is *remembered by* or *passed on to* agent a, or that the decision of agent b is embedded in the information of agent a. By (3.39) and (3.4), we have that

$$\mathcal{D}_{\langle a \rangle_{\mathfrak{D}}} \subset \mathcal{I}_a \ . \tag{3.41}$$

By comparing the definition of the decision-memory relation $\mathfrak{D}$ with that of the precedence relation $\mathfrak{P}$ in Definition 12, and by the definition (3.4) of $\mathcal{D}_C$, we see that

$$\mathcal{D}_{\langle a \rangle_{\mathfrak{D}}} = \mathcal{U}_{\langle a \rangle_{\mathfrak{D}}} \otimes \{\emptyset, \Omega\} \subset \mathcal{I}_a \subset \mathcal{U}_{\langle a \rangle_{\mathfrak{P}}} \otimes \mathcal{F} \ . \tag{3.42}$$

Therefore, we conclude that

$$\langle a \rangle_{\mathfrak{D}} \subset \langle a \rangle_{\mathfrak{P}} \ , \ \forall a \in A \ , \tag{3.43}$$

or, equivalently, that

$$\mathfrak{D} \subset \mathfrak{P} \ . \tag{3.44}$$

Remark 22. *If $b \, \mathfrak{D} \, a$, the decision made by agent b decision is passed on to agent a and, by the fact that $\mathfrak{D} \subset \mathfrak{P}$, b is a predecessor of a. However, the agent b can be a predecessor of a, but his influence may happen without passing on his decision to a.*

3.3.4 Typology of systems

Using the intrinsic model for information representation, Witsenhausen provided a typology of systems Witsenhausen (1971, 1975a). For each type of systems we provide an illustration in case of two interacting agents. We also replace this system typology in the framework of Game Theory in § 3.5.2.

Station

In Witsenhausen's formalism, a *station* is a subset of agents such that the set of information fields of these agents is totally ordered under inclusion (i.e., nested). In other words, a subset C of agents in A is a station iff the information-memory relation $\mathfrak{M}$ induces a total order on C (i.e., it consists of a chain of length $m = \mathrm{card}(C)$) iff there exists an ordering $(a_1, \ldots, a_m)$ of C such that

$$\mathfrak{I}_{a_1} \subset \cdots \subset \mathfrak{I}_{a_k} \subset \mathfrak{I}_{a_{k+1}} \subset \cdots \subset \mathfrak{I}_{a_m} \ , \tag{3.45}$$

or, equivalently, that $a_k \in \langle a_{k+1} \rangle_{\mathfrak{M}}$, for $k = 1, \ldots, m$.

Example 23. *For two agents a, b, this means that the information fields can be ordered so that: $\mathfrak{I}_a \subset \mathfrak{I}_b$ or $\mathfrak{I}_b \subset \mathfrak{I}_a$. One of the agents has private information that is not disclosed to the other agent. In economics, such problems are said as having* information asymmetry *because one of the agents has some kind of advantage on the other regarding information access.*

Partially nested systems

A system is *partially nested* Ho and Chu (1972, 1974) iff the precedence relation $\mathfrak{P}$ is included in the information-memory relation $\mathfrak{M}$ that is, $\mathfrak{P} \subset \mathfrak{M}$, namely $\langle a \rangle_{\mathfrak{P}} \subset \langle a \rangle_{\mathfrak{M}}$ for any agent $a \in A$. In a partially nested system, any agent knows what its predecessors know or, in other words, any predecessor of a given agent passes its information to that agent.

Example 24. *For two agents a, b, this means that $b \ \mathfrak{P} \ a$ (i.e., b is a predecessor of a) implies that $\mathfrak{I}_b \subset \mathfrak{I}_a$.*

Partially nested systems can be seen as a special form of *Principal-agents models* that will be described in § 4.1.1, but in such models the information reported by the predecessors of the Principal might be biased. Witsenhausen does not deal with such considerations in its definition of information fields.

A static team

A *static team* is a subset C of A such that $\langle C \rangle_{\mathfrak{P}} = \emptyset$ or, equivalently, that agents in C have no predecessor. When the whole set A of agents is a static team, any agent $a \in A$ has no predecessor: $\langle a \rangle_{\mathfrak{P}} = \emptyset, \forall a \in A$.

A system is *static* if the set A of agents is a static team.

Example 25. *Two agents a, b form a static team iff :*

$$\mathfrak{I}_a \subset \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathfrak{F} \ , \ \ \mathfrak{I}_b \subset \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathfrak{F} \ .$$

There is no interdependence between the decisions of the agents, just a dependence upon states of Nature.

Monic system

A system is *monic* iff it is reduced to a singleton, that is, a static team: $A = \{a\}$ and $\mathfrak{I}_a \subset \{\emptyset, \mathbb{U}_a\} \otimes \mathfrak{F}$.

Sequential systems

In *sequential* systems, there exists an ordering $(a_1, \dots, a_n)$ of A such that each agent a_k is influenced at most by the previous agents $a_1, \dots, a_{k-1}$ that is:

$$\langle a_1 \rangle_{\mathfrak{P}} = \emptyset \text{ and } \langle a_k \rangle_{\mathfrak{P}} \subset \{a_1, \dots, a_{k-1}\} \; , \; \forall k = 2, \dots, n \; . \tag{3.46}$$

Example 26. *The set of agents $A = \{a, b\}$ with information fields given by*

$$\mathfrak{I}_a = \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathfrak{F} \; , \; \; \mathfrak{I}_b = \mathfrak{U}_a \otimes \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega\}$$

forms a sequential system where agent a precedes agent b ($\langle a \rangle_{\mathfrak{P}} = \emptyset$ and $\langle b \rangle_{\mathfrak{P}} = \{a\}$), but $\mathfrak{I}_a$ and $\mathfrak{I}_b$ are not comparable: agent a observes only the state of Nature, whereas agent b observes only agent a's decision.

Quasiclassical systems

A system is *quasiclassical* Witsenhausen (1975a) iff it is sequential and partially nested. Equivalently, there exists an ordering $(a_1, \dots, a_n)$ of A such that $\langle a_1 \rangle_{\mathfrak{P}} = \emptyset$ and, for $k = 2, \dots, n$,

$$\langle a_k \rangle_{\mathfrak{P}} \subset \{a_1, \dots, a_{k-1}\} \text{ and } \langle a_k \rangle_{\mathfrak{P}} \subset \langle a_k \rangle_{\mathfrak{M}} \; . \tag{3.47}$$

In a quasiclassical system, there exists an ordering such that any agent is influenced at most by the previous agents and knows what his predecessors know.

Classical systems

Systems are called *classical* iff there exists an ordering $(a_1, \dots, a_n)$ of A for which it is both sequential and such that $\mathfrak{I}_{a_k} \subset \mathfrak{I}_{a_{k+1}}$ for $k = 1, \dots, n-1$ (station property). Equivalently, there exists an ordering $(a_1, \dots, a_n)$ of A such that $\langle a_1 \rangle_{\mathfrak{P}} = \emptyset$ and, for $k = 2, \dots, n$,

$$\langle a_k \rangle_{\mathfrak{P}} \subset \{a_1, \dots, a_{k-1}\} \subset \{a_1, \dots, a_{k-1}, a_k\} \subset \langle a_k \rangle_{\mathfrak{M}} \tag{3.48}$$

A classical system is necessarily partially nested, because $\langle a_k \rangle_{\mathfrak{P}} \subset \langle a_k \rangle_{\mathfrak{M}}$ for $k = 1, \dots, n$; hence, a classical system is quasiclassical. In a classical system, there exists an ordering such that any agent is influenced at most by the previous agents and knows what they know.

Example 27. *The set of agents $A = \{a, b\}$ with information fields given by*

$$\mathfrak{I}_a = \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathfrak{F} \; , \; \; \mathfrak{I}_b = \mathfrak{U}_a \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathfrak{F} \; ,$$

forms a classical system. First, the system is sequential, a precedes b ($\langle a \rangle_{\mathfrak{P}} = \emptyset$ and $a \in \langle b \rangle_{\mathfrak{P}}$): agent a observes the state of Nature and makes its decision accordingly; agent b observes both agent a's decision and the state of Nature and makes its decision based on that information. Second, one has that $\mathfrak{I}_a \subset \mathfrak{I}_b$ ($a \in \langle b \rangle_{\mathfrak{M}}$), which may be interpreted in different ways: one may say that agent a communicates her own information to agent b.

Causal but non sequential systems

We consider a set of agents $A = \{a, b\}$ with $\mathbb{U}_a = \{a_1, a_2\}$, $\mathbb{U}_b = \{b_1, b_2\}$, $\Omega = \{\omega_-, \omega_+\}$. The agents' information fields are given by

$$\begin{aligned} \mathfrak{I}_a &= \gamma(\{a_1, a_2\} \times \{b_1, b_2\} \times \{\omega_+\}, \{a_1, a_2\} \times \{b_1\} \times \{\omega_-\}) \; , \\ \mathfrak{I}_b &= \gamma(\{a_1, a_2\} \times \{b_1, b_2\} \times \{\omega_-\}, \{a_1\} \times \{b_1, b_2\} \times \{\omega_+\}) \end{aligned}$$

forms a causal but non sequential system.

When the state of Nature is ω_+, agent a only sees ω_+, whereas agent b sees ω_+ and the decision of a: thus a acts first, then b. The reverse holds true when the state of Nature is ω_-. Thus, there are history-ordering mappings φ from $\mathbb{H}$ towards $\{(a,b),(b,a)\}$, but they differ according to history: $\varphi\Big((u_a,u_b,\omega_+)\Big)=(a,b)$ and $\varphi\Big((u_a,u_b,\omega_-)\Big)=(b,a)$. The system is causal but not sequential.

Non-causal systems

The set of agents $A=\{a,b\}$ have information fields given by:

$$\mathcal{I}_a=\{\emptyset,\mathbb{U}_a\}\otimes\mathcal{U}_b\otimes\{\emptyset,\Omega\}\ ,\ \ \mathcal{I}_b=\mathcal{U}_a\otimes\{\emptyset,\mathbb{U}_b\}\otimes\{\emptyset,\Omega\}\ .$$

In *non-causal systems*, the decision process may have no solution, one solution, or multiple solutions.

Example 28. *Agent a observes agent b's decision (agent a's feedback), whereas agent b observes agent a's decision (agent b's feedback). Thus, agent a precedes agent b ($a\in\langle b\rangle_{\mathfrak{P}}$) and agent b precedes agent a ($b\in\langle a\rangle_{\mathfrak{P}}$).*

Theorem 29 (Witsenhausen (1975a))**.** *Any of the properties static team, monicity, sequentiality, quasiclassicality, classicality, causality, non-causality of a system is shared by all its subsystems.*

Hierarchical systems

Ho and Chu in Ho and Chu (1974) consider the case when the set A of agents can be partitioned in (nonempty) disjoint sets A_0,..., A_K as follows. Agents in A_0 are Nature's agents whose decisions depend only upon ω:

$$A_0=\{a\in A\mid \mathcal{I}_a\subset\bigotimes_{b\in A}\{\emptyset,\mathbb{U}_b\}\otimes\mathcal{F}\}\ . \tag{3.49}$$

Agents in A_0 thus form the largest static team ($\langle A_0\rangle_{\mathfrak{P}}=\emptyset$). Then,

$$A_1=\{a\in A\mid a\notin A_0 \text{ and } \langle a\rangle_{\mathfrak{P}}\subset A_0\} \tag{3.50}$$

and

$$A_{k+1}=\{a\in A\mid a\notin\bigcup_{i=1}^{k}A_j \text{ and } \langle a\rangle_{\mathfrak{P}}\subset\bigcup_{i=1}^{k}A_j\}\ . \tag{3.51}$$

Smart grids can be modeled as hierarchical systems made of three level of agents namely: generators, suppliers and micro grids. In Le Cadre and Bedo (2016), Le Cadre and Bedo study the interactions between agents composing a hierarchical systems. They consider renewable energy producers, energy suppliers, and captive micro grids. At each time period, each renewable producers generates a variable amount of power and communicates its price to the suppliers. Each supplier decides the quantity of power it needs from all generators, and the price it fixes toward its micro grid. Besides, each micro grid decides the power order that it buys from the supplier in order to maximize its benefit.

Parallel coordinated systems

We consider the case when the set A of agents can be partitioned in (nonempty) disjoint sets A_0, A_1, ..., A_K as follows. Agents in A_0 are Nature's agents whose decisions depend only upon ω:

$$A_0=\{a\in A\mid \mathcal{I}_a\subset\bigotimes_{b\in A}\{\emptyset,\mathbb{U}_b\}\otimes\mathcal{F}\}\ . \tag{3.52}$$

Agents in A_0 thus form the largest static team ($\langle A_0\rangle_{\mathfrak{P}}=\emptyset$). Then, A_1, ..., A_K have the property that every subset $A_1\cup A_0$, ..., $A_K\cup A_0$ is a subsystem.

3.4 Multiple agents, one criterion (team optimization)

3.4.1 Static team optimization

3.4.2 Multi-stage stochastic optimization

Ingredients for a stochastic sequential optimization problem

- A set $\{t_0, t_0+1, \ldots, T\} \subset \mathbb{N}$ of discrete times,
  with generic element t
- Control sets $\mathbb{U}_t$ containing control variable $u_t \in \mathbb{U}_t$,
  for $t = t_0, t_0+1, \ldots, T$
- Constraints of the form $u_t \in \mathbb{U}_t^{ad} \subset \mathbb{U}_t$
- A set Ω of scenarios, or states of Nature, with generic element ω
  (without temporal structure, a priori)
- A pre-criterion $j : \mathbb{U}_{t_0} \times \cdots \times \mathbb{U}_T \times \Omega \to \mathbb{R}$,
  with generic value $j(u_{t_0}, \ldots, u_T, \omega)$

Two-stage problem Times $t \in \{0, 1\}$ (and pre-criterion $L_0\big(u_0\big) + L_1\big(u_1, \omega\big)$)

- Stochastic optimization deals with risk attitudes:
  mathematical expectation $\mathbb{E}$, risk measure $\mathbb{F}$ (including worst case), probability or robust constraints
- Stochastic dynamic optimization emphasizes
  the handling of online information,
  and especially the nonanticipativity constraints

For the purpose of handling online information, we introduce fields and subfields

1. $(\Omega, \mathcal{F})$ a measurable space (uncertainties, states of Nature)
2. $(\mathbb{U}_{t_0}, \mathcal{U}_{t_0})$,..., $(\mathbb{U}_T, \mathcal{U}_T)$ measurable spaces (decision spaces)
3. Subfield $\mathcal{I}_t \subset \mathcal{U}_{t_0} \otimes \cdots \otimes \mathcal{U}_{t-1} \otimes \mathcal{F}$, for $t = t_0, \ldots, T$ (information)

The inclusion

$$\underbrace{\mathcal{I}_t}_{\text{information}} \subset \underbrace{\mathcal{U}_{t_0} \otimes \cdots \otimes \mathcal{U}_{t-1}}_{\text{past controls}} \otimes \mathcal{F}$$

captures the fact that the information at time t is made at most of past controls and of the state of Nature (causality)

Static team Subfield $\mathcal{I}_t \subset \mathcal{F}$ for $t = t_0, \ldots, T$ (no dynamic flow of information)

We introduce strategies

Definition 1. *Decision rule, policy, strategy A strategy is a sequence* $\gamma = \{\gamma_t\}_{t=t_0,\dots,T}$ *of measurable mappings from past histories to decision sets*

$$\begin{aligned}
&\gamma_{t_0} : (\Omega, \mathcal{F}) \to (\mathbb{U}_{t_0}, \mathcal{U}_{t_0}) \\
&\dots \\
&\ \gamma_t : (\mathbb{U}_{t_0} \times \dots \times \mathbb{U}_{t-1} \times \Omega, \mathcal{U}_{t_0} \otimes \dots \otimes \mathcal{U}_{t-1} \otimes \mathcal{F}) \to (\mathbb{U}_t, \mathcal{U}_t) \\
&\dots
\end{aligned}$$

With obvious notations, the set of strategies is denoted by

$$\Lambda_{t_0,\dots,T} = \prod_{t=t_0,\dots,T} \Lambda_t$$

We introduce admissible strategies to account for the interplay between decision and information

Definition 2. *Admissible strategy An admissible strategy is a strategy* $\gamma = \{\gamma_t\}_{t=t_0,\dots,T}$

$$\begin{aligned}
&\gamma_{t_0} : (\Omega, \mathcal{F}) \to (\mathbb{U}_{t_0}, \mathcal{U}_{t_0}) \\
&\dots \\
&\ \gamma_t : (\mathbb{U}_{t_0} \times \dots \times \mathbb{U}_{t-1} \times \Omega, \mathcal{U}_{t_0} \otimes \dots \otimes \mathcal{U}_{t-1} \otimes \mathcal{F}) \to (\mathbb{U}_t, \mathcal{U}_t) \\
&\dots
\end{aligned}$$

satisfying, for $t = t_0, \dots, T$*, the information constraints*

$$\gamma_t^{-1}(\mathcal{U}_t) \subset \underbrace{\mathcal{I}_t}_{\textit{information}}$$

With obvious notations, the set of admissible strategies is denoted by

$$\Lambda^{ad}_{t_0,\dots,T} = \prod_{t=t_0,\dots,T} \Lambda^{ad}_t$$

The solution map is attached to a strategy, and maps a scenario towards a history

Definition 3. *Solution map With a strategy* γ*, we associate the mapping*

$$S_\gamma : \Omega \to \underbrace{\mathbb{U}_{t_0} \times \dots \times \mathbb{U}_T \times \Omega}_{\textit{history space}}$$

called solution map, and defined by

$$(u_{t_0}, \dots, u_T, \omega) = S_\gamma(\omega) \iff \begin{cases} u_{t_0} & = \gamma_{t_0}(\omega) \\ u_{t_0+1} & = \gamma_{t_0+1}(u_{t_0}, \omega) \\ \vdots & \vdots \\ u_T & = \gamma_T(u_{t_0}, \cdots, u_{T-1}, \omega) \end{cases}$$

By composing the pre-criterion with the solution map, we move forward the design of a criterion

- With a strategy γ, we associate the solution map

$$S_\gamma : \Omega \to \underbrace{\mathbb{U}_{t_0} \times \cdots \times \mathbb{U}_T \times \Omega}_{\text{history space}}$$

  that maps a scenario towards a history

- The pre-criterion

$$j : \mathbb{U}_{t_0} \times \cdots \times \mathbb{U}_T \times \Omega \to \mathbb{R}$$

  maps a a history towards the real numbers

- Therefore, by composing the pre-criterion with the solution map, we obtain

$$j \circ S_\gamma : \Omega \to \mathbb{R}$$

  that maps a scenario towards the real numbers

For the purpose of building a criterion (and of handling risk attitudes), we introduce a risk measure As $j \circ S_\gamma \in \mathbb{R}^\Omega$, all we need is a risk measure

$$\mathbb{F} : \mathbb{R}^\Omega \to \mathbb{R} \cup \{+\infty\}$$

to build a criterion that maps a strategy γ towards the (extended) real numbers

$$\gamma \in \Lambda_{t_0,\ldots,T} \mapsto \mathbb{F} \circ j \circ S_\gamma \in \mathbb{R} \cup \{+\infty\}$$

where we recall that $\Lambda_{t_0,\ldots,T}$ denotes the set of strategies

We can now formulate an optimization problem under uncertainty

Definition 4. *Optimization problem under uncertainty When $\mathbb{F}$ is a risk measure on Ω,*

$$\mathbb{F} : \mathbb{R}^\Omega \to \mathbb{R} \cup \{+\infty\} \ ,$$

the corresponding optimization problem under uncertainty is

$$\min_{\gamma \in \Lambda^{ad}_{t_0,\ldots,T}} \mathbb{F}\Big(j\big(S_\gamma(\cdot)\big)\Big)$$

where we recall that $\Lambda^{ad}_{t_0,\ldots,T}$ denotes the set of admissible strategies, those such that

$$\gamma_t^{-1}(\mathcal{U}_t) \subset \mathcal{I}_t \ , \ \ \forall t = t_0, \ldots, T$$

Risk neutral and robust optimization appear as special cases

Definition 5. *When $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space, the risk-neutral stochastic optimization problem is*

$$\min_{\gamma \in \Lambda^{ad}_{t_0,\dots,T}} \mathbb{E}_{\mathbb{P}}\Big(j\big(S_\gamma(\cdot)\big)\Big)$$

Definition 6. *When $\overline{\Omega} \subset \Omega$, the robust optimization problem is*

$$\min_{\gamma \in \Lambda^{ad}_{t_0,\dots,T}} \sup_{\omega \in \overline{\Omega}} j\big(S_\gamma(\omega)\big)$$

3.5 Multiple agents, multiple criteria (non-cooperative game theory)

In previous chapters we showed how decentralized stochastic control problems can be formulated with Witsenhausen intrinsic model. This chapter is an extension of the intrinsic model to game theoretical settings.

We first present some standard classes of games. The aim of this first section is twofold: first it could serve as an introduction to game theory for people coming from the fields of optimization and control, second the class of games we introduce serves as a benchmark for the intrinsic model. We then complete the intrinsic model with the standard notions required for the study of games. Last, we present some preliminary results and point out some open questions.

3.5.1 Definitions

To extend Witsenhausen intrinsic model Witsenhausen (1971, 1975a) to game theory, we need to translate the standard notions of the field in this framework. Games defined in this framework will be designed as Witsenhausen games or, equivalently, intrinsic games.

In what follows we assume that the solvability/measurability property holds true.

Agents and Players

A **player** p is defined as a subset A_p of A such that the criterion j_a and the probability $\mathbb{P}_a$ are the same for all $a \in A_p$. We hence introduce the notation j_p and $\mathbb{P}_p$. If P is the set of all player, then $(A_p)_{p \in P}$ form a partition of A. Observe that the information fields of the agents do not need to be completely ordered.

Strategies and payoff

For a player $p \in P$ a **(pure) strategy** λ_p is an element of $\Pi_p := \prod_{a \in A_p} \Lambda^{ad}_a$. A **mixed strategy** μ_p is an element of $\Delta\Pi_p$, the set of distributions over Π_p. We use the term strategy (resp. mixed strategy) profile to designate a vector $(\gamma_p)_{p \in P}$ (resp. $(\mu_p)_{p \in P}$). For a given mixed strategy profile μ and a player p, the **expected payoff** is defined as:

$$\int_\Lambda \mathbb{E}_p[j_p \circ S_\gamma] d\mu(\gamma),$$

where $\mathbb{E}_p$ denote the expectation with respect to $\mathbb{P}_p$. We observe that we could have restricted the study to pure strategy, since as Witsenhausen stated in Witsenhausen (1975a).

"The possibility of mixed (i.e., randomized) decisions is implicitly included, with the randomizing devices included as factors (in Ω)."

Yet, we consider that mixed strategies are interesting mathematical tools (allow for the convexification of the strategy set) and the simplify the economical interpretations.

Intrinsic form game

To summarize an intrinsic form game is defined by:

- a set of **agents** partitioned by a **player** equivalence relation
- a **random set** Ω and for each agent, an **action set** $\mathbb{U}_a$, all equipped with σ**-fields**
- for each agent an **information field**,
- for each agent, a set of measurable and **admissible policies**, which are mapping from the history to the decision set
- the solvability/measurability property needs to be satisfied, and all for the introduction of a **measurable solution map** S_γ
- for each player, a measurable and bounded **criterion** and a **prior probability distribution**

Nash equilibrium

Pure Nash Equilibrium A **(pure) Nash Equilibrium** is a strategy profile λ^* such that, for any player $p \in P$ and any strategy $\lambda_p \in \Pi_p$

$$\mathbb{E}_p[j_p \circ S_{(\lambda_p^*, \lambda_{-p}^*)}] \geq \mathbb{E}_p[j_p \circ S_{(\lambda_p, \lambda_{-p}^*)}].$$

Mixed Nash Equilibrium A **mixed Nash Equilibrium** is a mixed strategy profile μ^* such that, for any player $p \in P$ and any mixed strategy $\mu_p \in \Delta\Pi_p$

$$\int_{\Lambda_p}\int_{\Lambda_{-p}} \mathbb{E}_p[j_p \circ S_\lambda] d\mu_{-p}^*(\lambda) d\mu_p^*(\lambda) \geq \int_{\Lambda_p}\int_{\Lambda_{-p}} \mathbb{E}_p[j_p \circ S_\lambda] d\mu_{-p}^*(\lambda) d\mu_p(\lambda).$$

3.5.2 Nash equilibrium under uncertainty

We denote real-valued random variables on $(\Omega, \mathcal{F})$ by

$$\mathbb{L}(\Omega, \mathcal{F}) = \{\mathbf{X} : (\Omega, \mathcal{F}) \to (\mathbb{R}, \mathcal{B}_{\mathbb{R}}) \ , \ \ \mathbf{X}^{-1}(\mathcal{B}_{\mathbb{R}}) \subset \mathcal{F}\} \ . \tag{3.53}$$

To each agent $a \in A$, we attach three features:

- a measurable *criterion*
$$j_a : \mathbb{H} \to \mathbb{R} \ , \tag{3.54}$$
- a *risk measure*
$$\mathbb{G}_a : \mathbb{L}(\Omega, \mathcal{F}) \to \mathbb{R} \cup \{+\infty\} \ , \tag{3.55}$$
where $\mathrm{dom}\mathbb{G}_a = \{\mathbf{X} \in \mathbb{L}(\Omega, \mathcal{F}) \ , \ \ \mathbb{G}_a(\mathbf{X}) < +\infty\}$ can account for restrictions on the random variables to be evaluated by the risk measure $\mathbb{G}_a$,

- an *information field*

$$\mathcal{I}_a \subset \mathcal{H} \ . \tag{3.56}$$

We suppose that the stochastic system $\{\mathbb{U}_a, \mathcal{U}_a, \mathcal{I}_a\}_{a \in A}$ displays no self-information and displays the solvability property. Therefore, by Definition 9, there is a solution map $S_\gamma : \Omega \to \mathbb{H}$, for any admissible policy $\gamma \in \Gamma_A^{ad}$. We suppose in addition that the solution map is measurable.

Definition 30. *We say that the admissible policy $\overline{\gamma} \in \Gamma_A^{ad}$ is a* Nash equilibrium under uncertainty *if*

$$\mathbb{G}_a\Big[j_a \circ S_{\overline{\gamma}_a, \overline{\gamma}_{-a}}\Big] \leq \mathbb{G}_a\Big[j_a \circ S_{\gamma_a, \overline{\gamma}_{-a}}\Big] \ , \ \ \forall a \in A \ , \ \ \forall \gamma_a \in \Gamma_a^{ad} \ . \tag{3.57}$$

Classical Nash equilibrium

The classical Nash equilibrium can be seen as a special case where

$$\Omega = \{\omega\} \ , \ \ \mathcal{I}_a = \bigotimes_{b \in A} \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega\} \ , \ \ \forall a \in A \ . \tag{3.58}$$

This setting is deterministic because Ω is reduced to a singleton. As all information fields $\mathcal{I}_a$ are trivial, admissible strategies are reduced to pure constant strategies, that is, to constant mappings identified with decision profiles in $\mathbb{U}_A$. As a consequence, the solvability property trivially holds true.

Stackelberg equilibrium

A Stackelberg equilibrium is a Nash equilibrium under uncertainty when there exists a (nonempty) subsystem, whose agents are called *leaders*.

When there is a subsystem, we know by Proposition 17 that the solution map displays a co-cycle property. This allows to display a Nash equilibrium under uncertainty as the solution of a bi-level optimization problem.

As an illustration, consider the case ot two agents $A = \{a, b\}$ with information fields given by

$$\mathcal{I}_a \subset \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathcal{F} \ , \ \ \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathcal{F} \subsetneq \mathcal{I}_b \subset \mathcal{U}_a \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathcal{F} \ .$$

Agent a is the leader since $\langle a \rangle_{\mathfrak{P}} = \emptyset$ (agent a has no predecessor), hence $\overline{\{a\}} = \{a\}$ by (3.27). The co-cycle property (3.30) of the solution map writes

$$M_{(\gamma_a, \gamma_b)}(\omega) = \Big(M_{\gamma_a}(\omega), M_{\gamma_b\big(M_{\gamma_a}(\omega), \cdot\big)}(\omega)\Big) \ , \ \ \forall \omega \in \Omega \ . \tag{3.59}$$

The definition (3.57) of a Nash equilibrium under uncertainty becomes (where we abusively keep ω)

$$\mathbb{G}_a\Big[j_a\Big(\overline{\gamma}_a(\omega), \overline{\gamma}_b\big(\overline{\gamma}_a(\omega)\big), \omega\Big)\Big] \leq \mathbb{G}_a\Big[j_a\Big(\gamma_a(\omega), \overline{\gamma}_b\big(\gamma_a(\omega)\big), \omega\Big)\Big] \ , \ \ \forall \gamma_a \in \Gamma_a^{ad} \ , \tag{3.60}$$

$$\mathbb{G}_b\Big[j_b\Big(\overline{\gamma}_a(\omega), \overline{\gamma}_b\big(\overline{\gamma}_a(\omega)\big), \omega\Big)\Big] \leq \mathbb{G}_b\Big[j_b\Big(\overline{\gamma}_a(\omega), \gamma_b\big(\overline{\gamma}_a(\omega)\big), \omega\Big)\Big] \ , \ \ \forall \gamma_b \in \Gamma_b^{ad} \ . \tag{3.61}$$

In the deterministic case, this gives

$$j_a\Big(\overline{u}_a, \overline{\gamma}_b\big(\overline{u}_a\big)\Big) \leq j_a\Big(u_a, \overline{\gamma}_b\big(u_a\big)\Big) \ , \ \ \forall u_a \in \mathbb{U}_a \ , \tag{3.62}$$

$$j_b\Big(\overline{u}_a, \overline{\gamma}_b\big(\overline{u}_a\big)\Big) \leq j_b\Big(\overline{u}_a, \gamma_b\big(\overline{u}_a\big)\Big) \ , \ \ \forall \gamma_b \in \Gamma_b^{ad} \ . \tag{3.63}$$

Types and Bayesian games

The private information of agent $a \in A$ is captured through its type. Agent a type set is Ω_a. We introduce the product of the agents' type sets Ω as follows

$$\Omega = \prod_{a \in A} \Omega_a \ . \tag{3.64}$$

In Bayesian games, $\mathbb{G}_a$ is an expectation.

Time

We introduce *players* $i \in \{1, \ldots, I\}$ and *time* $t \in \{0, 1, \ldots, T\}$. *Agents* are couples $(i, t) \in A = \{1, \ldots, I\} \times \{0, 1, \ldots, T\}$. We have

$$\langle (i, 0) \rangle_{\mathfrak{P}} = \emptyset \ , \ \ \langle (i, t) \rangle_{\mathfrak{P}} \subset \{1, \ldots, I\} \times \{0, 1, \ldots, t-1\} \ , \ \ \forall t \in \{1, \ldots, T\} \ . \tag{3.65}$$

Moral Hazard

Moral hazard (hidden action) occurs when decisions of the agent b are hidden to the principal a, that is, $\langle a \rangle_{\mathfrak{P}} = \emptyset$ or, equivalently,

$$\mathfrak{I}_a \subset \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \mathbb{U}_a\} \otimes \mathcal{F} \ . \tag{3.66}$$

In case of moral hazard, the system is sequential with the principal a as first player (which does not preclude to choose the agent b as first player in some special cases, as in a static team situation). An insurance company cannot observe the efforts of the insured to avoid risky behavior: the firm faces the hazard that insured persons behave immorally.

Adverse Selection

Adverse selection occurs when the agent b knows the state of nature, but the principal a, does not:

$$\{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \mathbb{U}_a\} \otimes \mathcal{F} \subsetneq \mathfrak{I}_b \subset \{\emptyset, \mathbb{U}_b\} \otimes \mathcal{U}_a \otimes \mathcal{F} \ , \ \ \mathfrak{I}_a \subset \mathcal{U}_b \otimes \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \Omega\} \ . \tag{3.67}$$

In the absence of observable information on potential customers, an insurance company offers a unique price for a contract, hence screens and selects the "bad" ones.

When $\langle a \rangle_{\mathfrak{P}} = \emptyset$, we have both moral hazard and adverse selection, and the principal has no information whatsoever. The system is sequential with the principal as first player.

Signaling

When $\langle a \rangle_{\mathfrak{P}} = \{b\}$, the agent b may reveal the state of nature by her decision which is observable by the principal a: this the so-called *signaling*. The most interesting case is when $\langle b \rangle_{\mathfrak{P}} = \{a\}$ yielding a non causal system (when $\langle b \rangle_{\mathfrak{P}} = \emptyset$, the system is sequential with the agent as first player).

In evolutionary biology, animals signal their genotype by their phenotype as in the *handicap paradox* Zahavi (1975).

The first application of signaling games to economic problems was Spence's model of job market signaling Spence (1974). Spence describes a game where workers have a certain ability (high or low) that the employer does not know. The workers send a signal by their choice of education. The cost of the education is higher for a low ability worker than for a high ability worker. The employers observe the workers' education but not their ability, and choose to offer the worker a high or low wage. In this model it is assumed that the level of education does not cause the high ability of the worker, but rather, only workers with high ability are able to attain a specific level of education without it being more costly than their increase in wage. In other words, the benefits of education are only greater than the costs for workers with a high level of ability, so only workers with a high ability will get an education.

Information may be given by mappings called *signals*, which capture what agent a observes from the history

$$\mathbf{S}_a : (\mathbb{H}, \mathcal{H}) \to (\mathbb{S}_a, \mathcal{S}_a)$$

where $(\mathbb{S}_a, \mathcal{S}_a)$ is some measurable space. Assuming that $\mathbf{S}_a$ is measurable from $(\mathbb{H}, \mathcal{H})$ to $(\mathbb{S}_a, \mathcal{S}_a)$, the connection with the "field" approach is given by the σ-field generated by the mapping:

$$\mathcal{I}_a = \mathbf{S}_a^{-1}(\mathcal{S}_a)$$

3.5.3 Extensive form game representation

As pointed out by Witsenhausen in Witsenhausen (1971), the difficulties in specifying the information structure of a game were faced and overcomed in the early days of Game Theory through the introduction of *extensive form* games Myerson (1997). The extensive form is the most richly structured way to describe game situations. The definition of the extensive form that is now standard in most of the literature on Game Theory, is due to Kuhn Kuhn (1997), who modified the earlier definition used by von Neumann and Morgenstern Myerson (1997). The *strategic* form and its generalization, the *Bayesian* form, are conceptually simpler forms that are more convenient for purposes of general analysis but are generally viewed as being derived from the extensive form.

3.5.4 Sub-game perfect equilibrium (SPE)

(...) This motivate our tentative to define a notion of subgame perfect equilibrium for intrinsic games. In this section we consider the game has a **sequential** structure. We denote by $\gamma_{\langle a \rangle_{\mathfrak{P}}}$ the vector of agent a's predecessors policies, and by $\gamma_{\langle a \rangle_{\mathfrak{P}-1}}$ the vector of agent a's successors policies. In the following, we propose two definitions of increasing generality. We show that those definitions are coherent with Kuhn's conception of SPE.

Tentative definitions

Sequential and deterministic A strategy profile λ^* is a **sub-game perfect equilibrium** iff for any $p \in P$, $a \in A_p$, $\gamma_{\langle a \rangle_{\mathfrak{P}}} \in \Lambda_{\langle a \rangle_{\mathfrak{P}}}$ and $\gamma_a \in \Lambda$,

$$j_p \circ S_{\gamma_{\langle a \rangle_{\mathfrak{P}}}, \gamma_a^*, \gamma^*_{\langle a \rangle_{\mathfrak{P}-1}}} \geq j_p \circ S_{\gamma_{\langle a \rangle_{\mathfrak{P}}}, \gamma_a, \gamma^*_{\langle a \rangle_{\mathfrak{P}-1}}}.$$

Sequential and stochastic A strategy profile λ^* is a **sub-game perfect equilibrium** iff for any $p \in P$, $a \in A_p$, $\gamma_{\langle a \rangle_{\mathfrak{P}}} \in \Lambda_{\langle a \rangle_{\mathfrak{P}}}$ and $\gamma_a \in \Lambda$,

$$\mathbb{E}_p[j_p \circ S_{\gamma_{\langle a \rangle_{\mathfrak{P}}}, \gamma_a^*, \gamma^*_{\overline{a, \langle a \rangle_{\mathfrak{P}}}}}] \geq \mathbb{E}_p[j_p \circ S_{\gamma_{\langle a \rangle_{\mathfrak{P}}}, \gamma_a, \gamma^*_{\overline{a, \langle a \rangle_{\mathfrak{P}}}}}].$$

3.5.5 Backward induction

For extensive form games (Kuhn), the notion of sub-game perfect equilibrium presents two interesting properties. First it is stronger than Nash equilibrium, therefore where we have several Nash equilibria, we may have only one SPE. Second, SPE, as opposed to Nash equilibrium, can be systematically computed, using backward induction. Moreover we could argue that from the perspective of rational agents, this notion is finer, since rational agents could do this backward induction. We point out that this is debatable, as psychological experiments show that SPE is not always the solution preferred by individuals.

See Chapter 9 of Carpentier et al. (2015) for a reference on the notion of **policy independence of conditional expectations (PICE)**, which was first introduced in Witsenhausen (1975b) with the following remarks:

> ”If an observer of a stochastic control system observes both the decision taken by an agent in the system and the data that was available for this decision, then the conclusions that the observer can draw do not depend on the functional relation (policy, control law) used by this agent to reach his decision.”

3.5.6 Comparison of Witsenhausen's intrinsic model with Kuhn extensive form

	Kuhn's extensive form	Witsenhausen intrinsic model
locus of decision	nodes	agents
information structure	decision sets	subfields
timing	decisions taken sequentially	intrinsic structure
representation of a game	not unique	”essentially unique”
expression of the order of play	tree	definition of predecessors and successors

3.5.7 Perfect recall and extended station

Extended station

3.6 Multiple agents, multiple criteria (cooperative game theory)

The formation of coalitions is fundamental in Game Theory. In all interactions with the other agents, coalition participants act as one unit (it may be useful to think of a "representative agent" taking their place); however, this arrangement will continue only as long as each player finds it desirable to act this way. Further bargaining occurs among the members of each coalition on how to divide what they obtained together. Thus, following Hart and Kurz Hart and Kurz (1983), we can state that the existence of coalitions implies that the interactions among the agents will be conducted on two levels: first, *among* the coalitions, and second, *within* each coalition.

The goal of this subsection is to introduce the basic notions of cooperative Game Theory, which might be useful to understand the concept of coalition in energy management problems. We will begin by formally defining *characteristic function games* and some of their subclasses, and then present the standard solution concepts for such games.

Characteristic function games

A game in the sense of Game Theory is an abstract mathematical model of a scenario in which self-interested agents interact.

We consider a non-empty set A of agents: the players of the game. A *coalition* is simply a subset of the players A. We will use $\mathcal{C}$, $\mathcal{C}'$, ..., to denote coalitions. The *grand coalition* is the set A of all players.

Definition 31. *A* characteristic function game $\mathcal{G}$ *is given by a pair* (A, v)*, where* A *is a finite, non-empty set of agents and* $v : 2^A \to \mathbb{R}$ *is a* characteristic function*, which maps each coalition* $\mathcal{C} \subseteq A$ *to a real number* $v(\mathcal{C})$*. The number* $v(\mathcal{C})$ *is usually referred to as the* value *of the coalition* $\mathcal{C}$*.*

Note that characteristic function games assign the value of a coalition to the coalition as a whole, and not its individual members. In fact, the question of how to divide coalitional value is a fundamental research topic in cooperative Game Theory, and we will see some answers to this question, in the form of solution concepts such as the Shapley value and the nucleolus. Note that an implicit assumption in characteristic form games is that the coalitional value $v\big(\mathcal{C}\big)$ can be divided amongst the participants in $\mathcal{C}$ in any way that the participants in $\mathcal{C}$ choose and furthermore that there is no loss in the coalition value caused by transfers. Formally, games with this property are said to the *Transferable Utility* (TU) games Chalkiadakis et al. (2011); Myerson (1997). We will restrict in this chapter to this class of games.

An outcome of a characteristic function game consists of two parts:

- a partition of players into coalitions, called a *coalitional structure*,
- a *payoff vector*, which distributes the value of each coalition among its participants.

Definition 32. *Given a characteristic function game* $\mathcal{G} = \big(A, v\big)$*, a* coalition structure *over* A *is a collection of non-empty subsets* $\mathcal{CS} = \Big\{\mathcal{C}^1, ..., \mathcal{C}^K\Big\}$ *such that:*

- $\cup_{j=1}^{K} \mathcal{C}^j = A$,
- $\mathcal{C}^i \cap \mathcal{C}^j = \emptyset \quad \forall i, j \in \{1, ..., K\}, i \neq j$.

This is equivalent to say that a coalition structure coincides with a partition of A.
A vector $\boldsymbol{j}$ *is a* payoff vector *for a coalition structure* $\mathcal{CS} = \Big\{\mathcal{C}^1, ..., \mathcal{C}^K\Big\}$ *over* A *if:*

- $j_a \geq 0 \quad \forall a \in A$,
- $\sum_{a \in \mathcal{C}^j} j_a \leq v\big(\mathcal{C}^j\big) \quad \forall j \in \{1, ..., K\}$.

The first requirement means that every player must appear in some coalition. The second, which is a feasability requirement, says that a player cannot appear in more than one coalition.

An *outcome* of $\mathcal{G}$ is a pair $\big(\mathcal{CS}, \boldsymbol{j}\big)$, where $\mathcal{CS}$ is a coalition structure over $\mathcal{G}$ and $\boldsymbol{j}$ is a payoff vector for $\mathcal{CS}$. Given a payoff vector $\boldsymbol{j}$, we write $\boldsymbol{j}(\mathcal{C})$ to denote the total payoff $\sum_{a \in \mathcal{C}} j_a$ of a coalition $\mathcal{C} \subseteq A$ under $\boldsymbol{j}$.

We will often assume an efficiency requirement: a payoff vector $\boldsymbol{j}$ is *efficient* if all the payoff obtained by a coalition is distributed amongst coalition participants i.e., $\sum_{a \in \mathcal{C}^j} j_a = v(\mathcal{C}^j) \quad \forall j \in \{1, ..., K\}$.

We define $v(\mathcal{CS}) = \sum_{\mathcal{C} \in \mathcal{CS}} v(\mathcal{C})$ the *social welfare* of the coalition structure $\mathcal{CS}$.

A payoff vector $\boldsymbol{j}$ for a coalition structure $\mathcal{CS}$ is said to be an *allocation* (sometimes also called *imputation* Chalkiadakis et al. (2011)) if it is efficient and moreover satisfies the *individual rationality* condition i.e., $j_a \geq v(\{a\}) \quad \forall a \in A$.

Subclasses of characteristic function games

We will now define four important subclasses of coalitional games: monotone games, superadditive games, convex games, and simple games.

Monotone games In such games, adding an agent to an existing coalition can only increase the overall productivity of this coalition.

Definition 33. *A characteristic function game* $\mathcal{G} = (A, v)$ *is said to be* monotone *if it satisfies* $v(\mathcal{C}) \leq v(\mathcal{C}')$ *for every pair of coalitions* $\mathcal{C}, \mathcal{C}' \subseteq A$ *such that* $\mathcal{C} \subseteq \mathcal{C}'$.

Note that non-monotonicity might be caused by communication and coordination costs Chalkiadakis et al. (2011).

Superadditive Games In such games, it is always profitable for two groups of players to join forces.

Definition 34. *A characteristic function game* $\mathcal{G} = (A, v)$ *is said to be* superadditive *if it satisfies* $v(\mathcal{C} \cup \mathcal{C}') \geq v(\mathcal{C}) + v(\mathcal{C}')$ *for every pair of disjoint coalitions* $\mathcal{C}, \mathcal{C}' \subseteq A$.

In superadditive games, there is no reason for agents to form a coalition structure consisting of multiple coalitions: the agents can earn at least as much profit by working together within the grand coalition.

Note that any non-superadditive game can be transformed into a superadditive game by computing, for each coalition, the maximum amount this coalition can earn by splitting into sub-coalitions. Formally, given a (non-superadditive) game $\mathcal{G} = (A, v)$ we can define a new game $\mathcal{G}^\star = (A, v^\star)$ by setting:

$$v^\star(\mathcal{C}) = \max_{\mathcal{CS} \in \mathcal{CS}_\mathcal{C}} v(\mathcal{CS}) ,$$

for every coalition $\mathcal{C} \subseteq A$, where $\mathcal{CS}_\mathcal{C}$ denotes the space of all coalition structures over $\mathcal{C}$. The game $\mathcal{G}^\star$ is called the *superadditive cover* of $\mathcal{G}$. It is superadditive even if $\mathcal{G}$ is not Chalkiadakis et al. (2011).

Convex Games

The superadditivity places a restriction on the behavior of the characteristic function v on disjoint coalitions. By placing a similar restriction on v's behavior on non-disjoint coalitions, we obtain the class of *convex* games.

Definition 35. *A characteristic function* v *is said to be* supermodular *if it satisfies:*

$$v(\mathcal{C} \cup \mathcal{C}') + v(\mathcal{C} \cap \mathcal{C}') \geq v(\mathcal{C}) + v(\mathcal{C}')$$

for every pair of coalitions $\mathcal{C}, \mathcal{C}' \subseteq A$. *A game with a supermodular characteristic function is said* convex.

Convex games have a very intuitive characterization in terms of players' marginal contributions: in a convex game, a player is more useful when he joins a bigger coalition.

Proposition 36 (Chalkiadakis et al. (2011))**.** *A characteristic function* $\mathcal{G} = (A, v)$ *is convex iff for every pair of coalitions* $\mathcal{C}, \mathcal{C}'$ *such that* $\mathcal{C} \subset \mathcal{C}'$ *and very player* $i \in A \setminus \mathcal{C}'$ *it holds that:*

$$v(\mathcal{C}' \cup \{a\}) - v(\mathcal{C}') \geq v(\mathcal{C} \cup \{a\}) - v(\mathcal{C})$$

Simple games

A game $\mathcal{G} = (A, v)$ is said to be *simple* if it is monotone and its characteristic function only takes values 0 and 1 i.e., $v(\mathcal{C}) \in \{0; 1\}$ for any $\mathcal{C} \subseteq A$.

3.6.1 Solutions

One can evaluate the outcome of a characteristic function game $\mathcal{G} = (A, v)$ according to two sets of criteria:

1) *stability* i.e., what are the incentives for the agents to stay in a coalition structure,

2) *fairness* i.e., how well each agent's payoff reflects his marginal contribution to the coalition he belongs.

Stability issue: the core solution

Consider a characteristic function game $\mathcal{G} = (A, v)$ and an outcome $(\mathcal{CS}, \boldsymbol{j})$ of this game. $\boldsymbol{j}(\mathcal{C})$ denotes the total payoff of coalition $\mathcal{C}$ under $\boldsymbol{j}$. Now, if $\boldsymbol{j}(\mathcal{C}) < v(\mathcal{C})$ for some $\mathcal{C} \subseteq A$, the agents in $\mathcal{C}$ could do better by abandoning the coalition structure $\mathcal{CS}$ and forming a coalition of their own.

The set of stable outcomes i.e., outcomes where no subset of players has an incentive to deviate is called the *core* of $\mathcal{G}$.

Definition 37. *The* core *of a characteristic function game* $\mathcal{G} = (A, v)$ *is the set of all outcomes* $(\mathcal{CS}, \boldsymbol{j})$ *such that* $\boldsymbol{j}(\mathcal{C}) \geq v(\mathcal{C})$ *for every* $\mathcal{C} \subseteq A$ *and then* $\boldsymbol{j}(\mathcal{C}) = v(\mathcal{C})$ *for any* $\mathcal{C} \in \mathcal{CS}$.

As a solution concept, the core suffers from three main drawbacks:

- it can be empty,
- it can be quite large, hence selecting a suitable core allocation can be difficult,
- in many cases, the allocations that lie in the core can be unfair to one or more players i.e., the player's resulting payoff does not reflect his marginal contribution to the game.

These drawbacks motivated the search for a solution concept which can associate with every characteristic function game $\mathcal{G} = (A, v)$, a *unique* payoff vector known as the value of the game (which is quite different from the value of a coalition).

Proposition 38. *Chalkiadakis et al. (2011) A characteristic function game* $\mathcal{G} = (A, v)$ *has a non-empty core iff its superadditive cover* $\mathcal{G}^\star = (A, v^\star)$ *has a non-empty core.*

We now skip from the concept of stability to the concept of fairness.

Fairness issues: Shapley value, Banzhaf power index and the nucleolus

Shapley value

The Shapley value is usually formulated with respect to the grand coalition. It defines a way of distributing the value $v(A)$ for the grand coalition.

Shapley approaches the problem of searching for a solution concept which could avoid the drawbacks of the core by defining a set of desirable properties and he characterized a unique collection of mappings (one for each agent) that satisfies the four axioms listed below, known later as the *Shapley value*:

- efficiency axiom: $\sum_{a \in A} \Psi_a(v) = v(A)$,
- symmetry axiom: if agent a and b are such that $v(\mathcal{C} \cup \{a\}) = v(\mathcal{C} \cup \{b\})$ for every coalition $\mathcal{C}$ not containing agent a and b, then $\Psi_a(v) = \Psi_b(v)$,
- dummy axiom: if agent a is such that $v(\mathcal{C}) = v(\mathcal{C} \cup \{a\})$ for every coalition $\mathcal{C}$ not containing a, then $\Psi_a(v) = 0$,
- additivity axiom: if v and $\widetilde{v}$ are characteristic functions, then $\Psi_a(v + \widetilde{v}) = \Psi_a(v) + \Psi_a(\widetilde{v}) \quad \forall a \in A$.

The Shapley value has an interpretation that takes into account the order in which the players join the grand coalition A. In the event where the players join the grand coalition in a random order, the payoff allotted by the Shapley value to an agent $a \in A$ is the expected marginal contribution of agent

a when he joins the grand coalition. Given any Transferable Utility (TU) game $\mathcal{G} = (A, v)$, for every agent $a \in A$ the Shapley value $\Psi_a(v)$ assigns the payoff $\Psi_a(v)$ given by:

$$\Psi_a(v) = \sum_{\mathcal{C} \subseteq A \setminus \{a\}} \frac{card(\mathcal{C})!\big(card(A) - card(\mathcal{C}) - 1\big)!}{card(A)!} \Big[v\big(\mathcal{C} \cup \{a\}\big) - v\big(\mathcal{C}\big)\Big] . \tag{3.68}$$

In general, the Shapley value is unrelated to the core. However, in some applications, one can show that the Shapley value lies in the core; Such a result is of interest, since if such an allocation is found, it combines both the stability of the core as well as the axioms and fairness of the Shapley value. An interesting result is that for convex games the Shapley value lies in the core.

Banzhaf power index

Another solution concept that is motivated by fairness considerations is the *Banzhaf power index*. Just like the Shapley value, the Banzhaf index measures agents' expected marginal contributions. However, instead of averaging over all permutations of players, it averages over all coalitions in the game.

Definition 39. *Given a characteristic function game $\mathcal{G} = (A, v)$, the* Banzhaf power index *of an agent $a \in A$ is denoted $\beta_a(\mathcal{G})$ and is given by*

$$\beta_a(\mathcal{G}) = \frac{1}{2^{A-1}} \sum_{\mathcal{C} \subseteq A \setminus \{a\}} \Big[v(\mathcal{C} \cup \{a\}) - v(\mathcal{C})\Big] . \tag{3.69}$$

Since efficiency is surely a very desirable property of a payoff distribution scheme, a rescaled version of the Banzhaf index has been proposed. Formally, the *normalized Banzhaf index* $\eta_a(\mathcal{G})$ is defined as

$$\eta_a(\mathcal{G}) = \frac{\beta_a(\mathcal{G})}{\sum_{a \in A} \beta_a(\mathcal{G})} . \tag{3.70}$$

Nucleolus

The *nucleolus* is a solution concept that defined a unique outcome for a game. The nucleolus is based on the notion of *deficit*. Formally, given a super-additive game $\mathcal{G} = (A, v)$, a coalition $\mathcal{C} \subseteq A$, and a payoff vector $\boldsymbol{j}$ for this game, the *deficit* of $\mathcal{C}$ with respect to $\boldsymbol{j}$ is defined as

$$d(\boldsymbol{j}, \mathcal{C}) = v(\mathcal{C}) - \boldsymbol{j}(\mathcal{C}) . \tag{3.71}$$

This quantity measures $\mathcal{C}$'s incentive to deviate under $\boldsymbol{j}$. Any payoff vector $\boldsymbol{j}$ generates a 2^A-dimensional *deficit vector* $\boldsymbol{d(j)} = \big(d(\boldsymbol{j}, \mathcal{C}_k)\big)_{k=1,\dots,2^A}$ where $\big(\mathcal{C}_k\big)_{k=1,\dots,2^A}$ is the list of all subsets of A ordered by their deficit under $\boldsymbol{j}$, from the largest to the smallest: $v(\mathcal{C}_k) - \boldsymbol{j}(\mathcal{C}_k) \geq v(\mathcal{C}_l) - \boldsymbol{j}(\mathcal{C}_l)$ for any $1 \leq k < l \leq 2^A$. Two deficit vectors can be compared lexicographically: given two payoff vectors $\boldsymbol{j}, \boldsymbol{j'}$, we say that $\boldsymbol{d(j)}$ is lexicographically smaller than $\boldsymbol{d(j')}$ if there exists $a \in \{1, \dots, 2^A\}$ such that the $a-1$ entries of $\boldsymbol{d(j)}$ and $\boldsymbol{d(j')}$ are equal, but the a-th entry of $\boldsymbol{d(j)}$ is smaller than the a-th entry of $\boldsymbol{d(j')}$; if this is the case, we write $\boldsymbol{d(j)} <_{lex} \boldsymbol{d(j')}$. We extend this notation by setting $\boldsymbol{d(j)} \leq_{lex} \boldsymbol{d(j')}$ if $\boldsymbol{d(j)} <_{lex} \boldsymbol{d(j')}$ or $\boldsymbol{d(j)} = \boldsymbol{d(j')}$.

The nucleolus is then defined as the set of all allocations that have the lexicographically smallest deficit vector.

Definition 40. *The* nucleolus $\mathcal{N}(\mathcal{G})$ *of a super-additive game $\mathcal{G} = (A, v)$ is the set*

$$\mathcal{N}(\mathcal{G}) = \Big\{\boldsymbol{j} \in \mathcal{A}(A) | \boldsymbol{d(j)} \leq_{lex} \boldsymbol{d(j')} \quad \textit{for all} \quad \boldsymbol{j'} \in \mathcal{A}(A)\Big\} . \tag{3.72}$$

where $\mathcal{A}(A)$ is the set of all allocations for the grand coalition.

An interesting property of the nucleolus is that it always belongs to the core.

3.6.2 Games with communication

There are many games, like the Prisoners' dilemma, in which the Nash equilibria yield very low payoff for the players, relative to other non equilibrium outcomes. In such situations, the players would want to transform the game, if possible, to extend the set of equilibria to include better outcomes. Players might seek to transform a game by trying to communicate with each other and coordinate their moves, perhaps even by formulating contractual agreements.

When we say that a given game is *with contracts*, we mean that, in addition to the options that are given to the players in the formal structure of the game, the players also have very wide options to bargain with each other and to sign contracts, where each contract binds the players who sign it to some correlated strategy that may depend on the set of players who sign.

Formally, given any strategic form game $\mathcal{G} = \big(A, (u_a)_{a\in A}, (j_a)_{a\in A}\big)$, a *correlated strategy* for a set of players is any probability distribution over the set of possible combinations of pure strategies that these players can choose in $\mathcal{G}$. Given any $C \subseteq A$, a correlated strategy for C is any probability distribution in $\Delta(\mathbb{U}_C)$, where $\mathbb{U}_C = \prod_{a\in C} \mathbb{U}_a$.

A set of players C might implement a correlated strategy τ_C by having a centralizator who generates randomly a profile of pure strategies in $\mathbb{U}_C$ in such a way that the probability of designing any $u_C = (u_a)_{a\in C}$ in $\mathbb{U}_C$ is $\tau_C(u_C)$. Then the mediator would tell each player a in C to implement the strategy u_a that is the a-th component of the designated profile u_C.

Given any correlated strategy τ in $\Delta(\mathbb{U})$ for all players, for each a, let $J_a(\tau)$ denote the *expected payoff* to player a when τ is implemented in the game $\mathcal{G}$

$$J_a(\tau) = \sum_{u\in\mathbb{U}} \tau(u) j_a(u) \; . \tag{3.73}$$

Let $\boldsymbol{J}(\tau) = \Big(J_a(\tau)\Big)_{a\in A}$ denote the expected payoff allocation to the players in A that would result from implementing τ. With this notation, a *contract* is represented by any vector $\tau = (\tau_C)_{C\subseteq A}$ in $\prod_{C\subseteq A} \Delta(\mathbb{U}_C)$. For any such contract τ, τ_C represents the correlated strategy that would be implemented by the players in C if C were the set of players who sign the contract. So for any expected payoff vector in the set $\{\boldsymbol{J}(\tau) | \tau \in \Delta(\mathbb{U})\}$ there exists a contract such that, if the players all signed this contract, then they would get this expected payoff allocation. This set of possible expected allocations is closed and convex in $\mathbb{R}^A$.

However, not all such contracts could actually be signed by everyone in an equilibrium of the implicit contract-signing game. For any player a, the *max-min value* also called *security level*, in game $\mathcal{G}$

$$v_a = \min_{\tau_{A\setminus a}\in\Delta(\mathbb{U}_{A\setminus a})} \Big(\max_{u_a\in\mathbb{U}_a} \sum_{u_{A\setminus a}\in\mathbb{U}_{A\setminus a}} \tau_{A\setminus a}(u_{A\setminus a}) j_a(u_{A\setminus a}, u_a) \Big) \; . \tag{3.74}$$

The minimax value for agent a is the best expected payoff that agent a could get against the worst (for him) correlated strategy that the other players could use against him. A *minimax strategy* against player a is any correlated strategy in $\Delta(\mathbb{U}_{A\setminus a})$ that achieves the minimum in (3.74). The theory of two-person zero sum game then implies that the minimax value also satisfies

$$v_a = \max_{\tau_a\in\Delta(\mathbb{U}_a)} \Big(\min_{u_{A\setminus a}\in\mathbb{U}_{A\setminus a}} \sum_{u_a\in\mathbb{U}_a} \tau_a(u_a) j_a(u_{A\setminus a}, u_a) \Big) \; . \tag{3.75}$$

So player a has a randomized strategy that achieves the above maximum and guarantees him an expected payoff that is not less than his minimax value, no matter what the other players may do.

A correlated strategy $\tau \in \Delta(\mathbb{U})$ is *individual rational* if

$$J_a(\tau) \geq v_a \quad \forall a \in A \; . \tag{3.76}$$

This condition is also called *participation constraints*.

In general, for any finite strategy form game $\mathcal{G}$ a mediator who was trying to help coordinate the agents' actions would at least need to let each agent a know which strategy in $\mathbb{U}_a$ was recommended for him. In the game with mediated communication, each agent a would actually have an enlarged set of communication strategies that would include all mappings from $\mathbb{U}_a$ into $\mathbb{U}_a$, each of which represents a possible rule for choosing an element of $\mathbb{U}_a$, as a function of the mediator's recommendation in $\mathbb{U}_a$.

Suppose it is common knowledge that the mediator will determine his recommendations according to the probability distribution τ in $\Delta(\mathbb{U})$; so $\tau(u)$ denotes the probability that any given pure strategy profile $u = (u_a)_{a \in A}$ would be recommended by the mediator. Then it would be an equilibrium for all players to obey the mediator's recommendation if

$$J_a(\tau) \geq \sum_{u \in \mathbb{U}} \tau(u) j_a(u_{A \setminus a}, \delta_a(u_a)) \quad \forall a \in A, \forall \delta_a : \mathbb{U}_a \to \mathbb{U}_a \ . \tag{3.77}$$

Following Aumann Aumann and Dreze (1974); Aumann (1987), we say that τ is a *correlated equilibrium* of $\mathcal{G}$ if $\tau \in \Delta(\mathbb{U})$ and τ satisfies condition described in Equation (3.77). A correlated equilibrium is any correlated strategy for the players in $\mathcal{G}$ that could be self-enforcingly implemented with the help of a mediator who can make non-binding confidential recommendations to each player.

It can be shown Myerson (1997) that Equation (3.77) is equivalent to the following system of inequalities

$$\sum_{u_{A \setminus a} \in \mathbb{U}_{A \setminus a}} \tau(u) \Big(j_a(u) - j_a(u_{A \setminus a}, u'_a) \Big) \geq 0 \quad \forall a \in A, \forall u_a \in \mathbb{U}_a, \forall u'_a \in \mathbb{U}_a \ . \tag{3.78}$$

No agent a could expect to increase his expected payoff by using some disobedient action u'_a after getting any recommendation u_a from the mediator.

Chapter 4

Mechanism Design (Hélène Le Cadre, Benjamin Heymann)

Contents

4.1 Mechanism design

4.1.1 Mechanism Design by an informed principal

We consider a *general Bayesian incentive* problem. We allow for both informational (adverse selection) and strategic (moral hazard) constraints on the ability of these individuals to coordinate themselves.

As in Bayesian games, for each agent $a \in A$, the set of possible types is Ω_a.

A *mechanism* is any rule determining the agents' actions as a function of their types. The set of feasible mechanisms is limited by two factors

- the incentives for each agent to report his private information honestly (types are unverifiable),
- the incentive of agent to control some private decisions that cannot be cooperatively coordinated with the others (for example, the agent's level of effort).

This gives rise to two classes of actions: those which are publicly observable and enforceable $\mathbb{U}^0$ (that we will call *collective actions*), and those that must be privately controlled. Any $u^0 \in \mathbb{U}^0$ represents a combination of actions which the agents can commit to carry out even if it may turn out ex post to be harmful to any or all of the agents. For each agent a, we let $\mathbb{U}_a$ represent the set of all possible private actions controlled by agent a.

We let $\Omega = \prod_{a\in A} \Omega_a$ denote the set of all possible combinations of individual types with $\omega = (\omega_a)_{a\in A}$ denoting a typical types-vector or state in Ω.We let Ω_{-a} denotes the set of possible combinations of types of the agents other than a. Similarly, we let

$$\mathbb{U} = \mathbb{U}^0 \times \prod_{a\in A} \mathbb{U}_a \ , \tag{4.1}$$

denote the set of all possible combinations of public and private actions with $u = \Big(u_0, (u_a)_{a\in A}\Big)$ denoting a vector of actions or outcomes in $\mathbb{U}$. In this §we assume that $\mathbb{U}$ and Ω are (non empty) finite sets.

In this framework, given any vector of types ω and actions u, we let $j_a(u, \omega)$ denote the payoff to agent a, when u is the outcome and ω is the state of the game. We let $p_a(\omega_{-a}|\omega_a) > 0$ denote the *conditional probability* that agent a would assign to the event that $\omega = (\omega_a)_{a\in A}$ is the actual state of the state, given that he knows his actual type to be ω_a.

We now describe the set of feasible mechanisms for coordinating the public and private actions, as a function of the individuals' types:

- each agent simultaneously and confidentially reports his type to a trustworthy mediator,
- the mediator then chooses an outcomes $u = \Big(u_0, (u_a)_{a\in A}\Big)$ in $\mathbb{U}$ as a (possibly random) function of the vector of reported types,
- the enforceable action u_0 is carried out, and each agent is confidentially informed that u_a is the private action recommended for him.

A mechanism is any function $\tau : \mathbb{U} \times \Omega \to \mathbb{R}$ such that

$$\begin{aligned} &\sum_{u\in\mathbb{U}} \tau(u,\omega) = 1 \,, \\ &\tau(u,\omega) \geq 0 \quad \forall u \in \mathbb{U} \quad \forall \omega \in \Omega \,. \end{aligned} \tag{4.2}$$

Here $\tau(u, \omega)$ is interpreted as the probability that u will be the outcome chosen by the mediator if ω is the reported state in agents' types. For any possible types ω_a, ω_a' of agent a, any function $\delta_a : \mathbb{U}_a \to \mathbb{U}_a$, and any mechanism τ, we make the following definitions.

Definition 41.

$$J_a(\tau|\omega_a) = \sum_{\omega_{-a}\in\Omega_{-a}} p_a(\omega_{-a}|\omega_a) \sum_{u\in\mathbb{U}} \tau(u,\omega) j_a(u,\omega) \tag{4.3}$$

and

$$J_a^\star(\tau,\delta_a,\omega_a'|\omega_a) = \sum_{\omega_{-a}\in\Omega_{-a}} p_a(\omega_{-a}|\omega_a) \sum_{u\in\mathbb{U}} \tau(u,\omega_{-a},\omega_a') j_a\Big(\big(u_{-a},\delta_a(u_a)\big),\omega\Big) \tag{4.4}$$

In this paper, whenever ω, ω_a, and ω_{-a} appears in the same formula, ω_{-a} denotes the vector of all components other than ω_a in the vector $\omega = \Big(\omega_1, ..., \omega_A\Big)$. Also, (ω_{-a}, ω_a') and $(u_{-a}, \delta_a(u_a))$ are respectively the vectors that differ from ω and u in that ω_a' replaces ω_a and $\delta_a(u_a)$ replaces u_a.

Thus, $J_a(\tau|\omega_a)$ is the conditionally expected utility for individual a, given that his type is ω_a, if all individuals report their types honestly and carry on their recommended private actions obediently, when the mediator uses mechanism τ.

On the other hand, if agent a reports ω_a' and plans to use private action $\delta_a(u_a)$ when u_a is recommended, while all other agents are honest and obedient, then $J_a^\star(\tau, \delta_a, \omega_a'|\omega_a)$ is agent a's conditionally expected utility from mechanism τ, given that a's true type is ω_a. Notice that the mediator's recommendation may convert information to agent a about the others' types, so that a might rationally choose his private actions as some function $\delta_a(.)$ of his recommended action.

Definition 42. *The mechanism τ is* incentive compatible *if*

$$J_a(\tau|\omega_a) > J_a^\star(\tau,\delta_a,\omega_a'|\omega_a) \quad \forall \omega_a \in \Omega_a, \forall \omega_a' \in \Omega_a, \forall \delta_a : \mathbb{U}_a \to \mathbb{U}_a \,. \tag{4.5}$$

Condition (4.5) asserts that honest and obedient participation in the mechanism τ must be a Bayesian Nash equilibrium for all the agents in A. For any Bayesian equilibrium of any other coordination game which the individuals might play, there exists an equivalent incentive-compatible mechanism satisfying (4.5). This idea is called the *revelation principle*.

Example 43. *Suppose that each agent's set of private actions is simply $\mathbb{U}_a = \{accept; reject\}$, and that all utility payoffs will be zero if any agent chooses his* reject *option. Suppose also that there is an enforceable action (*fire everyone*) that also makes all payoffs zero. Then, without loss of generality,* *we need only consider mechanism in which no agent is ever asked to* reject, *since the* fire everyone *action may be used instead. Then the incentive constraints (4.5) reduce to*

$$J_a(\tau|\omega_a) > \sum_{\omega_{-a}\in\Omega_{-a}} \sum_{u\in\mathbb{U}} p_a(\omega_{-a}|\omega_a)\tau(u,\omega_{-a},\omega'_a)j_a(u,\omega) \quad \forall a, \forall \omega_a \in \Omega_a, \forall \omega'_a \in \Omega_a \ , \tag{4.6}$$

and

$$J_a(\tau|\omega_a) > 0 \quad \forall a, \forall \omega_a \in \Omega_a \ . \tag{4.7}$$

This means that no agent should have any incentive to lie or reject in the mechanism.

If an outside agent with no private information were given the authority to control all communication between agents and to determine the enforceable actions in $\mathbb{U}^0$, then he could implement any incentive-compatible mechanism satisfying constraints (4.2) and (4.5). But if one of the informed agents in A can influence the selection of mechanism when he already knows his own type, then a fundamental issue arises to constrain the choice of mechanism: if the selection of coordination mechanism depends in any way on one individual's type, the selection of the mechanism itself will convey information about his type to the other agents. Then a mechanism is feasible if it is incentive compatible after all other agents have inferred whatever information might be implicit in the establishment of the mechanism itself.

In this §we assume that an agent, let's call him a^P can effectively control all communications and can dictate how the action in $\mathbb{U}^0$ is to be determined without any need to bargain or compromise with any of the other $A\backslash\{a^P\}$ agents[1]. That is, agent a^P has complete authority to select any mechanism for coordinating the enforceable and private actions of the agents in A. Agent a^P is called the *principal* of the system. The other agents in $A \setminus a^P$ are referred to as *subordinates*.

We assume that the principal already knows his type at the time when he selects the mechanism, and that this is not a repeated situation. Thus the best incentive-compatible mechanism for the principal maximizes his conditional expected utility $J_{a^P}(\tau|\omega_{a^P})$, then his choice will depend on his true type ω_{a^P}, and so the subordinate agents may be able to infer (partially or completely) the principal's type from his choice of τ. With this new information, the subordinates may find new opportunities to gain by dishonesty or disobedience. So a mechanism might not be incentive-compatible eventhough Equation (41) is satisfied because the fact that τ is used allows the subordinates to learn about the principal's type.

Let Ω^P be any non-empty subset of Ω_{a^P}. We say that a mechanism τ is *incentive compatible* given Ω^P if τ is incentive compatible for the principal i.e., τ satisfies (4.5) for $a = a^P$ and

$$\begin{aligned}
& \sum_{\omega_{-a}\in\Omega_{-a},\omega_{a^P}\in\Omega^P} \sum_{u\in\mathbb{U}} p_a(\omega_{-a}|\omega_a)\tau(u,\omega)j_a(u,\omega) \\
> \ & \sum_{\omega_{-a}\in\Omega_{-a},t_{a^P}\in\Omega^P} \sum_{u\in\mathbb{U}} p_a(\omega_{-a}|\omega_a)\tau(u,\omega_{-a},\omega'_a)j_a\Big(\big(u_{-a},\delta_a(u_a)\big),\omega\Big) \\
& \forall a \in A \setminus a^P, \forall \omega_a \in \Omega_a, \forall \omega'_a \in \Omega'_a, \forall \delta_a : \mathbb{U}_a \to \mathbb{U}_a \ .
\end{aligned} \tag{4.8}$$

Condition (4.8) asserts that no subordinate a should expect to gain by reporting ω'_a and by disobeying his instructions according to Δ_a, when he knows that ω_a is his true type and the principal's type is in Ω^P. Thus, if the subordinates expected that the principal would propose mechanism τ if his type were in Ω^P, but otherwise would propose some other mechanism, then τ could be successfully implemented only if it were incentive compatible given Ω^P.

[1]The difference between $\mathbb{U}^0$ and $\mathbb{U}^1$ is that the action in $\mathbb{U}^1$ is subject to moral hazard, in the incentive constraints, but the ection in $\mathbb{U}^0$ is not.

This concept of conditional incentive compatibility describes what the principal could achieve if some information were revealed. However, as we try to construct a theory to determine which mechanism the principal should implement, there is no loss of generality in assuming that all types of the principal should choose the same mechanism, so that his actual choice of mechanism will convey no information. We may refer to this claim as the principle of *inscrutability*. Its essential justification is that the principal should never need to communicate any information to the subordinates by hys choice of mechanism, because he can always build such communication into the process of the mechanism itself.

Formally, suppose that there are some mechanisms $\left(\tau_k\right)_{k=1,...,K}$ and sets of types $\left(\Omega_k^P\right)_{k=1,...,K}$ forming a partition of Ω_{a^P}, such that the types in Ω_k^P are expected to implement τ_k for every k in $\{1, 2, ..., K\}$. Since the subordinates would rationally infer that the principal's type is in Ω_k^P when τ_k is proposed, each τ_k must be incentive compatible given Ω_k^P. Since the principal already knows his type, he would choose to implement these mechanisms in this fashion only if they satisfy

$$J_{a^P}(u_k|\omega_{a^P}) > J_{a^P}(u_j|\omega_{a^P}) \quad \forall j, \forall k, \forall \omega_{a^P} \in \Omega^P \ . \tag{4.9}$$

and are incentive compatible for him separately. But now consider the mechanism $\tau^\star$ defined by

$$\tau^\star(u, \omega) = \tau_k(u, \omega) \quad \text{if} \quad \omega_{a^P} \in \Omega^P \ . \tag{4.10}$$

This mechanism $\tau^\star$ is completely equivalent to the system of mechanisms $\{\tau_1, ..., \tau_K\}$ on the partition $\{\Omega_1^P, ..., \Omega_K^P\}$, giving the same distribution of outcomes in every state. That is, saying that for each k, if the principal's type is in Ω_k^P then he will implement τ_k is empirically indistinguishable from saying that the principal will implement $\tau^\star$, no matter what his type is. It is straightforward to verify that $\tau^\star$ is incentive compatible, using Equation (4.8) (with $\tau = \tau_k$ and $\Omega = \Omega_k^P$) and (4.9) to prove that 4.5 holds for $\tau = \tau^\star$.

We aim to predict which mechanism a principal with private information might select. For inscrutability, any mechanism that we predict must be reasonable for all of his types to select. If the principal's different types would actually prefer different incentive-compatible mechanisms, then the predicted mechanism must be some kind of compromise between the different goals of the principal's possible types.

4.2 Mechanism design presented in the intrinsic framework

4.2.1 Definitions

A **mechanism** is a *set of rules* that: (1) allows participants to communicate with a central operator, (2) maps any sequence of messages with an outcome that affects all the participants. When a designer (a company, an institution or a person for instance) is looking for a mechanism satisfying some criteria, he needs to solve a **mechanism design problem**. This section is devoted to a formal presentation of a generic mechanism design problem.

The actors: agents, players and Nature A situation where mechanism design theory could be applied brings together two kinds of actors. (1) The **principal** (or **designer**) is in charge of the *design* of the mechanism itself. For this reason, he will be referred to with the letter $\mathfrak{d}$. (2) The second category of actors is made of **players** $p \in P$ who interact through the mechanism chosen by the designer. From those interactions will result an output pre-defined in the mechanism. If we take the point of view of a player, every possible choice of mechanism by the designer constitutes a game (this is obviously why we call them players). In most of our examples there is exactly one agent per player, but generically (as in the previous chapter), one player may be embodied by a collection of **agents** A_p.

Information and decisions One of the most fundamental feature of a mechanism design problem is that *the agents knows things that the principal does not.* These things can be either **private information** (such as the willingness to pay for a good, health problems, preferences concerning either a presidential candidate, a schedules or a university) or **hidden actions** (for instant, the quantity of effort an agent invests in a job). The latter will be modeled with measurable **hidden actions set** $(\mathbb{U}_a^h, \mathcal{D}_a^h)$, while the former should be formalized with the help of a random set $(\Omega_a, \mathcal{F}_a)$ for each agent a. A random set $(\Omega_0, \mathcal{F}_0)$ aggregates the stochasticity observed by the designer. When $(\mathbb{U}_a^h, \mathcal{D}_a^h)$ is trivial, we say that there is no **moral hazard**. The random set thus writes as a product:

$$\Omega = \Omega_0 \times \prod_{a \in A} \Omega_a .$$

We write $\mathcal{F}$ its associated σ-field. Despite the fact that the designer is not per say an agent, we still model its knowledge with an information field $\mathcal{I}_{\mathfrak{d}}$. The information available to an actor (agent or designer) can be decomposed into two parts:

1. what is known by the **direct observation of Nature** $\mathcal{I}_a^{\clubsuit}$ (or $\mathcal{I}_{\mathfrak{d}}^{\clubsuit}$), for instance the private information. The designer does not have any control on this information, which corresponds to Nature choices. Those field are subfield of $\mathcal{F}$.

2. what is learned through the **messages** permitted by the mechanism. We will clarify this second aspect in the sequel through the definition of a mechanism.

Since the agents have access to their private information, it is true that

$$\{\Omega_0, \emptyset\} \otimes_{\widetilde{a} \neq a} \{\Omega_{\widetilde{a}}, \emptyset\} \otimes \mathcal{F}_a \subseteq \mathcal{I}_a^{\clubsuit} .$$

Since the designer does not know the private information, $\mathcal{I}_{\mathfrak{d}}^{\clubsuit}$ satisfies

$$\mathcal{I}_{\mathfrak{d}}^{\clubsuit} = \mathcal{F}_0 \otimes_{a \in A} \{\Omega_a, \emptyset\} .$$

Players preferences An **outcome set** $\mathbb{O}$ contains the possible issues of the interactions. It corresponds to the set of possible social choice (for instance the winner of an auction or of a presidential election), over which every player has some preferences. Since we allow monetary transfers, we model the players preferences with quasi-linear utility functions

$$U_p(\mathfrak{o}, t, \omega_p, u_p) := \widetilde{U}_p(\mathfrak{o}, \omega_p, u_p) + t$$

for $\mathfrak{o} \in \mathbb{O}$, $\omega_p \in \Omega_p := \prod_{a \in A_p} \Omega_a$ and $t \in \mathbb{R}$, where $\widetilde{U}_p : \mathbb{O} \to \mathbb{R}$ aggregates the player non monetary preferences. Observe that we could have aggregated the transfers in the outcomes $\mathbb{O}$, but we think it is worth distinguishing between monetary and non-monetary allocations.

Mechanism We call **mechanism** $M := ((\mathbb{U}_a^m, \mathcal{D}_a^m)_{a \in A}, (\mathcal{I}_a^m)_{a \in A}, \kappa)$ the choice of:

1. a collection of **measurable message sets** $(\mathbb{U}_a^m, \mathcal{D}_a^m)_{a \in A}$; writing $\mathbb{U}_a := \mathbb{U}_a^h \times \mathbb{U}_a^m$ we define as in the previous chapter the **history set** H

2. a collection of **information fields** $(\mathcal{I}_a^m)$ such that setting

   $$\mathcal{I}_a := \mathcal{I}_a^{\clubsuit} \otimes \mathcal{I}_a^m$$

   yields a **solvable** decision problem with measurable solution map S. We set $\mathcal{I}_{\mathfrak{d}}^m := \otimes_{a \in A} \mathcal{D}_a^m$ and $\mathcal{I}_{\mathfrak{d}} := \mathcal{I}_{\mathfrak{d}}^{\clubsuit} \otimes \mathcal{I}_{\mathfrak{d}}^m$.

3. a **contract function** $\kappa = (\kappa_{\mathbb{O}}, (\kappa_p)_{p \in P}) : H \to \mathbb{O} \times \mathbb{R}^P$, measurable with respect to $\mathcal{I}_{\mathfrak{d}}$

Prior representations The actors - players and designer - have a prior on the state of Nature, $\mathbb{P}_p$ (or $\mathbb{P}_{\mathfrak{d}}$). It is a probability distribution over Ω.

Game of a mechanism Consider a mechanism $M = ((\mathbb{U}^m_a, \mathcal{D}^m_a)_{a\in A}, (\mathcal{I}^m_a)_{a\in A}, \kappa)$. We set for any player $p \in P$, $h \in H$,

$$j_p(h) := U_p(\kappa_{\mathbb{O}}(h), \kappa_p(h), u_p, \omega_p).$$

We get an intrinsic game. We call it **game of M** and denote it by $G(M)$.

Social choice function A **social choice function** is an application f from $\Omega \times \prod_{a\in A} \mathbb{U}^h_a$ to $\mathbb{O} \times \mathbb{R}^P$ measurable with respect to $\mathcal{F} \otimes_{a\in A} \mathcal{D}^h_a$. We denote by $f_{\mathbb{O}}$ its first coordinate.

Game solution concept A **game solution concept** SC maps any game G with a subset (possibly empty) of its strategy profiles $SC(G)$.

NB: This includes (pure) Nash , Bayes-Nash, sub-game perfect and dominant strategy equilibria, and exclude mixed strategy Nash equilibria.

Implementation of a social choice function Let[2] SC be a solution concept, M a mechanism, f a social choice function. We say that M SC**-implementes** f when there exists $\gamma \in SC(G(M))$ such that for any $\omega \in \Omega$,

$$\kappa(S_\gamma(\omega)) = f(\pi(S_\gamma(\omega))),$$

where π is the projection of H on $\Omega \times \prod_{a\in A} \mathbb{U}^h_a$. We say that f is SC**-implementable** whenever such (M, γ) exists.

Direct mechanism [3] A **direct mechanism** is a mechanism such that $\mathbb{U}^m_a = \Omega_a \times \mathbb{U}^h_a$.

Revelation principle A **revelation principle** is a claim that any SC-implementable function f can be SC**-implemented** by a direct mechanism.

4.2.2 Criteria for the designer

To choose a mechanism, the designer is usually given a set of quantitative (i.e. we want to maximize something) and qualitative criteria (i.e. we want to satisfy a constraint) on the transfers and the individual incitations.

Budget balance This is a qualitative criteria on the transfers. Generically, it means that the designer will not need to bring money **from outside**. We assume that the mechanism M SG-implements a social choice function f with equilibrium strategy γ^*. There are four versions of this criteria:

1. weak ex post budget balance: $\sum_{p\in P} k_p(S_{\gamma^*}(\omega)) \geq 0$ for all $\omega \in \Omega$
2. weak ex ante budget balance: $\mathbb{E} \sum_{p\in P} k_p(S_{\gamma^*}(\omega)) \geq 0$
3. strong ex post budget balance: $\sum_{p\in P} k_p(S_{\gamma^*}(\omega)) = 0$
4. strong ex ante budget balance:$\mathbb{E} \sum_{p\in P} k_p(S_{\gamma^*}(\omega)) = 0$.

[2] tentative definition
[3] revoir partie Moral hazard

Profit This is a quantitative criteria on the transfers. The designer want to maximize the (expected) quantity of money he can get out of the system (think of an auction for instance), which is

$$\mathbb{E}\sum_{p\in P} k_p(S_{\gamma^*}(\omega))$$

No side payments This is a qualitative criteria on the transfers, it means that

$$k_p = 0$$

Efficiency This is a quantitative criteria. *Assume* $\prod_{a\in A}\mathbb{U}_a^h$ *is trivial.* A social choice function f is said to be **efficient** when it maximizes

$$\sum_{p\in P} u_p(f_{\mathbb{O}}(\omega, u), \omega_p),$$

for any $(\omega, u) \in \Omega \times \prod_{a\in A}\mathbb{U}_a^h$. A mechanism is said **efficient** when it implements an efficient social choice function.

Individual Rationality This a qualitative criteria, also called **participation constraint**. When the players have outside opportunities, the designer may want to ensure that they do not have any better option. If we denote by $\overline{v}_p(\omega_p)$ the best payoff the player can get from not participating in the mechanism, then we can define three increasingly strong notions of IR:

1. **ex ante individual rationality**:
2. **interim individual rationality**:
3. **ex post individual rationality**:

Truth revealing

Chapter 5

Some Local Agents Problems

Contents

5.1 Optimal control of a Microgrid with Combined Heat and Power Generator

François Pacaud, Efficacity and École des Ponts.

Most of European countries must produce more than 20 % of their electrical energy with renewable energies by 2020, and smart and micro-grids are more and more put forward to achieve this goal. These new technologies allow utility managers to control in real time the consumption of consumers and the production of different power plants.

Deterministic controls, such as Model Predictive Control (MPC), are the most used methods to manage a micro-grid. But consumptions and renewable energy productions are hardly foreseeable, and it is often difficult to satisfy the adequation between demand and production in deterministic framework. That is why we focus on stochastic optimal management to control a micro-grid.

We consider here a domestic micro-grid, composed of a smart home equipped with smart devices (thermostat, controller) and whose energy is produced by renewable sources (micro-cogeneration, solar panels). This system is modelled with two state variables, and we will consider thermal and electrical demands as stochastic variables. Stochastic optimal control will be used to manage the energy in this system, and the control will be tested upon a realistic numerical model. We will put emphasis on the algorithms used (stochastic dynamic programming and stochastic dual dynamic programming) and the numerical results obtained. A benchmark with other methods, such as MPC and heurisitics, will be presented.

This work is part of a larger program, aiming to control a micro-grid where several houses and decentralized power sources are connected together through the local network. As the size of the problem increases, other methods must be investigated to tackle the curse of dimensionality. Decomposition and coordination schemes have proved their effectiveness in deterministic settings, and DADP (Dual Approximate Dynamic Programming) offers promising results in the stochastic framework. We will sketch some perspectives to apply such algorithms to large-scale smart-grid problems.

5.1.1 Problem statement and mathematical formulation

We give in Figure 5.1 a schematic representation of a house equipped with a CHP:

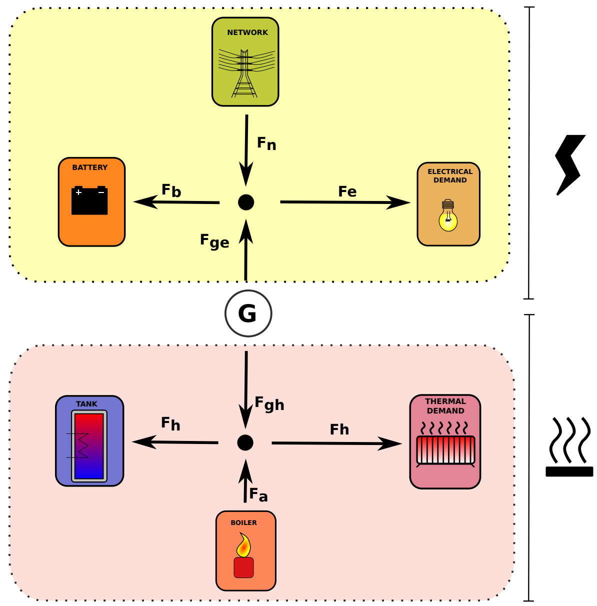

Figure 5.1: A house equiped with a CHP.

We give below the electrical equations:

- Dynamic constraints $B_{t+1} = \alpha_B B_t - \beta_B F_{B,t}$
- Capacity constraints $B^\flat \leq B_t \leq B^\sharp$
- Max charge/discharge constraint

$$\begin{aligned} & \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\ \iff & \frac{\Delta B^\flat + (1-\alpha_B)B_t}{\beta_B} \leq F_{B,t} \leq \frac{\Delta B^\sharp + (1-\alpha_B)B_t}{\beta_B} \end{aligned}$$

Offer=Demand $\underbrace{F_{GE,t}}_{CHP} + \underbrace{F_{B,t}}_{Battery} + \underbrace{F_{NE,t+1}}_{Network} = \underbrace{D^E_{t+1}}_{Demand}$

- Dynamic constraints: $H_{t+1} = \alpha_H H_t + \beta_H \left[F_{A,t} + F_{GH,t} - F_{H,t+1}\right]$
  where: $F_{H,t+1} = \min\left(D^T_{t+1}, \dfrac{\alpha_H H_t - H^\flat}{\beta_H} + \left[F_{A,t} + F_{GH,t}\right]\right)$
- Capacity constraints $H^\flat \leq H_t \leq H^\sharp$

We give the optimization criterion:

- Two instantaneous linear costs:
  - Using gas for auxiliary burner: $\pi_{gas} F_{A,t}$
  - Using the CHP generator: $\pi_{gas} P_G Y_t$
- Two instantaneous convex costs:

– selling/buying electricity from/to the network: $-\underbrace{b_E \max\{0, -F_{NE,t+1}\}}_{\text{selling}} + \underbrace{h_E \max\{0, F_{NE,t+1}\}}_{\text{buying}}$

convex because we assume that $b_E < h_E$

– Missing heat demand: $\underbrace{b_T\left(D^T_{t+1} - F_{H,t+1}\right)}_{\text{uncomfort}}$

5.1.2 Stochastic optimal control of the microgrid

5.1.3 Using decomposition for managing several houses

We use decomposition/coordination algorithms to control an urban neighbourhood as illustrated in Figure 5.2. Algorithms such as SDDP, DADP will be used:

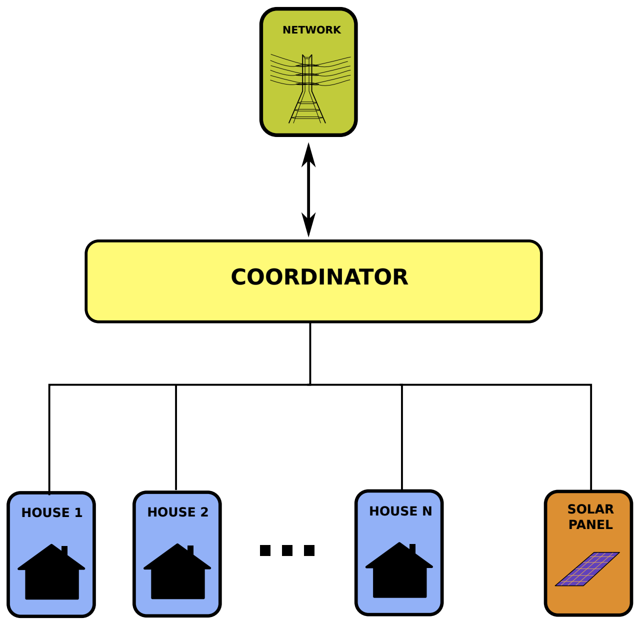

Figure 5.2: Decomposition-coordination to manage an eco-neighborhood.

5.2 Energy Management in a Solar Microgrid Using Stochastic Programming and Data Clustering

Rodrigo A. Carrasco, Slavia Vojkovic and Isabel Weber, Universidad Adolfo Ibáñez, Chile

The increasing interest in integrating intermittent renewable energy sources into the grid, like photovoltaic (PV) or wind based generation, presents several major challenges from the viewpoint of reliability and control. Energy storage enables large-scale integration of intermittent sources, allowing the penetration of distributed generation technologies to increase at a reasonable economic and environmental cost Kok et al. (2010). Despite its benefits, energy storage has not been fully utilized. Among the limiting factors is, besides the cost, the lack of appropriate control and management strategies, since now users need to decide when to store, consume and, if possible, inject that energy back to the grid Olivares et al. (2014).

In this work we present a novel approach for designing energy control policies for a PV microgrid, based on stochastic programming and data clustering to deal with the uncertainty of the generation. We use data clustering to determine the regimes in which the microgrid works around the year, and also to generate the scenarios used in the stochastic program. Next, we simplify the solutions obtained from the stochastic program to define rules that are easy to implement. The output of our procedure is a simple control policy that can be implemented in small microcomputers to control the flow of energy, with the objective of minimizing the total cost of consumption. Finally, our simulation based experimental results show that the resulting policy obtains costs that are very similar, in average, to the ones obtained by solving a multi-stage stochastic program.

5.3 A subway station with energy breaking saving

Tristan Rigaut, Efficacity and Ecole des Ponts, France

Urban railway stations are responsible for a significant amount of energy consumption of cities transportation facilities. Their efficiency could be significantly improved by harnessing different unexploited renewable or recoverable energy resources. Residual regenerative braking energy, geothermal or waste heat and local renewable energies represent each a potential we shouldn't neglect in subway stations. However, to fully exploit these energies potential we need energetic buffers in order to handle their intermittency. We present hereby a strategy relying on electrical storage systems as well as multi-physical building inertia to tackle the variability issue of such energetic potentials. It requires a proper electrical equipments control in a demand response fashion. At the crossroad between microgrids and smart buildings management, the aim of this work is to provide a methodology to control in real time the energetic characteristics of a subway station under different comfort and service constraints. Optimal control policies are calculated using approximate dynamic programming methods such as model predictive control, computational stochastic dynamic programming or stochastic dual dynamic programming. We apply the obtained policies tonumerical simulations based on real historical sensors data and we present results to compare the quality of our methods. A discussion about models simplifications and the use of various heuristic methods to compute near-optimal policies online is carried out. The aim is to spark a debate about whether we should look for suboptimal policies adapted to highly detailed models or conversely optimal solutions to simplified optimization models.

5.4 Urban microgrid with pumping station and batteries

Bernardo Pagoncelli, Universidad Adolfo Ibáñez, Chile

In most countries energy is generated in a centralized fashion, whether by capital-intensive companies or controlled by governments. In most cases (Brazil for example) the dispatch decision is done by a governmental agency that tries to minimize the overall cost of production and transmission. Consumers are passive entities that simply receive the energy generated, and pay the corresponding fee, which in the case of industrial consumers is usually done via contracts.

There are several forces in the society that are pushing for changes in the current energy model. First there is the issue of climate change, linked to the use of fossil fuels. Second, it has been shown that the energy efficiency of the current forms of generation is around 12%, while the potential for renewable energies is around 40%. Linked with the scarcity of fossil fuels, it is of utmost importance to find alternative ways of generating energy more efficiently. Finally, in the last 20 years or so the technology for generating energy through renewables (solar, wind, biofuels, etc) has improved significantly.

The emergence of renewables is creating a new paradigm, in which consumers can also generate energy for their own consumption or sell into the grid. Therefore, traditional models that only allowed for a centralized producer need to be revised. Households, small communities or small towns will be regarded as a small generating unit, and the role of the central planner might change from producer to coordinator. Big energy companies are now moving into renewables and services for the new decentralized grids that are emerging. The giant energy company E.On broke into two, creating a new company called Uniper to deal with microgrids.

From a modelling viewpoint, several changes need to be implemented, and we are going to highlight two in this Section. First, decentralized models with renewables have to incorporate generation uncertainty as an important part of the formulation. Wind and solar energy are intermittent sources and therefore assuming that production in a given period is deterministic would be a gross simplification. In order to obtain accurate representations for such randomness, it is necessary to have data such as total radiation in some specific area or hourly wind availability. Having this type of data allows for a better representation of randomness, which could be modelled as a random variable following a specific distribution, or through some type of robust estimation when data is scarce.

Another key aspect of such models, that comes entangled with the randomness of renewables, is storage. Due to the intrinsic variability of those sources, batteries need to be part of the system to generate the necessary stability. Microgrids need to be able to satisfy their demand on cloudy days, or in days with no wind. Additionally, from a financial perspective batteries can be a relevant source of income when prices are high in the network. If the microgrid is connected to a larger grid, energy can be sold when prices are convenient, and bought when the storage level is low or when the weather forecast is not favorable.

Both aspects mentioned have important implications for the model of a decentralized microgrid. Regarding the inclusion of uncertainty, the role of risk has to be addressed somehow in the model. The simples case is to optimize the system operation on average, that is, the objective is to minimize the expected cost of operation. However, risk-averse decision makers might want to avoid blackouts at any cost, or want to keep the cost tat every stage below some threshold with high probability. In summary there are different ways of including risk into the model, and specialized algorithms need to be used for each case. Regarding storage, batteries be as small as a home appliance, or as big as reservoirs used for pumping water (the so-called pumping stations). From a modelling viewpoint any battery is the same: the changes are given by the total storage capacity, and the charge/discharge speed. It is interesting to include more than one battery in a model and analyze the optimal policy regarding the use of different batteries.

5.5 Gestion du risque de l'agrégateur d'énergie renouvelable intermittente sur les marchés de l'électricité

Ariel Waserhole, ENSTA ParisTech, UMA and Sun'R Smart Energy, France

Jusqu'à présent, les énergies Renouvelables intermittentes (EnR) comme le photovoltaïque ou l'éolien étaient rémunérées par tarif d'achat garanti. Ces tarifs d'achat assuraient au producteur EnR une rémunération fixe quel que soit la demande en électricité et quel que soit le prix du marché. Avec l'augmentation de la production EnR prévue, cette décorrélation de la production ENR avec les besoins du système électrique et les signaux prix du marché constitue un risque de déséquilibre. C'est pourquoi la loi de transition énergétique (en application des objectifs fixés par la commission européenne) met en place à partir du 1er janvier 2016 un système de rémunération des producteurs EnR sur la base d'un complément de rémunération. Ce nouveau dispositif oblige les producteurs EnR à vendre leur électricité sur les marchés et à recevoir le cas échéant une rémunération complémentaire pour couvrir si besoin leurs coûts. C'est dans le cadre de cette évolution réglementaire que se développe le métier d'agrégateur EnR de Sun'R Smart Energy : il s'agit de vendre l'électricité issue d'énergies renouvelables sur les marchés, pour le compte des producteurs.

Les figures ci-dessous illustre le rôle de l'agrégateur valorisant de l'EnR sur les marchés de l'électricité :

1. Chaque heure, il estime tout d'abord la prévision de production EnR sur un horizon de 48 heures.

2. Ensuite il estime les prix de vente sur les différents marchés afin de faire un arbitrage entre vendre l'énergie la veille sur le marché Day-Ahead ou, jusqu'à une heure avant livraison sur le marché Intraday (les prévisions de prix ne sont pas représentées sur le schéma).

3. La production EnR n'étant pas prévisible, des écarts existent entre l'énergie vendue et l'énergie produite. Ces écarts seront pénalisés à un prix variant chaque heure qu'il faut également estimer.

4. Enfin l'agrégateur doit payer les producteurs pour l'EnR produite à un prix, spécifié par contrat, qui peut dépendre ou non des prix de marché. Le fait que le producteur soit rémunéré pour l'énergie produite et non pour l'énergie vendue, implique un risque marché (incertitude des prix) et un risque volume (incertitude de la production EnR) pour l'agrégateur.

En raison du caractère fortement aléatoire de la production EnR et des prix de marché, nous modélisons le problème de l'agrégateur EnR dans une cadre stochastique. Nous discutons des hypothèses de modélisation concernant les prix de marché et des limites du paradigme *price taker*, *i.e.* il est possible d'acheter/vendre n'importe quelle quantité d'énergie sans influencer le prix de marché. Cela nous amène à ne plus considérer comme critère d'optimisation uniquement l'espérance de gain mais également des notions de risque. Nous comparons différentes manières de maitriser le risque : des techniques simples, basées sur des notions de robustesse, et des techniques plus évoluées comme la cVaR Ángeles Moreno et al. (2012).

Nous présentons un protocole expérimental pour estimer le risque marché et le risque volume. Des expériences sur des données *open data* avec des prévisions reproductibles sont discutées afin d'aider le décideur à choisir la stratégie d'optimisation la plus adaptée à ses besoins.

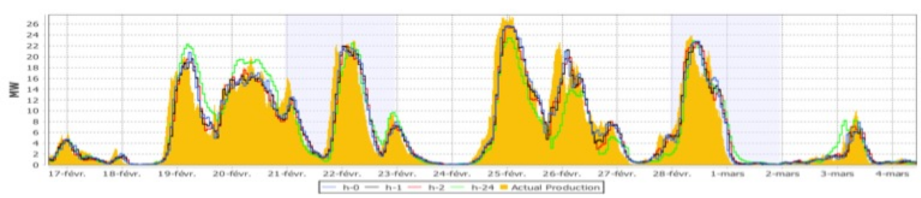

Figure 5.3: Production éolienne et prévisions.

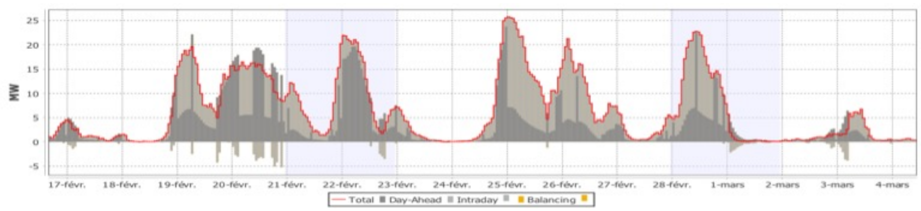

Figure 5.4: Vente d'énergie sur les marchés Day-Ahead et Intraday se basant sur les prévisions de production.

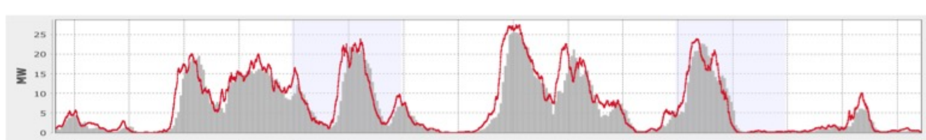

Figure 5.5: L'énergie vendue sur les marchés est différente de l'énergie produite.

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