Optimal Control and Sizing of a Combined Heat and Power Microgrid

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Efficacity - CERMICS - CERMICS

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Outline

1. Model and Objectives
2. Moving from Deterministic to Stochastic Optimization
3. Obtaining Control Policies to Drive the Microgrid
4. Assessing Control Policies

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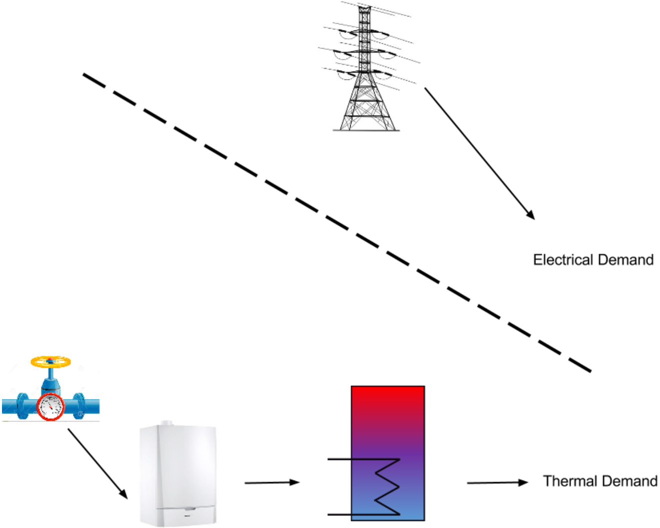
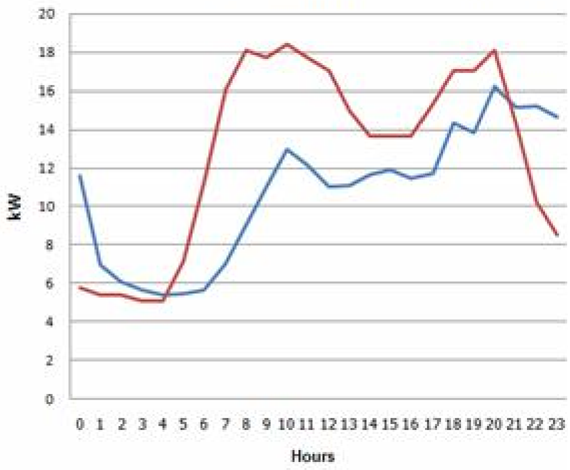
In a "classical" energy system, thermal and electrical energy management are usually treated apart

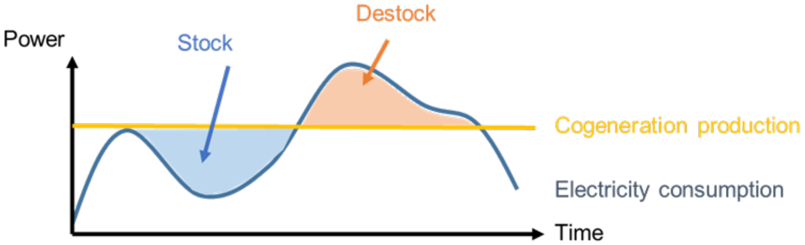


As electrical and thermal consumptions are correlated, a coupled management could be envisaged

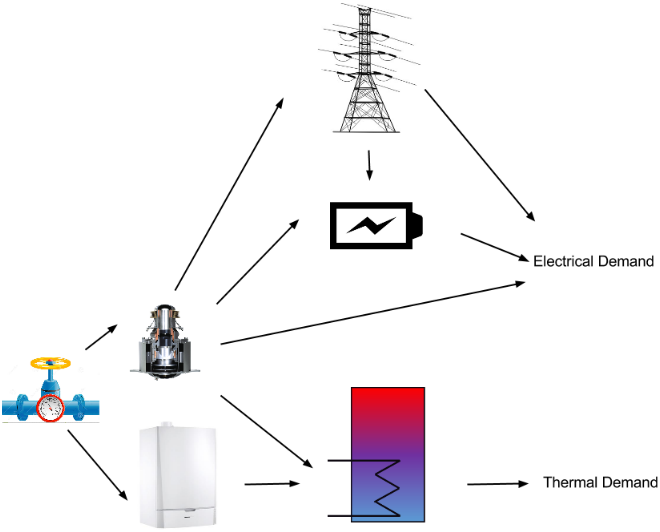


We add two new elements to the system

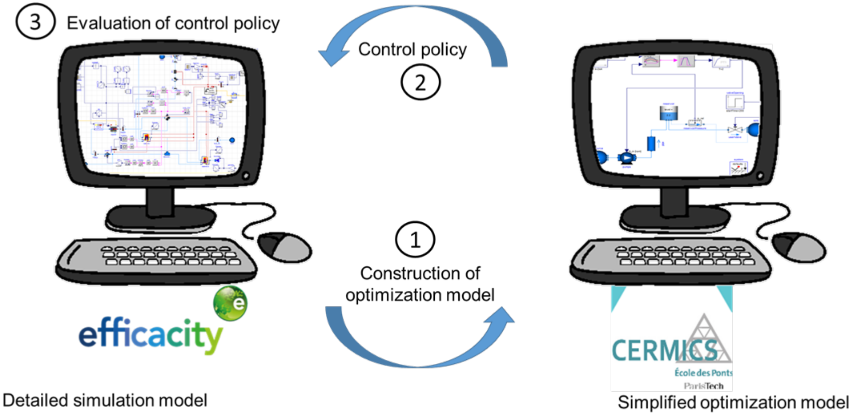
- A Combined Heat and Power (CHP) generator
- A battery



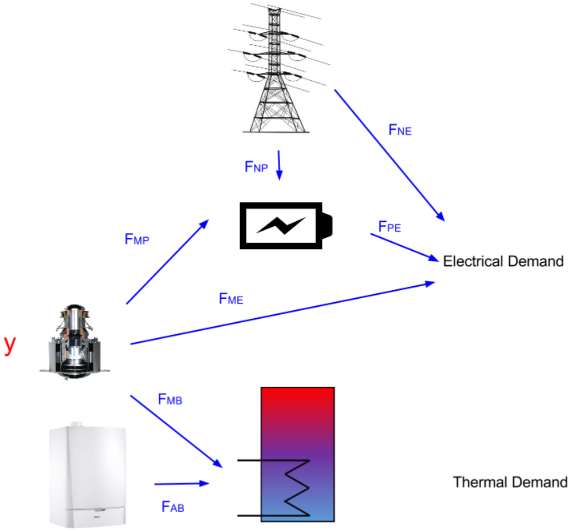
The new microgrid offers "Efficacity opportunities" !



We now focus on developping an optimization model to produce strategies to be tested on a simulation/assessment model



First, we start by identifying control variables



Then, we lay out a zoology of variables

- We consider discrete periods $t \in \{0, 1, 2, ..., \tau\}$, corresponding to timestep δt
- We have two stock variables:
  - B_t for the battery level
  - H_t for the hot water storage
- Control variables are:
  - the boolean ON/OFF CHP generator control variable Y_t
  - all the positive energy flows F_t
- Control constraints are the following:

$$Y_t \in \{0, 1\}$$

$$F_t = (F_{GB,t}, F_{GEl,t}, F_{BEl,t}, F_{NB,t}, F_{AH,t}, F_{GH,t}, F_{HTh,t}) \geq 0$$

- Slack variables are

$$F_{N,t} = (F_{NEl,t}, F_{NTh,t})$$

More about the flow variables

- Electrical flows
  - from the Combined Heat and Power (CHP) generator G to the battery B: $F_{GB,t}$
  - from the Combined Heat and Power (CHP) generator G to the electrical demand El: $F_{GEl,t}$
  - from the battery B to the electrical demand El: $F_{BEl,t}$
  - from the electricity network N to the battery B: $F_{NB,t}$
  - from and to the electricity network N to the electrical demand El: $F_{NEl,t}$
- Thermal flows
  - from the Combined Heat and Power (CHP) generator G to the hot water storage H: $F_{GH,t}$
  - from the auxiliary burner A to the hot water storage H: $F_{AH,t}$
  - from the hot water storage H to the thermal demand Th: $F_{HTh,t}$
  - from and to the "thermal network" N to the thermal demand Th: $F_{NTh,t}$

We write electrical constraints

- Dynamic constraints

$$B_{t+1} = B_t + \alpha_B(\underbrace{F_{GB,t}}_{CHP} + \underbrace{F_{NB,t}}_{Network}) - \underbrace{F_{BEI,t}}_{goes\ \ out} \quad \text{with } \alpha_B \leq 1$$

- Capacity constraints

$$B^\flat \leq B_t \leq B^\sharp$$

- Max charge/discharge constraint

$$\Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp$$

- CHP generator constraint

$$F_{GB,t} + F_{GEI,t} \leq Y_t \times power^{El}$$

- Offer=Demand

$$\underbrace{F_{GEI,t}}_{CHP} + \underbrace{F_{BEI,t}}_{Battery} + \underbrace{F_{NEI,t+1}}_{Network} = \underbrace{D^{El}_{t+1}}_{Demand}$$

We write thermal and CHP generator constraints

- Dynamic constraints

$$H_{t+1} = \beta_H H_t + \underbrace{F_{AH,t}}_{Boiler} + \underbrace{F_{GH,t}}_{CHP} - \underbrace{F_{HTh,t}}_{goes\ \ out} \quad \text{with } \beta_H \leq 1$$

- Capacity constraints

$$H^\flat \leq H_t \leq H^\sharp$$

- Auxiliary boiler constraint

$$F_{AH,t} \leq F^\sharp_{AH}$$

- CHP generator constraint

$$F_{GH,t} \leq Y_t \times power^{Th}$$

- Offer=Demand

$$\underbrace{F_{HTh,t}}_{Tank} + \underbrace{F_{NTh,t+1}}_{Network} = \underbrace{D^{Th}_{t+1}}_{Demand}$$

Finally, we introduce costs to make up a criterion

- There are three instantaneous linear costs:
  - Using gas for auxiliary burner: $p_{gas} F_{AH,t}$
  - Buying electricity for the battery: $p_{el} F_{NB,t}$
  - Using the CHP generator: $p_{chp} Y_t$
- There are two instantaneous convex costs:
  - selling/buying electricity from/to the network:

$$\underbrace{-b_{El} \max\{0, -F_{NEl,t+1}\}}_{\text{selling}} + \underbrace{h_{El} \max\{0, F_{NEl,t+1}\}}_{\text{buying}}$$

    with $b_{El} < h_{El}$
  - wasting heat/missing heat demand:

$$\underbrace{b_{Th} \max\{0, -F_{NTh,t+1}\}}_{\text{wasting heat}} + \underbrace{h_{Th} \max\{0, F_{NTh,t+1}\}}_{\text{missing heat demand}}$$

Instantaneous and final costs

- The instantaneous costs are

$$\begin{aligned} \mathcal{C}(Y_t, F_t, F_{N,t+1}) = & \underbrace{p_{chp} Y_t}_{CHP} + \underbrace{p_{gas} F_{AH,t}}_{Aux\ \ Burner} + \underbrace{p_{el} F_{NB,t}}_{Electricity} \\ & - b_{El} \max\{0, -F_{NEl,t+1}\} + h_{El} \max\{0, F_{NEl,t+1}\} \\ & + b_{Th} \max\{0, -F_{NTh,t+1}\} + h_{Th} \max\{0, F_{NTh,t+1}\} \end{aligned}$$

- We add a final linear cost:

$$-p_H H_\tau - p_B B_\tau$$

to avoid empty stocks at the final horizon τ

We are now able to formulate an optimization problem

$$\min_{Y_\cdot \in \{0,1\}, F_\cdot \geq 0} \sum_{t=0}^{\tau-1} \underbrace{\mathcal{C}(Y_t, F_t, F_{N,t+1})}_{instant.\ cost} \underbrace{- p_H H_\tau - p_B B_\tau}_{final\ cost}$$

$$\begin{aligned}
s.t. \quad Y_\cdot = (Y_0, \dots, Y_{\tau-1}), \quad F_\cdot = (F_0, \dots, F_{\tau-1}) \\
B^\flat \leq B_t \leq B^\sharp \\
H^\flat \leq H_t \leq H^\sharp \\
B_{t+1} = B_t + \alpha_B (F_{GB,t} + F_{NB,t}) - F_{BEl,t} \\
F_{GEl,t} + F_{BEl,t} + F_{NEl,t+1} = D^{El}_{t+1} \\
\Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
H_{t+1} = \beta_H H_t + F_{AH,t} + F_{GH,t} - F_{HTh,t} \\
F_{HTh,t} + F_{NTh,t+1} = D^{Th}_{t+1} \\
F_{AH,t} \leq F^\sharp_{AH} \\
F_{GH,t} \leq Y_t \times power^{Th} \\
F_{GB,t} + F_{GEl,t} \leq Y_t \times power^{El}
\end{aligned}$$

What are our objectives?

- We have to deal with the issue of stochasticity (of the demands)
- For this purpose, we will have to reformulate the problem
- and to shift from deterministic solutions — that is, deterministic trajectories — to adapted processes

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Uncertainty brings difficulties

$$\min_{Y_\cdot \in \{0,1\}, F_\cdot \geq 0} \sum_{t=0}^{\tau-1} \mathcal{C}(Y_t, F_t, F_{N,t+1}) - p_H H_\tau - p_B B_\tau$$

$$\begin{aligned}
s.t. \quad Y_\cdot = (Y_0, \ldots, Y_{\tau-1}), \quad F_\cdot = (F_0, \ldots, F_{\tau-1}) \\
B^\flat \leq B_t \leq B^\sharp \\
H^\flat \leq H_t \leq H^\sharp \\
B_{t+1} = B_t + \alpha_B(F_{GB,t} + F_{NB,t}) - F_{BEl,t} \\
F_{GEl,t} + F_{BEl,t} + F_{NEl,t+1} = D^{El}_{t+1} \\
\Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
H_{t+1} = \beta_H H_t + F_{AH,t} + F_{GH,t} - F_{HTh,t} \\
F_{HTh,t} + F_{NTh,t+1} = D^{Th}_{t+1} \\
F_{AH,t} \leq F^\sharp_{AH} \\
F_{GH,t} \leq Y_t \times power^{Th} \\
F_{GB,t} + F_{GEl,t} \leq Y_t \times power^{El}
\end{aligned}$$

We model demands as random variables

- We introduce a probabilistic setting, with $(\Omega, \mathcal{A}, \mathbb{P})$ a probability space
  - Ω is the sample space, or the scenario space
  - $\mathbb{P}$ is a probability
  - $\mathbb{E}$ is the mathematical expectation attached to the probability $\mathbb{P}$
- Then, we model demands as random variables

$$D_t^{El} : \Omega \to \mathbb{R}$$
$$D_t^{Th} : \Omega \to \mathbb{R}$$

  so that $(D_1^{El}, D_1^{Th}, \dots, D_T^{El}, D_T^{Th})$ forms a stochastic process
- Here, D_{t+1}^{El} and D_{t+1}^{Th} stand for the demands during the time interval $[t, t+1[$

Stochasticity is contaminating and all variables turn into random variables!

$$\min_{Y_\cdot \in \{0,1\}, F_\cdot \geq 0} \mathbb{E}\left[\sum_{t=0}^{\tau-1} \mathcal{C}(Y_t, F_t, F_{N,t+1}) - p_H H_\tau - p_B B_\tau\right]$$

$$
\begin{aligned}
s.t. \qquad && B^\flat \leq B_t \leq B^\sharp \\
&& H^\flat \leq H_t \leq H^\sharp \\
&& B_{t+1} = B_t + \alpha_B(F_{GB,t} + F_{NB,t}) - F_{BEl,t} \\
&& F_{GEl,t} + F_{BEl,t} + F_{NEl,t+1} = D^{El}_{t+1} \\
&& \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
&& H_{t+1} = \beta_H H_t + F_{AH,t} + F_{GH,t} - F_{HTh,t} \\
&& F_{HTh,t} + F_{NTh,t+1} = D^{Th}_{t+1} \\
&& F_{AH,t} \leq F^\sharp_{AH} \\
&& F_{GH,t} \leq Y_t \times power^{Th} \\
&& F_{GB,t} + F_{GEl,t} \leq Y_t \times power^{El}
\end{aligned}
$$

And we need to add the nonanticipativity constraints

- Filtration

$$\mathcal{A}_t = \sigma(D_1^{El}, D_1^{Th}, \dots, D_t^{El}, D_t^{Th})$$

- Control variables measurability

$$(Y_t, F_t) = (Y_t, F_{GB,t}, F_{GEl,t}, F_{BEl,t}, F_{NB,t}, F_{AH,t}, F_{GH,t}, F_{HTh,t})$$

is $\mathcal{A}_t$–measurable

- Slack variables measurability

$$F_{N,t} = (F_{NEl,t}, F_{NTh,t}) \text{ is } \mathcal{A}_t\text{–measurable}$$

What comes next?

- A special case of adapted process is given by control policies
- Control policies, or decision rules,
  are a natural notion of solution for micro-grid management
- Indeed, we want to control the system in real-time,
  by means of proper control policies
  that take into account the past information through current states,
  that is, stocks in battery and hot water storage

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We look for control policies to drive the system

- At each time t, if we know the stocks H_t and B_t, what are the best decisions Y_t and F_t?
- We define a control policy as a mapping

$$\pi_t : (H_t, B_t) \to (Y_t, F_t)$$

- By construction, a control policy π_t induces an $\mathcal{A}_t$-adapted stochastic process

$$(Y_t, F_t) = \pi_t(H_t, B_t) \text{ is } \mathcal{A}_t\text{–measurable}$$

Stochastic dynamic programming yields control policies

- To use stochastic dynamic programming, with state (H_t, B_t) — and obtain control policies feeding on the stocks (H_t, B_t) — we need to make a rather strong assumption: the demands stochastic process $(D_t^{El}, D_t^{Th})_{t=1,\dots,\tau}$ is a white noise (sequence of independent random variables)
- Then, the future cost functions $V_t(H, B)$ satisfy a Bellman equation

$$V_t(H_t, B_t) = \min_{Y_t \in \{0,1\}, F_t \geq 0} \mathbb{E}\big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V_{t+1}(H_{t+1}, B_{t+1})\big]$$

- And we obtain a control policy by solving an optimization problem

$$\pi_{SDP,t} : (H_t, B_t) \mapsto \operatorname*{arg\,min}_{Y_t \in \{0,1\}, F_t \geq 0} \mathbb{E}\big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V_{t+1}(H_{t+1}, B_{t+1})\big]$$

More on the Bellman equation

$$V_t(H_t, B_t) = \min_{Y_t \in \{0,1\}, F_t \geq 0} \mathbb{E}_{(D^{El}_{t+1}, D^{Th}_{t+1})} \left[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V_{t+1}(H_{t+1}, B_{t+1}) \right]$$

where $H_{t+1}, B_{t+1}, F_{N,t+1}$ are given by

$$\begin{aligned}
B^\flat \leq B_t &\leq B^\sharp \\
H_t &\leq H^\sharp \\
B_{t+1} = B_t + \alpha_B(F_{GB,t} + F_{NB,t}) &- F_{BEl,t} \\
F_{GEl,t} + F_{BEl,t} + F_{NEl,t+1} &= D^{El}_{t+1} \\
\Delta B^\flat \leq B_{t+1} - B_t &\leq \Delta B^\sharp \\
H_{t+1} = \beta_H H_t + F_{AH,t} + F_{GH,t} &- F_{HTh,t} \\
F_{HTh,t} + F_{NTh,t+1} &= D^{Th}_{t+1} \\
F_{AH,t} &\leq F^\sharp_{AH} \\
F_{GH,t} &\leq Y_t \times power^{Th} \\
F_{GB,t} + F_{GEl,t} &\leq Y_t \times power^{El}
\end{aligned}$$

More on the Bellman equation numerics

- As the state (H, B) is 2-dimensional, we can discretize the stocks (H, B) on a grid, and then
  - store the values $V_t(H, B)$ of the Bellman function on that grid
  - with the Bellman equation approximated by interpolation
- Then, we obtain the optimal policies $\pi_{SDP,t}(H, B)$, not by storing their values, but by solving "on the fly"

$$\pi_{SDP,t}(H_t, B_t) \in \underset{Y_t \in \{0,1\}, F_t \geq 0}{\arg\min} \mathbb{E}\big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V_{t+1}(H_{t+1}, B_{t+1})\big]$$

- However, storing Bellman functions on a grid is no longer possible when the dimension of the state is more than four
- Now, we present an approach where the Bellman functions are replaced by functions belonging to an "easy-to-parameterize" family: Stochastic Dual Dynamic Programming

We introduce another Bellman equation

$$\widetilde{V}_t(H_t, B_t) = \min_{Y_t \in [0,1], F_t \geq 0} \mathbb{E}\left[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + \widetilde{V}_{t+1}(H_{t+1}, B_{t+1})\right]$$

- In the above equation, compared with the original Bellman equation, we have relaxed the integrity constraint on Y_t: $Y_t \in \{0, 1\} \to Y_t \in [0, 1]$
- With these assumptions, we obtain that

$$\widetilde{V}_t = V_{SDDP,t} \leq V_{SDP,t} = V_t$$

since — as we make decisions in a larger set — we obtain lower costs

Second, we obtain a convex lower Bellman function

- As costs $\mathcal{C}(Y_t, F_t, F_{N,t+1})$ are jointly convex in states (H_t, B_t) and controls (Y_t, F_t)
- As the constraints are affine in states and controls
- As the dynamics is affine in states and controls
- We obtain a convex lower bound $V_{SDDP,t}$

$$V_{SDDP,t} \leq V_{SDP,t}$$

$$V_{SDDP,t}(H_t, B_t) = \min_{Y_t \in [0,1], F_t \geq 0} \mathbb{E}\big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V_{SDDP,t+1}(H_{t+1}, B_{t+1})\big]$$

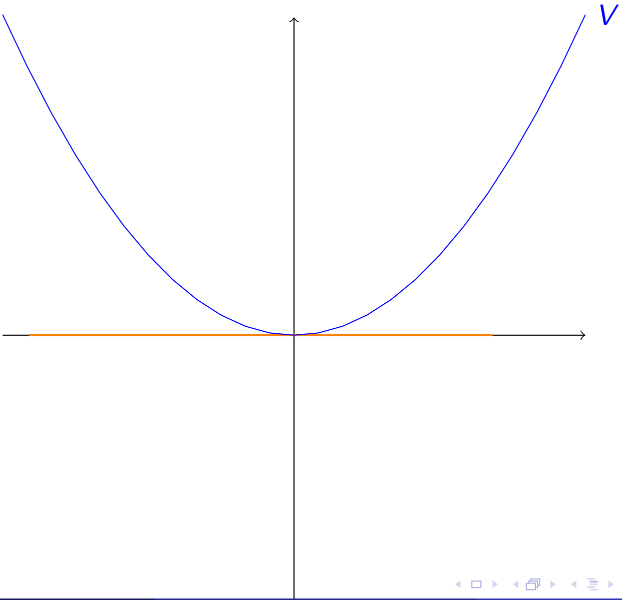
Third, we use Stochastic Dual Dynamic Programming (SDDP)

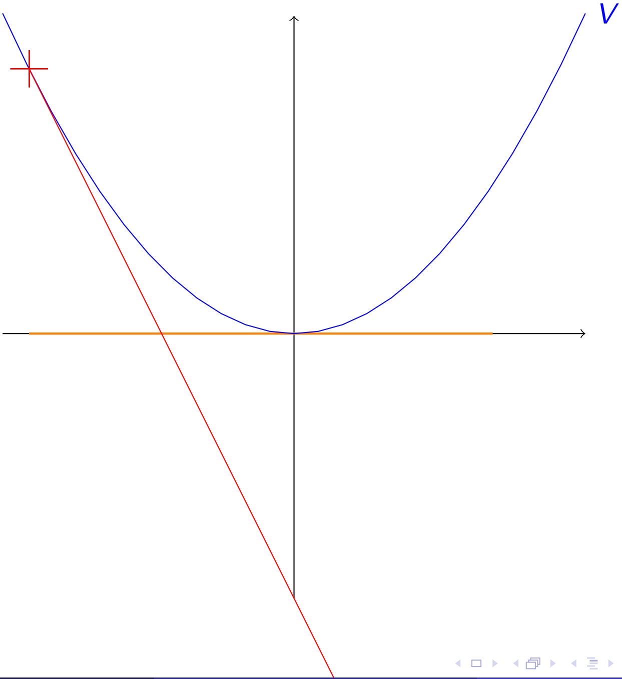
- Instead of computing the Bellman function $V_{SDDP,t}$ at every point, the SDDP algorithm constructs a polyhedral lower approximation
- For this, we use the property that any (closed s.c.i.) convex function coincides with the supremum of its minorizing affine functions
- As the lower bound $V_{SDDP,t}$ is convex,
  we look for lower piecewise linear approximations $V_{SDDP,t}$

$$V^{\flat}_{SDDP,t} \leq V_{SDDP,t} \leq V_{SDP,t}$$

- These approximations are improved by adding "cuts" (affine functions), computed by linear programming
- These cuts are added only at certain states (H, B), those likely to be visited by the state stochastic process

Thanks to Vincent Leclère

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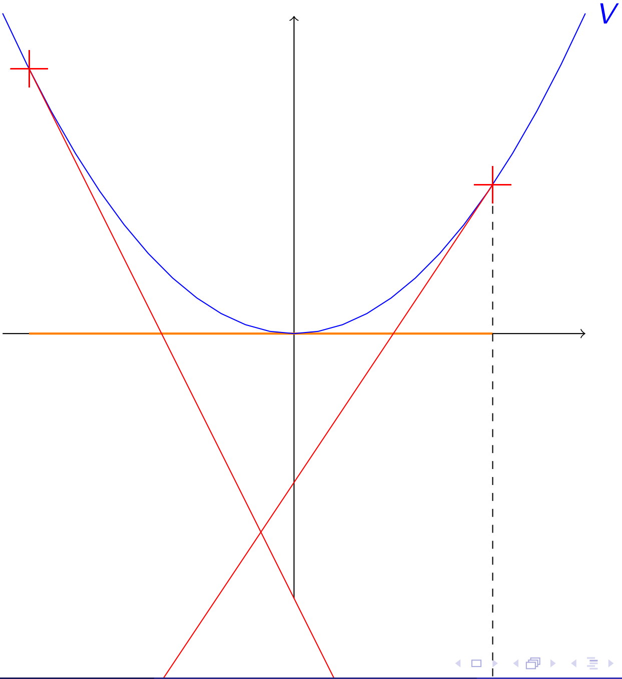
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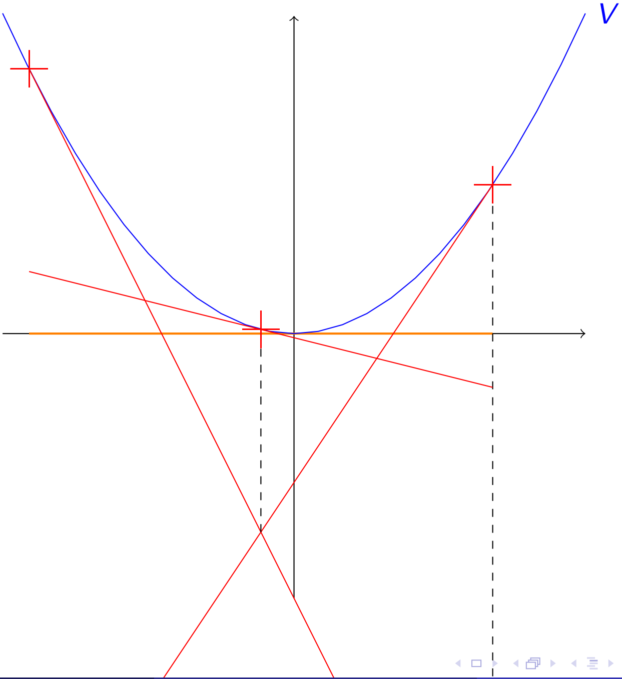
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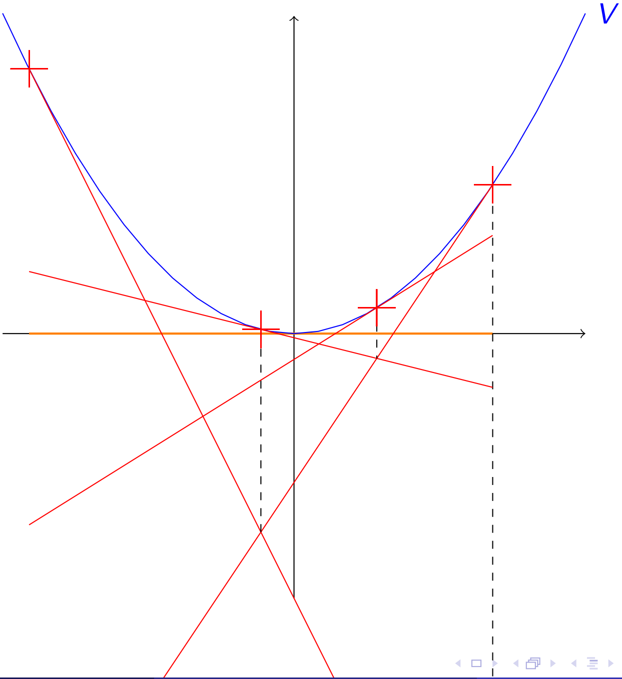
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More on the SDDP algorithm

At the beginning of step k, we suppose that we have, for each time t, an approximation $\check{V}_t^{(k)}$ of V_t satisfying

- $\check{V}_t^{(k)} \leq V_t$
- $\check{V}_T^{k} = \text{final cost}$
- $\check{V}_t^{(k)}$ is convex

More on the SDDP algorithm

- Forward path
  - We select a noise trajectory $t \mapsto w_t$
  - By a forward loop in time, we define a state trajectory $t \mapsto x_t^{(k)}$ with controls computed by an online optimization with $\check{V}_t^{(k)}$ at time t in the Bellman equation

More on the SDDP algorithm

- Forward path
  - We select a noise trajectory $t \mapsto w_t$
  - By a forward loop in time, we define a state trajectory $t \mapsto x_t^{(k)}$ with controls computed by an online optimization with $\check{V}_t^{(k)}$ at time t in the Bellman equation
- We improve the approximation of V_t by adding a cut at $x_t^{(k)}$
- Backward path
  - We obtain dual variables $\widehat{\lambda}_t^{(k+1)}(w)$ and $\widehat{\beta}_t^{(k+1)}(w)$
  - We define the new lower approximation

$$\check{V}_t^{(k+1)}(x) = \max \left\{ \check{V}_t^{(k)}(x), \beta_t^{(k+1)} + \left\langle \lambda_t^{(k+1)}, x - x_t^{(k)} \right\rangle \right\}$$

  - We easily check that $\check{V}_t^{(k+1)}$ is a lower convex approximation of V_t, and go one step back in time: $t \leftarrow t - 1$

Upon reaching $t = 0$, we have completed step k of the algorithm

Finally, we obtain admissible control policies

$$\pi_t(H_t, B_t) \in \underset{Y_t \in \{0,1\}, F_t \geq 0}{\arg\min} \mathbb{E}\Big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V^{\flat}_{SDDP,t+1}(H_{t+1}, B_{t+1})\Big]$$

Finally, we obtain admissible control policies

$$\pi_t(H_t, B_t) \in \underset{Y_t \in \{0,1\}, F_t \geq 0}{\arg\min} \mathbb{E}\Big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V^{\flat}_{SDDP,t+1}(H_{t+1}, B_{t+1})\Big]$$

To be compared with

$$V_{SDDP,t}(H_t, B_t) = \min_{Y_t \in [0,1], F_t \geq 0} \mathbb{E}\Big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V_{SDDP,t+1}(H_{t+1}, B_{t+1})\Big]$$

and with

$$\pi_{SDP,t}(H_t, B_t) \in \underset{Y_t \in \{0,1\}, F_t \geq 0}{\arg\min} \mathbb{E}\Big[\mathcal{C}(Y_t, F_t, F_{N,t+1}) + V_{t+1}(H_{t+1}, B_{t+1})\Big]$$

SDP vs SDDP: drawbacks and advantages

SDP vs *SDDP*

- Exact Future Cost Functions $V_{SDP,t}$
- Discretization of the state space and interpolation of Future Cost Functions
- Limited number of state variables (3 or 4)

- Lower approximation $V^{\flat}_{SDDP,t}$ of Future Cost Functions
- No need to discretize the state space
- Less limited number of state variables

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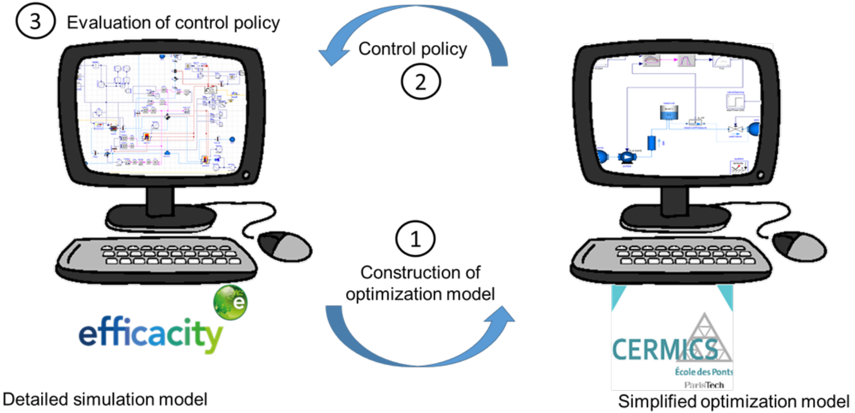
We have control policies, how should we test them?

- We have a sophisticated simulator for the system:
  - Nonlinear model representing in detail the dynamic of each element of the system
  - Real consumptions simulator taking into account the time correlations in the demands
- We run simulations to test our control policies

How the optimization and simulation/assessment models should interact



Feedback on this cooperation Efficacity-CERMICS